

# LECTURE NOTES

*ON*

## POWER SYSTEM ANALYSIS

**2019 – 2020**

**III B. Tech II Semester (JNTUA-R15)**

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## **SYLLABUS**

**UNIT -I POWER SYSTEM NETWORK MATRICES** Representation of Power System Elements, Graph Theory: Definitions, Bus Incidence Matrix, Ybus Formation by Direct and Singular Transformation Methods, Numerical Problems. Formation of ZBus: Partial Network, Algorithm for the Modification of ZBus Matrix for Addition Element for the Following Cases: Addition of Element from a New Bus to Reference, Addition of Element from a New Bus to an Old Bus, Addition of Element Between an Old Bus to Reference and Addition of Element Between Two Old Buses (Derivations and Numerical Problems).- Modification of ZBus for the Changes in Network ( Problems )

**UNIT – II SHORT CIRCUIT ANALYSIS** Per-Unit System of Representation. Per-Unit Equivalent Reactance Network of a Three Phase Power System, Numerical Problems.

Symmetrical Fault Analysis: Short Circuit Current and MVA Calculations, Fault Levels, Application of Series Reactors, Numerical Problems. Symmetrical Component Theory: Symmetrical Component Transformation, Positive, Negative and Zero Sequence Components: Voltages, Currents and Impedances. Sequence Networks: Positive, Negative and Zero sequence Networks, Numerical Problems. Unsymmetrical Fault Analysis: LG, LL, LLG faults with and without Fault Impedance, Numerical Problems.

**UNIT – III POWER FLOW STUDIES-I** Necessity of Power Flow Studies – Data for Power Flow Studies – Derivation of Static Load Flow Equations – Load Flow Solutions using Gauss Seidel Method: Acceleration Factor, Load Flow Solution with and without P-V Buses, Algorithm and Flowchart. Numerical Load flow Solution for Simple Power Systems (Max. 3-Buses): Determination of Bus Voltages, Injected Active and Reactive Powers (Sample One Iteration only) and Finding Line Flows/Losses for the given Bus Voltages.

**UNIT – IV POWER FLOW STUDIES-II** Newton Raphson Method in Rectangular and Polar Co-Ordinates Form: Load Flow Solution with or without PV Buses- Derivation of Jacobian Elements, Algorithm and Flowchart. Decoupled and Fast Decoupled Methods. - Comparison of Different Methods – DC Load Flow

**UNIT – V POWER SYSTEM STABILITY ANALYSIS** Elementary Concepts of Steady State, Dynamic and Transient Stabilities - Description of: Steady State Stability Power Limit, Transfer Reactance, Synchronizing Power Coefficient, Power Angle Curve and Determination of Steady State Stability and Methods to Improve Steady State Stability - Derivation of Swing Equation - Determination of Transient Stability by Equal Area Criterion, Application of Equal Area Criterion, Critical Clearing Angle Calculation. Solution of Swing Equation by 4th Order Runge Kutta Method (up to 2 iterations) - Methods to improve Stability - Application of Auto Reclosing and Fast Operating Circuit Breakers.

Course Outcomes: At the end of the course the student should be able to:  Form the Zbus and Ybus of a given power system network  Compare different methods used for obtaining load flow solution  Conduct load flow studies on a given system  Make fault calculations for various types of faults  Determine the transient stability by equal area criterion  Determine steady state stability power limit  Distinguish between different types of buses used in load flow solution

**TEXT BOOKS:** 1. Power Systems Analysis, Grainger and Stevenson, Tata Mc Graw-hill, 2005. 2. Modern Power system Analysis 2nd edition, I.J.Nagrath & D.P.Kothari: Tata McGrawHill Publishing Company, 2003.

**REFERENCE BOOKS:** 1. Computer Techniques in Power System Analysis 2nd Edition,, M A Pai, TMH, 2005. 2. Computer Techniques and Models in Power Systems, K. Uma Rao, I. K. International, 2007. 3. Electric Power Systems 1st Edition, S. A. Nasar, Schaum's Outline Series, TMH, 1997. 4. Computer Methods in Power System Analysis, E. Stagg and El-Abiad, Tata Mc Graw Hill, 1969

.....**CHAPTER- 1-A**

**INCIDENCE AND NETWORK MATRICES**

[**CONTENTS:** Definitions of important terms, Incidence matrices: Element node incidence matrix and Bus incidence matrix, Primitive networks and matrices, Performance of primitive networks, Frames of reference, Singular transformation analysis, Formation of bus admittance matrix, examples]

**INTRODUCTION**

The solution of a given linear network problem requires the formation of a set of equations describing the response of the network. The mathematical model so derived, must describe the characteristics of the individual network components, as well as the relationship which governs the interconnection of the individual components. In the bus frame of reference the variables are the node voltages and node currents.

The independent variables in any reference frame can be either currents or voltages. Correspondingly, the coefficient matrix relating the dependent variables and the independent variables will be either an impedance or admittance matrix. The formulation of the appropriate relationships between the independent and dependent variables is an integral part of a digital computer program for the solution of power system problems. The formulation of the network equations in different frames of reference requires the knowledge of graph theory. Elementary graph theory concepts are presented here, followed by development of network equations in the bus frame of reference.

**ELEMENTARY LINEAR GRAPH THEORY: IMPORTANT TERMS**

The geometrical interconnection of the various branches of a network is called the *topology* of the network. The connection of the network topology, shown by replacing all its elements by lines is called a *graph*. A *linear graph* consists of a set of objects called *nodes* and another set called *elements* such that each element is identified with an ordered pair of nodes. An *element* is defined as any line segment of the graph irrespective of the characteristics of the components involved. A graph in which a

direction is assigned to each element is called an *oriented graph* or a *directed graph*. It is to be noted that the directions of currents in various elements are arbitrarily assigned and the network equations are derived, consistent with the assigned directions. Elements are indicated by numbers and the nodes by encircled numbers. The ground node is taken as the reference node. In electric networks the convention is to use associated directions for the voltage drops. This means the voltage drop in a branch is taken to be in the direction of the current through the branch. Hence, we need not mark the voltage polarities in the oriented graph.

**Connected Graph :** This is a graph where at least one path (disregarding orientation) exists between any two nodes of the graph. A representative power system and its oriented graph are as shown in Fig 1, with:

$$\begin{array}{ll} e = \text{number of elements} = 6 & l = \text{number of links} = e - b = 3 \\ n = \text{number of nodes} = 4 & \text{Tree} = T(1,2,3) \text{ and} \\ b = \text{number of branches} = n - 1 = 3 & \text{Co-tree} = T(4,5,6) \end{array}$$

**Sub-graph :**  $sG$  is a sub-graph of  $G$  if the following conditions are satisfied:

- $sG$  is itself a graph
- Every node of  $sG$  is also a node of  $G$
- Every branch of  $sG$  is a branch of  $G$

For eg.,  $sG(1,2,3)$ ,  $sG(1,4,6)$ ,  $sG(2)$ ,  $sG(4,5,6)$ ,  $sG(3,4)$ ,... are all valid sub-graphs of the oriented graph of Fig.1c.

**Loop :** A sub-graph  $L$  of a graph  $G$  is a loop if

- $L$  is a connected sub-graph of  $G$
- Precisely two and not more/less than two branches are incident on each node in  $L$

In Fig 1c, the set  $\{1,2,4\}$  forms a loop, while the set  $\{1,2,3,4,5\}$  is not a valid, although the set  $\{1,3,4,5\}$  is a valid loop. The KVL (Kirchhoff's Voltage Law) for the loop is stated as follows: *In any lumped network, the algebraic sum of the branch voltages around any of the loops is zero.*

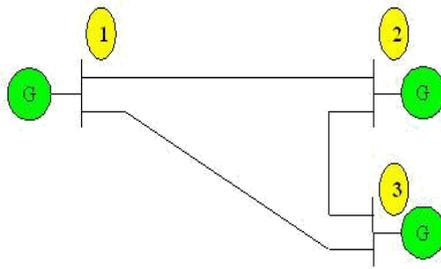


Fig 1a. Single line diagram of a power system

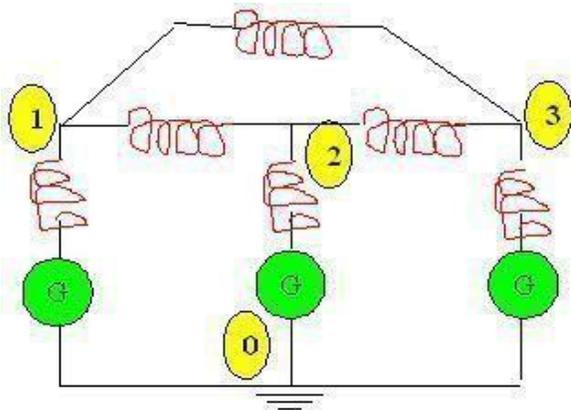


Fig 1b. Reactance diagram

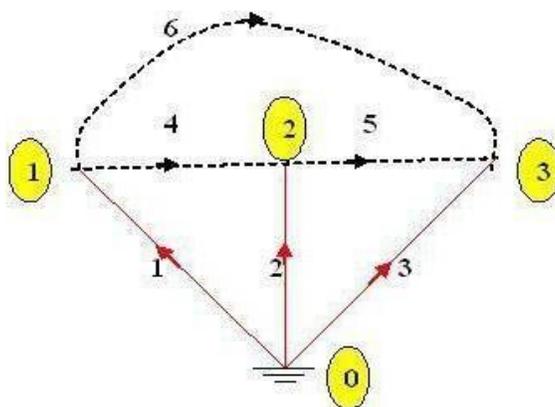


Fig 1c. Oriented Graph

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**Cutset :** It is a set of branches of a connected graph G which satisfies the following conditions :

- The removal of all branches of the cutset causes the remaining graph to have two separate unconnected sub-graphs.
- The removal of all but one of the branches of the set, leaves the remaining graph connected.

Referring to Fig 1c, the set {3,5,6} constitutes a cutset since removal of them isolates node 3 from rest of the network, thus dividing the graph into two unconnected sub-graphs. However, the set(2,4,6) is not a valid cutset! The KCL (Kirchhoff's Current Law) for the cutset is stated as follows: *In any lumped network, the algebraic sum of all the branch currents traversing through the given cutset branches is zero.*

**Tree:** It is a connected sub-graph containing all the nodes of the graph G, but without any closed paths (loops). There is one and only one path between every pair of nodes in a tree. The elements of the tree are called twigs or branches. In a graph with  $n$  nodes,

$$\text{The number of branches: } b = n - 1 \tag{1}$$

For the graph of Fig 1c, some of the possible trees could be T(1,2,3), T(1,4,6), T(2,4,5), T(2,5,6), etc.

**Co-Tree :** The set of branches of the original graph G, not included in the tree is called the *co-tree*. The co-tree could be connected or non-connected, closed or open. The branches of the co-tree are called *links*. By convention, the tree elements are shown as solid lines while the co-tree elements are shown by dotted lines as shown in Fig.1c for tree T(1,2,3). With  $e$  as the total number of elements,

$$\text{The number of links: } l = e - b = e - n + 1 \tag{2}$$

For the graph of Fig 1c, the co-tree graphs corresponding to the various tree graphs are as shown in the table below:

<b>Tree</b>	T(1,2,3)	T(1,4,6)	T(2,4,5)	T(2,5,6)
<b>Co-Tree</b>	T(4,5,6)	T(2,3,5)	T(1,3,6)	T(1,3,4)

**Basic loops:** When a link is added to a tree it forms a closed path or a loop. Addition of each subsequent link forms the corresponding loop. A loop containing only one link and remaining branches is called a *basic loop* or a *fundamental loop*. These loops are defined for a particular tree. Since each link is associated with a basic loop, the number of basic loops is equal to the number of links.

**Basic cut-sets:** Cut-sets which contain only one branch and remaining links are called *basic cutsets* or fundamental cut-sets. The basic cut-sets are defined for a particular tree. Since each branch is associated with a basic cut-set, the number of basic cut-sets is equal to the number of branches.

**Examples on Basics of LG Theory:**

**Example-1:** Obtain the oriented graph for the system shown in Fig. E1. Select any four possible trees. For a selected tree show the basic loops and basic cut-sets.

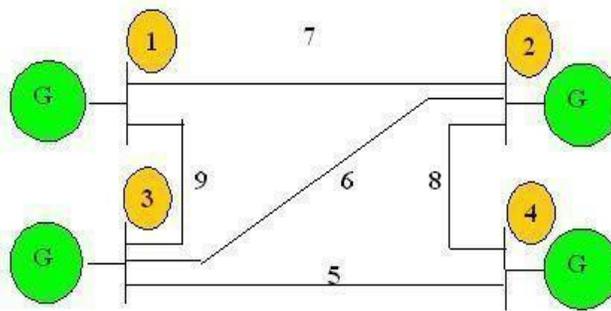


Fig. E1a. Single line diagram of Example System

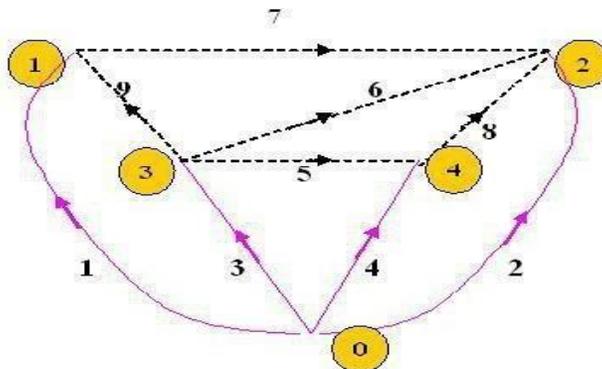


Fig. E1b. Oriented Graph of Fig. E1a.

For the system given, the oriented graph is as shown in figure E1b. some of the valid Tree graphs could be  $T(1,2,3,4)$ ,  $T(3,4,8,9)$ ,  $T(1,2,5,6)$ ,  $T(4,5,6,7)$ , etc. The basic cut-sets (A,B,C,D) and basic loops (E,F,G,H,I) corresponding to the oriented graph of Fig.E1a and tree,  $T(1,2,3,4)$  are as shown in Figure E1c and Fig.E1d respectively.

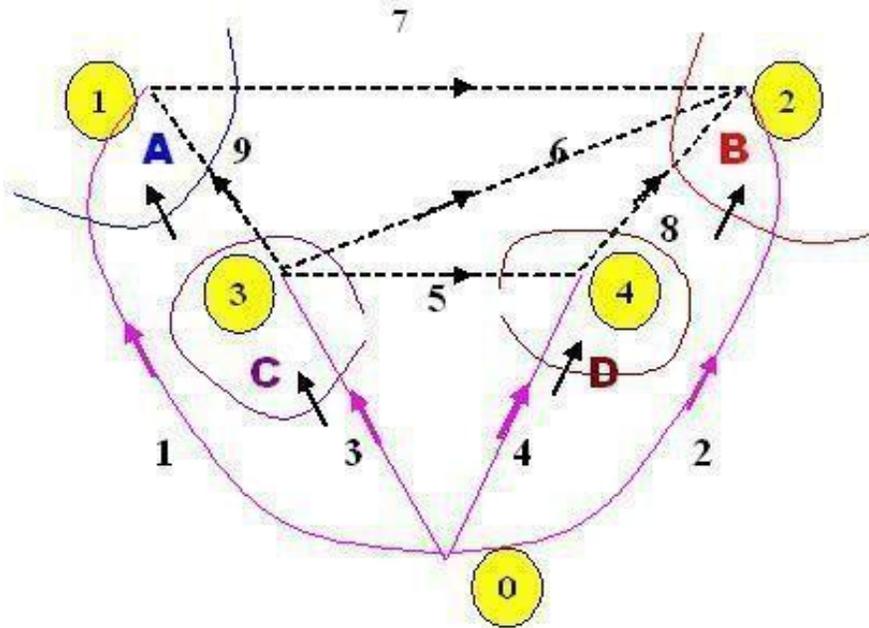


Fig. E1c. Basic Cutsets of Fig. E1a.

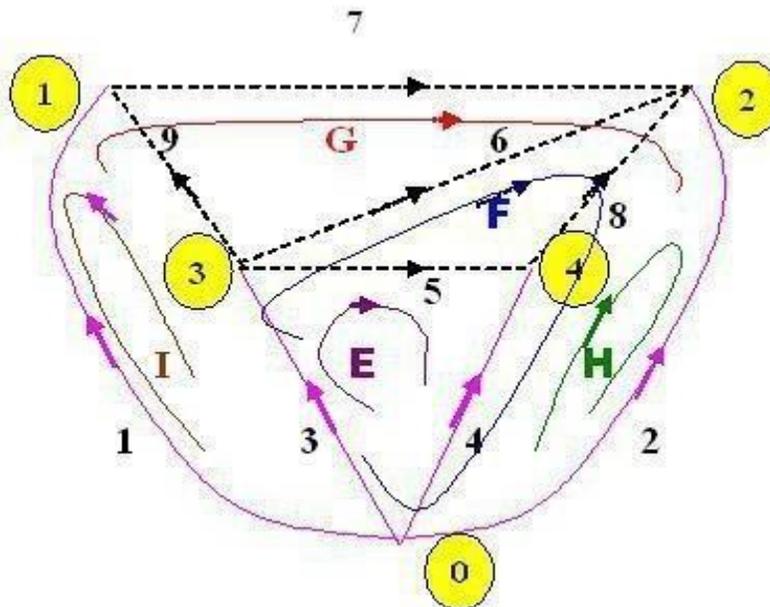


Fig. E1d. Basic Loops of Fig. E1a.

**INCIDENCE MATRICES**

**Element–node incidence matrix:**  $\hat{A}$

The incidence of branches to nodes in a connected graph is given by the element-node

incidence matrix,  $\hat{A}$ . An element  $a_{ij}$  of  $\hat{A}$  is defined as under:

- $a_{ij} = 1$  if the branch- $i$  is incident to and oriented away from the node- $j$ .
- $= -1$  if the branch- $i$  is incident to and oriented towards the node- $j$ .
- $= 0$  if the branch- $i$  is not at all incident on the node- $j$ .

Thus the dimension of  $\hat{A}$  is  $e \times n$ , where  $e$  is the number of elements and  $n$  is the number of nodes in the network. For example, consider again the sample system with its oriented graph as in fig. 1c. the corresponding element-node incidence matrix, is obtained as under:

$$\hat{A} = \begin{array}{c|cccc} & \text{Nodes} & & & & \\ \hline \text{Elements} & 0 & 1 & 2 & 3 & \\ \hline 1 & 1 & -1 & & & \\ 2 & 1 & & -1 & & \\ \hline 3 & 1 & & & -1 & \\ 4 & & 1 & -1 & & \\ 5 & & & 1 & -1 & \\ 6 & & 1 & & -1 & \end{array}$$

It is to be noted that the first column and first row are not part of the actual matrix and they only indicate the element number node number respectively as shown. Further, the sum of every row is found to be equal to zero always. Hence, the rank of the matrix is less than  $n$ . Thus in general, the matrix  $\hat{A}$  satisfies the identity:

$$\sum_{j=1}^n a_{ij} = 0 \quad \forall i = 1, 2, \dots, e. \tag{3}$$

**Bus incidence matrix: A**

By selecting any one of the nodes of the connected graph as the reference node, the corresponding column is deleted from  $A$  to obtain the bus incidence matrix,  $A$ . The dimensions of  $A$  are  $e \times (n-1)$  and the rank is  $n-1$ . In the above example, selecting node-0 as reference node, the matrix  $A$  is obtained by deleting the column corresponding to node-0, as under:

$$A = \begin{matrix} & \begin{matrix} \text{Buses} \\ \text{Elements} \end{matrix} & \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{matrix} = \begin{matrix} \begin{matrix} A_b \\ A_l \end{matrix} & \begin{matrix} \text{Branches} \\ \text{Links} \end{matrix} \end{matrix}$$

	<b>Buses</b>		<b>1</b>	<b>2</b>	<b>3</b>
	<b>Elements</b>				
<b>1</b>			-1		
<b>2</b>				-1	
<b>3</b>					-1
<b>4</b>			1	-1	
<b>5</b>				1	-1
<b>6</b>			1		-1

<b>A<sub>b</sub></b>	<b>Branches</b>
<b>A<sub>l</sub></b>	<b>Links</b>

It may be observed that for a selected tree, say, T(1,2,3), the bus incidence matrix can be so arranged that the branch elements occupy the top portion of the A-matrix followed by the link elements. Then, the matrix-A can be partitioned into two sub matrices  $A_b$  and  $A_l$  as shown, where,

- (i)  $A_b$  is of dimension (bxb) corresponding to the branches and
- (ii)  $A_l$  is of dimension (lxb) corresponding to links.

$A$  is a rectangular matrix, hence it is singular.  $A_b$  is a non-singular square matrix of dimension-b. Since  $A$  gives the incidence of various elements on the nodes with their direction of incidence, the KCL for the nodes can be written as

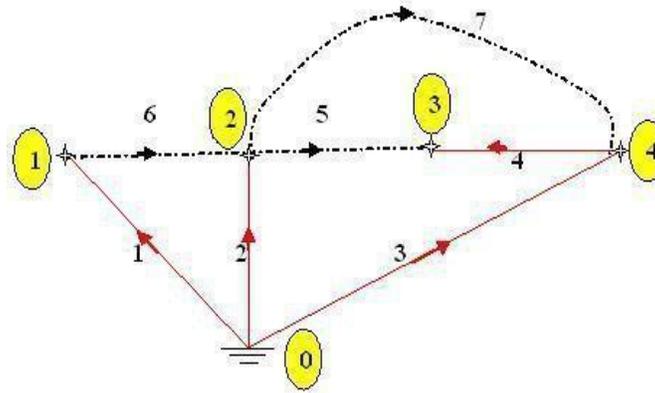
$$A^T i = 0 \tag{4}$$

where  $A^T$  is the transpose of matrix  $A$  and  $i$  is the vector of branch currents. Similarly for the branch voltages we can write,

$$v = A \overline{E}_{bus} \tag{5}$$

**Examples on Bus Incidence Matrix:**

**Example-2:** For the sample network-oriented graph shown in Fig. E2, by selecting a tree,  $T(1,2,3,4)$ , obtain the incidence matrices  $A$  and  $\hat{A}$ . Also show the partitioned form of the matrix- $A$ .



**Fig. E2. Sample Network-Oriented Graph**

nodes

$\hat{A}$  = Elements

$e \backslash n$	0	1	2	3	4
1	0	1	-1	0	0
2	0	1	0	-1	0
3	0	1	0	0	0
4	0	0	0	0	-1
5	0	0	0	1	-1
6	0	0	1	-1	0
7	0	0	0	1	0

buses

$A$  = Elements

$e \backslash b$	1	2	3	4
1	-1	0	0	0
2	0	-1	0	0
3	0	0	0	-1
4	0	0	-1	1
5	0	1	-1	0
6	1	-1	0	0
7	0	1	0	-1

Corresponding to the Tree, T(1,2,3,4), matrix-A can be partitioned into two sub-matrices as under:

$$A_b = \text{branch} \begin{matrix} \text{buses} \\ \begin{bmatrix} b \backslash b & 1 & 2 & 3 & 4 \\ 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & 0 & -1 \\ \hline & 4 & 0 & 0 & -1 & 1 \end{bmatrix} \end{matrix}$$

$$A_l = \text{link} \begin{matrix} \text{buses} \\ \begin{bmatrix} l \backslash b & 1 & 2 & 3 & 4 \\ 5 & 0 & 1 & -1 & 0 \\ \hline & 7 & 0 & 1 & 0 & -1 \end{bmatrix} \end{matrix}$$

**Example-3:** For the sample-system shown in Fig. E3, obtain an oriented graph. By selecting a tree, T(1,2,3,4), obtain the incidence matrices A and A Also show the partitioned form of the matrix-A.

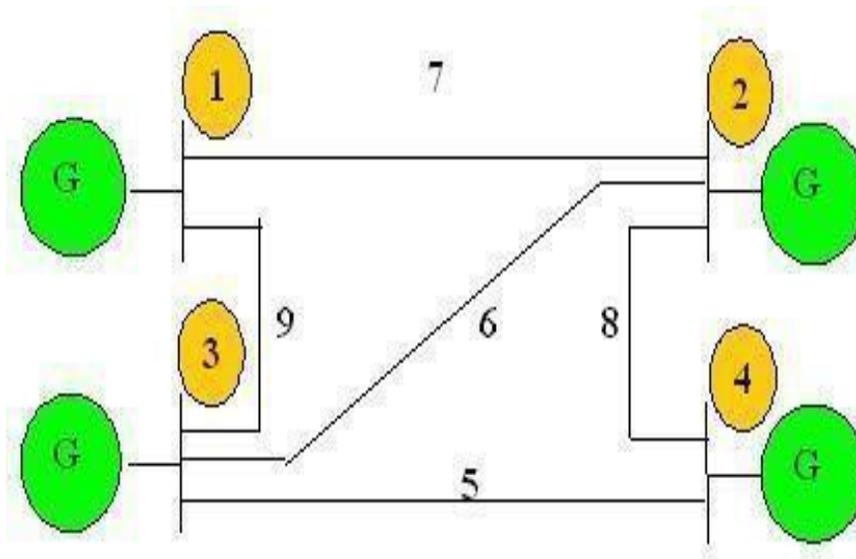


Fig. E3a. Sample Example network

Consider the oriented graph of the given system as shown in figure E3b, below.

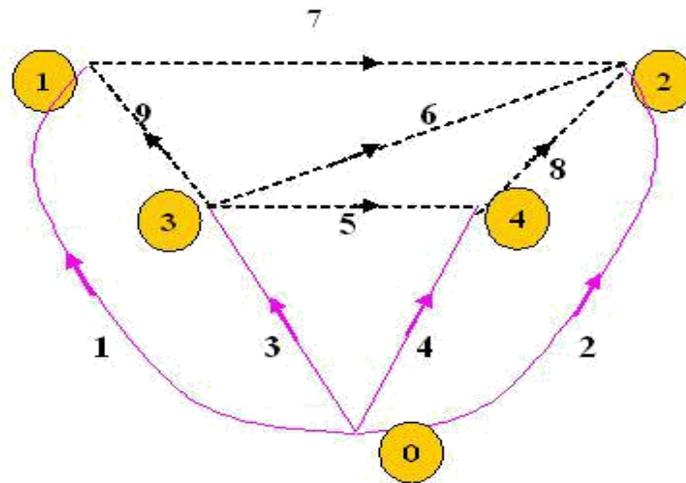


Fig. E3b. Oriented Graph of system of Fig-E3a.

Corresponding to the oriented graph above and a Tree,  $T(1,2,3,4)$ , the incidence matrices  $\hat{A}$  and  $A$  can be obtained as follows:

$$\hat{A} = \begin{array}{c|cccc} \text{e/n} & 0 & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & -1 & & & \\ 2 & 1 & & -1 & & \\ 3 & 1 & & & -1 & \\ 4 & 1 & & & & -1 \\ 5 & & & & 1 & -1 \\ 6 & & & -1 & 1 & \\ 7 & & 1 & -1 & & \\ 8 & & & -1 & & 1 \\ 9 & & -1 & & 1 & \end{array}$$

$$A = \begin{array}{c|cccc} \text{e/b} & 1 & 2 & 3 & 4 \\ \hline 1 & -1 & & & \\ 2 & & -1 & & \\ 3 & & & -1 & \\ 4 & & & & -1 \\ 5 & & & 1 & -1 \\ 6 & & -1 & 1 & \\ 7 & 1 & -1 & & \\ 8 & & -1 & & 1 \\ 9 & -1 & & 1 & \end{array}$$

Corresponding to the Tree,  $T(1,2,3,4)$ , matrix- $A$  can be partitioned into two sub-matrices as under:

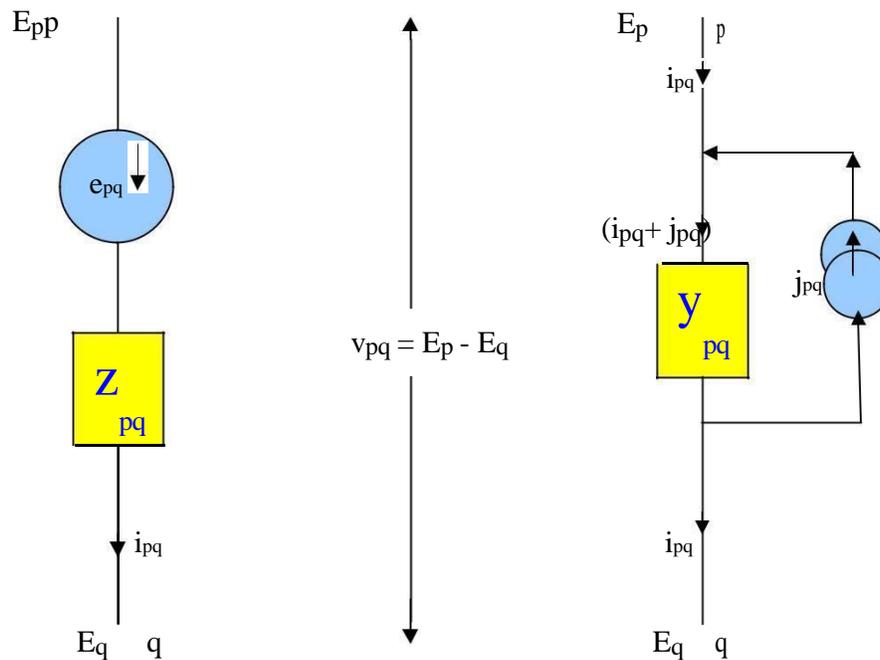
$$A_b = \begin{array}{c|cccc} \text{e/b} & 1 & 2 & 3 & 4 \\ \hline 1 & -1 & & & \\ 2 & & -1 & & \\ 3 & & & -1 & \\ 4 & & & & -1 \end{array}$$

$$A_l = \begin{array}{c|cccc} \text{e/b} & 1 & 2 & 3 & 4 \\ \hline 5 & & & 1 & -1 \\ 6 & & -1 & 1 & \\ 7 & 1 & -1 & & \\ 8 & & -1 & & 1 \\ 9 & -1 & & 1 & \end{array}$$

**PRIMITIVE NETWORKS**

So far, the matrices of the interconnected network have been defined. These matrices contain complete information about the network connectivity, the orientation of current, the loops and cutsets. However, these matrices contain no information on the nature of the elements which form the interconnected network. The complete behaviour of the network can be obtained from the knowledge of the behaviour of the individual elements which make the network, along with the incidence matrices. An element in an electrical network is completely characterized by the relationship between the current through the element and the voltage across it.

**General representation of a network element:** In general, a network element may contain active or passive components. Figure 2 represents the alternative impedance and admittance forms of representation of a general network component.



**Fig.2 Representation of a primitive network element**  
 (a) Impedance form (b) Admittance form

The network performance can be represented by using either the impedance or the admittance form of representation. With respect to the element, p-q, let,

$v_{pq}$  = voltage across the element p-q,

$e_{pq}$  = source voltage in series with the element p-

q,  $i_{pq}$  = current through the element p-q,

$j_{pq}$  = source current in shunt with the element p-

q,  $z_{pq}$  = self impedance of the element p-q and

$y_{pq}$  = self admittance of the element p-q.

**Performance equation:** Each element p-q has two variables,  $v_{pq}$  and  $i_{pq}$ . The performance of the given element p-q can be expressed by the performance equations as under:

$$\begin{aligned} v_{pq} + e_{pq} &= z_{pq} i_{pq} && \text{(in its impedance form)} \\ i_{pq} + j_{pq} &= y_{pq} v_{pq} && \text{(in its admittance form)} \end{aligned} \quad (6)$$

Thus the parallel source current  $j_{pq}$  in admittance form can be related to the series source voltage,  $e_{pq}$  in impedance form as per the identity:

$$j_{pq} = -y_{pq} e_{pq} \quad (7)$$

A set of non-connected elements of a given system is defined as a *primitive Network* and an element in it is a fundamental element that is not connected to any other element. In the equations above, if the variables and parameters are replaced by the corresponding vectors and matrices, referring to the complete set of elements present in a given system, then, we get the performance equations of the primitive network in the form as under:

$$\begin{aligned} v + e &= [z] i \\ i + j &= [y] v \end{aligned} \quad (8)$$

### Primitive network matrices:

A diagonal element in the matrices,  $[z]$  or  $[y]$  is the self impedance  $z_{pq-pq}$  or self admittance,  $y_{pq-pq}$ . An off-diagonal element is the mutual impedance,  $z_{pq-rs}$  or mutual admittance,  $y_{pq-rs}$ , the value present as a mutual coupling between the elements p-q and r-s. The primitive network admittance matrix,  $[y]$  can be obtained also by

inverting the primitive impedance matrix, [z]. Further, if there are no mutually coupled elements in the given system, then both the matrices, [z] and [y] are diagonal. In such cases, the self impedances are just equal to the reciprocal of the corresponding values of self admittances, and vice-versa.

**Examples on Primitive Networks:**

**Example-4:** Given that the self impedances of the elements of a network referred by the bus incidence matrix given below are equal to:  $Z_1=Z_2=0.2$ ,  $Z_3=0.25$ ,  $Z_4=Z_5=0.1$  and  $Z_6=0.4$  units, draw the corresponding oriented graph, and find the primitive network matrices. Neglect mutual values between the elements.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

**Solution:**

The element node incidence matrix,  $A^e$  can be obtained from the given A matrix, by pre-augmenting to it an extra column corresponding to the reference node, as under.

$$A^e = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

Based on the conventional definitions of the elements of  $A$ , the oriented graph can be formed as under:

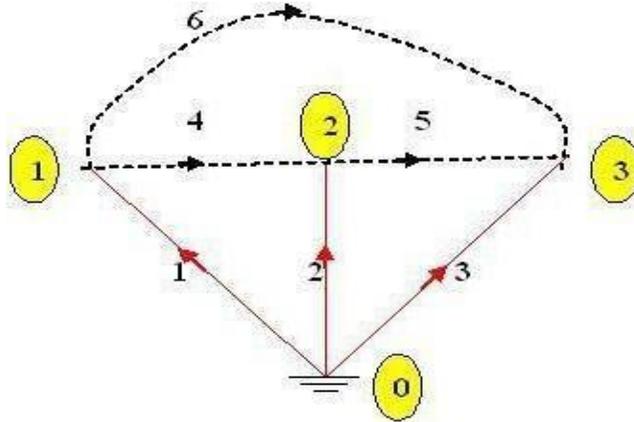


Fig. E4 Oriented Graph

Thus the primitive network matrices are square, symmetric and diagonal matrices of order  $e = \text{no. of elements} = 6$ . They are obtained as follows.

$$[z] = \begin{bmatrix} 0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4 \end{bmatrix}$$

And

$$[y] = \begin{bmatrix} 5.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.5 \end{bmatrix}$$

**Example-5:** Consider three passive elements whose data is given in Table E5 below.  
Form the primitive network impedance matrix.

**Table E5**

Element number	Self impedance ( $Z_{pq-pq}$ )		Mutual impedance, ( $Z_{pq-rs}$ )	
	Bus-code, (p-q)	Impedance in p.u.	Bus-code, (r-s)	Impedance in p.u.
1	1-2	j 0.452		
2	2-3	j 0.387	1-2	j 0.165
3	1-3	j 0.619	1-2	j 0.234

**Solution:**

$$[z] = \begin{matrix} & \begin{matrix} 1-2 & 2-3 & 1-3 \end{matrix} \\ \begin{matrix} 1-2 \\ 2-3 \\ 1-3 \end{matrix} & \begin{bmatrix} j\ 0.452 & j\ 0.165 & j\ 0.234 \\ j\ 0.165 & j\ 0.387 & 0 \\ j\ 0.234 & 0 & j\ 0.619 \end{bmatrix} \end{matrix}$$

**Note:**

- The size of  $[z]$  is  $e \times e$ , where  $e$ = number of elements,
- The diagonal elements are the self impedances of the elements
- The off-diagonal elements are mutual impedances between the corresponding elements.
- Matrices  $[z]$  and  $[y]$  are inter-invertible.

## FORMATION OF Y<sub>BUS</sub> AND Z<sub>BUS</sub>

The bus admittance matrix, Y<sub>BUS</sub> plays a very important role in computer aided power system analysis. It can be formed in practice by either of the methods as under:

1. Rule of Inspection
2. Singular Transformation
3. Non-Singular Transformation
4. Z<sub>BUS</sub> Building Algorithms, etc.

The performance equations of a given power system can be considered in three different frames of reference as discussed below:

### Frames of Reference:

*Bus Frame of Reference:* There are b independent equations (b = no. of buses) relating the bus vectors of currents and voltages through the bus impedance matrix and bus admittance matrix:

$$\begin{aligned} E_{BUS} &= Z_{BUS} I_{BUS} \\ I_{BUS} &= Y_{BUS} E_{BUS} \end{aligned} \quad (9)$$

*Branch Frame of Reference:* There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$$\begin{aligned} E_{BR} &= Z_{BR} I_{BR} \\ I_{BR} &= Y_{BR} E_{BR} \end{aligned} \quad (10)$$

*Loop Frame of Reference:* There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$$\begin{aligned} E_{LOOP} &= Z_{LOOP} I_{LOOP} \\ I_{LOOP} &= Y_{LOOP} E_{LOOP} \end{aligned} \quad (11)$$

Of the various network matrices referred above, the bus admittance matrix (Y<sub>BUS</sub>) and the bus impedance matrix (Z<sub>BUS</sub>) are determined for a given power system by the rule of inspection as explained next.

### Rule of Inspection

Consider the 3-node admittance network as shown in figure5. Using the basic branch relation:  $I = (YV)$ , for all the elemental currents and applying Kirchhoff's Current Law principle at the nodal points, we get the relations as under:

$$\begin{aligned} \text{At node 1: } I_1 &= Y_1 V_1 + Y_3 (V_1 - V_3) + Y_6 (V_1 - V_2) \\ \text{At node 2: } I_2 &= Y_2 V_2 + Y_5 (V_2 - V_3) + Y_6 (V_2 - V_1) \\ \text{At node 3: } 0 &= Y_3 (V_3 - V_1) + Y_4 V_3 + Y_5 (V_3 - V_2) \end{aligned} \quad (12)$$

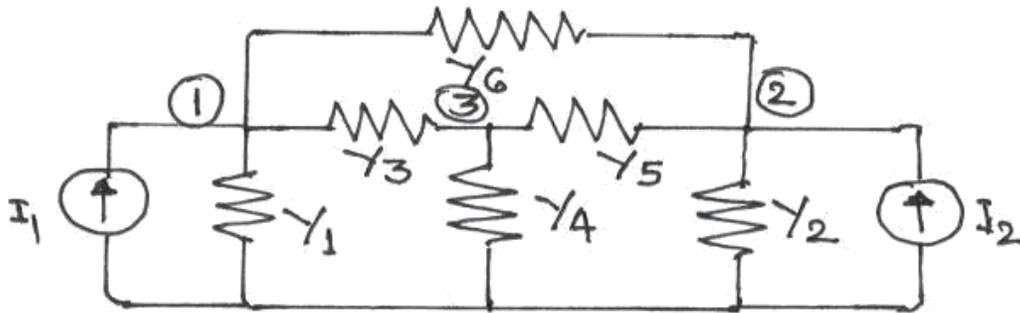


Fig. 3 Example System for finding YBUS

These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \end{bmatrix} = \begin{bmatrix} (Y_1+Y_3+Y_6)-Y_6 & -Y_3 \\ -Y_6 & (Y_2+Y_5+Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3+Y_4+Y_5) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \quad (13)$$

In other words, the relation of equation (9) can be represented in the form

$$I_{BUS} = Y_{BUS} E_{BUS} \quad (14)$$

Where, Y<sub>BUS</sub> is the bus admittance matrix, I<sub>BUS</sub> & E<sub>BUS</sub> are the bus current and bus voltage vectors respectively.

By observing the elements of the bus admittance matrix, Y<sub>BUS</sub> of equation (13), it is observed that the matrix elements can as well be obtained by a simple inspection of the given system diagram:

*Diagonal elements:* A diagonal element (Y<sub>ii</sub>) of the bus admittance matrix, Y<sub>BUS</sub>, is equal to the sum total of the admittance values of all the elements incident at the bus/node i,

*Off Diagonal elements:* An off-diagonal element (Y<sub>ij</sub>) of the bus admittance matrix, Y<sub>BUS</sub>, is equal to the negative of the admittance value of the connecting element present between the buses I and j, if any.

This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

$$\begin{aligned} Y_{ii} &= \sum y_{ij} \quad (j = 1,2,\dots,n) \\ Y_{ij} &= -y_{ij} \quad (j = 1,2,\dots,n) \end{aligned} \quad (15)$$

For  $i = 1, 2, \dots, n$ ,  $n =$  no. of buses of the given system,  $y_{ij}$  is the admittance of element connected between buses  $i$  and  $j$  and  $y_{ii}$  is the admittance of element connected between bus  $i$  and ground (reference bus).

**Bus impedance matrix**

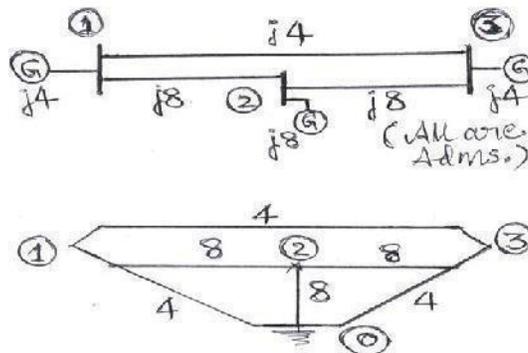
In cases where, the bus impedance matrix is also required, it cannot be formed by direct inspection of the given system diagram. However, the bus admittance matrix determined by the rule of inspection following the steps explained above, can be inverted to obtain the bus impedance matrix, since the two matrices are inter-invertible.

**Note:** It is to be noted that the rule of inspection can be applied only to those power systems that do not have any mutually coupled elements.

**Examples on Rule of Inspection:**

**Example 6:** Obtain the bus admittance matrix for the admittance network shown aside by the rule of inspection

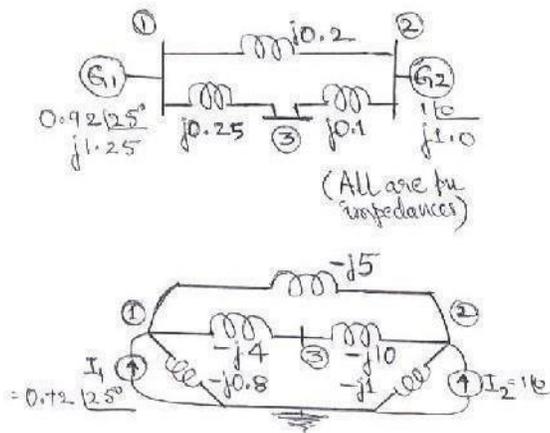
$$Y_{BUS} = j \begin{vmatrix} 16 & -8 & -4 \\ -8 & 24 & -8 \\ -4 & -8 & 16 \end{vmatrix}$$

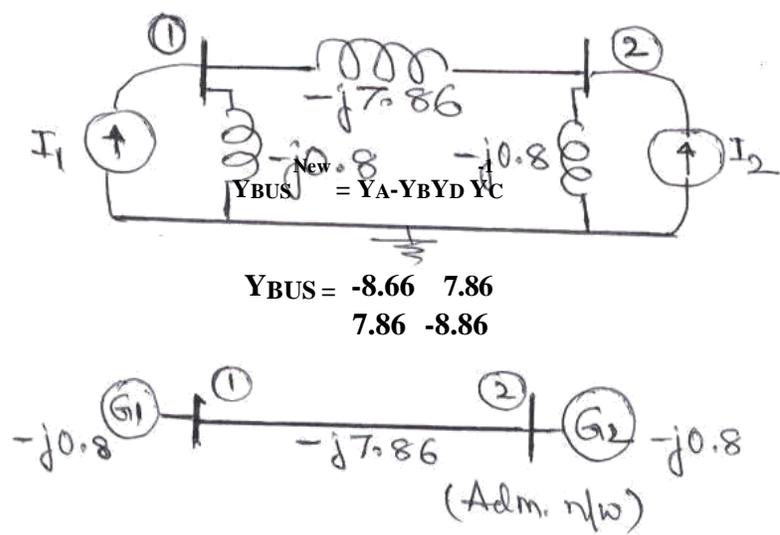


**Example 7:** Obtain  $Y_{BUS}$  for the impedance network shown aside by the rule of inspection. Also, determine  $Y_{BUS}$  for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.

$$Y_{BUS} = j \begin{vmatrix} -9.3 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1}$$





$$Y_{BUS} = \begin{bmatrix} -8.66 & 7.86 \\ 7.86 & -8.86 \end{bmatrix}$$

$$Y_{BUS}^{New} = Y_A - Y_B Y_D Y_C$$

$$Y_{BUS} = j \begin{vmatrix} -8.66 & 7.86 \\ 7.86 & -8.66 \end{vmatrix}$$

**SINGULAR TRANSFORMATIONS**

The primitive network matrices are the most basic matrices and depend purely on the impedance or admittance of the individual elements. However, they do not contain any information about the behaviour of the interconnected network variables. Hence, it is necessary to transform the primitive matrices into more meaningful matrices which can relate variables of the interconnected network.

**Bus admittance matrix, YBUS and Bus impedance matrix, ZBUS**

In the bus frame of reference, the performance of the interconnected network is described by *n* independent nodal equations, where *n* is the total number of buses (*n*+1 nodes are present, out of which one of them is designated as the reference node). For example a 5-bus system will have 5 external buses and 1 ground/ ref. bus). The

performance equation relating the bus voltages to bus current injections in bus frame of reference in admittance form is given by

$$I_{BUS} = Y_{BUS} E_{BUS} \quad (17)$$

Where  $E_{BUS}$  = vector of bus voltages measured with respect to reference bus

$I_{BUS}$  = Vector of currents injected into the bus

$Y_{BUS}$  = bus admittance matrix

The performance equation of the primitive network in admittance form is given by

$$i + j = [y] v$$

Pre-multiplying by  $A^t$  (transpose of A), we obtain

$$A^t i + A^t j = A^t [y] v \quad (18)$$

However, as per equation (4),

$$A^t i = 0,$$

since it indicates a vector whose elements are the algebraic sum of element currents incident at a bus, which by Kirchoff's law is zero. Similarly,  $A^t j$  gives the algebraic sum of all source currents incident at each bus and this is nothing but the total current injected at the bus. Hence,

$$A^t j = I_{BUS} \quad (19)$$

Thus from (18) we have,  $I_{BUS} = A^t [y] v$  (20)

However, from (5), we have

$$v = A E_{BUS}$$

And hence substituting in (20) we get,

$$I_{BUS} = A^t [y] A E_{BUS} \quad (21)$$

Comparing (21) with (17) we obtain,

$$Y_{BUS} = A^t [y] A \quad (22)$$

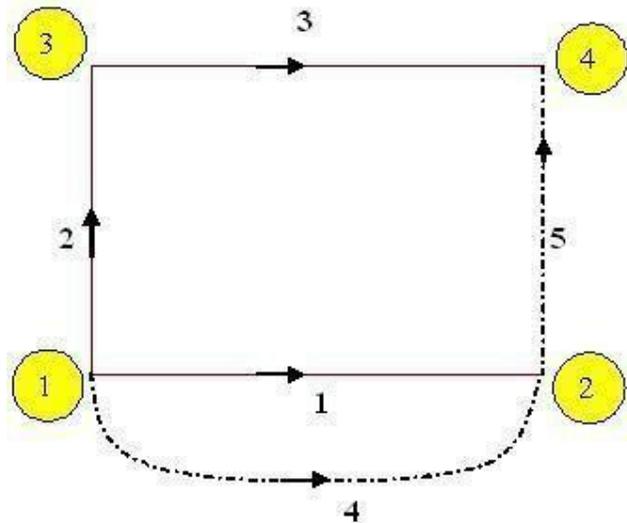
The bus incidence matrix is rectangular and hence singular. Hence, (22) gives a singular transformation of the primitive admittance matrix [y]. The bus impedance matrix is given by ,

$$Z_{BUS} = Y_{BUS}^{-1} \quad (23)$$

Note: This transformation can be derived using the concept of power invariance, however, since the transformations are based purely on KCL and KVL, the transformation will obviously be power invariant.

**Examples on Singular Transformation:**

**Example 8:** For the network of Fig E8, form the primitive matrices  $[z]$  &  $[y]$  and obtain the bus admittance matrix by singular transformation. Choose a Tree  $T(1,2,3)$ . The data is given in Table E8.



**Fig E8 System for Example-8**

**Table E8: Data for Example-8**

Elements	Self impedance	Mutual impedance
1	$j 0.6$	-
2	$j 0.5$	$j 0.1$ (with element 1)
3	$j 0.5$	-
4	$j 0.4$	$j 0.2$ (with element 1)
5	$j 0.2$	-

**Solution:**

The bus incidence matrix is formed taking node 1 as the reference bus.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

The primitive incidence matrix is given by,

$$[z] = \begin{bmatrix} j0.6 & j0.1 & 0.0 & j0.2 & 0.0 \\ j0.1 & j0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & j0.5 & 0.0 & 0.0 \\ j0.2 & 0.0 & 0.0 & j0.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & j0.2 \end{bmatrix}$$

The primitive admittance matrix  $[y] = [z]^{-1}$  and given by,

$$[y] = \begin{bmatrix} -j2.0833 & j0.4167 & 0.0 & j1.0417 & 0.0 \\ j0.4167 & -j2.0833 & 0.0 & -j0.2083 & 0.0 \\ 0.0 & 0.0 & -j2.0 & 0.0 & 0.0 \\ j1.0417 & -j0.2083 & 0.0 & -j3.0208 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -j5.0 \end{bmatrix}$$

The bus admittance matrix by singular transformation is obtained as

$$Y_{BUS} = A [y] A^t = \begin{bmatrix} -j8.0208 & j0.2083 & j5.0 \\ j0.2083 & -j4.0833 & j2.0 \\ j5.0 & j2.0 & -j7.0 \end{bmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1} = \begin{bmatrix} j0.2713 & j0.1264 & j0.2299 \\ j0.1264 & j0.3437 & j0.1885 \\ j0.2299 & j0.1885 & j0.3609 \end{bmatrix}$$

**SUMMARY**

The formulation of the mathematical model is the first step in obtaining the solution of any electrical network. The independent variables can be either currents or voltages. Correspondingly, the elements of the coefficient matrix will be impedances or admittances.

Network equations can be formulated for solution of the network using graph theory, independent of the nature of elements. In the graph of a network, the tree-branches and links are distinctly identified. The complete information about the interconnection of the network, with the directions of the currents is contained in the bus incidence matrix.

The information on the nature of the elements which form the interconnected network is contained in the primitive impedance matrix. A primitive element can be represented in impedance form or admittance form. In the bus frame of reference, the performance of the interconnected system is described by  $(n-1)$  nodal equations, where  $n$  is the number of nodes. The bus admittance matrix and the bus impedance matrix relate the bus voltages and currents. These matrices can be obtained from the primitive impedance and admittance matrices.

Chapter-1-BFORMATION OF BUS IMPEDANCE MATRIX

[CONTENTS: Node elimination by matrix algebra, generalized algorithms for ZBUS building, addition of BRANCH, addition of LINK, special cases of analysis, removal of elements, changing the impedance value of an element, examples]

**NODE ELIMINATION BY MATRIX ALGEBRA**

Nodes can be eliminated by the matrix manipulation of the standard node equations. However, *only those nodes at which current does not enter or leave the network can be considered for such elimination*. Such nodes can be eliminated either in one group or by taking the eligible nodes one after the other for elimination, as discussed next.

**CASE-A: Simultaneous Elimination of Nodes:**

Consider the performance equation of the given network in bus frame of reference in admittance form for a n-bus system, given by:

$$\mathbf{I}_{\text{BUS}} = \mathbf{Y}_{\text{BUS}} \mathbf{E}_{\text{BUS}} \quad (1)$$

Where  $\mathbf{I}_{\text{BUS}}$  and  $\mathbf{E}_{\text{BUS}}$  are n-vectors of injected bus current and bus voltages and  $\mathbf{Y}_{\text{BUS}}$  is the square, symmetric, coefficient bus admittance matrix of order n.

Now, of the n buses present in the system, let p buses be considered for node-elimination so that the reduced system after elimination of p nodes would be retained with m (= n-p) nodes only. Hence the corresponding performance equation would be similar to (1) except that the coefficient matrix would be of order m now, i.e.,

$$\mathbf{I}_{\text{BUS}} = \mathbf{Y}_{\text{BUS}}^{\text{new}} \mathbf{E}_{\text{BUS}} \quad (2)$$

Where  $\mathbf{Y}_{\text{BUS}}$  is the bus admittance matrix of the reduced network and the vectors  $\mathbf{I}_{\text{BUS}}$  and  $\mathbf{E}_{\text{BUS}}$  are of order m. It is assumed in (1) that  $\mathbf{I}_{\text{BUS}}$  and  $\mathbf{E}_{\text{BUS}}$  are obtained with their elements arranged such that the elements associated with p nodes to be eliminated are in the lower portion of the vectors. Then the elements of  $\mathbf{Y}_{\text{BUS}}$  also get located accordingly so that (1) after matrix partitioning yields,

$$\begin{bmatrix} \mathbf{I}_{\text{BUS-m}} \\ \mathbf{I}_{\text{BUS-p}} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_A & \mathbf{Y}_B \\ \mathbf{Y}_C & \mathbf{Y}_D \end{bmatrix} \begin{bmatrix} \mathbf{E}_{\text{BUS-m}} \\ \mathbf{E}_{\text{BUS-p}} \end{bmatrix} \quad (3)$$

Where the self and mutual values of  $\mathbf{Y}_A$  and  $\mathbf{Y}_D$  are those identified only with the nodes to be retained and removed respectively and  $\mathbf{Y}_C = \mathbf{Y}_B^t$  is composed of only the corresponding mutual admittance values, that are common to the nodes  $m$  and  $p$ .

Now, for the  $p$  nodes to be eliminated, it is necessary that, each element of the vector  $\mathbf{I}_{\text{BUS-p}}$  should be zero. Thus we have from (3):

$$\begin{aligned} \mathbf{I}_{\text{BUS-m}} &= \mathbf{Y}_A \mathbf{E}_{\text{BUS-m}} + \mathbf{Y}_B \mathbf{E}_{\text{BUS-p}} \\ \mathbf{I}_{\text{BUS-p}} &= \mathbf{Y}_C \mathbf{E}_{\text{BUS-m}} + \mathbf{Y}_D \mathbf{E}_{\text{BUS-p}} = 0 \end{aligned} \quad (4)$$

Solving, 
$$\mathbf{E}_{\text{BUS-p}} = -\mathbf{Y}_D^{-1} \mathbf{Y}_C \mathbf{E}_{\text{BUS-m}} \quad (5)$$

Thus, by simplification, we obtain an expression similar to (2) as,

$$\mathbf{I}_{\text{BUS-m}} = \{ \mathbf{Y}_A - \mathbf{Y}_B \mathbf{Y}_D^{-1} \mathbf{Y}_C \} \mathbf{E}_{\text{BUS-m}} \quad (6)$$

Thus by comparing (2) and (6), we get an expression for the new bus admittance matrix in terms of the sub-matrices of the original bus admittance matrix as:

$$\mathbf{Y}_{\text{BUS}}^{\text{new}} = \{ \mathbf{Y}_A - \mathbf{Y}_B \mathbf{Y}_D^{-1} \mathbf{Y}_C \} \quad (7)$$

This expression enables us to construct the given network with only the necessary nodes retained and all the unwanted nodes/buses eliminated. However, it can be observed from (7) that the expression involves finding the inverse of the sub-matrix  $\mathbf{Y}_D$  (of order  $p$ ). This would be computationally very tedious if  $p$ , the nodes to be eliminated is very large, especially for real practical systems. In such cases, it is more advantageous to eliminate the unwanted nodes from the given network by considering one node only at a time for elimination, as discussed next.

**CASE-B: Separate Elimination of Nodes:**

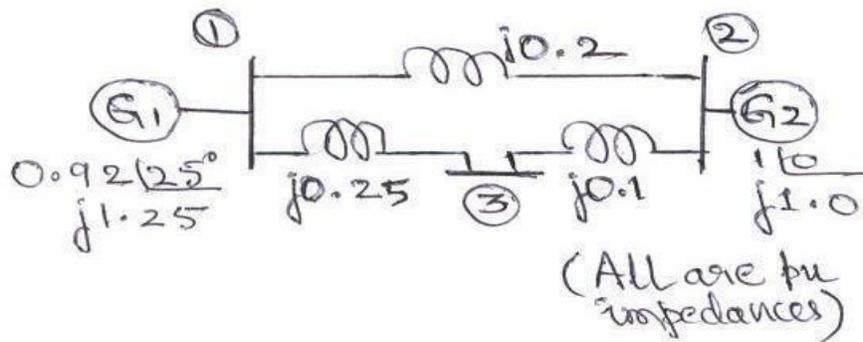
Here again, the system buses are to be renumbered, if necessary, such that the node to be removed always happens to be the last numbered one. The sub-matrix  $Y_D$  then would be a single element matrix and hence its inverse would be just equal to its own reciprocal value. Thus the generalized algorithmic equation for finding the elements of the new bus admittance matrix can be obtained from (6) as,

$$Y_{ij}^{new} = Y_{ij}^{old} - Y_{in} Y_{nj} / Y_{nn} \quad \forall i, j = 1, 2, \dots, n. \quad (8)$$

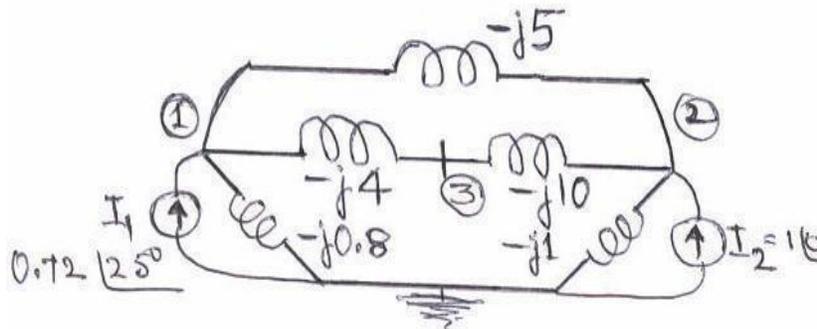
Each element of the original matrix must therefore be modified as per (7). Further, this procedure of eliminating the last numbered node from the given system of  $n$  nodes is to be iteratively repeated  $p$  times, so as to eliminate all the unnecessary  $p$  nodes from the original system.

**Examples on Node elimination:**

**Example-1:** Obtain  $Y_{BUS}$  for the impedance network shown below by the rule of inspection. Also, determine  $Y_{BUS}$  for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.



The admittance equivalent network is as follows:



The bus admittance matrix is obtained by RoI as:

$$Y_{BUS} = j \begin{vmatrix} -9.8 & 5.4 \\ & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$

The reduced matrix after elimination of node 3 from the given system is determined as per the equation:

$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

$$Y_{BUS}^{new} = \begin{matrix} n/n & 1 & 2 \\ 1 & \mathbf{-j8.66} & \mathbf{j7.86} \\ 2 & \mathbf{j7.86} & \mathbf{-j8.66} \end{matrix}$$

Alternatively,

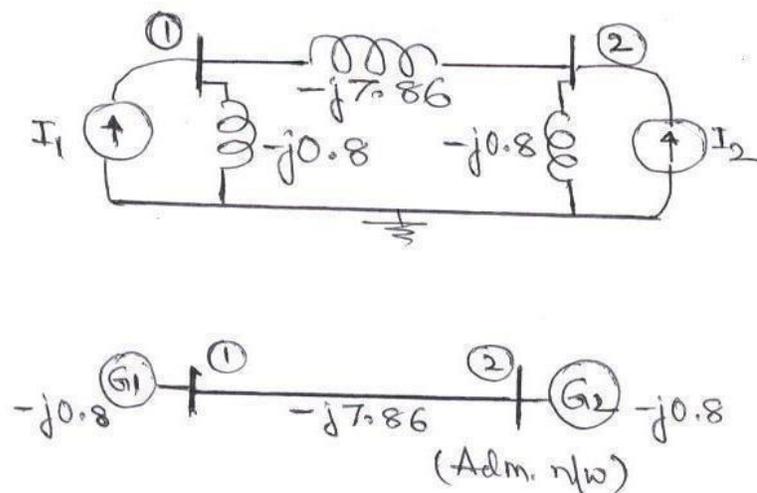
$$Y_{ij}^{new} = Y_{ij}^{old} - Y_{i3} Y_{3j} / Y_{33} \quad \forall i, j = 1, 2.$$

$$Y_{11} = Y_{11} - Y_{13} Y_{31} / Y_{33} = \mathbf{-j8.66}$$

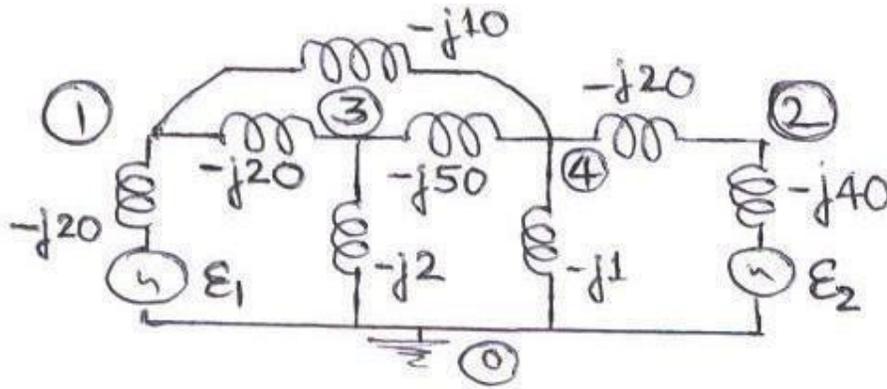
$$Y_{22} = Y_{22} - Y_{23} Y_{32} / Y_{33} = \mathbf{-j8.66}$$

$$Y_{12} = Y_{21} = Y_{12} - Y_{13} Y_{32} / Y_{33} = \mathbf{j7.86}$$

Thus the reduced network can be obtained again by the rule of inspection as shown below.



**Example-2:** Obtain  $Y_{BUS}$  for the admittance network shown below by the rule of inspection. Also, determine  $Y_{BUS}$  for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.

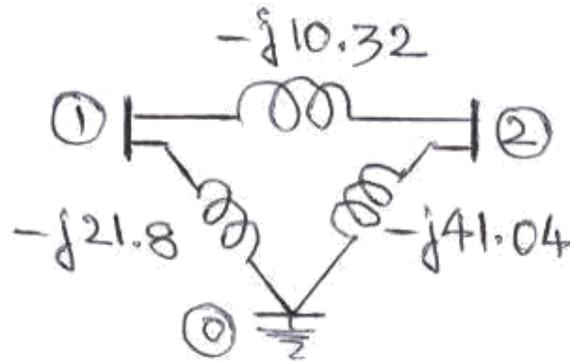


$$Y_{BUS} = \begin{matrix} n/n & 1 & 2 & 3 & 4 \\ 1 & -j50 & 0 & j20 & j10 \\ 2 & 0 & -j60 & 0 & j72 \\ 3 & j20 & 0 & -j72 & j50 \\ 4 & j10 & j72 & j50 & -j81 \end{matrix} = \begin{matrix} Y_A & Y_B \\ Y_C & Y_D \end{matrix}$$

$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

$$Y_{BUS}^{new} = \begin{matrix} n/n & 1 & 2 \\ 1 & -j32.12 & j10.32 \\ 2 & j10.32 & -j51.36 \end{matrix}$$

Thus the reduced system of two nodes can be drawn by the rule of inspection as under:



**ZBUS building**

**FORMATION OF BUS IMPEDANCE MATRIX**

The bus impedance matrix is the inverse of the bus admittance matrix. An alternative method is possible, based on an algorithm to form the bus impedance matrix directly from system parameters and the coded bus numbers. The bus impedance matrix is formed adding one element at a time to a partial network of the given system. The performance equation of the network in bus frame of reference in impedance form using the currents as independent variables is given in matrix form by

$$E_{bus} = [Z] I_{bus} \tag{9}$$

When expanded so as to refer to a  $n$  bus system, (9) will be of the form

$$E_1 = Z_{11} I_1 + Z_{12} I_2 + \dots + Z_{1k} I_k + \dots + Z_{1n} I_n$$

$$E_k = Z_{k1} I_1 + Z_{k2} I_2 + \dots + Z_{kk} I_k + \dots + Z_{kn} I_n$$

$$E_n = Z_{n1} I_1 + Z_{n2} I_2 + \dots + Z_{nk} I_k + \dots + Z_{nn} I_n \tag{10}$$

Now assume that the bus impedance matrix  $Z_{bus}$  is known for a partial network of  $m$  buses and a known reference bus. Thus,  $Z_{bus}$  of the partial network is of dimension  $m \times m$ . If now a new element is added between buses  $p$  and  $q$  we have the following two possibilities:

- (i)  $p$  is an existing bus in the partial network and  $q$  is a new bus; in this case  $p$ - $q$  is a **branch** added to the  $p$ -network as shown in Fig 1a, and
- (ii) both  $p$  and  $q$  are buses existing in the partial network; in this case  $p$ - $q$  is a **link** added to the  $p$ -network as shown in Fig 1b.

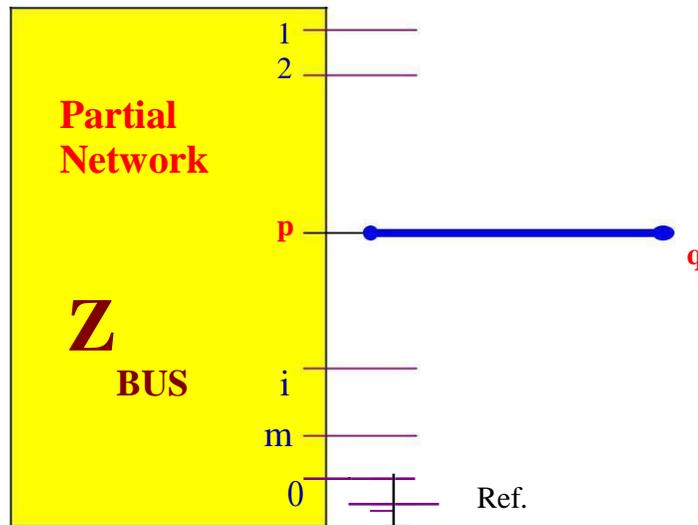


Fig 1a. Addition of branch p-q

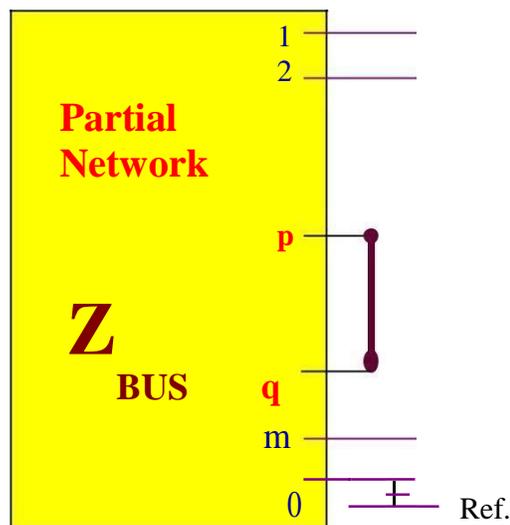


Fig 1b. Addition of link p-q

If the added element is a branch, p-q, then the new bus impedance matrix would be of order m+1, and the analysis is confined to finding only the elements of the new row and column (corresponding to bus-q) introduced into the original matrix.

If the added element is a link, p-q, then the new bus impedance matrix will remain unaltered with regard to its order. However, all the elements of the original matrix are updated to take account of the effect of the link added.

**ADDITION OF A BRANCH**

Consider now the performance equation of the network in impedance form with the added branch p-q, given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1p} & \dots & Z_{1m} \\ Z_{21} & Z_{22} & \dots & Z_{2p} & \dots & Z_{2m} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{p1} & Z_{p2} & \dots & Z_{pp} & \dots & Z_{pm} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{m1} & Z_{m2} & \dots & Z_{mp} & \dots & Z_{mm} \\ Z_{q1} & Z_{q2} & \dots & Z_{qp} & \dots & Z_{qm} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_q \end{bmatrix} \tag{11}$$

It is assumed that the added branch p-q is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

**Vector  $y_{pq-rs}$  is not equal to zero and  $Z_{ij} = Z_{ji} \forall i, j = 1, 2, \dots, m, q$**  (12)

**To find  $Z_{qi}$ :**

The elements of last row-q and last column-q are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.2. Since all other bus currents are zero, we have from (11) that

$$E_k = Z_{ki} I_i = Z_{ki} \quad \forall k = 1, 2, \dots, i, \dots, p, \dots, m, q \tag{13}$$

Hence,  $E_q = Z_{qi}$ ;  $E_p = Z_{pi}$  .....

Also,  $E_q = E_p - v_{pq}$ ; so that  $Z_{qi} = Z_{pi} - v_{pq} \forall i = 1, 2, \dots, i, \dots, p, \dots, m, \neq q$  (14)

**To find  $v_{pq}$ :**

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} \bar{i} \\ i_{pq} \\ \bar{i} \\ i_{rs} \end{bmatrix} = \begin{bmatrix} \bar{y} \\ y_{pq,pq} \\ \bar{y} \\ y_{rs,pq} \end{bmatrix} \begin{bmatrix} \bar{v} \\ v_{pq} \\ \bar{v} \\ v_{rs} \end{bmatrix} \quad (15)$$

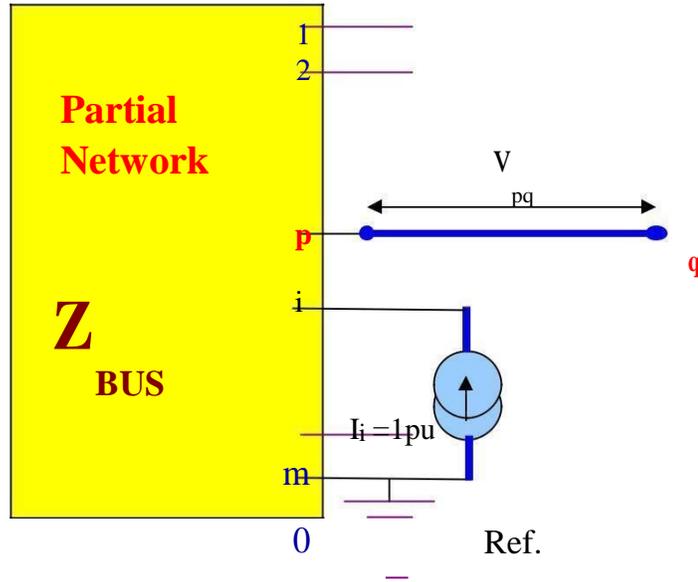


Fig.2 Calculation for \$Z\_{qi}\$

where \$i\_{pq}\$ is current through element \$p-q\$

\$\bar{i}\_{rs}\$ is vector of currents through elements of the partial network

\$v\_{pq}\$ is voltage across element \$p-q\$

\$y\_{pq,pq}\$ is self – admittance of the added element

\$\bar{y}\_{pq,rs}\$ is the vector of mutual admittances between the added elements \$p-q\$ and elements \$r-s\$ of the partial network.

\$\bar{v}\_{rs}\$ is vector of voltage across elements of partial network.

\$\bar{y}\_{rs,pq}\$ is transpose of \$\bar{y}\_{pq,rs}\$.

\$\bar{y}\_{rs,rs}\$ is the primitive admittance of partial network.

Since the current in the added branch \$p-q\$, is zero, \$i\_{pq} = 0\$. We thus have from (15),

$$i_{pq} = y_{pq,pq} v_{pq} + \bar{y}_{pq,rs} v_{rs} = 0 \quad (16)$$

Solving, 
$$v_{pq} = -\frac{y_{pq,rs} \bar{V}_{rs}}{y_{pq,pq}} \quad \text{or}$$

$$v_{pq} = -\frac{y_{pq,rs} \bar{E}_r - \bar{E}_s}{y_{pq,pq}} \quad (17)$$

Using (13) and (17) in (14), we get

$$Z_{qi} = Z_{pi} + \frac{y_{pq,rs} Z_{ri} - Z_{si}}{y_{pq,pq}} \quad i = 1, 2, \dots, m; i \neq q \quad (18)$$

**To find  $Z_{qq}$ :**

The element  $Z_{qq}$  can be computed by injecting a current of 1pu at bus-q,  $I_q = 1.0$  pu.

As before, we have the relations as under:

$$E_k = Z_{kq} I_q = Z_{kq} \quad \forall k = 1, 2, \dots, i, \dots, p, \dots, m, q \quad (19)$$

Hence,  $E_q = Z_{qq} I_q$ ;  $E_p = Z_{pq} I_q$ ; Also,  $E_q = E_p - v_{pq}$ ; so that  $Z_{qq} = Z_{pq} - v_{pq}$  (20)

Since now the current in the added element is  $i_{pq} = -I_q = -1.0$ , we have from(15)

$$i_{pq} = y_{pq,pq} v_{pq} + y_{pq,rs} \bar{V}_{rs} = -1$$

Solving, 
$$v_{pq} = -1 + \frac{y_{pq,rs} \bar{V}_{rs}}{y_{pq,pq}}$$

$$v_{pq} = -1 + \frac{y_{pq,rs} \bar{E}_r - \bar{E}_s}{y_{pq,pq}} \quad (21)$$

Using (19) and (21) in (20), we get

$$Z_{qq} = Z_{pq} + \frac{1 + y_{pq,rs} Z_{rq} - Z_{sq}}{y_{pq,pq}} \quad (22)$$

**Special Cases**

The following special cases of analysis concerning ZBUS building can be considered with respect to the addition of branch to a p-network.

**Case (a):** If there is no mutual coupling then elements of  $y_{pq,rs}$  are zero. Further, if p is the reference node, then  $E_p=0$ . thus,

$$Z_{pi} = 0 \quad i = 1, 2, \dots, m : i \neq q$$

And  $Z_{pq} = 0$ .

Hence, from (18) (22)  $Z_{qi} = 0 \quad i = 1, 2, \dots, m; i \neq q$

And  $Z_{qq} = z_{pq,pq} \quad \backslash (23)$

Case (b): If there is no mutual coupling and if p is not the ref. bus, then, from (18) and (22), we again have,

$$Z_{qi} = Z_{pi}, \quad i = 1, 2, \dots, m; i \neq q$$

$$Z_{qq} = Z_{pq} + z_{pq, pq} \quad (24)$$

**ADDITION OF A LINK**

Consider now the performance equation of the network in impedance form with the added link p-l, (p-l being a fictitious branch and l being a fictitious node) given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ \vdots \\ E_l \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1p} & \dots & Z_{1m} \\ Z_{21} & Z_{22} & \dots & Z_{2p} & \dots & Z_{2m} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{p1} & Z_{p2} & \dots & Z_{pp} & \dots & Z_{pm} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{m1} & Z_{m2} & \dots & Z_{mp} & \dots & Z_{mm} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{l1} & Z_{l2} & \dots & Z_{li} & \dots & Z_{lm} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ \vdots \\ I_l \end{bmatrix} \quad (25)$$

It is assumed that the added branch p-q is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

**Vector  $y_{pq-rs}$  is not equal to zero and  $Z_{ij} = Z_{ji} \quad \forall i, j = 1, 2, \dots, m, l.$**  (26)

**To find  $Z_{li}$ :**

The elements of last row-l and last column-l are determined by injecting a current of pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.3. Further, the current in the added element is made

zero by connecting a voltage source,  $e_l$  in series with element p-q, as shown. Since all other bus currents are zero, we have from (25) that

$$E_k = Z_{ki} I_i = Z_{ki} \quad \forall k = 1, 2, \dots, i, \dots, p, \dots, m, l \quad (27)$$

Hence,  $e_l = E_l = Z_{li}$ ;  $E_p = Z_{pi}$ ;  $E_p = Z_{pi} \dots \dots \dots$

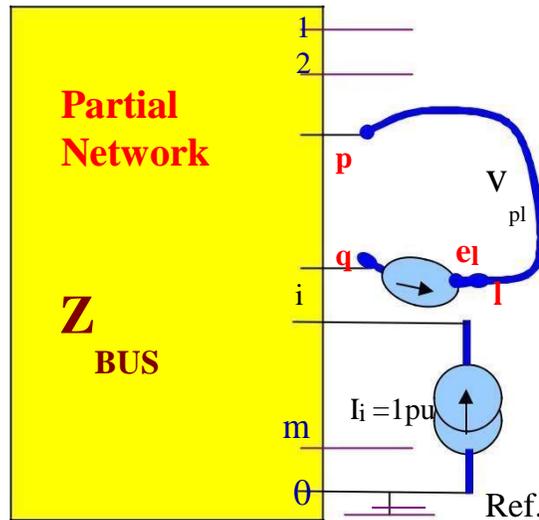
Also,  $e_l = E_p - E_q - v_{pq}$ ;

So that  $Z_{li} = Z_{pi} - Z_{qi} - v_{pq} \quad \forall i = 1, 2, \dots, i, \dots, p, \dots, q, \dots, m, \neq l$  (28)

**To find  $v_{pq}$ :**

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i \\ \vdots \\ i_{pl} \\ \vdots \\ i_{rs} \end{bmatrix} = \begin{bmatrix} y_{pl,pl} & \bar{y}_{pl,rs} \\ \bar{y}_{rs,pl} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pl} \\ \bar{v}_{rs} \end{bmatrix} \quad (29)$$



**Fig.3 Calculation for  $Z_{ii}$**

where  $i_{pl}$  is current through element  $p-q$

$i_{rs}$  is vector of currents through elements of the partial network

$v_{pl}$  is voltage across element  $p-q$

$y_{pl,pl}$  is self – admittance of the added element

$\bar{y}_{pl,rs}$  is the vector of mutual admittances between the added elements  $p-q$  and elements  $r-s$  of the partial network.

$\bar{v}_{rs}$  is vector of voltage across elements of partial network.

$\bar{y}_{rs,pl}$  is transpose of  $\bar{y}_{pl,rs}$ .

$\bar{y}_{rs,rs}$  is the primitive admittance of partial network.

Since the current in the added branch p-l, is zero,  $i_{pl} = 0$ . We thus have from (29),

$$i = y_{pl,rs} v_{pl} - y_{pl,rs} v_{rs} = 0 \tag{30}$$

Solving,  $v_{pl} = \frac{y_{pl,rs} v_{rs}}{y_{pl,pl}}$  or

$$v_{pl} = \frac{y_{pl,rs} (E_r - E_s)}{y_{pl,pl}} \tag{31}$$

However,

$$\bar{y}_{pl,rs} = \bar{y}_{pq,rs}$$

$$y_{pl,pl} = y_{pq,pq}$$

And Using (27), (31) and (32) in (28), we get  $Z_{li} = Z_{li} - Z_{pi} + \frac{y_{pq,rs} Z_{ri} Z_{si}}{y_{pq,pq}}$   $i = 1, 2, \dots, m; i \neq l$  (32)

$$Z_{li} = Z_{li} - Z_{pi} + \frac{y_{pq,rs} Z_{ri} Z_{si}}{y_{pq,pq}} \quad i = 1, 2, \dots, m; i \neq l \tag{33}$$

**To find Z<sub>ll</sub>:**

The element Z<sub>ll</sub> can be computed by injecting a current of 1pu at bus-l, I<sub>l</sub> = 1.0 pu.

As before, we have the relations as under:

$$E_k = Z_{kl} I_l = Z_{kl} \quad \forall k = 1, 2, \dots, i, \dots, p, \dots, q, \dots, m, l \tag{34}$$

Hence, e<sub>l</sub> = E<sub>l</sub> = Z<sub>ll</sub>; E<sub>p</sub> = Z<sub>pl</sub>;

Also, e<sub>l</sub> = E<sub>p</sub> - E<sub>q</sub> - v<sub>pl</sub>;

$$\text{So that } Z_{ll} = Z_{pl} - Z_{ql} - v_{pl} \quad \forall i=1, 2, \dots, i, \dots, p, \dots, q, \dots, m, \neq l \tag{35}$$

Since now the current in the added element is  $i_{pl} = -I_l = -1.0$ , we have from (29)

$$i_{pl} = y_{pl,rs} v_{pl} - y_{pl,rs} v_{rs} = -1$$

Solving,  $v_{pl} = -1 + \frac{y_{pl,rs} v_{rs}}{y_{pl,pl}}$

$$v_{pl} = -1 + \frac{y_{pl,rs} (E_r - E_s)}{y_{pl,pl}} \tag{36}$$

However,

$$\bar{y}_{pl,rs} = \bar{y}_{pq,rs}$$

$$y_{pl,pl} = y_{pq,pq}$$

And Using (34), (36) and (37) in (35), we get (37)

Using (34), (36) and (37) in (35), we get

$$Z_{ll} = Z_{pp} - Z_{pq} + \frac{1 + y_{pq,rs}}{y_{pq,pq}} \left( \frac{Z_{rl}}{Z_{rl}} - \frac{Z_{sl}}{Z_{sl}} \right) \quad (38)$$

**Special Cases Contd....**

The following special cases of analysis concerning Z<sub>BUS</sub> building can be considered with respect to the addition of link to a p-network.

**Case (c):** If there is no mutual coupling, then elements of  $\bar{y}_{pq,rs}$  are zero. Further, if p is the reference node, then E<sub>p</sub>=0. thus,

$$Z_{li} = -Z_{qi}, \quad i = 1, 2, \dots, m; i \neq l$$

$$Z_{ll} = -Z_{pq} + z_{pq,pq} \quad (39)$$

From (39), it is thus observed that, when a link is added to a ref. bus, then the situation is similar to adding a branch to a fictitious bus and hence the following steps are followed:

1. The element is added similar to addition of a branch (case-b) to obtain the new matrix of order m+1.
2. The extra fictitious node, l is eliminated using the node elimination algorithm.

**Case (d):** If there is no mutual coupling, then elements of  $\bar{y}_{pq,rs}$  are zero. Further, if p is not the reference node, then

$$Z_{li} = Z_{pi} - Z_{qi}$$

$$Z_{ll} = Z_{pl} - Z_{ql} - Z_{pq,pq} = Z_{pp} + Z_{qq} - 2Z_{pq} + Z_{pq,pq} \quad (40)$$

**MODIFICATION OF Z<sub>BUS</sub> FOR NETWORK CHANGES**

An element which is not coupled to any other element can be removed easily. The Z<sub>bus</sub> is modified as explained in sections above, by adding in parallel with the element (to be removed), a link whose impedance is equal to the negative of the impedance of the element to be removed. Similarly, the impedance value of an element which is not coupled to any other element can be changed easily. The Z<sub>bus</sub> is modified again as explained in sections above, by adding in parallel with the element (whose impedance is to be changed), a link element of impedance value chosen such that the parallel equivalent impedance is equal to the desired value of impedance. When mutually coupled elements are removed, the Z<sub>bus</sub> is modified by introducing appropriate changes in the bus currents of the original network to reflect the changes introduced due to the removal of the elements.

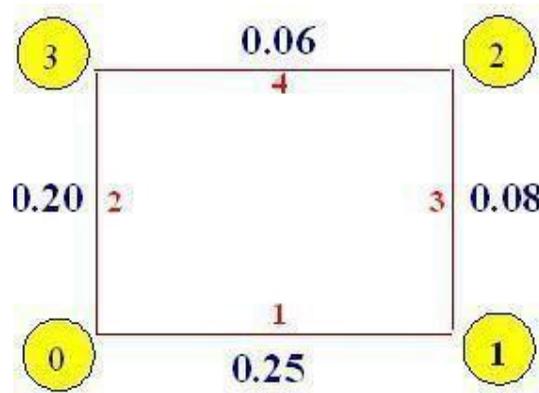
**Examples on ZBUS building**

**Example 1:** For the positive sequence network data shown in table below, obtain ZBUS by building procedure.

Sl. No.	P-q (nodes)	Pos. seq. reactance in pu
1	0-1	0.25
2	0-3	0.20
3	1-2	0.08
4	2-3	0.06

**Solution:**

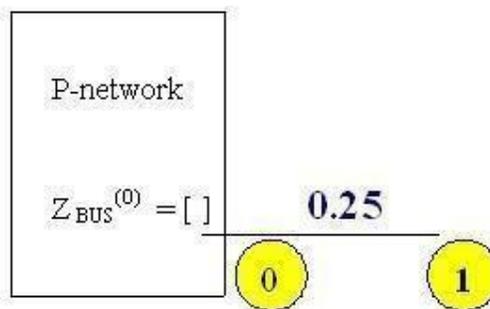
The given network is as shown below with the data marked on it. Assume the elements to be added as per the given sequence: 0-1, 0-3, 1-2, and 2-3.



**Fig. E1: Example System**

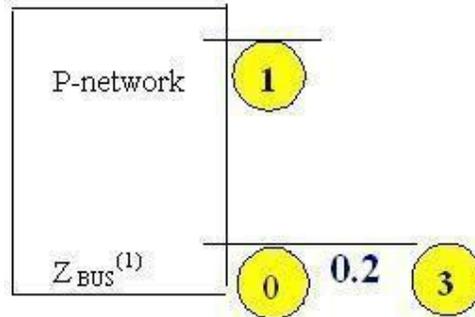
Consider building ZBUS as per the various stages of building through the consideration of the corresponding partial networks as under:

**Step-1:** Add element-1 of impedance 0.25 pu from the external node-1 (q=1) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



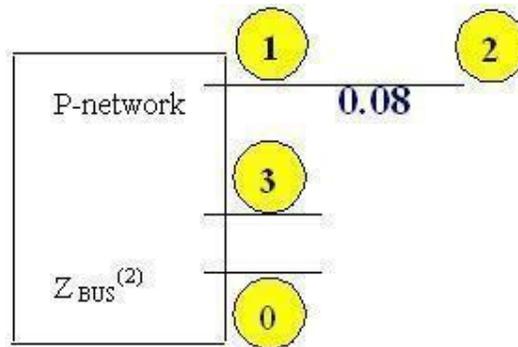
$$Z_{BUS}^{(1)} = \begin{matrix} & 1 \\ 1 & \boxed{0.25} \end{matrix}$$

**Step-2:** Add element-2 of impedance 0.2 pu from the external node-3 (q=3) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



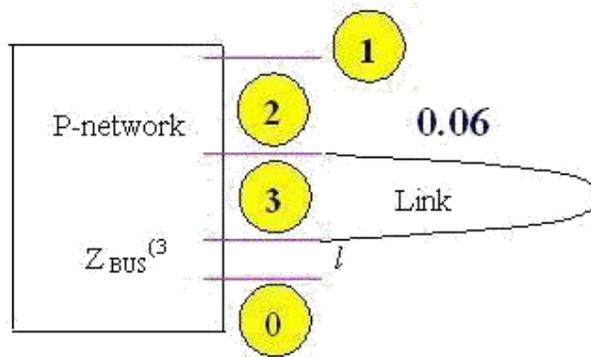
$$Z_{BUS}^{(2)} = \begin{matrix} & \begin{matrix} 1 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 3 \end{matrix} & \begin{bmatrix} 0.25 & 0 \\ 0 & 0.2 \end{bmatrix} \end{matrix}$$

**Step-3:** Add element-3 of impedance 0.08 pu from the external node-2 (q=2) to internal node-1 (p=1). (Case-b), as shown in the partial network;



$$Z_{BUS}^{(3)} = \begin{matrix} & \begin{matrix} 1 & 3 & 2 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 2 \end{matrix} & \begin{bmatrix} 0.25 & 0 & 0.25 \\ 0 & 0.2 & 0 \\ 0.25 & 0 & 0.33 \end{bmatrix} \end{matrix}$$

**Step-4:** Add element-4 of impedance 0.06 pu between the two internal nodes, node-2 (p=2) to node-3 (q=3). (Case-d), as shown in the partial network;



$$Z_{BUS}^{(4)} = \begin{matrix} & \begin{matrix} 1 & 3 & 2 & l \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 2 \\ l \end{matrix} & \begin{bmatrix} 0.25 & 0 & 0.25 & 0.25 \\ 0 & 0.2 & 0 & -0.2 \\ 0.25 & 0 & 0.33 & 0.33 \\ 0.25 & -0.2 & 0.33 & 0.59 \end{bmatrix} \end{matrix}$$

The fictitious node  $l$  is eliminated further to arrive at the final impedance matrix as under:

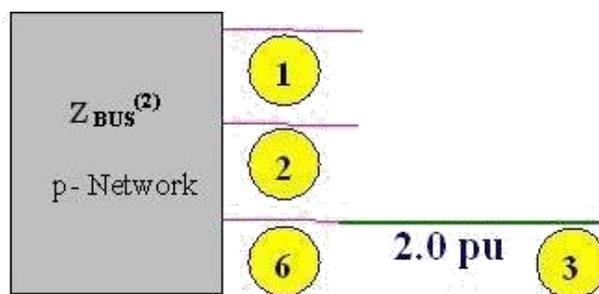
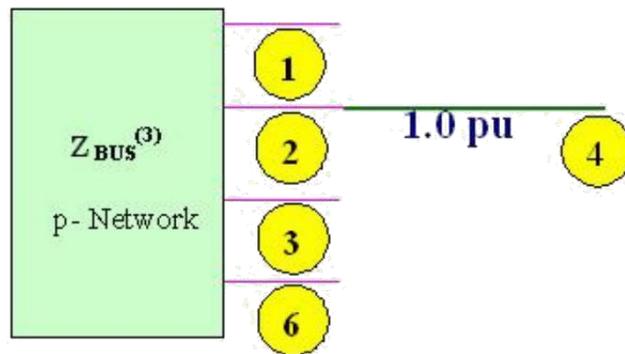
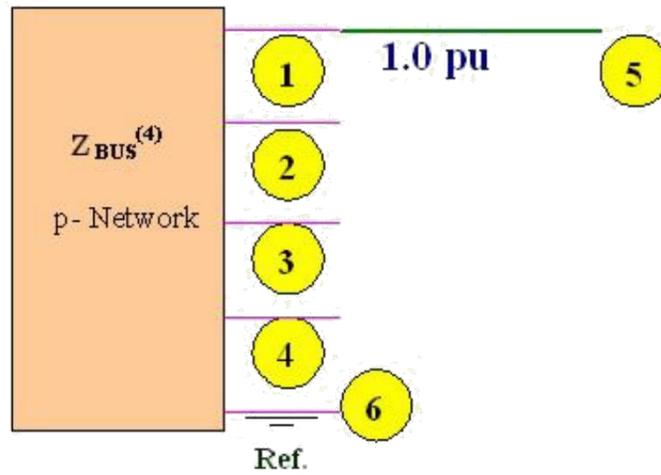
$$Z_{BUS}^{(final)} = \begin{matrix} & \begin{matrix} 1 & 3 & 2 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 2 \end{matrix} & \begin{bmatrix} 0.1441 & 0.0847 & 0.1100 \\ 0.0847 & 0.1322 & 0.1120 \\ 0.1100 & 0.1120 & 0.1454 \end{bmatrix} \end{matrix}$$

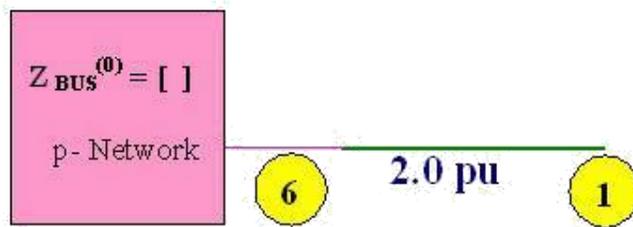
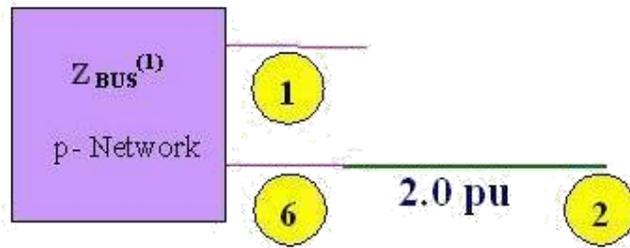
**Example 2:** The  $Z_{BUS}$  for a 6-node network with bus-6 as ref. is as given below. Assuming the values as pu reactances, find the topology of the network and the parameter values of the elements involved. Assume that there is no mutual coupling of any pair of elements.

$$Z_{BUS} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 2 & 0 & 0 & 0 & 2 \\ 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 3 & 0 \\ 2 & 0 & 0 & 0 & 3 \end{bmatrix} \end{matrix}$$

**Solution:**

The specified matrix is so structured that by its inspection, we can obtain the network by backward analysis through the various stages of  $Z_{BUS}$  building and p-networks as under:





Thus the final network is with 6 nodes and 5 elements connected as follows with the impedance values of elements as indicated.

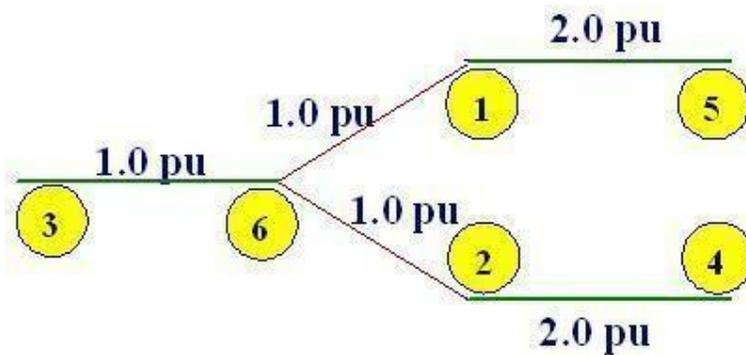
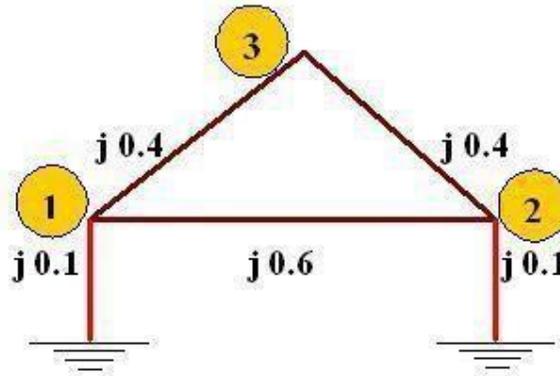


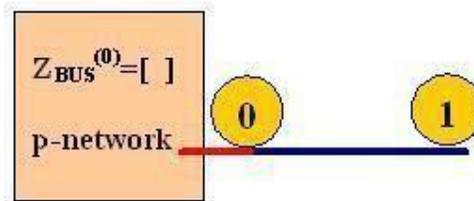
Fig. E2: Resultant network of example-2

**Example 3:** Construct the bus impedance matrix for the system shown in the figure below by building procedure. Show the partial networks at each stage of building the matrix. Hence arrive at the bus admittance matrix of the system. How can this result be verified in practice?



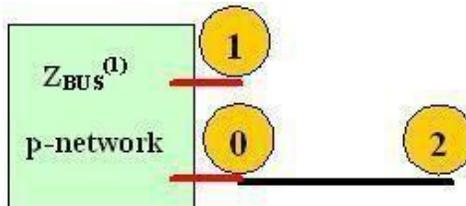
**Solution:** The specified system is considered with the reference node denoted by node-0. By its inspection, we can obtain the bus impedance matrix by building procedure by following the steps through the p-networks as under:

**Step1:** Add branch 1 between node 1 and reference node. (q =1, p = 0)



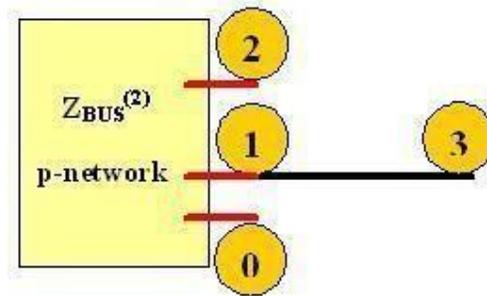
$$Z_{bus}^{(1)} = 1 [j0.1]$$

**Step2:** Add branch 2, between node 2 and reference node. (q = 2, p = 0).



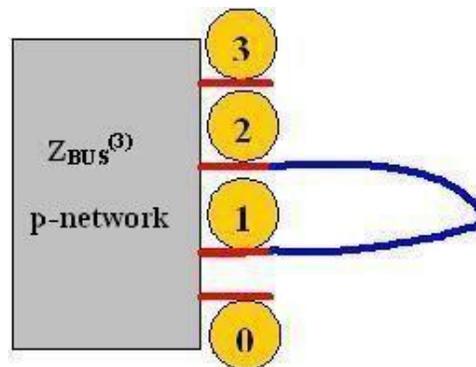
$$Z_{BUS} = \begin{matrix} & \begin{matrix} 1 & 2 \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{bmatrix} j0.1 & 0 \\ 0 & j0.15 \end{bmatrix} \end{matrix}$$

**Step3:** Add branch 3, between node 1 and node 3 (p = 1, q = 3)



$$Z_{BUS} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} j0.1 & 0 & j0.1 \\ 0 & j0.15 & 0 \\ j0.1 & 0 & j0.5 \end{bmatrix} \end{matrix}$$

**Step 4:** Add element 4, which is a link between node 1 and node 2. (p = 1, q = 2)



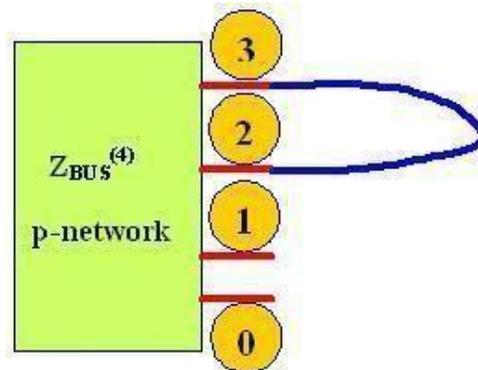
$$Z_{bus} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & l \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ l \end{matrix} & \begin{bmatrix} j0.1 & 0 & j0.1 & j0.1 \\ 0 & j0.15 & 0 & -j0.15 \\ j0.1 & 0 & j0.5 & j0.1 \\ j0.1 & -j0.15 & j0.1 & j0.85 \end{bmatrix} \end{matrix}$$

Now the extra node-*l* has to be eliminated to obtain the new matrix of step-4, using the algorithmic relation:

$$Y_{ij}^{new} = Y_{ij}^{old} - Y_{in} Y_{nj} / Y_{nn} \quad \forall i,j = 1,2,3.$$

$$Z_{bus} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} j0.08823 & j0.01765 & j0.08823 \\ j0.01765 & j0.12353 & j0.01765 \\ j0.08823 & j0.01765 & j0.48823 \end{bmatrix} \end{matrix}$$

**Step 5:** Add link between node 2 and node 3 (p = 2, q=3)



$$\begin{aligned}
 Z_{11} &= Z_{21} - Z_{31} = j0.01765 - j0.08823 = -j0.07058 \\
 Z_{12} &= Z_{22} - Z_{32} = j0.12353 - j0.01765 = j0.10588 \\
 Z_{13} &= Z_{23} - Z_{33} = j0.01765 - j0.48823 = -j0.47058 \\
 Z_{1l} &= Z_{2l} - Z_{3l} + Z_{23,23} \\
 &= j0.10588 - (-j0.47058) + j0.4 = j0.97646
 \end{aligned}$$

Thus, the new matrix is as under:

$$Z_{bus} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & l \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ l \end{matrix} & \begin{bmatrix} j0.08823 & j0.01765 & j0.08823 & -j0.07058 \\ j0.01765 & j0.12353 & j0.01765 & j0.10588 \\ j0.08823 & j0.01765 & j0.48823 & -j0.47058 \\ -j0.07058 & j0.10588 & -j0.47058 & j0.97646 \end{bmatrix} \end{matrix}$$

Node  $l$  is eliminated as shown in the previous step:

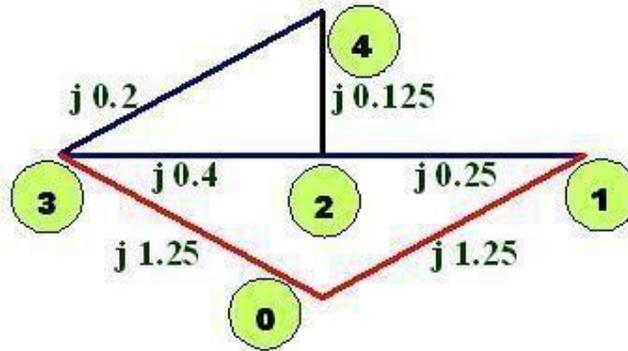
$$Z_{bus} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} j0.08313 & j0.02530 & j0.05421 \\ j0.02530 & j0.11205 & j0.06868 \\ j0.05421 & j0.06868 & j0.26145 \end{bmatrix} \end{matrix}$$

Further, the bus admittance matrix can be obtained by inverting the bus impedance matrix as under:

$$Y_{bus} = [Z_{bus}]^{-1} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} -j14.1667 & j1.6667 & j2.5 \\ j1.6667 & -j10.8334 & j2.5 \\ j2.5 & j2.5 & -j5.0 \end{bmatrix} \end{matrix}$$

As a check, it can be observed that the bus admittance matrix,  $Y_{BUS}$  can also be obtained by the rule of inspection to arrive at the same answer.

**Example 4:** Form the bus impedance matrix for the network shown below.



**Solution:**

Add the elements in the sequence, 0-1, 1-2, 2-3, 0-3, 3-4, 2-4, as per the various steps of building the matrix as under:

**Step1:** Add element 1, which is a branch between node-1 and reference node.

$$Z_{bus} = 1 \begin{bmatrix} j1.25 \end{bmatrix}$$

**Step2:** Add element 2, which is a branch between nodes 1 and 2.

$$Z_{bus} = 2 \begin{bmatrix} 1 & 2 \\ j1.25 & j1.25 \\ j1.25 & j1.5 \end{bmatrix}$$

**Step3:** Add element 3, which is a branch between nodes 2 and 3

$$Z_{bus} = 3 \begin{bmatrix} 1 & 2 & 3 \\ j1.25 & j1.25 & j1.25 \\ j1.25 & j1.5 & j1.5 \\ j1.25 & j1.5 & j1.9 \end{bmatrix}$$

**Step4:** Add element 4, which is a link from node 3 to reference node.

$$Z_{bus} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & l \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ l \end{matrix} & \begin{bmatrix} j1.25 & j1.25 & j1.25 & j1.25 \\ j1.25 & j1.5 & j1.5 & j1.5 \\ j1.25 & j1.5 & j1.9 & j1.9 \\ j1.25 & j1.5 & j1.9 & j3.15 \end{bmatrix} \end{matrix}$$

Eliminating node  $l$ ,

$$Z_{bus} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} j0.75397 & j0.65476 & j0.49603 \\ j0.65476 & j0.78571 & j0.59524 \\ j0.49603 & j0.59524 & j0.75397 \end{bmatrix} \end{matrix}$$

**Step5:** Add element 5, a branch between nodes 3 and 4.

$$Z_{bus} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} j0.75397 & j0.65476 & j0.49603 & j0.49603 \\ j0.65476 & j0.65476 & j0.59524 & j0.59524 \\ j0.49603 & j0.59524 & j0.75397 & j0.75397 \\ j0.49603 & j0.59524 & j0.75397 & j0.95397 \end{bmatrix} \end{matrix}$$

**Step 6:** Add element 6, a link between nodes 2 & 4.

$$Z_{bus} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & l \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ l \end{matrix} & \begin{bmatrix} j0.75397 & j0.65476 & j0.49603 & j0.49603 & j0.15873 \\ j0.65476 & j0.65476 & j0.59524 & j0.59524 & j0.19047 \\ j0.49603 & j0.59524 & j0.75397 & j0.75397 & -j0.15873 \\ j0.49603 & j0.59524 & j0.75397 & j0.95397 & -j0.35873 \\ j0.15873 & j0.19047 & -j0.15873 & -j0.35873 & j0.67421 \end{bmatrix} \end{matrix}$$

Eliminating node  $l$  we get the required bus impedance , matrix

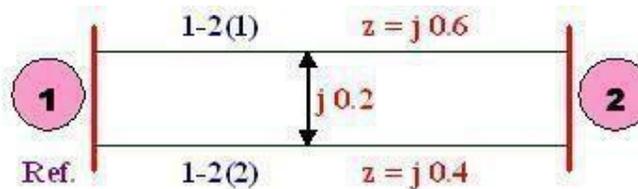
$$Z_{bus} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} j0.7166 & j0.6099 & j0.5334 & j0.5805 \\ j0.6099 & j0.7319 & j0.6401 & j0.6966 \\ j0.5334 & j0.6401 & j0.7166 & j0.6695 \\ j0.5805 & j0.6966 & j0.6695 & j0.7631 \end{bmatrix} \end{matrix}$$

**Example 5:** Form the bus impedance matrix for the network data given below.

Element	Self Impedance		Mutual Impedance	
	Bus p-q	Z <sub>pq, pq</sub> (pu)	Bus r-s	Z <sub>pq, rs</sub> (pu)
1	1-2(1)	j0.6		
2	1-2(2)	j0.4	1-2(1)	j0.2

**Solution:**

Let bus-1 be the reference. Add the elements in the sequence 1-2(1), 1-2(2). Here, in the step-2, there is mutual coupling between the pair of elements involved.



**Step1:** Add element 1 from bus 1 to 2, element 1-2(1). ( p=1, q=2, p is the reference node)

$$Z_{bus} = \begin{matrix} & 2 \\ 2 & [j0.6] \end{matrix}$$

**Step2:** Add element 2, element 1-2(2), which is a link from bus1 to 2, mutually coupled with element 1, 1-2(1).

$$Z_{bus} = \begin{matrix} & 2 & 1 \\ 2 & \begin{bmatrix} j0.6 & Z_{21} \\ Z_{12} & Z_{22} \end{bmatrix} \\ 1 & \end{matrix}$$

Where,

$$Z_{21} = Z_{12} = -Z_{22} + \frac{y_{12(2),12(1)}(Z_{12} - Z_{22})}{y_{12(2),12(2)}}$$

$$Z_{12} = Z_{11} = 0 \text{ (as bus 1 is reference)}$$

Consider the primitive impedance matrix for the two elements given by

$$[z] = \begin{matrix} & \begin{matrix} 1-2(1) & 1-2(2) \end{matrix} \\ \begin{matrix} 1-2(1) \\ 1-2(2) \end{matrix} & \begin{bmatrix} j0.6 & j0.2 \\ j0.2 & j0.4 \end{bmatrix} \end{matrix}$$

Thus the primitive admittance matrix is obtained by taking the inverse of [z] as

$$[y] = \begin{matrix} & \begin{matrix} 1-2(1) & 1-2(2) \end{matrix} \\ \begin{matrix} 1-2(1) \\ 1-2(2) \end{matrix} & \begin{bmatrix} -j2.0 & j1.0 \\ j1.0 & -j3.0 \end{bmatrix} \end{matrix}$$

Thus,

$$y_{12(1),12(2)} = j1.0; \quad y_{12(2),12(2)} = -j3.0$$

So that we have,

$$Z_{21} = Z_{12} = -j0.6 + \frac{(j1.0)(-j0.6)}{-j3.0} = -j0.4$$

$$Z_{11} = -Z_{21} + \frac{1 + y_{12(2),12(1)}(Z_{11} - Z_{21})}{y_{12(2),12(2)}} = j0.4 + \frac{1 + (j1.0)(j0.4)}{-j3.0} = j0.6$$

$$Z_{bus} = \begin{matrix} & \begin{matrix} 2 & 1 \end{matrix} \\ \begin{matrix} 2 \\ 1 \end{matrix} & \begin{bmatrix} j0.6 & -j0.4 \\ -j0.4 & j0.6 \end{bmatrix} \end{matrix}$$

Thus, the network matrix corresponding to the 2-node, 1-bus network given, is obtained after eliminating the extra node-1 as a single element matrix, as under:

$$Z_{bus} = \begin{matrix} & 2 \\ 2 & \begin{bmatrix} j0.3333 \end{bmatrix} \end{matrix}$$

## CHAPTER 2

### LOAD FLOW ANALYSIS

[**CONTENTS:** Review of solution of equations, direct and iterative methods, classification of buses, importance of slack bus and YBUS based analysis, constraints involved, load flow equations, GS method: algorithms for finding the unknowns, concept of acceleration of convergence, NR method- algorithms for finding the unknowns, tap changing transformers, Fast decoupled load flow, illustrative examples]

#### **REVIEW OF NUMERICAL SOLUTION OF EQUATIONS**

The numerical analysis involving the solution of algebraic simultaneous equations forms the basis for solution of the performance equations in computer aided electrical power system analyses, such as during linear graph analysis, load flow analysis (nonlinear equations), transient stability studies (differential equations), etc. Hence, it is necessary to review the general forms of the various solution methods with respect to all forms of equations, as under:

##### **Solution Linear equations:**

###### **Direct methods:**

Cramer's (Determinant) Method,  
Gauss Elimination Method (only for smaller systems),  
LU Factorization (more preferred method), etc.

###### **Iterative methods:**

Gauss Method  
Gauss-Siedel Method (for diagonally dominant systems)

- **Solution of Nonlinear equations:**

###### **Iterative methods only:**

Gauss-Siedel Method (for smaller systems)  
Newton-Raphson Method (if corrections for variables are small)

- **Solution of differential equations:**

###### **Iterative methods only:**

- Euler and Modified Euler method,
- RK IV-order method,
- Milne's predictor-corrector method, etc.

It is to be observed that the nonlinear and differential equations can be solved only by the iterative methods. The iterative methods are characterized by the various performance features as under:

- ⑩ Selection of initial solution/ estimates
- ⑩ Determination of fresh/ new estimates during each iteration
- ⑩ Selection of number of iterations as per tolerance limit
- ⑩ Time per iteration and total time of solution as per the solution method selected
- ⑩ Convergence and divergence criteria of the iterative solution
- ⑩ Choice of the Acceleration factor of convergence, etc.

**A comparison of the above solution methods is as under:**

- ⑩ In general, the direct methods yield exact or accurate solutions. However, they are suited for only the smaller systems, since otherwise, in large systems, the possible round-off errors make the solution process inaccurate.
- ⑩ The iterative methods are more useful when the diagonal elements of coefficient matrix are large in comparison with the off diagonal elements. The round-off errors in these methods are corrected at the successive steps of the iterative process.
- ⑩ The Newton-Raphson method is very much useful for solution of non – linear equations, if all the values of the corrections for the unknowns are very small in magnitude and the initial values of unknowns are selected to be reasonably closer to the exact solution.

## **LOAD FLOW STUDIES**

**Introduction:** Load flow studies are important in planning and designing future expansion of power systems. The study gives steady state solutions of the voltages at all the buses, for a particular load condition. Different steady state solutions can be obtained, for different operating conditions, to help in planning, design and operation of the power system.

Generally, load flow studies are limited to the transmission system, which involves bulk power transmission. The load at the buses is assumed to be known. Load flow

studies throw light on some of the important aspects of the system operation, such as: violation of voltage magnitudes at the buses, overloading of lines, overloading of generators, stability margin reduction, indicated by power angle differences between buses linked by a line, effect of contingencies like line voltages, emergency shutdown of generators, etc. Load flow studies are required for deciding the economic operation of the power system. They are also required in transient stability studies. Hence, load-flow studies play a vital role in power system studies.

Thus the load flow problem consists of finding the power flows (real and reactive) and voltages of a network for given bus conditions. At each bus, there are four quantities of interest to be known for further analysis: the real and reactive power, the voltage magnitude and its phase angle. Because of the nonlinearity of the algebraic equations, describing the given power system, their solutions are obviously, based on the iterative methods only. The constraints placed on the load flow solutions could be:

- ⑩ The Kirchhoff's relations holding good,
- ⑩ Capability limits of reactive power sources,
- ⑩ Tap-setting range of tap-changing transformers,
- ⑩ Specified power interchange between interconnected systems,
- ⑩ Selection of initial values, acceleration factor, convergence limit, etc.

**Classification of buses for LFA:** Different types of buses are present based on the specified and unspecified variables at a given bus as presented in the table below:

**Table 1. Classification of buses for LFA**

Sl. No.	Bus Types	Specified Variables	Unspecified variables	Remarks
1	Slack/ Swing Bus	$ V , \delta$	$P_G, Q_G$	$ V , \delta$ : are assumed if not specified as 1.0 and $0^0$
2	Generator/ Machine/ PV Bus	$P_G,  V $	$Q_G, \delta$	A generator is present at the machine bus
3	Load/ PQ Bus	$P_G, Q_G$	$ V , \delta$	About 80% buses are of PQ type
4	Voltage Controlled Bus	$P_G, Q_G,  V $	$\delta, a$	'a' is the % tap change in tap-changing transformer

**Importance of swing bus:** The slack or swing bus is usually a PV-bus with the largest capacity generator of the given system connected to it. The generator at the swing bus supplies the power difference between the “specified power into the system at the other buses” and the “total system output plus losses”. Thus swing bus is needed to supply the additional real and reactive power to meet the losses. Both the magnitude and phase angle of voltage are specified at the swing bus, or otherwise, they are assumed to be equal to 1.0 p.u. and  $0^0$ , as per flat-start procedure of iterative solutions. The real and reactive powers at the swing bus are found by the computer routine as part of the load flow solution process. It is to be noted that the source at the swing bus is a perfect one, called the swing machine, or slack machine. It is voltage regulated, i.e., the magnitude of voltage fixed. The phase angle is the system reference phase and hence is fixed. The generator at the swing bus has a torque angle and excitation which vary or swing as the demand changes. This variation is such as to produce fixed voltage.

Importance of  $Y_{BUS}$

**based LFA:**  
flow programs employ

The majority of load

methods using the bus admittance matrix, as this method is found to be more economical. The bus admittance matrix plays a very important role in load flow analysis. It is a complex, square and symmetric matrix and hence only  $n(n+1)/2$  elements of  $Y_{BUS}$  need to be stored for a n-bus system. Further, in the  $Y_{BUS}$  matrix,  $Y_{ij} = 0$ , if an incident element is not present in the system connecting the buses ‘i’ and ‘j’. since in a large power system, each bus is connected only to a fewer buses through an incident element, (about 6-8), the coefficient matrix,  $Y_{BUS}$  of such systems would be highly sparse, i.e., it will have many zero valued elements in it. This is defined by the sparsity of the matrix, as under:

$$\begin{aligned} \text{Percentage sparsity of a} & & \text{Total no. of zero valued elements of } Y_{BUS} \\ \text{given matrix of } n^{\text{th}} \text{ order:} & = & \text{Total no. of entries of } Y_{BUS} \\ S & = & (Z / n^2) \times 100 \% \end{aligned} \quad (1)$$

The percentage sparsity of  $Y_{BUS}$ , in practice, could be as high as 80-90%, especially for very large, practical power systems. This sparsity feature of  $Y_{BUS}$  is extensively

used in reducing the load flow calculations and in minimizing the memory required to store the coefficient matrices. This is due to the fact that only the non-zero elements YBUS can be stored during the computer based implementation of the schemes, by adopting the suitable optimal storage schemes. While YBUS is thus highly sparse, its inverse, ZBUS, the bus impedance matrix is not so. It is a FULL matrix, unless the optimal bus ordering schemes are followed before proceeding for load flow analysis.

### THE LOAD FLOW PROBLEM

Here, the analysis is restricted to a balanced three-phase power system, so that the analysis can be carried out on a single phase basis. The per unit quantities are used for all quantities. The first step in the analysis is the formulation of suitable equations for the power flows in the system. The power system is a large interconnected system, where various buses are connected by transmission lines. At any bus, complex power is injected into the bus by the generators and complex power is drawn by the loads. Of course at any bus, either one of them may not be present. The power is transported from one bus to other via the transmission lines. At any bus  $i$ , the complex power  $S_i$  (injected), shown in figure 1, is defined as

$$S_i = S_{Gi} - S_{Di} \quad (2)$$

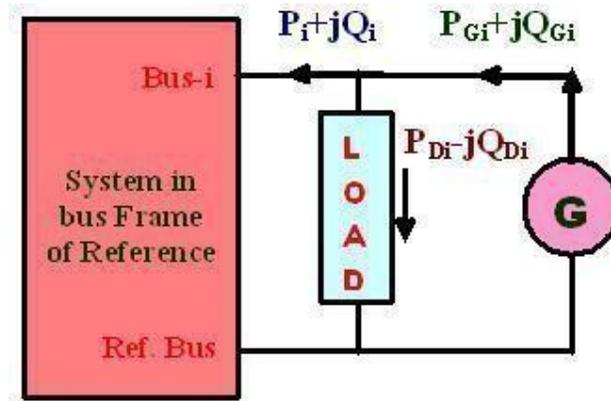


Fig.1 power flows at a bus-i

where  $S_i$  = net complex power injected into bus  $i$ ,  $S_{Gi}$  = complex power injected by the generator at bus  $i$ , and  $S_{Di}$  = complex power drawn by the load at bus  $i$ . According to conservation of complex power, at any bus  $i$ , the complex power injected into the bus must be equal to the sum of complex power flows out of the bus via the transmission lines. Hence

$S_i = \sum S_{ij} \quad \forall i = 1, 2, \dots, n$  (3) where  $S_{ij}$  is the sum over all lines connected to the bus and  $n$  is the number of buses in the system (excluding the ground). The bus current injected at the bus- $i$  is defined as

$$I_i = I_{Gi} - I_{Di} \quad \forall i = 1, 2, \dots, n \quad (4)$$

where  $I_{Gi}$  is the current injected by the generator at the bus and  $I_{Di}$  is the current drawn by the load (demand) at that bus. In the bus frame of reference

$$I_{BUS} = Y_{BUS} V_{BUS} \quad (5)$$

where

$$I_{BUS} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} \text{ is the vector of currents injected at the buses,}$$

$Y_{BUS}$  is the bus admittance matrix, and

$$V_{BUS} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \text{ is the vector of complex bus voltages.}$$

Equation (5) can be considered as

$$I_i = \sum_{j=1}^n Y_{ij} V_j \quad \forall i = 1, 2, \dots, n \quad (6)$$

The complex power  $S_i$  is given by

$$S_i = V_i I_i^* = V_i \left( \sum_{j=1}^n Y_{ij} V_j \right)^* = V_i \sum_{j=1}^n Y_{ij}^* V_j^* \quad (7)$$

Let  $V_i = V_i \angle \delta_i = V_i (\cos \delta_i + j \sin \delta_i)$

$$\delta_{ij} = \delta_i - \delta_j$$

$$Y_{ij} = G_{ij} + jB_{ij}$$

Hence from (7), we get,

$$S_i = \sum_{j=1}^n |V_i| |V_j| (\cos \delta_{ij} + j \sin \delta_{ij}) (G_{ij} - jB_{ij}) \tag{8}$$

Separating real and imaginary parts in (8) we obtain,

$$P_i = \sum_{j=1}^n V_i V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \tag{9}$$

$$Q_i = \sum_{j=1}^n |V_i| |V_j| (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \tag{10}$$

An alternate form of  $P_i$  and  $Q_i$  can be obtained by representing  $Y_{ij}$  also in polar form

as  $Y_{ij} = \left| \frac{Y}{ij} \right| \angle \theta_{ij} \tag{11}$

Again, we get from (7),

$$S_i = |V_i| \angle \delta_i \sum_{j=1}^n |Y_{ij}| \angle -\theta_{ij} |V_j| \angle -\delta_j \tag{12}$$

The real part of (12) gives  $P_i$ .

$$\begin{aligned} P_i &= |V_i| \sum_{j=1}^n |Y_{ij}| |V_j| \cos(-\theta_{ij} + \delta_i - \delta_j) \\ &= |V_i| \sum_{j=1}^n |Y_{ij}| |V_j| \cos(\theta_{ij} - \delta_i + \delta_j) \quad \text{or} \\ P_i &= \sum_{j=1}^n |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \quad \forall i = 1, 2, \dots, n, \end{aligned} \tag{13}$$

Similarly,  $Q_i$  is imaginary part of (12) and is given by

$$\begin{aligned} Q_i &= |V_i| \sum_{j=1}^n |Y_{ij}| |V_j| \sin -(\theta_{ij} - \delta_i + \delta_j) \quad \text{or} \\ Q_i &= -\sum_{j=1}^n |V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \quad \forall i = 1, 2, \dots, n \end{aligned} \tag{14}$$

Equations (9)-(10) and (13)-(14) are the ‘power flow equations’ or the ‘load flow equations’ in two alternative forms, corresponding to the n-bus system, where each bus- $i$  is characterized by four variables,  $P_i$ ,  $Q_i$ ,  $|V_i|$ , and  $\delta_i$ . Thus a total of 4n variables are involved in these equations. The load flow equations can be solved for

any  $2n$  unknowns, if the other  $2n$  variables are specified. This establishes the need for classification of buses of the system for load flow analysis into: PV bus, PQ bus, etc.

### **DATA FOR LOAD FLOW**

Irrespective of the method used for the solution, the data required is common for any load flow. All data is normally in pu. The bus admittance matrix is formulated from these data. The various data required are as under:

**System data:** It includes: number of buses- $n$ , number of PV buses, number of loads, number of transmission lines, number of transformers, number of shunt elements, the slack bus number, voltage magnitude of slack bus (angle is generally taken as  $0^\circ$ ), tolerance limit, base MVA, and maximum permissible number of iterations.

**Generator bus data:** For every PV bus  $i$ , the data required includes the bus number, active power generation  $P_{Gi}$ , the specified voltage magnitude  $|V_{sp}|$ , minimum reactive power limit  $Q_{i,min}$ , and maximum reactive power limit  $Q_{i,max}$ .

**Load data:** For all loads the data required includes the the bus number, active power demand  $P_{Di}$ , and the reactive power demand  $Q_{Di}$ .

**Transmission line data:** For every transmission line connected between buses  $i$  and  $k$  the data includes the starting bus number  $i$ , ending bus number  $k$ , resistance of the line, reactance of the line and the half line charging admittance.

#### **Transformer data:**

For every transformer connected between buses  $i$  and  $k$  the data to be given includes: the starting bus number  $i$ , ending bus number  $k$ , resistance of the transformer, reactance of the transformer, and the off nominal turns-ratio  $a$ .

**Shunt element data:** The data needed for the shunt element includes the bus number where element is connected, and the shunt admittance ( $G_{sh} + j B_{sh}$ ).

### **GAUSS – SEIDEL (GS) METHOD**

The GS method is an iterative algorithm for solving non linear algebraic equations. An initial solution vector is assumed, chosen from past experiences, statistical data or from practical considerations. At every subsequent iteration, the solution is updated

till convergence is reached. The GS method applied to power flow problem is as discussed below.

**Case (a): Systems with PQ buses only:**

Initially assume all buses to be PQ type buses, except the slack bus. This means that (n-1) complex bus voltages have to be determined. For ease of programming, the slack bus is generally numbered as bus-1. PV buses are numbered in sequence and PQ buses are ordered next in sequence. This makes programming easier, compared to random ordering of buses. Consider the expression for the complex power at bus-*i*, given from (7), as:

$$S_i = V_i \left( \sum_{j=1}^n Y_{ij} V_j \right)^*$$

This can be written as

$$S_i^* = V_i^* \left( \sum_{j=1}^n Y_{ij} V_j \right) \tag{15}$$

Since  $S_i^* = P_i - jQ_i$ , we get,

$$\frac{P_i - jQ_i}{V_i^*} = \sum_{j=1}^n Y_{ij} V_j$$

So that,

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{j=1 \\ j \neq i}}^n Y_{ij} V_j \tag{16}$$

Rearranging the terms, we get,

$$V_i = \frac{1}{Y_{ii}} \left[ \frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^n Y_{ij} V_j \right] \quad \forall i = 2, 3, \dots, n \tag{17}$$

Equation (17) is an implicit equation since the unknown variable, appears on both sides of the equation. Hence, it needs to be solved by an iterative technique. Starting from an initial estimate of all bus voltages, in the RHS of (17) the most recent values of the bus voltages is substituted. One iteration of the method involves computation of all the bus voltages. In Gauss-Seidel method, the value of the updated voltages are used in the computation of subsequent voltages in the same iteration, thus speeding up

convergence. Iterations are carried out till the magnitudes of all bus voltages do not change by more than the tolerance value. Thus the algorithm for GS method is as under:

**Algorithm for GS method**

- Prepare data for the given system as required.
- Formulate the bus admittance matrix  $Y_{BUS}$ . This is generally done by the rule of inspection.
- Assume initial voltages for all buses, 2,3,...n. In practical power systems, the magnitude of the bus voltages is close to 1.0 p.u. Hence, the complex bus voltages at all (n-1) buses (except slack bus) are taken to be  $1.0 \angle 0^0$ . This is normally referred as the *flat start* solution.

4. Update the voltages. In any  $(k + 1)^{st}$  iteration, from (17) the voltages are given by

$$V_i^{(k+1)} = \frac{1}{Y_{ii}} \left[ \frac{P_i - jQ_i}{V_i^{(k)}} - \sum_{j=1, j \neq i}^n Y_{ij} V_j^{(k+1)} - \sum_{j=i+1}^n Y_{ij} V_j^{(k)} \right] \quad \forall i=2,3,\dots,n \quad (18)$$

Here note that when computation is carried out for bus-i, updated values are already available for buses 2,3,...(i-1) in the current  $(k+1)^{st}$  iteration. Hence these values are used. For buses  $(i+1) \dots n$ , values from previous,  $k^{th}$  iteration are used.

5. Continue iterations till

$$\left| \frac{\Delta V_i^{(k+1)}}{V_i^{(k)}} \right| \quad \forall i = 2,3,\dots,n \quad \left| \right. = V_i^{(k+1)} - V_i^{(k)} < \epsilon \quad (19)$$

Where,  $\epsilon$  is the tolerance value. Generally it is customary to use a value of 0.0001 pu.

- Compute slack bus power after voltages have converged using (15) [assuming bus 1 is slack bus].

$$S_1^* = P_1 - jQ_1 = V_1 \left( \sum_{j=1}^n Y_{1j} V_j \right) \quad (20)$$

- Compute all line flows.
- The complex power loss in the line is given by  $S_{ik} + S_{ki}$ . The total loss in the system is calculated by summing the loss over all the lines.

**Case (b): Systems with PV buses also present:**

At PV buses, the magnitude of voltage and not the reactive power is specified. Hence it is needed to first make an estimate of  $Q_i$  to be used in (18). From (15) we have

$$Q_i = -\text{Im} \left\{ V_i \sum_{j=1}^n Y_{ij}^* V_j \right\}$$

Where  $\text{Im}$  stands for the imaginary part. At any  $(k+1)^{\text{st}}$  iteration, at the PV bus- $i$ ,

$$Q_{i(k+1)} = -\text{Im} \left\{ (V_i^{(k)})^* \sum_{j=1}^{i-1} Y_{ij} V_j^{(k+1)} + (V_i^{(k)})^* \sum_{j=i}^n Y_{ij} V_j^{(k)} \right\} \quad (21)$$

The steps for  $i^{\text{th}}$  PV bus are as follows:

- Compute  $Q_i^{(k+1)}$  using (21)
  - Calculate  $V_i$  using (18) with  $Q_i = Q_i^{(k+1)}$
3. Since  $|V_i|$  is specified at the PV bus, the magnitude of  $V_i$  obtained in step 2

has to be modified and set to the specified  $|V_i|$ . Therefore,

$$V_i^{(k+1)} = \left| V_{i,sp} \right| \angle \delta_i^{(k+1)} \quad (22)$$

The voltage computation for PQ buses does not change.

**Case (c): Systems with PV buses with reactive power generation limits specified:**

In the previous algorithm if the  $Q$  limit at the voltage controlled bus is violated during any iteration, i.e  $Q_i^{(k+1)}$  computed using (21) is either less than  $Q_{i,min}$  or greater than

$Q_{i,max}$ , it means that the voltage cannot be maintained at the specified value due to lack of reactive power support. This bus is then treated as a PQ bus in the  $(k+1)^{\text{st}}$  iteration and the voltage is calculated with the value of  $Q_i$  set as follows:

$$\begin{aligned} \text{If } Q_i < Q_{i,min} & \qquad \qquad \text{If } Q_i > Q_{i,max} \\ \text{Then } Q_i = Q_{i,min}. & \qquad \qquad \text{Then } Q_i = Q_{i,max}. \end{aligned} \quad (23)$$

If in the subsequent iteration, if  $Q_i$  falls within the limits, then the bus can be switched back to PV status.

**Acceleration of convergence**

It is found that in GS method of load flow, the number of iterations increase with increase in the size of the system. The number of iterations required can be reduced if

the correction in voltage at each bus is accelerated, by multiplying with a constant  $\alpha$ , called the acceleration factor. In the  $(k+1)^{st}$  iteration we can let

$$V_i^{(k+1)}(\text{accelerated}) = V_i^{(k)} + \alpha (V_i^{(k+1)} - V_i^{(k)}) \quad (24)$$

where  $\alpha$  is a real number. When  $\alpha = 1$ , the value of  $V_i^{(k+1)}$  is the computed value. If  $1 < \alpha < 2$ , then the value computed is extrapolated. Generally  $\alpha$  is taken between 1.2 to 1.6, for GS load flow procedure. At PQ buses (pure load buses) if the voltage magnitude violates the limit, it simply means that the specified reactive power demand cannot be supplied, with the voltage maintained within acceptable limits.

**Examples on GS load flow analysis:**

**Example-1:** Obtain the voltage at bus 2 for the simple system shown in Fig 2, using the Gauss–Seidel method, if  $V_1 = 1 \angle 0^\circ$  pu.

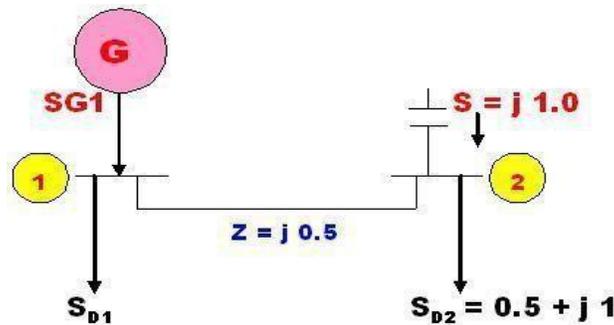


Fig : System of Example 1

**Solution:**

Here the capacitor at bus 2, injects a reactive power of 1.0 pu. The complex power injection at bus 2 is

$$S_2 = j1.0 - (0.5 + j 1.0) = - 0.5$$

$$\text{pu. } V_1 = 1 \angle 0^\circ$$

$$Y_{BUS} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

$$V^{(k+1)} = \begin{bmatrix} 1 & P-jQ \\ \frac{22}{(V^{(k)})^*} & -Y \end{bmatrix} V_1$$

Since  $V_1$  is specified it is a constant through all the iterations. Let the initial voltage at bus 2,  $V_2^0 = 1 + j 0.0 = 1 \angle 0^\circ$  pu.

$$\begin{aligned}
 V_2^1 &= \frac{1}{2} \left[ \frac{-0.5}{1 \angle 0^\circ} - (j2 \times 1 \angle 0^\circ) \right] \\
 &= \frac{1}{2} \left[ \frac{-0.5}{1 \angle 0^\circ} - j2 \right] \\
 &= 1.0 - j0.25 = 1.030776 \angle -14.036^\circ \\
 V_2^2 &= \frac{1}{2} \left[ \frac{-0.5}{1.030776 \angle 14.036^\circ} - (j2 \times 1 \angle 0^\circ) \right] \\
 &= \frac{1}{2} \left[ \frac{-0.5}{1.030776 \angle 14.036^\circ} - j2 \right] \\
 &= 0.94118 - j0.23529 = 0.970145 \angle -14.036^\circ \\
 V_2^3 &= \frac{1}{2} \left[ \frac{-0.5}{0.970145 \angle 14.036^\circ} - (j2 \times 1 \angle 0^\circ) \right] \\
 &= \frac{1}{2} \left[ \frac{-0.5}{0.970145 \angle 14.036^\circ} - j2 \right] \\
 &= 0.9375 - j0.249999 = 0.970261 \angle -14.931^\circ \\
 V_2^4 &= \frac{1}{2} \left[ \frac{-0.5}{0.970261 \angle 14.931^\circ} - (j2 \times 1 \angle 0^\circ) \right] \\
 &= \frac{1}{2} \left[ \frac{-0.5}{0.970261 \angle 14.931^\circ} - j2 \right] \\
 &= 0.933612 - j0.248963 = 0.966237 \angle -14.931^\circ \\
 V_2^5 &= \frac{1}{2} \left[ \frac{-0.5}{0.966237 \angle 14.931^\circ} - (j2 \times 1 \angle 0^\circ) \right] \\
 &= \frac{1}{2} \left[ \frac{-0.5}{0.966237 \angle 14.931^\circ} - j2 \right] \\
 &= 0.933335 - j0.25 = 0.966237 \angle -14.995^\circ
 \end{aligned}$$

Since the difference in the voltage magnitudes is less than  $10^{-6}$  pu, the iterations can be stopped. To compute line flow

$$\begin{aligned}
 I_{12} &= \frac{V_1 - V_2}{Z_{12}} = \frac{1 \angle 0^\circ - 0.966237 \angle -14.995^\circ}{j0.5} \\
 &= 0.517472 \angle -14.931^\circ
 \end{aligned}$$

$$\begin{aligned}
 S_{12} &= V_1 I_{12}^* = 1 \angle 0^\circ \times 0.517472 \angle 14.931^\circ \\
 &= 0.5 + j0.133329 \text{ pu}
 \end{aligned}$$

$$\begin{aligned}
 I_{21} &= \frac{V_2 - V_1}{Z_{12}} = \frac{0.966237 \angle -14.995^\circ - 1 \angle 0^\circ}{j0.5} \\
 &= 0.517472 \angle -14.931^\circ
 \end{aligned}$$

$$\begin{aligned}
 S_{21} &= V_2 I_{21}^* = -0.5 + j0.0 \text{ pu}
 \end{aligned}$$

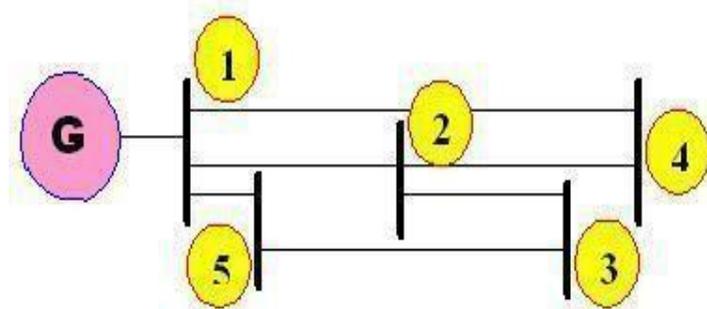
The total loss in the line is given by

$$S_{12} + S_{21} = j0.133329 \text{ pu}$$

Obviously, it is observed that there is no real power loss, since the line has no resistance.

**Example-2:**

For the power system shown in fig. below, with the data as given in tables below, obtain the bus voltages at the end of first iteration, by applying GS method.



**Power System of Example 2**

**Line data of example 2**

SB	EB	R (pu)	X (pu)	$B_c$
				2
1	2	0.10	0.40	-
1	4	0.15	0.60	-
1	5	0.05	0.20	-
2	3	0.05	0.20	-
2	4	0.10	0.40	-
3	5	0.05	0.20	-

**Bus data of example 2**

Bus No.	PG (pu)	QG (pu)	PD (pu)	QD (pu)	$ V_{sp} $ (pu)	$\delta$
1	-	-	-	-	1.02	$0^\circ$
2	-	-	0.60	0.30	-	-
3	1.0	-	-	-	1.04	-
4	-	-	0.40	0.10	-	-
5	-	-	0.60	0.20	-	-

**Solution:** In this example, we have,

- Bus 1 is slack bus, Bus 2, 4, 5 are PQ buses, and Bus 3 is PV bus
- The lines do not have half line charging admittances  $P_2$   
 $+ jQ_2 = PG_2 + jQG_2 - (PD_2 + jQD_2) = -0.6 - j0.3$

$$P_3 + jQ_3 = P_{G3} + jQ_{G3} - (P_{D3} + jQ_{D3}) = 1.0 + jQ_{G3}$$

Similarly  $P_4 + jQ_4 = -0.4 - j0.1$ ,  $P_5 + jQ_5 = -0.6 - j0.2$

The  $Y_{bus}$  formed by the rule of inspection is given by:

$$Y_{bus} = \begin{bmatrix} 2.15685 & -0.58823 & 0.0+j0.0 & -0.39215 & -1.17647 \\ -j8.62744 & +j2.35294 & & +j1.56862 & +j4.70588 \\ -0.58823 & 2.35293 & -1.17647 & -0.58823 & 0.0+j0.0 \\ +j2.35294 & -j9.41176 & +j4.70588 & +j2.35294 & \\ 0.0+j0.0 & -1.17647 & 2.35294 & 0.0+j0.0 & -1.17647 \\ +j4.70588 & +j4.70588 & -j9.41176 & & +j4.70588 \\ -0.39215 & -0.58823 & 0.0+j0.0 & 0.98038 & 0.0+j0.0 \\ +j1.56862 & +j2.35294 & & -j3.92156 & \\ -1.17647 & 0.0+j0.0 & -1.17647 & 0.0+j0.0 & 2.35294 \\ +j4.70588 & & +j4.70588 & & -j9.41176 \end{bmatrix}$$

The voltages at all PQ buses are assumed to be equal to  $1+j0.0$  pu. The slack bus voltage is taken to be  $V_1^0 = 1.02+j0.0$  in all iterations.

$$V_2^1 = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{V_2^0} - Y_{21} V_1^0 - Y_{23} V_3^0 - Y_{24} V_4^0 - Y_{25} V_5^0 \right]$$

$$= \frac{1}{Y_{22}} \left[ \frac{-0.6 + j0.3}{1.0 - j0.0} - \{(-0.58823 + j2.35294) \times 1.02 \angle 0^\circ\} \right]$$

$$- \{(-1.17647 + j4.70588) \times 1.04 \angle 0^\circ\} - \{(-0.58823 + j2.35294) \times 1.0 \angle 0^\circ\}]$$

$$= 0.98140 \angle -3.0665^\circ = 0.97999 - j0.0525$$

Bus 3 is a PV bus. Hence, we must first calculate  $Q_3$ . This can be done as under:

$$Q_3 = V_3 V_1 (G_{31} \sin \delta_{31} - B_{31} \cos \delta_{31}) + V_3 V_2 (G_{32} \sin \delta_{32} - B_{32} \cos \delta_{32})$$

$$+ V_3 V_3 (G_{33} \sin \delta_{33} - B_{33} \cos \delta_{33}) + V_3 V_4 (G_{34} \sin \delta_{34} - B_{34} \cos \delta_{34})$$

$$+ V_3 V_5 (G_{35} \sin \delta_{35} - B_{35} \cos \delta_{35})$$

We note that  $\delta_1 = 0^\circ$ ;  $\delta_2 = -3.0665^\circ$ ;  $\delta_3 = 0^\circ$ ;  $\delta_4 = 0^\circ$  and  $\delta_5 = 0^\circ$

$$\therefore \delta_{31} = \delta_{33} = \delta_{34} = \delta_{35} = 0^\circ (\delta_{ik} = \delta_i - \delta_k); \quad \delta_{32} = 3.0665^\circ$$

$$Q_3 = 1.04 [1.02 (0.0+j0.0) + 0.9814 \{-1.17647 \times \sin(3.0665^\circ) - 4.70588 \times \cos(3.0665^\circ)\} + 1.04\{-9.41176 \times \cos(0^\circ)\} + 1.0 \{0.0 + j0.0\} + 1.0\{-4.70588 \times \cos(0^\circ)\}]$$

$$= 1.04 [-4.6735 + 9.78823 - 4.70588] = 0.425204 \text{ pu.}$$

$$V_3^1 = \frac{1}{Y_{33}} \left[ \frac{P_3 - jQ_3}{V_3^0} - Y_{31} V_1^0 - Y_{32} V_2^0 - Y_{34} V_4^0 - Y_{35} V_5^0 \right]$$

$$= \frac{1}{Y_{33}} \left[ \begin{array}{c} 1.0 - j0.425204 \\ 1.04 - j0.0 \end{array} - \left\{ \begin{array}{c} (-1.7647 + j4.70588) \times (0.98140 \angle -3.0665^\circ) \\ (-1.17647 + j4.70588) \times (1 \angle 0^\circ) \end{array} \right\} \right]$$

$$2. 1.05569 \angle 3.077^\circ = 1.0541 + j0.05666 \text{ pu.}$$

Since it is a PV bus, the voltage magnitude is adjusted to specified value and  $V_3^1$  is computed as:  $V_3^1 = 1.04 \angle 3.077^\circ \text{ pu}$

$$V_4 = \frac{1}{Y_{44}} \left[ \begin{array}{c} P - jQ \\ V_4 \end{array} - Y_{41} V_1^0 - Y_{42} V_2^1 - Y_{43} V_3^1 - Y_{45} V_5^0 \right]$$

$$= \frac{1}{Y_{44}} \left[ \begin{array}{c} -0.4 + j0.1 \\ 1.0 - j0.0 \end{array} - \left\{ \begin{array}{c} (-0.39215 + j1.56862) \times 1.02 \angle 0^\circ \\ (-0.58823 + j2.35294) \times (0.98140 \angle -3.0665^\circ) \end{array} \right\} \right]$$

$$2. \frac{0.45293 - j3.8366}{j0.12149 \ 0.98038 - j3.92156} = 0.955715 \angle -7.303^\circ \text{ pu} = 0.94796 - j0.15156$$

$$V_5^1 = \frac{1}{Y_{55}} \left[ \begin{array}{c} P - jQ \\ V_5 \end{array} - Y_{51} V_1^0 - Y_{52} V_2^1 - Y_{53} V_3^1 - Y_{54} V_4^1 \right]$$

$$= \frac{1}{Y_{55}} \left[ \begin{array}{c} -0.6 + j0.2 \\ 1.0 - j0.0 \end{array} - \left\{ \begin{array}{c} (-1.17647 + j4.70588) \times 1.02 \angle 0^\circ \\ (-1.17647 + j4.70588) \times 1.04 \angle 3.077^\circ \end{array} \right\} \right]$$

$$5. 0.994618 \angle -1.56^\circ = 0.994249 - j0.027$$

Thus at end of 1<sup>st</sup> iteration, we have,

$$\begin{aligned} V_1 &= 1.02 \angle 0^\circ \text{ pu} & V_2 &= 0.98140 \angle -3.066^\circ \text{ pu} \\ V_3 &= 1.04 \angle 3.077^\circ \text{ pu} & V_4 &= 0.955715 \angle -7.303^\circ \text{ pu} \\ \text{and} & & V_5 &= 0.994618 \angle -1.56^\circ \text{ pu} \end{aligned}$$

### Example-3:

Obtain the load flow solution at the end of first iteration of the system with data as given below. The solution is to be obtained for the following cases

7. All buses except bus 1 are PQ Buses
8. Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu
9. Bus 2 is PV bus, with voltage magnitude specified as 1.04 and  $0.25 \leq Q_2 \leq 1.0$  pu.

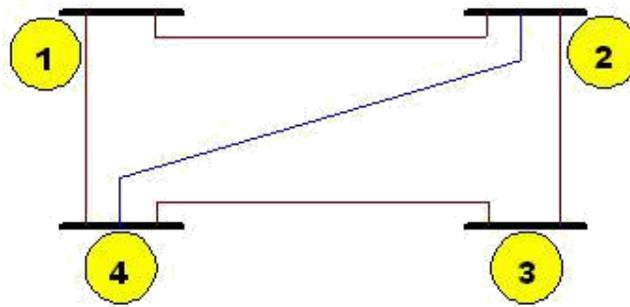


Fig. System for Example 3

**Table: Line data of example 3**

SB	EB	R (pu)	X (pu)
1	2	0.05	0.15
1	3	0.10	0.30
2	3	0.15	0.45
2	4	0.10	0.30
3	4	0.05	0.15

**Table: Bus data of example 3**

Bus No.	P <sub>i</sub> (pu)	Q <sub>i</sub> (pu)	V <sub>i</sub>
1	-	-	1.04 ∠ 0°
2	0.5	-0.2	-
3	-1.0	0.5	-
4	-0.3	-0.1	-

**Solution:** Note that the data is directly in terms of injected powers at the buses. The bus admittance matrix is formed by inspection as under:

$$Y_{BUS} = \begin{bmatrix} 3.0 - j9.0 & -2.0 + j6.0 & -1.0 + j3.0 & 0 \\ -2.0 + j6.0 & 3.666 - j11.0 & -0.666 + j2.0 & -1.0 + j3.0 \\ -1.0 + j3.0 & -0.666 + j2.0 & 3.666 - j11.0 & -2.0 + j6.0 \\ 0 & -1.0 + j3.0 & -2.0 + j6.0 & 3.0 - j9.0 \end{bmatrix}$$

**Case(i): All buses except bus 1 are PQ Buses**

Assume all initial voltages to be 1.0 ∠ 0° pu.

$$V_2^1 = \frac{1}{Y} \begin{bmatrix} P - jQ \\ -Y_{21}V_1^0 - Y_{23}V_3^0 - Y_{24}V_4^0 \\ V_2 \end{bmatrix}$$

$$\begin{aligned}
 &= \frac{1}{Y} \left[ \frac{1}{\begin{matrix} 0.5 + j0.2 \\ 22 \end{matrix}} - \{(-2.0 + j6.0) \times (1.04 \angle 0^\circ)\} \right. \\
 &\quad \left. - \{(-0.666 + j2.0) \times (1.0 \angle 0^\circ)\} - \{(-1.0 + j3.0) \times (1.0 \angle 0^\circ)\} \right] \\
 &= 1.02014 \angle 2.605^\circ \\
 V_3^1 &= \frac{1}{Y_{33}} \left[ \frac{P - jQ}{V_3^*} - Y_{31} V_1^0 - Y_{32} V_2^1 - Y_{34} V_4^0 \right] \\
 &= \frac{1}{\begin{matrix} 1 \\ 33 \end{matrix}} \left[ \frac{-1.0 - j0.5}{1.0 - j0.0} - \{(-1.0 + j3.0) \times (1.04 \angle 0.0^\circ)\} \right. \\
 &\quad \left. - \{(-0.666 + j2.0) \times (1.02014 \angle 2.605^\circ)\} - \{(-2.0 + j6.0) \times (1.0 \angle 0^\circ)\} \right] \\
 &= -1.03108 \angle -4.831^\circ \\
 V_4^1 &= \frac{1}{Y_{44}} \left[ \frac{P - jQ}{V_4^*} - Y_{41} V_1^0 - Y_{42} V_2^1 - Y_{43} V_3^1 \right] \\
 &= \frac{1}{\begin{matrix} 1 \\ 44 \end{matrix}} \left[ \frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0) \times (1.02014 \angle 2.605^\circ)\} \right. \\
 &\quad \left. - \{(-2.0 + j6.0) \times (1.03108 \angle -4.831^\circ)\} \right] \\
 &= -1.02467 \angle -0.51^\circ
 \end{aligned}$$

Hence

$$\begin{aligned}
 V_1^1 &= 1.04 \angle 0^\circ \text{ pu} & V_2^1 &= 1.02014 \angle 2.605^\circ \text{ pu} \\
 V_3^1 &= 1.03108 \angle -4.831^\circ \text{ pu} & V_4^1 &= 1.02467 \angle -0.51^\circ \text{ pu}
 \end{aligned}$$

**Case(ii): Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu**

We first compute Q2.

$$\begin{aligned}
 Q_2 &= V_2 \left[ Y_{21} (\cos \delta_{21} - B_{21} \sin \delta_{21}) + V_2 (G_{22} \sin \delta_{22} - B_{22} \cos \delta_{22}) \right. \\
 &\quad \left. + V_3 (G_{23} \sin \delta_{23} - B_{23} \cos \delta_{23}) + V_4 (G_{24} \sin \delta_{24} - B_{24} \cos \delta_{24}) \right]
 \end{aligned}$$

$$\begin{aligned}
 V_2^1 &= \frac{1}{\begin{matrix} 1 \\ 22 \end{matrix}} \left[ \frac{0.5 - j0.20}{1.04 \angle 0} - \{(-2.0 + j6.0) \times (1.04 \angle 0^\circ)\} \right. \\
 &\quad \left. - \{(-0.666 + j2.0) \times (1.0 \angle 0^\circ)\} - \{(-1.0 + j3.0) \times (1.0 \angle 0^\circ)\} \right] \\
 &= 15.1051288 + j0.033883
 \end{aligned}$$

The voltage magnitude is adjusted to 1.04. Hence  $V_2^1 = 1.04 \angle 1.846^\circ$

$$V_3^1 = \frac{1}{Y_{33}} \left[ \frac{-1.0 - j0.5}{1.0 \angle 0} - \{(-1.0 + j3.0) \times (1.04 \angle 0.0^\circ)\} \right. \\ \left. - \{(-0.666 + j2.0) \times (1.04 \angle 1.846^\circ)\} - \{(-2.0 + j6.0) \times (1.0 \angle 0^\circ)\} \right]$$

$$V_4^1 = \frac{1}{Y_{44}} \left[ \frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0) \times (1.04 \angle 1.846^\circ)\} \right. \\ \left. - \{(-2.0 + j6.0) \times (1.035587 \angle -4.951^\circ)\} \right]$$

$$= 0.9985 \angle -0.178^\circ$$

Hence at end of 1<sup>st</sup> iteration we have:

$$V_1^1 = 1.04 \angle 0^\circ \text{ pu} \qquad V_2^1 = 1.04 \angle 1.846^\circ \text{ pu}$$

$$V_3^1 = 1.035587 \angle -4.951^\circ \text{ pu} \qquad V_4^1 = 0.9985 \angle -0.178^\circ \text{ pu}$$

**Case (iii):** Bus 2 is PV bus, with voltage magnitude specified as 1.04 &  $0.25 \leq Q_2 \leq 1$  pu. If  $0.25 \leq Q_2 \leq 1.0$  pu then the computed value of  $Q_2 = 0.208$  is less than the lower limit. Hence,  $Q_2$  is set equal to 0.25 pu. Iterations are carried out with this value of  $Q_2$ . The voltage magnitude at bus 2 can no longer be maintained at 1.04. Hence, there is no necessity to adjust for the voltage magnitude. Proceeding as before we obtain at the end of first iteration,

$$V_1^1 = 1.04 \angle 0^\circ \text{ pu} \qquad V_2^1 = 1.05645 \angle 1.849^\circ \text{ pu}$$

$$V_3^1 = 1.038546 \angle -4.933^\circ \text{ pu} \qquad V_4^1 = 1.081446 \angle 4.896^\circ \text{ pu}$$

### **Limitations of GS load flow analysis:**

GS method is very useful for very small systems. It is easily adoptable, it can be generalized and it is very efficient for systems having less number of buses. However, GS LFA fails to converge in systems with one or more of the features as under:

- Systems having large number of radial lines
- Systems with short and long lines terminating on the same bus
- Systems having negative values of transfer admittances
- Systems with heavily loaded lines, etc.

GS method successfully converges in the absence of the above problems. However, convergence also depends on various other set of factors such as: selection of slack bus, initial solution, acceleration factor, tolerance limit, level of accuracy of results needed, type and quality of computer/ software used, etc.

CHAPTER 3

NEWTON –RAPHSON METHOD

Newton-Raphson (NR) method is used to solve a system of non-linear algebraic equations of the form  $f(x) = 0$ . Consider a set of  $n$  non-linear algebraic equations given by

$$f_i(x_1, x_2, \dots, x_n) = 0 \quad i = 1, 2, \dots, n \tag{25}$$

Let  $x_1^0, x_2^0, \dots, x_n^0$ , be the initial guess of unknown variables and  $\Delta x_1^0, \Delta x_2^0, \dots, \Delta x_n^0$  be the respective corrections. Therefore,

$$f_i(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \dots, x_n^0 + \Delta x_n^0) = 0 \quad i = 1, 2, \dots, n \tag{26}$$

The above equation can be expanded using Taylor's series to give

$$f_i(x_1^0, x_2^0, \dots, x_n^0) + \left[ \left( \frac{\partial f_i}{\partial x_1} \right)_0 \Delta x_1^0 + \left( \frac{\partial f_i}{\partial x_2} \right)_0 \Delta x_2^0 + \dots + \left( \frac{\partial f_i}{\partial x_n} \right)_0 \Delta x_n^0 \right] + \text{Higher order terms} = 0 \quad \forall i = 1, 2, \dots, n \tag{27}$$

Where,  $\left( \frac{\partial f_i}{\partial x_1} \right)_0, \left( \frac{\partial f_i}{\partial x_2} \right)_0, \dots, \left( \frac{\partial f_i}{\partial x_n} \right)_0$  are the partial derivatives of  $f_i$  with respect

to  $x_1, x_2, \dots, x_n$  respectively, evaluated at  $(x_1^0, x_2^0, \dots, x_n^0)$ . If the higher order terms are neglected, then (27) can be written in matrix form as

$$\begin{bmatrix} f_1^0 \\ f_2^0 \\ \vdots \\ f_n^0 \end{bmatrix} + \begin{bmatrix} \left( \frac{\partial f_1}{\partial x_1} \right)_0 & \left( \frac{\partial f_1}{\partial x_2} \right)_0 & \dots & \left( \frac{\partial f_1}{\partial x_n} \right)_0 \\ \left( \frac{\partial f_2}{\partial x_1} \right)_0 & \left( \frac{\partial f_2}{\partial x_2} \right)_0 & \dots & \left( \frac{\partial f_2}{\partial x_n} \right)_0 \\ \vdots & \vdots & \ddots & \vdots \\ \left( \frac{\partial f_n}{\partial x_1} \right)_0 & \left( \frac{\partial f_n}{\partial x_2} \right)_0 & \dots & \left( \frac{\partial f_n}{\partial x_n} \right)_0 \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix} = 0 \tag{28}$$

In vector form (28) can be written as

$$F^0 + J^0 \Delta X^0 = 0$$

Or  $F^0 = -J^0 \Delta X^0$

Or  $\Delta X^0 = -[J^0]^{-1} F^0 \tag{29}$

And  $X^1 = X^0 + \Delta X^0 \tag{30}$

Here, the matrix [J] is called the **Jacobian** matrix. The vector of unknown variables is updated using (30). The process is continued till the difference between two successive iterations is less than the tolerance value.

### NR method for load flow solution in polar coordinates

In application of the NR method, we have to first bring the equations to be solved, to the form  $f_i(x_1, x_2, \dots, x_n) = 0$ , where  $x_1, x_2, \dots, x_n$  are the unknown variables to be determined. Let us assume that the power system has  $n_1$  PV buses and  $n_2$  PQ buses. In polar coordinates the unknown variables to be determined are:

□  $\delta_i$ , the angle of the complex bus voltage at bus  $i$ , at all the PV and PQ buses. This gives us  $n_1 + n_2$  unknown variables to be determined.

□  $|V_i|$ , the voltage magnitude of bus  $i$ , at all the PQ buses. This gives us  $n_2$  unknown variables to be determined.

Therefore, the total number of unknown variables to be computed is:  $n_1 + 2n_2$ , for which we need  $n_1 + 2n_2$  consistent equations to be solved. The equations are given by,

$$\Delta P_i = P_{i,sp} - P_{i,cal} = 0 \quad (31)$$

$$\Delta Q_i = Q_{i,sp} - Q_{i,cal} = 0 \quad (32)$$

Where  $P_{i,sp}$  = Specified active power at bus  $i$

$Q_{i,sp}$  = Specified reactive power at bus  $i$

$P_{i,cal}$  = Calculated value of active power using voltage estimates.

$Q_{i,cal}$  = Calculated value of reactive power using voltage estimates

$\Delta P$  = Active power residue

$\Delta Q$  = Reactive power residue

The real power is specified at all the PV and PQ buses. Hence (31) is to be solved at all PV and PQ buses leading to  $n_1 + n_2$  equations. Similarly the reactive power is specified at all the PQ buses. Hence, (32) is to be solved at all PQ buses leading to  $n_2$  equations.

We thus have  $n_1 + 2n_2$  equations to be solved for  $n_1 + 2n_2$  unknowns. (31) and (32) are of the form  $F(x) = 0$ . Thus NR method can be applied to solve them. Equations (31) and (32) can be written in the form of (30) as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = - \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} \quad (33)$$

Where  $J_1, J_2, J_3, J_4$  are the negated partial derivatives of  $\Delta P$  and  $\Delta Q$  with respect to corresponding  $\delta, V$ . The negated partial derivative of  $\Delta P$ , is same as the partial derivative of  $P_{cal}$ , since  $P_{sp}$  is a constant. The various computations involved are discussed in detail next.

### Computation of $P_{cal}$ and $Q_{cal}$ :

The real and reactive powers can be computed from the load flow equations as:

$$\begin{aligned} P_{i,cal} &= P_i = \sum_{k=1}^n V_i V_k (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}) \\ &= G_{ii} V_i^2 + \sum_{\substack{k=1 \\ k \neq i}}^n V_i V_k (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}) \end{aligned} \quad (34)$$

$$\begin{aligned} Q_{i,cal} &= Q_i = \sum_{k=1}^n V_i V_k (G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}) \\ &= -B_{ii} V_i^2 + \sum_{\substack{k=1 \\ k \neq i}}^n V_i V_k (G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}) \end{aligned} \quad (35)$$

The powers are computed at any  $(r + 1)^{st}$  iteration by using the voltages available from previous iteration. The elements of the Jacobian are found using the above equations as:

### Elements of $J_1$

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^n V_i V_k \{ G_{ik} (-\sin \delta_{ik}) + B_{ik} \cos \delta_{ik} \}$$

$$= -Q_i - B_{ii} V_i^2$$

$$\frac{\partial P_i}{\partial \delta_{ik}} = \sum_{k=1}^n V_i V_k \{ G_{ik} (-\sin \delta_{ik})(-1) + B_{ik} (\cos \delta_{ik})(-1) \}$$

**Elements of J3**

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{k=1, k \neq i}^n V_i V_k (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}) = P_i - G_{ii} V_i^2$$

$$\frac{\partial Q_i}{\partial \delta_{ik}} = -V_i V_k (G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik})$$

**Elements of J2**

$$\frac{\partial P_i}{\partial V_i} = 2 V_i G_{ii} + \sum_{k=1, k \neq i}^n V_k (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}) = P_i + V_i^2 G_{ii}$$

$$\frac{\partial P_i}{\partial V_k} = V_k (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik})$$

**Elements of J4**

$$\frac{\partial P_i}{\partial V_i} = -2 V_i B_{ii} + \sum_{k=1, k \neq i}^n V_k (G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}) = Q_i - V_i^2 B_{ii}$$

$$\frac{\partial P_i}{\partial V_k} = V_k (G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik})$$

Thus, the linearized form of the equation could be considered again as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix}$$

The elements are summarized below:

(i)  $H_{ii} = \frac{\partial P_i}{\partial V_i} = P_i + G_{ii} V_i^2$

(ii)  $H_{ik} = \frac{\partial P_i}{\partial \delta_{ik}} = -V_i V_k (G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik})$

(iii)  $N_{ii} = \frac{\partial P_i}{\partial V_i} = P_i + G_{ii} V_i^2$

(iv)  $N_{ik} = \frac{\partial P_i}{\partial V_k} = V_k (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik})$

(v)  $M_{ii} = \frac{\partial Q_i}{\partial \delta_{ii}} = P_i - G_{ii} V_i^2$

$$(vi) M_{ik} = \frac{\partial Q_i}{\partial \delta^k} = -(a_{ki} e + b_{ki} f) = -N_{ik}$$

$$(vii) L_{ii} = \frac{\partial Q_i}{\partial V_i} = \frac{Q_i}{V_i} - B_{ii} V_i$$

$$(viii) L_{ik} = \frac{\partial Q_i}{\partial V_k} = a_{ki} f - b_{ki} e = H_{ik}$$

In the above equations,

$$Y = G$$

$$e_k + jf_k = V_k (\cos \delta_k + j \sin \delta_k)$$

$$\text{And } a_k + jb_k = (G_{ik} + jB_{ik})(e_k + jf_k) \tag{36}$$

If  $Y_{ik} = 0.0 + j0.0$  (if there is no line between buses  $i$  and  $k$ ) then the corresponding off-diagonal elements in the Jacobian matrix will also be zero. Hence, the Jacobian is also a sparse matrix.

**Size of the sub-matrices of the Jacobian:** The dimensions of the various sub-matrices are as per the table below:

Matrix	size
H	$(n_1+n_2) \times (n_1+n_2)$
N	$(n_1+n_2) \times (n_2)$
M	$(n_2) \times (n_1+n_2)$
L	$(n_2) \times (n_2)$
J	$(n_1+2n_2) \times (n_1+2n_2)$
$\Delta P$	$(n_1+n_2) \times 1$
$\Delta Q$	$n_2 \times 1$
$\Delta \delta$	$(n_1+n_2) \times 1$
$\Delta  V $	$n_2 \times 1$

**ALGORITHM FOR NR METHOD  
IN POLAR COORDINATES**

- Formulate the Y<sub>BUS</sub>
- Assume initial voltages as follows:

$$V_i = |V_{i,sp}| \angle 0^0 \quad (\text{at all PV buses})$$

$$V_i = 1 \angle 0^0 \quad (\text{at all PQ buses})$$

3. At  $(r+1)^{st}$  iteration, calculate  $P_i^{(r+1)}$  at all the PV and PQ buses and  $Q_i^{(r+1)}$  at all the PQ buses, using voltages from previous iteration,  $V_i^{(r)}$ . The formulae to be used are

$$P_{i,cal} = P_i = G_{ii} V_i^2 + \sum_{\substack{k=1 \\ k \neq i}}^n V_i V_k (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik})$$

$$Q_{i,cal} = Q_i = -B_{ii} V_i^2 + \sum_{\substack{k=1 \\ k \neq i}}^n V_i V_k (G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik})$$

4. Calculate the power mismatches (power residues)

$$\Delta P_i^{(r)} = P_i - P_{i,sp}^{(r+1)} \quad (\text{at PV and PQ buses})$$

$$\Delta Q_i^{(r)} = Q_i - Q_{i,cal}^{(r+1)} \quad (\text{at PQ buses})$$

5. Calculate the Jacobian  $[J^{(r)}]$  using  $V_i^{(r)}$  and its elements spread over H, N, M, L sub- matrices using the relations derived as in (36).

6. Compute

$$\begin{bmatrix} \Delta \delta^{(r)} \\ \Delta V^{(r)} \end{bmatrix} = [J^{(r)}]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$$

7. Update the variables as follows:

$$\delta_i^{(r+1)} = \delta_i^{(r)} + \Delta \delta_i^{(r)} \quad (\text{at all buses})$$

$$|V_i|_{r+1} = |V_i|^{(r)} + \Delta |V_i|^{(r)}$$

8. Go to step 3 and iterate till the power mismatches are within acceptable tolerance.

**DECOUPLED LOAD FLOW**

In the NR method, the inverse of the Jacobian has to be computed at every iteration. When solving large interconnected power systems, alternative solution methods are possible, taking into account certain observations made of practical systems. These are,

- = Change in voltage magnitude  $|V_i|$  at a bus primarily affects the flow of reactive power  $Q$  in the lines and leaves the real power  $P$  unchanged. This observation implies that  $\frac{\partial Q_i}{\partial |V_j|}$  is much larger than  $\frac{\partial P_i}{\partial |V_j|}$ . Hence, in the Jacobian, the elements of the sub-matrix  $[N]$ , which contains terms that are partial derivatives of real power with respect to voltage magnitudes can be made zero.
- (ii) Change in voltage phase angle at a bus, primarily affects the real power flow  $P$  over the lines and the flow of  $Q$  is relatively unchanged. This observation implies that  $\frac{\partial P_i}{\partial \delta_j}$  is much larger than  $\frac{\partial Q_i}{\partial \delta_j}$ . Hence, in the Jacobian the elements of the sub-matrix  $[M]$ , which contains terms that are partial derivatives of reactive power with respect to voltage phase angles can be made zero.

These observations reduce the NRLF linearised form of equation to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \frac{\Delta |V|}{|V|} \end{bmatrix} \quad (37)$$

From (37) it is obvious that the voltage angle corrections  $\Delta \delta$  are obtained using real power residues  $\Delta P$  and the voltage magnitude corrections  $\frac{\Delta |V|}{|V|}$  are obtained from reactive power residues  $\Delta Q$ . This equation can be solved through two alternate strategies as under:

**Strategy-1**

(i) Calculate  $\Delta P^{(r)}$ ,  $\Delta Q^{(r)}$  and  $J^{(r)}$

$$\text{Compute } \begin{bmatrix} \Delta \delta^{(r)} \\ \frac{V^{(r)} \Delta P^{(r)}}{V^{(r)}} \\ \frac{Q^{(r)}}{V^{(r)}} \end{bmatrix} \begin{bmatrix} \Delta \delta^{(r)} \\ \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$$

- Update  $\delta$  and  $V$   $\left[ J^{(r)} \right]_{-1}$
- Go to step (i) and iterate till convergence is reached.

In the first strategy, the variables are solved simultaneously. In the second strategy the iteration is conducted by first solving for  $\Delta \delta$  and using updated values of  $\delta$  to calculate  $V$ . Hence, the second strategy results in faster convergence, compared to the first strategy.

**FAST DECOUPLED LOAD FLOW**

If the coefficient  $\frac{1}{V^{(r)} + \Delta V^{(r)}}$  matrices are constant, the need to update the Jacobian at every iteration is eliminated. This has resulted in development of fast decoupled load flow (FDLF). Here, certain assumptions are made based on the observations of practical power systems as under:

5.  $B_{ij} \gg G_{ij}$  (Since the  $\frac{X}{R}$  ratio of transmission lines is high in well designed systems)

(ii) The voltage angle difference  $(\delta_i - \delta_j)$  between two buses in the system is very

small. This means  $\cos(\delta_i - \delta_j) \cong 1$  and  $\sin(\delta_i - \delta_j) = 0.0$

3.  $Q_i \ll |V_i|^2$

With these assumptions the elements of the Jacobian become

$$H_{ik} = L_{ik} = -\frac{1}{V_i} V_k \quad (i \neq k)$$

$$H_{ii} = L_{ii} = -\frac{1}{V_i} \sum_{k \neq i} V_k$$

The matrix (37) reduces to  $\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H \\ N \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix}$

$$[\Delta Q] = [V_i \quad V_j \quad B_{ij}] \begin{bmatrix} \Delta V_i \\ \Delta V_j \\ \Delta V_i \end{bmatrix} \quad (38)$$

Where  $B$  and  $B''$  are negative of the susceptances of respective elements of the bus admittance matrix.

In (38) if we divide LHS and RHS by  $V_i$  and assume we get,  $\frac{V_j}{V_i} \approx 1$ ,

$$\begin{bmatrix} \frac{\Delta P}{V_i} \\ \frac{\Delta Q}{V_i} \end{bmatrix} = [B_{ij}] \begin{bmatrix} \Delta V_j \\ \Delta V_i \end{bmatrix} \quad (39)$$

Equations (39) constitute the Fast Decoupled load flow equations. Further simplification is possible by:

- Omitting effect of phase shifting transformers
- Setting off-nominal turns ratio of transformers to 1.0
- In forming  $B_{ij}$ , omitting the effect of shunt reactors and capacitors which mainly affect reactive power
- Ignoring series resistance of lines in forming the  $Y_{bus}$ .

with these assumptions we obtain a loss-less network. In the FDLF method, the matrices  $[B]$  and  $[B']$  are constants and need to be inverted only once at the beginning of the iterations.

### REPRESENTATION OF TAP CHANGING TRANSFORMERS

Consider a tap changing transformer represented by its admittance connected in series with an ideal autotransformer as shown (a= turns ratio of transformer)

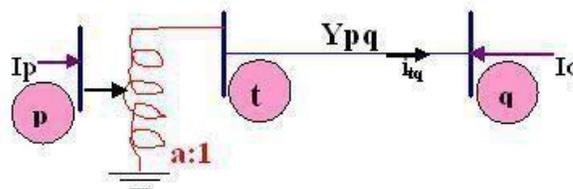


Fig. 2. Equivalent circuit of a tap setting transformer

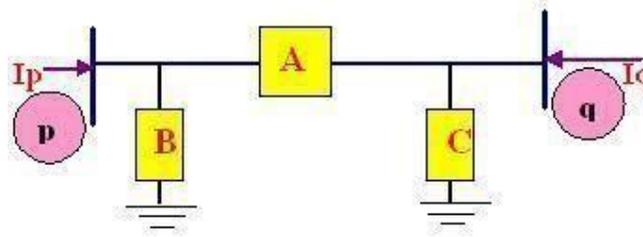


Fig. 3.  $\pi$ -Equivalent circuit of Fig.2 above.

By equating the bus currents in both the mutually equivalent circuits as above, it can be shown that the  $\pi$ -equivalent circuit parameters are given by the expressions as under:

**(i) Fixed tap setting transformers (on no load)**

$$A = Y_{pq} / a$$

$$B = 1/a (1/a - 1) Y_{pq}$$

**C = (1-1/a)  $Y_{pq}$  (i) Tap changing under load (TCUL) transformers (on load)**

$$A = Y_{pq}$$

$$B = (1/a - 1) (1/a + 1 - E_q/E_p) Y_{pq}$$

$$C = (1-1/a) (E_p/E_q) Y_{pq}$$

Thus, here, in the case of TCUL transformers, the shunt admittance values are observed to be a function of the bus voltages.

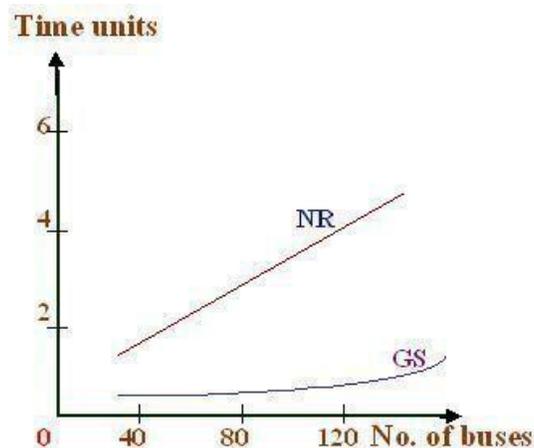
## COMPARISON OF LOAD FLOW METHODS

The comparison of the methods should take into account the computing time required for preparation of data in proper format and data processing, programming ease, storage requirements, computation time per iteration, number of iterations, ease and time required for modifying network data when operating conditions change, etc. Since all the methods presented are in the bus frame of reference in admittance form, the data preparation is same for all the methods and the bus admittance matrix can be formed using a simple algorithm, by the rule of inspection.

Due to simplicity of the equations, Gauss-Seidel method is relatively easy to program. Programming of NR method is more involved and becomes more

complicated if the buses are randomly numbered. It is easier to program, if the PV buses are ordered in sequence and PQ buses are also ordered in sequence.

The storage requirements are more for the NR method, since the Jacobian elements have to be stored. The memory is further increased for NR method using rectangular coordinates. The storage requirement can be drastically reduced by using sparse matrix techniques, since both the admittance matrix and the Jacobian are sparse matrices. The time taken for a single iteration depends on the number of arithmetic and logical operations required to be performed in a full iteration. The Gauss –Seidel method requires the fewest number of operations to complete iteration. In the NR method, the computation of the Jacobian is necessary in every iteration. Further, the inverse of the Jacobian also has to be computed. Hence, the time per iteration is larger than in the GS method and is roughly about 7 times that of the GS method, in large systems, as depicted graphically in figure below. Computation time can be reduced if the Jacobian is updated once in two or three iterations. In FDLF method, the Jacobian is constant and needs to be computed only once. In both NR and FDLF methods, the time per iteration increases directly as the number of buses.



**Figure 4. Time per Iteration in GS and NR methods**

The number of iterations is determined by the convergence characteristic of the method. The GS method exhibits a linear convergence characteristic as compared to the NR method which has a quadratic convergence. Hence, the GS method requires more number of iterations to get a converged solution as compared to the NR method. In the GS method, the number of iterations increases directly as the size of

the system increases. In contrast, the number of iterations is relatively constant in NR and FDLF methods. They require about 5-8 iterations for convergence in large systems. A significant increase in rate of convergence can be obtained in the GS method if an acceleration factor is used. All these variations are shown graphically in figure below. The number of iterations also depends on the required accuracy of the solution. Generally, a voltage tolerance of 0.0001 pu is used to obtain acceptable accuracy and the real power mismatch and reactive power mismatch can be taken as 0.001 pu. Due to these reasons, the NR method is faster and more reliable for large systems. The convergence of FDLF method is geometric and its speed is nearly 4-5 times that of NR method.

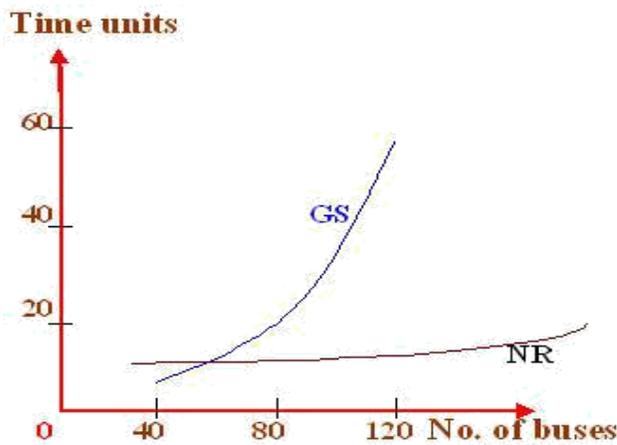


Figure 5. Total time of Iteration in GS and NR methods

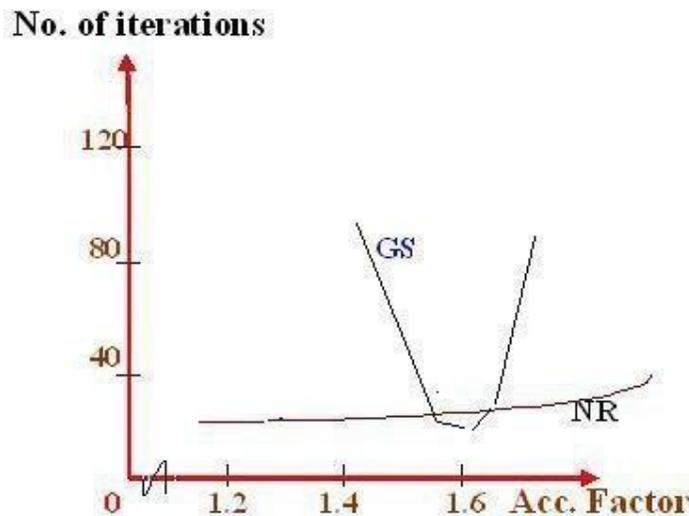


Figure 6. Influence of acceleration factor on load flow methods

**FINAL WORD**

In this chapter, the load flow problem, also called as the power flow problem, has been considered in detail. The load flow solution gives the complex voltages at all the buses and the complex power flows in the lines. Though, algorithms are available using the impedance form of the equations, the sparsity of the bus admittance matrix and the ease of building the bus admittance matrix, have made algorithms using the admittance form of equations more popular.

The most popular methods are the Gauss-Seidel method, the Newton-Raphson method and the Fast Decoupled Load Flow method. These methods have been discussed in detail with illustrative examples. In smaller systems, the ease of programming and the memory requirements, make GS method attractive. However, the computation time increases with increase in the size of the system. Hence, in large systems NR and FDLF methods are more popular. There is a trade off between various requirements like speed, storage, reliability, computation time, convergence characteristics etc. No single method has all the desirable features. However, NR method is most popular because of its versatility, reliability and accuracy

**CHAPTER 4****REPRESENTATION OF POWER SYSTEMS**

[CONTENTS: One line diagram, impedance diagram, reactance diagram, per unit quantities, per unit impedance diagram, formation of bus admittance & impedance matrices, examples]

**One Line Diagram**

In practice, electric power systems are very complex and their size is unwieldy. It is very difficult to represent all the components of the system on a single frame. The complexities could be in terms of various types of protective devices, machines (transformers, generators, motors, etc.), their connections (star, delta, etc.), etc. Hence, for the purpose of power system analysis, a simple single phase equivalent circuit is developed called, the one line diagram (OLD) or the single line diagram (SLD). *An SLD is thus, the concise form of representing a given power system.* It is to be noted that a given SLD will contain only such data that are relevant to the system analysis/study under consideration. For example, the details of protective devices need not be shown for load flow analysis nor it is necessary to show the details of shunt values for stability studies.

**Symbols used for SLD**

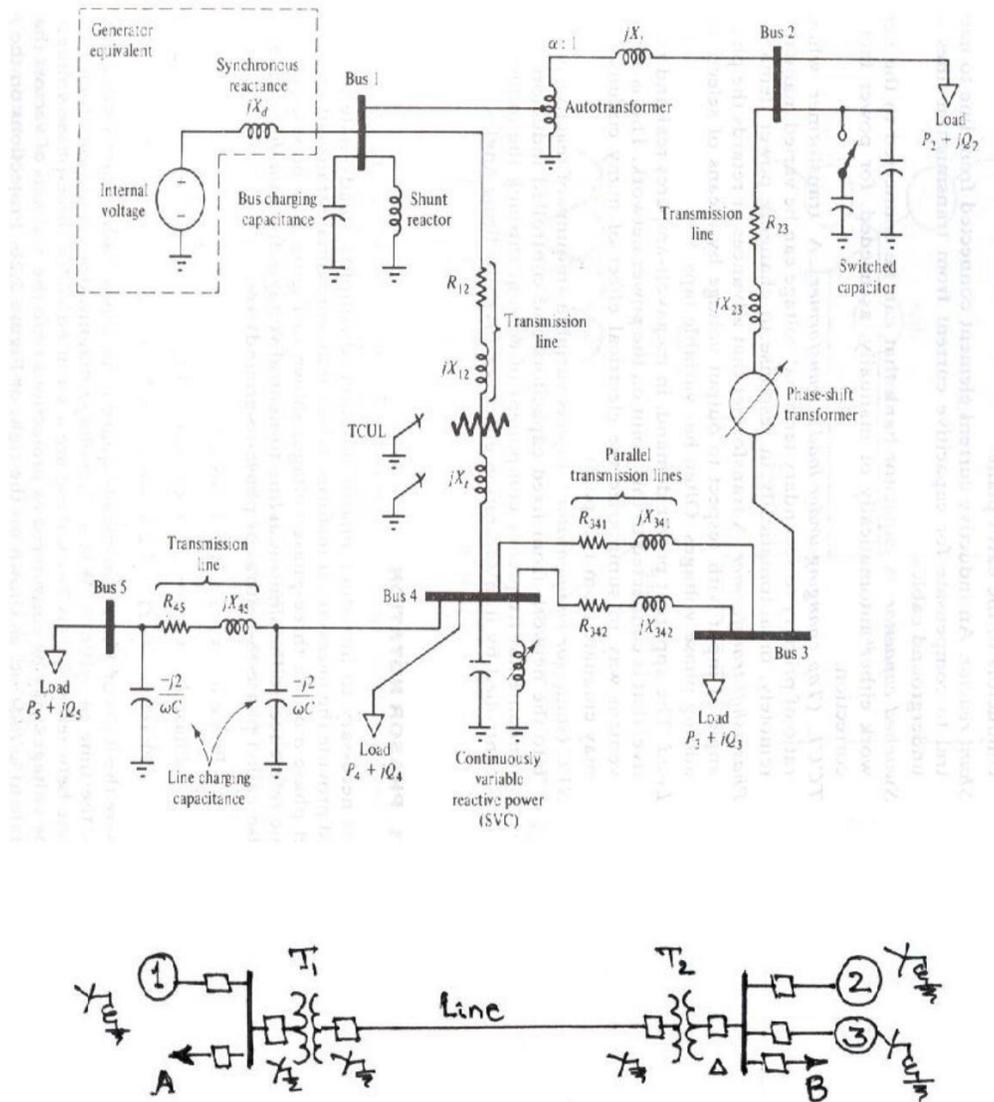
Various symbols are used to represent the different parameters and machines as single phase equivalents on the SLD,. Some of the important symbols used are as listed in the table of Figure 1.

MOTOR (M)	;	Generator (G)
Transformers:	2-Winding	$\frac{3E}{3E}$
	3-Winding	$\frac{33E}{33E}$
Power Circuit breaker		$\square$
3 $\phi$ Delta:	$\Delta$	star: $\gamma$
3 $\phi$ star-grounded neutral:	$\gamma$	$\frac{3E}{3E}$
Grounded thro' $X_n$	$\gamma$	$\frac{3E}{X_n}$
CT	$\text{---} \text{---}$	;
		PT $\text{---} \text{---}$

**Figure 1. TABLE OF SYMBOLS FOR USE ON SLDS**

**Example system**

Consider for illustration purpose, a sample example power system and data as under: Generator 1: 30 MVA, 10.5 KV,  $X''= 1.6$  ohms, Generator 2: 15 MVA, 6.6 KV,  $X''=$  ohms, Generator 3: 25 MVA, 6.6 KV,  $X''= 0.56$  ohms, Transformer 1 (3-phase): 15 MVA, 33/11 KV,  $X=15.2$  ohms/phase on HT side, Transformer 2 (3-phase): 15 MVA, 33/6.2 KV,  $X=16.0$  ohms/phase on HT side, Transmission Line: 20.5 ohms per phase, Load A: 15 MW, 11 KV, 0.9 PF (lag); and Load B: 40 MW, 6.6 KV, 0.85 PF (lag). The corresponding SLD incorporating the standard symbols can be shown as in figure 2.



**Figure 2. SAMPLE SYSTEM OLD**

It is observed here, that the generators are specified in 3-phase MVA, L-L voltage and per phase Y-equivalent impedance, transformers are specified in 3-phase MVA, L-L voltage transformation ratio and per phase Y-equivalent impedance on any one side and the loads are specified in 3-phase MW, L-L voltage and power factor.

### Impedance Diagram

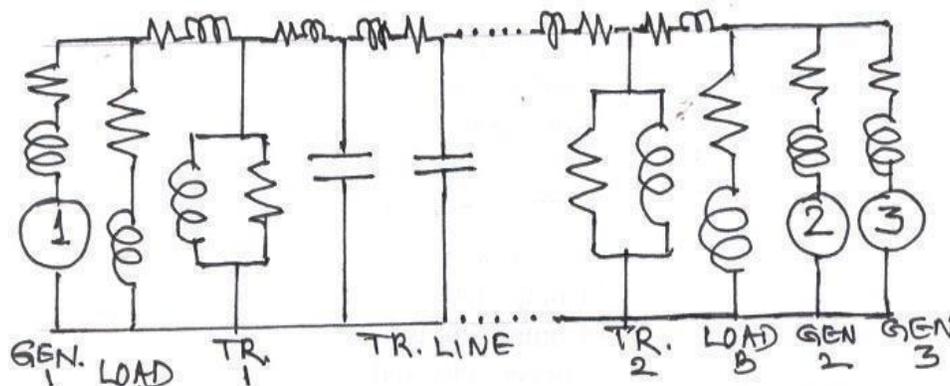
The impedance diagram on single-phase basis for use under balanced conditions can be easily drawn from the SLD. The following assumptions are made in obtaining the impedance diagrams.

#### Assumptions:

- The single phase transformer equivalents are shown as ideals with impedances on appropriate side (LV/HV),
- The magnetizing reactances of transformers are negligible,
- The generators are represented as constant voltage sources with series resistance or reactance,
- The transmission lines are approximated by their equivalent  $\pi$ -Models,
- The loads are assumed to be passive and are represented by a series branch of resistance or reactance and
- Since the balanced conditions are assumed, the neutral grounding impedances do not appear in the impedance diagram.

#### Example system

As per the list of assumptions as above and with reference to the system of figure 2, the impedance diagram can be obtained as shown in figure 3.



**Figure 3. IMPEDANCE DIAGRAM**

### Reactance Diagram

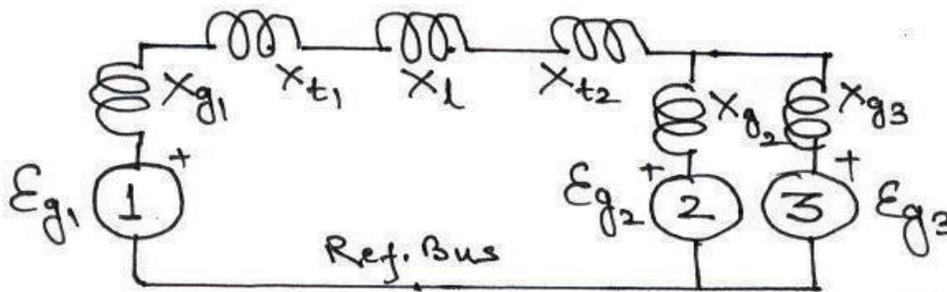
With some more additional and simplifying assumptions, the impedance diagram can be simplified further to obtain the corresponding reactance diagram. The following are the assumptions made.

#### **Additional assumptions:**

- Ⓟ The resistance is often omitted during the fault analysis. This causes a very negligible error since, resistances are negligible
- Ⓟ Loads are Omitted
- Ⓟ Transmission line capacitances are ineffective &
- Ⓟ Magnetizing currents of transformers are neglected.

#### **Example system**

as per the assumptions given above and with reference to the system of figure 2 and figure 3, the reactance diagram can be obtained as shown in figure 4.



**Figure 4. REACTANCE DIAGRAM**

Note: These impedance & reactance diagrams are also referred as the *Positive Sequence Diagrams/ Networks*.

### Per Unit Quantities

during the power system analysis, it is a usual practice to represent current, voltage, impedance, power, etc., of an electric power system in per unit or percentage of the base or reference value of the respective quantities. The numerical per unit (pu) value of any quantity is its ratio to a chosen base value of the same dimension. Thus a pu value is a normalized quantity with respect to the chosen base value.

**Definition:** Per Unit value of a given quantity is the ratio of the **actual value** in any given unit to the **base value** in the same unit. The percent value is 100 times the pu value. Both the pu and percentage methods are simpler than the use of actual values. Further, the main advantage in using the pu system of computations is that the result that comes out of the sum, product, quotient, etc. of two or more pu values is expressed in per unit itself.

In an electrical power system, the parameters of interest include the current, voltage, complex power (VA), impedance and the phase angle. Of these, the phase angle is dimensionless and the other four quantities can be described by knowing any two of them. Thus clearly, an arbitrary choice of any two base values will evidently fix the other base values.

Normally the nominal voltage of lines and equipment is known along with the complex power rating in MVA. Hence, in practice, the base values are chosen for complex power (MVA) and line voltage (KV). The chosen base MVA is the same for all the parts of the system. However, the base voltage is chosen with reference to a particular section of the system and the other base voltages (with reference to the other sections of the systems, these sections caused by the presence of the transformers) are then related to the chosen one by the turns-ratio of the connecting transformer.

If  $I_b$  is the base current in kilo amperes and  $V_b$ , the base voltage in kilovolts, then the base MVA is,  $S_b = (V_b I_b)$ . Then the base values of current & impedance are given by

$$\text{Base current (kA), } I_b = \frac{MVA_b}{KV_b} = \frac{S_b}{V_b} \tag{1.1}$$

$$\text{Base impedance, } Z_b = \frac{V_b}{I_b} = \frac{(KV_b)^2}{MVA_b} \tag{1.2}$$

Hence the per unit impedance is given by

$$Z_{pu} = \frac{Z_{ohms}}{Z_b} = Z_{ohms} \left( \frac{MVA_b}{KV_b^2} \right) \tag{1.3}$$

In 3-phase systems,  $KV_b$  is the line-to-line value &  $MVA_b$  is the 3-phase MVA. [1-phase MVA = (1/3) 3-phase MVA].

**Changing the base of a given pu value:**

It is observed from equation (3) that the pu value of impedance is proportional directly to the base MVA and inversely to the square of the base KV. If  $Z_{pu}^{new}$  is the pu impedance required to be calculated on a new set of base values:  $MVA_b^{new}$  &  $KV_b^{new}$  from the already given per unit impedance  $Z_{pu}^{old}$ , specified on the old set of base values,

$$Z_{pu}^{new} = Z_{pu}^{old} \left( \frac{MVA_b^{new}}{MVA_b^{old}} \right) \left( \frac{KV_b^{old}}{KV_b^{new}} \right)^2 \tag{1.4}$$

On the other hand, the change of base can also be done by first converting the given pu impedance to its ohmic value and then calculating its pu value on the new set of base values.

**Merits and Demerits of pu System**

Following are the advantages and disadvantages of adopting the pu system of computations in electric power systems:

**Merits:**

- ❑ The pu value is the same for both 1-phase and 3-phase systems
- ❑ The pu value once expressed on a proper base, will be the same when referred to either side of the transformer. Thus the presence of transformer is totally eliminated
- ❑ The variation of values is in a smaller range (nearby unity). Hence the errors involved in pu computations are very less.
- ❑ Usually the nameplate ratings will be marked in pu on the base of the nameplate ratings, etc.

**Demerits:**

If proper bases are not chosen, then the resulting pu values may be highly absurd (such as 5.8 pu, -18.9 pu, etc.). This may cause confusion to the user. However, this problem can be avoided by selecting the base MVA near the high-rated equipment and a convenient base KV in any section of the system.

**(iv) pu Impedance / Reactance Diagram**

for a given power system with all its data with regard to the generators, transformers, transmission lines, loads, etc., it is possible to obtain the corresponding impedance or reactance diagram as explained above. If the parametric values are shown in pu on the properly selected base values of the system, then the diagram is referred as the per unit impedance or reactance diagram. In forming a pu diagram, the following are the procedural steps involved:

- ❑ Obtain the one line diagram based on the given data
- ❑ Choose a common base MVA for the system
- ❑ Choose a base KV in any one section (Sections formed by transformers)
- ❑ Find the base KV of all the sections present
- ❑ Find pu values of all the parameters: R, X, Z, E, etc.
- ❑ Draw the pu impedance/ reactance diagram.

**Formation Of YBUS & zBUS**

The performance equations of a given power system can be considered in three different frames of reference as discussed below:

**Frames of Reference:**

*Bus Frame of Reference:* There are b independent equations (b = no. of buses) relating the bus vectors of currents and voltages through the bus impedance matrix and bus admittance matrix:

$$\begin{aligned} E_{BUS} &= Z_{BUS} I_{BUS} \\ I_{BUS} &= Y_{BUS} E_{BUS} \end{aligned} \quad (1.5)$$

*Branch Frame of Reference:* There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$$\begin{aligned} E_{BR} &= Z_{BR} I_{BR} \\ I_{BR} &= Y_{BR} E_{BR} \end{aligned} \quad (1.6)$$

*Loop Frame of Reference:* There are  $b$  independent equations ( $b = \text{no. of branches of a selected Tree sub-graph of the system Graph}$ ) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

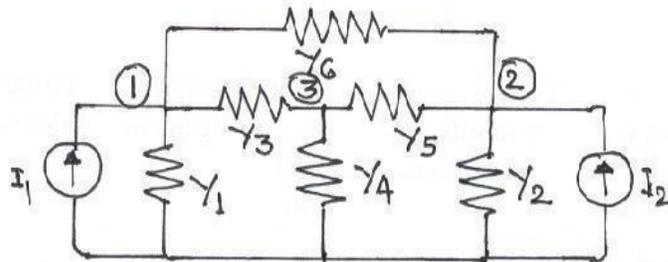
$$\begin{aligned} E_{\text{LOOP}} &= Z_{\text{LOOP}} I_{\text{LOOP}} \\ I_{\text{LOOP}} &= Y_{\text{LOOP}} E_{\text{LOOP}} \end{aligned} \quad (1.7)$$

Of the various network matrices referred above, the bus admittance matrix ( $Y_{\text{BUS}}$ ) and the bus impedance matrix ( $Z_{\text{BUS}}$ ) are determined for a given power system by the rule of inspection as explained next.

**Rule of Inspection**

Consider the 3-node admittance network as shown in figure 5. Using the basic branch relation:  $I = (YV)$ , for all the elemental currents and applying Kirchhoff's Current Law principle at the nodal points, we get the relations as under:

$$\begin{aligned} \text{At node 1: } I_1 &= Y_1 V_1 + Y_3 (V_1 - V_3) + Y_6 (V_1 - V_2) \\ \text{At node 2: } I_2 &= Y_2 V_2 + Y_5 (V_2 - V_3) + Y_6 (V_2 - V_1) \\ \text{At node 3: } 0 &= Y_3 (V_3 - V_1) + Y_4 V_3 + Y_5 (V_3 - V_2) \end{aligned} \quad (1.8)$$



**Figure 5. EXAMPLE SYSTEM FOR FINDING  $Y_{\text{BUS}}$**

These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \end{bmatrix} = \begin{bmatrix} (Y_1 + Y_3 + Y_6) - Y_6 & -Y_3 & \\ -Y_6 & (Y_2 + Y_5 + Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3 + Y_4 + Y_5) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \quad (1.9)$$

In other words, the relation of equation (9) can be represented in the form

$$I_{\text{BUS}} = Y_{\text{BUS}} E_{\text{BUS}} \quad (1.10)$$

Where,  $Y_{\text{BUS}}$  is the bus admittance matrix,  $I_{\text{BUS}}$  &  $E_{\text{BUS}}$  are the bus current and bus voltage vectors respectively.

By observing the elements of the bus admittance matrix,  $Y_{\text{BUS}}$  of equation (9), it is observed that the matrix elements can as well be obtained by a simple inspection of the given system diagram:

**Diagonal elements:** A diagonal element ( $Y_{ii}$ ) of the bus admittance matrix,  $Y_{BUS}$ , is equal to the sum total of the admittance values of all the elements incident at the bus/node  $i$ ,

**Off Diagonal elements:** An off-diagonal element ( $Y_{ij}$ ) of the bus admittance matrix,  $Y_{BUS}$ , is equal to the negative of the admittance value of the connecting element present between the buses  $i$  and  $j$ , if any.

This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

$$\begin{aligned} Y_{ii} &= \sum y_{ij} \quad (j = 1, 2, \dots, n) \\ Y_{ij} &= -y_{ij} \quad (j = 1, 2, \dots, n) \end{aligned} \quad (1.11)$$

For  $i = 1, 2, \dots, n$ ,  $n =$  no. of buses of the given system,  $y_{ij}$  is the admittance of element connected between buses  $i$  and  $j$  and  $y_{ii}$  is the admittance of element connected between bus  $i$  and ground (reference bus).

**Bus impedance matrix**

In cases where, the bus impedance matrix is also required, then it cannot be formed by direct inspection of the given system diagram. However, the bus admittance matrix determined by the rule of inspection following the steps explained above, can be inverted to obtain the bus impedance matrix, since the two matrices are inter-invertible.

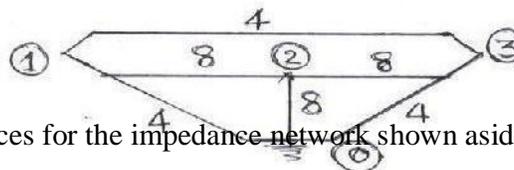
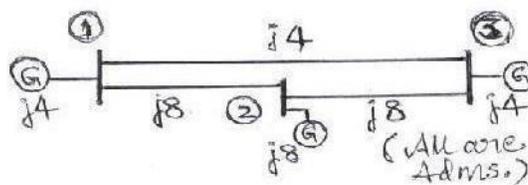
**Note:** It is to be noted that the rule of inspection can be applied only to those power systems that do not have any mutually coupled elements.

**Examples**

**I EXAMPLES ON RULE OF INSPECTION:**

**Problem #1:** Obtain the bus admittance matrix for the admittance network shown aside by the rule of inspection

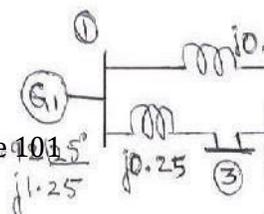
$$Y_{BUS} = \begin{vmatrix} 16 & -8 & -4 \\ -8 & 24 & -8 \\ -4 & -8 & 16 \end{vmatrix}$$



**Problem #2:** Obtain  $Y_{BUS}$  and  $Z_{BUS}$  matrices for the impedance network shown aside by the rule of inspection. Also, determine  $Y_{BUS}$

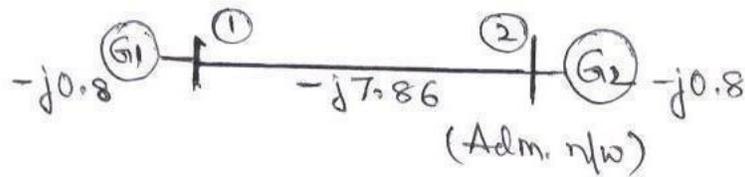
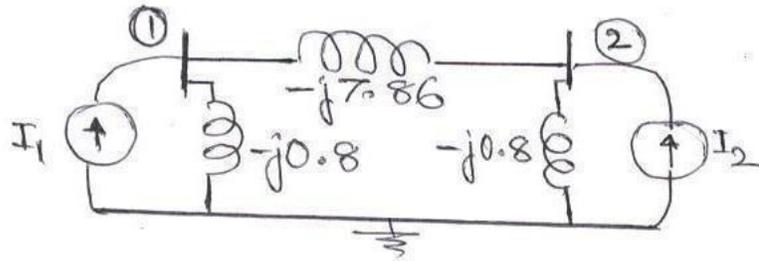
for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.

EMUIT



$$Y_{BUS} = \begin{vmatrix} -9.8 & 5.4 \\ 5 & -16.10 \\ 4 & 10 & -14 \end{vmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1}$$



$$Y_{BUS}^{New} = Y_A - Y_B Y_D Y_C$$

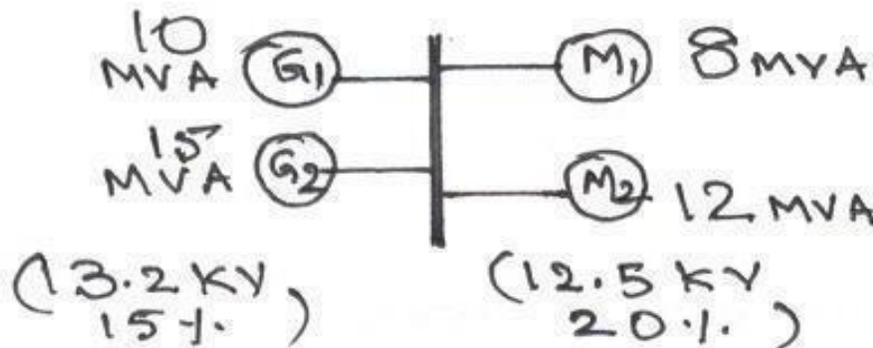
$$Y_{BUS} = \begin{vmatrix} -8.66 & 7.86 \\ 7.86 & -8.86 \end{vmatrix}$$

**II EXAMPLES ON PER UNIT ANALYSIS:****Problem #1:**

Two generators rated 10 MVA, 13.2 KV and 15 MVA, 13.2 KV are connected in parallel to a bus bar. They feed supply to 2 motors of inputs 8 MVA and 12 MVA respectively. The operating voltage of motors is 12.5 KV. Assuming the base quantities as 50 MVA, 13.8 KV, draw the per unit reactance diagram. The percentage reactance for generators is 15% and that for motors is 20%.

**Solution:**

The one line diagram with the data is obtained as shown in figure P1(a).



**Figure P1(a).**  
**OLD of the given system**

*Selection of base quantities: 50 MVA, 13.8 KV (Given)*

*Calculation of pu values:*

$$X_{G1} = j 0.15 (50/10) (13.2/13.8)^2 = j 0.6862 \text{ pu.}$$

$$X_{G2} = j 0.15 (50/15) (13.2/13.8)^2 = j 0.4574 \text{ pu.}$$

$$X_{m1} = j 0.2 (50/8) (12.5/13.8)^2 = j 1.0256 \text{ pu.}$$

$$X_{m2} = j 0.2 (50/12) (12.5/13.8)^2 = j 0.6837 \text{ pu.}$$

$$E_{g1} = E_{g2} = (13.2/13.8) = 0.9565 \angle 0^{\circ} \text{ pu}$$

$$E_{m1} = E_{m2} = (12.5/13.8) = 0.9058 \angle 0^{\circ} \text{ pu}$$

Thus the pu reactance diagram can be drawn as shown in figure P1(b).

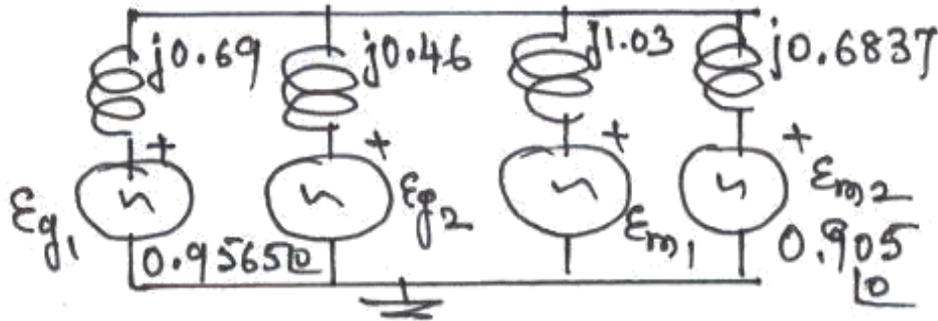


Figure P1(b).

Per Unit Reactance Diagram

**Problem #2:**

Draw the per unit reactance diagram for the system shown in figure below. Choose a base of 11 KV, 100 MVA in the generator circuit.

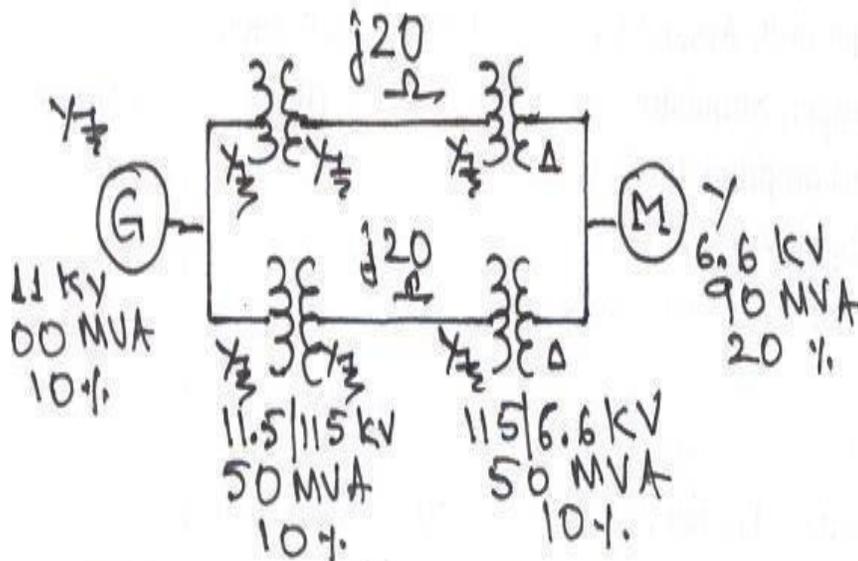


Figure P2(a). OLD of the given system

**Solution:**

The one line diagram with the data is considered as shown in figure.

*Selection of base quantities:*

**100 MVA, 11 KV** in the generator circuit(Given); the voltage bases in other sections are: 11 (115/11.5) = **110 KV** in the transmission line circuit and 110 (6.6/11.5) = **6.31 KV** in the motor circuit.

*Calculation of pu values:*

$$X_G = j 0.1 \text{ pu}, X_m = j 0.2 (100/90) (6.6/6.31)^2 = j 0.243 \text{ pu.}$$

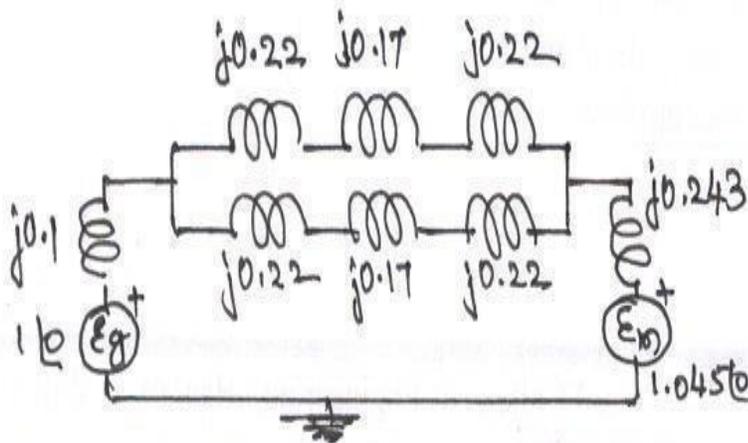
$$X_{t1} = X_{t2} = j 0.1 (100/50) (11.5/11)^2 = j 0.2185 \text{ pu.}$$

$$X_{t3} = X_{t4} = j 0.1 (100/50) (6.6/6.31)^2 = j 0.219 \text{ pu.}$$

$$X_{\text{lines}} = j 20 (100/110^2) = j 0.1652 \text{ pu.}$$

$$E_g = 1.0 \angle 0^\circ \text{ pu}, E_m = (6.6/6.31) = 1.045 \angle 0^\circ \text{ pu}$$

Thus the pu reactance diagram can be drawn as shown in figure P2(b).



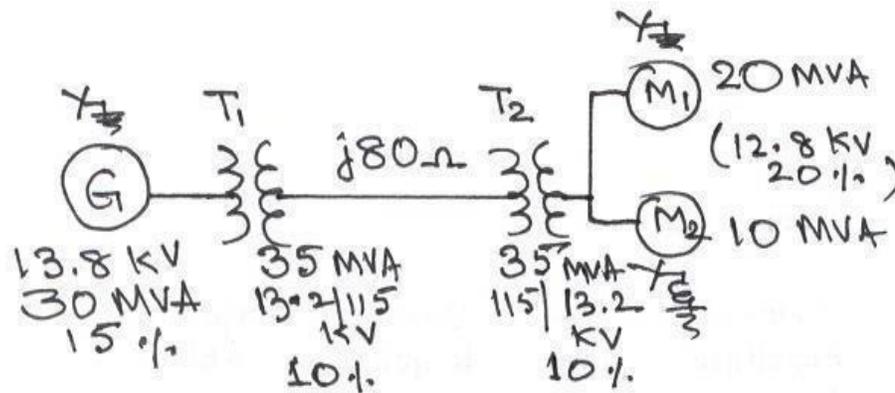
**Figure P2(b). Per Unit Reactance Diagram**

**Problem #3:**

A 30 MVA, 13.8 KV, 3-phase generator has a sub transient reactance of 15%. The generator supplies 2 motors through a step-up transformer - transmission line – step-down transformer arrangement. The motors have rated inputs of 20 MVA and 10 MVA at 12.8 KV with 20% sub transient reactance each. The 3-phase transformers are rated at 35 MVA, 13.2 KV- $\Delta$  /115 KV-Y with 10 % leakage reactance. The line reactance is 80 ohms. Draw the equivalent per unit reactance diagram by selecting the generator ratings as base values in the generator circuit.

**Solution:**

The one line diagram with the data is obtained as shown in figure P3(a).



**Figure P3(a). OLD of the given system**

*Selection of base quantities:*

**30 MVA, 13.8 KV** in the generator circuit(Given);

The voltage bases in other sections are:

13.8 (115/13.2) = **120.23 KV** in the transmission line circuit

and 120.23 (13.26/115) = **13.8 KV** in the motor circuit.

*Calculation of pu values:*

$$X_G = j 0.15 \text{ pu.}$$

$$X_{m1} = j 0.2 (30/20) (12.8/13.8)^2 = j 0.516 \text{ pu.}$$

$$X_{m2} = j 0.2 (30/10) (12.8/13.8)^2 = j 0.2581 \text{ pu.}$$

$$X_{t1} = X_{t2} = j 0.1 (30/35) (13.2/13.8)^2 = j 0.0784 \text{ pu.}$$

$$X_{\text{line}} = j 80 (30/120.23^2) = j 0.17 \text{ pu.}$$

$$E_g = 1.0 \angle 0^0 \text{ pu; } E_{m1} = E_{m2} = (6.6/6.31) = 0.93 \angle 0^0 \text{ pu}$$

Thus the pu reactance diagram can be drawn as shown in figure P3(b).

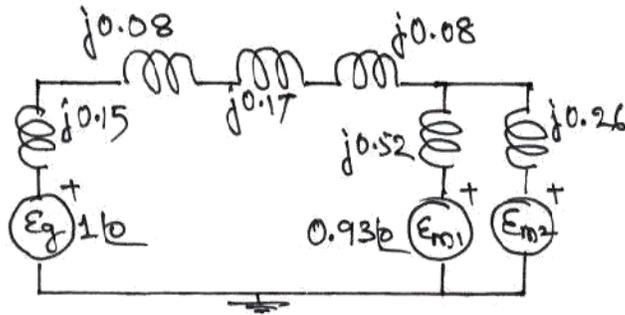


Figure P3(b). Per Unit Reactance Diagram

**Problem #4:**

A 33 MVA, 13.8 KV, 3-phase generator has a sub transient reactance of 0.5%. The generator supplies a motor through a step-up transformer - transmission line – step-down transformer arrangement. The motor has rated input of 25 MVA at 6.6 KV with 25% sub transient reactance. Draw the equivalent per unit impedance diagram by selecting 25 MVA (3φ), 6.6 KV (LL) as base values in the motor circuit, given the transformer and transmission line data as under:

*Step up transformer bank:* three single phase units, connected Δ–Y, each rated 10 MVA, 13.2/6.6 KV with 7.7 % leakage reactance and 0.5 % leakage resistance;

*Transmission line:* 75 KM long with a positive sequence reactance of 0.8 ohm/ KM and a resistance of 0.2 ohm/ KM; and

*Step down transformer bank:* three single phase units, connected Δ–Y, each rated 8.33 MVA, 110/3.98 KV with 8% leakage reactance and 0.8 % leakage resistance;

**Solution:**

The one line diagram with the data is obtained as shown in figure P4(a).

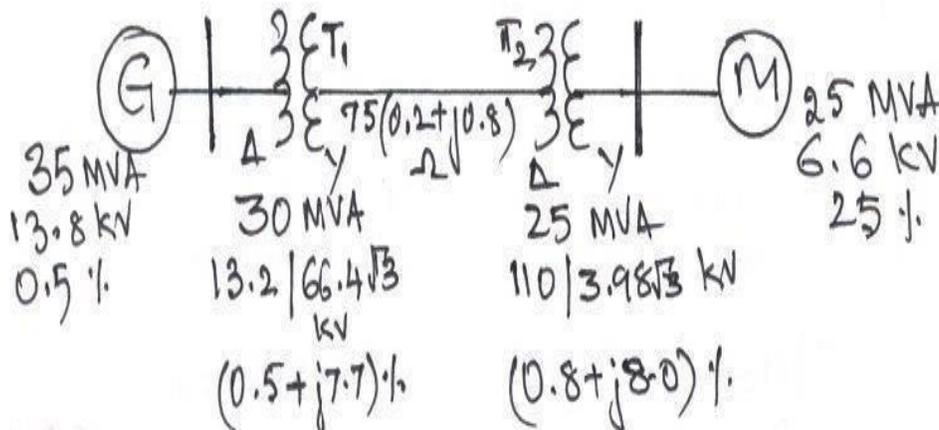


Figure P4(a). OLD of the given system

3-phase ratings of transformers:

T<sub>1</sub>: 3(10) = 30 MVA, 13.2/ 66.4√3 KV = 13.2/ 115 KV, X = 0.077, R = 0.005 pu. T<sub>2</sub>: 3(8.33) = 25 MVA, 110/ 3.98√3 KV = 110/ 6.8936 KV, X = 0.08, R = 0.008 pu.

Selection of base quantities:

25 MVA, 6.6 KV in the motor circuit (Given); the voltage bases in other sections are: 6.6 (110/6.8936) = **105.316** KV in the transmission line circuit and 105.316 (13.2/115) = **12.09** KV in the generator circuit.

Calculation of pu values:

$X_m = j 0.25$  pu;  $E_m = 1.0 \angle 0^0$  pu.

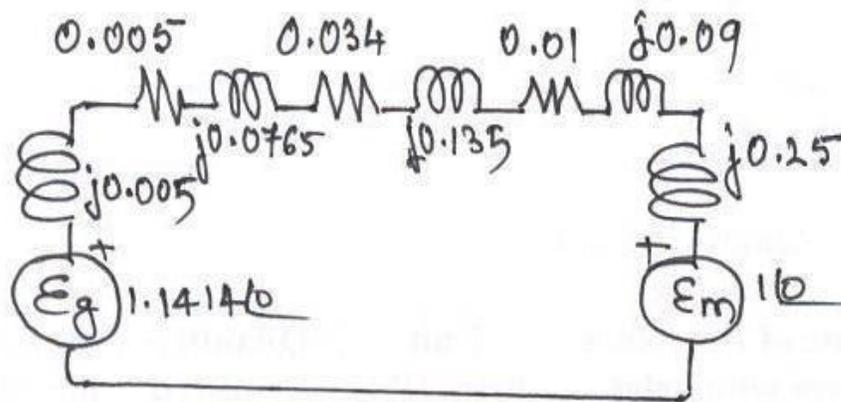
$X_G = j 0.005 (25/33) (13.8/12.09)^2 = j 0.005$  pu;  $E_g = 13.8/12.09 = 1.414 \angle 0^0$  pu.

$Z_{l1} = 0.005 + j 0.077 (25/30) (13.2/12.09)^2 = 0.005 + j 0.0765$  pu. (ref. to LV side)

$Z_{l2} = 0.008 + j 0.08 (25/25) (110/105.316)^2 = 0.0087 + j 0.0873$  pu. (ref. to HV side)

$Z_{line} = 75 (0.2 + j 0.8) (25/ 105.316^2) = 0.0338 + j 0.1351$  pu.

Thus the pu reactance diagram can be drawn as shown in figure P4(b).



**Figure P4(b).**  
**Per Unit Reactance Diagram**

**1.8 Exercises for Practice****Problems**

1. Determine the reactances of the three generators rated as follows on a common base of 200 MVA, 35 KV: Generator 1: 100 MVA, 33 KV, sub transient reactance of 10%; Generator 2: 150 MVA, 32 KV, sub transient reactance of 8% and Generator 3: 110 MVA, 30 KV, sub transient reactance of 12%.

[Answers:  $X_{G1} = j 0.1778$ ,  $X_{g2} = j 0.089$ ,  $X_{g3} = j 0.16$  all in per unit]

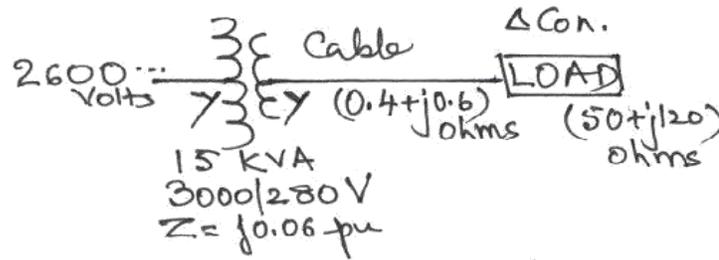
2. A 100 MVA, 33 KV, 3-phase generator has a sub transient reactance of 15%. The generator supplies 3 motors through a step-up transformer - transmission line – step-down transformer arrangement. The motors have rated inputs of 30 MVA, 20 MVA and 50 MVA, at 30 KV with 20% sub transient reactance each. The 3-phase transformers are rated at 100 MVA, 32 KV- $\Delta$  /110 KV-Y with 8 % leakage reactance. The line has a reactance of 50 ohms. By selecting the generator ratings as base values in the generator circuit, determine the base values in all the other parts of the system. Hence evaluate the corresponding pu values and draw the equivalent per unit reactance diagram.

[Answers:  $X_G = j 0.15$ ,  $X_{m1} = j 0.551$ ,  $X_{m2} = j 0.826$ ,  $X_{m3} = j 0.331$ ,  $E_{g1} = 1.0 \angle 0^\circ$ ,  $E_{m1} = E_{m2}$   
 $E_{m3} = 0.91 \angle 0^\circ$ ,  $X_{t1} = X_{t2} = j 0.0775$  and  $X_{line} = j 0.39$  all in per unit]

6. A 80 MVA, 10 KV, 3-phase generator has a sub transient reactance of 10%. The generator supplies a motor through a step-up transformer - transmission line – step-down transformer arrangement. The motor has rated input of 95 MVA, 6.3 KV with 15% sub transient reactance. The step-up 3-phase transformer is rated at 90 MVA, 11 KV-Y /110 KV-Y with 10% leakage reactance. The 3-phase step-down transformer consists of three single phase Y- $\Delta$  connected transformers, each rated at 33.33 MVA, 68/6.6 KV with 10% leakage reactance. The line has a reactance of 20 ohms. By selecting the 11 KV, 100 MVA as base values in the generator circuit, determine the base values in all the other parts of the system. Hence evaluate the corresponding pu values and draw the equivalent per unit reactance diagram.

[Answers:  $X_G = j 1.103$ ,  $X_m = j 0.165$ ,  $E_{g1} = 0.91 \angle 0^\circ$ ,  $E_m = 1.022 \angle 0^\circ$ ,  $X_{t1} = j 0.11$ ,  $X_{t2} = j 0.114$  and  $X_{line} = j 0.17$  all in per unit]

4. For the three-phase system shown below, draw an impedance diagram expressing all impedances in per unit on a common base of 20 MVA, 2600 V on the HV side of the transformer. Using this impedance diagram, find the HV and LV currents.



**Figure E3. OLD of the Example system**

[Answers:  $S_b = 20$  MVA;  $V_b = 2.6$  KV (HV) and  $0.2427$  KV (LV);  $V_t = 1.0 \angle 0^\circ$ ,  $X_t = j 0.107$ ,  $Z_{\text{cable}} = 0.136 + j 0.204$  and  $Z_{\text{load}} = 5.66 + j 2.26$ ,  $I = 0.158$  all in per unit,  $I$  (hv) =  $0.7$  A and  $I$  (lv) =  $7.5$  A]

### Objective type questions

- (iv) Under no load conditions the current in a transmission line is due to.  
 Corona effects  
 Capacitance of the line  
 Back flow from earth  
 None of the above
- (v) In the short transmission line which of the following is used?  
 $\pi$  - Model  
 T - Model  
 Both (a) and (b)  
 None of the above
- (vi) In the short transmission line which of the following is neglected?  
 $I^2 R$  loss  
 Shunt admittance  
 Series impedance  
 All of the above
- (vii) Which of the following loss in a transformer is zero even at full load?  
 Eddy current  
 Hysteresis  
 Core loss  
 Friction loss
- (viii) The transmission line conductors are transposed to  
 Balance the current  
 Obtain different losses  
 Obtain same line drops  
 Balance the voltage

[Ans.: 1(b), 2(a), 3(b), 4(d), 5(c)]

## **SYMMETRICAL THREE PHASE FAULTS**

[**CONTENTS:** Preamble, transients on a transmission line, short circuit of an unloaded synchronous machine- short circuit currents and reactances, short circuit of a loaded machine, selection of circuit breaker ratings, examples]

### **Preamble**

In practice, any disturbance in the normal working conditions is termed as a FAULT. The effect of fault is to load the device electrically by many times greater than its normal rating and thus damage the equipment involved. Hence all the equipment in the fault line should be protected from being overloaded. In general, overloading involves the increase of current up to 10-15 times the rated value. In a few cases, like the opening or closing of a circuit breaker, the transient voltages also may overload the equipment and damage them.

In order to protect the equipment during faults, fast acting circuit breakers are put in the lines. To design the rating of these circuit breakers or an auxiliary device, the fault current has to be predicted. By considering the equivalent per unit reactance diagrams, the various faults can be analyzed to determine the fault parameters. This helps in the protection and maintenance of the equipment.

Faults can be symmetrical or unsymmetrical faults. In symmetrical faults, the fault quantity rises to several times the rated value equally in all the three phases. For example, a 3-phase fault - a dead short circuit of all the three lines not involving the ground. On the other hand, the unsymmetrical faults may have the connected fault quantities in a random way. However, such unsymmetrical faults can be analyzed by using the *Symmetrical Components*. Further, the neutrals of the machines and equipment may or may not be grounded or the fault may occur through fault impedance. The three-phase fault involving ground is the most severe fault among the various faults encountered in electric power systems.

### **Transients on a transmission line**

Now, let us Consider a transmission line of resistance R and inductance L supplied by an ac source of voltage  $v$ , such that  $v = V_m \sin(\omega t + \alpha)$  as shown in figure 1. Consider the short circuit transient on this transmission line. In order to analyze this symmetrical 3-phase fault, the following assumptions are made:



The supply is a constant voltage source,



The short circuit occurs when the line is unloaded and

- The line capacitance is negligible.

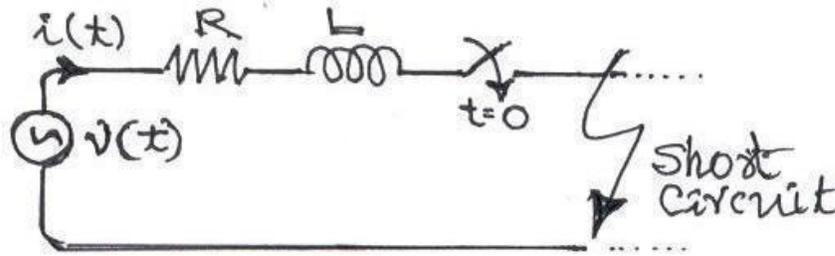


Figure 1. Short Circuit Transients on an Unloaded Line.

Thus the line can be modeled by a lumped R-L series circuit. Let the short circuit take place at  $t=0$ . The parameter,  $\alpha$  controls the instant of short circuit on the voltage wave. From basic circuit theory, it is observed that the current after short circuit is composed of the two parts as under:  $i = i_s + i_t$ , Where,  $i_s$  is the steady state current and  $i_t$  is the transient current. These component currents are determined as follows.

$$\begin{aligned} \text{Consider, } v &= V_m \sin(\omega t + \alpha) \\ &= iR + L \frac{di}{dt} \end{aligned} \quad (2.1)$$

$$\text{and } i = I_m \sin(\omega t + \alpha - \theta) \quad (2.2)$$

$$\text{Where } V_m = \sqrt{2}V; I_m = \sqrt{2}I; Z_{\text{mag}} = \sqrt{R^2 + (\omega L)^2} = \tan^{-1}(\omega L/R) \quad (2.3)$$

$$\text{Thus } i_s = [V_m/Z] \sin(\omega t + \alpha - \theta) \quad (2.4)$$

Consider the performance equation of the circuit of figure 1 under circuit as:

$$\begin{aligned} iR + L \frac{di}{dt} &= 0 \\ \text{i.e., } (R/L + d/dt)i &= 0 \end{aligned} \quad (2.5)$$

In order to solve the equation (5), consider the complementary function part of the solution as:  $CF = C_1 e^{(-t/\tau)}$  (2.6)

Where  $\tau (= L/R)$  is the time constant and  $C_1$  is a constant given by the value of steady state current at  $t = 0$ . Thus we have,

$$\begin{aligned} C_1 &= -i_s(0) \\ &= -[V_m/Z] \sin(\alpha - \theta) \\ &= [V_m/Z] \sin(\theta - \alpha) \end{aligned} \quad (2.7)$$

Similarly the expression for the transient part is given by:

$$\begin{aligned} i_t &= -i_s(0) e^{(-t/\tau)} \\ &= [V_m/Z] \sin(\theta - \alpha) e^{(-R/L)t} \end{aligned} \quad (2.8)$$

Thus the total current under short circuit is given by the solution of equation (1) as [combining equations (4) and (8)],

$$i = i_s + i_t$$

$$= [\sqrt{2}V/Z] \sin (\omega t + \alpha - \theta) + [\sqrt{2}V/Z] \sin (\theta - \alpha) e^{(-R/L)t} \quad (2.9)$$

Thus,  $i_s$  is the sinusoidal steady state current called as the *symmetrical short circuit current* and  $i_t$  is the unidirectional value called as the *DC off-set current*. This causes the total current to be unsymmetrical till the transient decays, as clearly shown in figure 2.

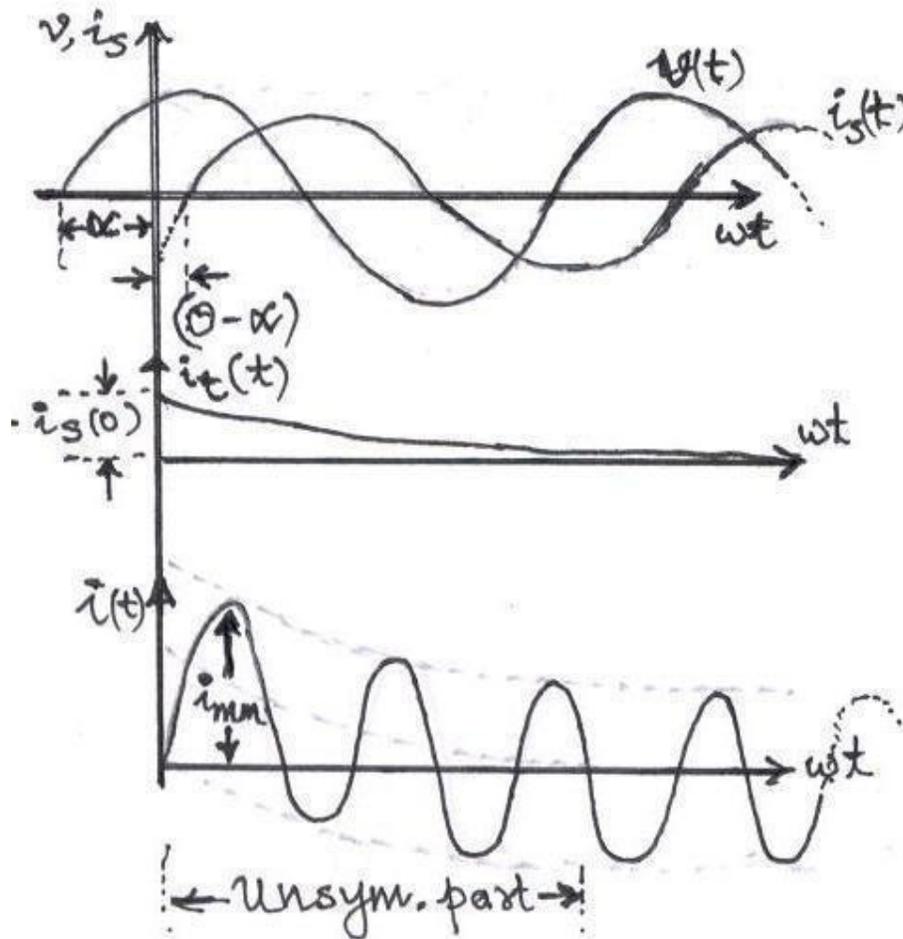


Figure 2. Plot of Symmetrical short circuit current,  $i(t)$ .

The maximum momentary current,  $i_{mm}$  thus corresponds to the first peak. Hence, if the decay in the transient current during this short interval of time is neglected, then we have (sum of the two peak values);

$$i_{mm} = [\sqrt{2}V/Z] \sin (\theta - \alpha) + [\sqrt{2}V/Z] \quad (2.10)$$

now, since the resistance of the transmission line is very small, the impedance angle  $\theta$ , can be taken to be approximately equal to  $90^\circ$ . Hence, we have

$$i_{mm} = [\sqrt{2}V/Z] \cos \alpha + [\sqrt{2}V/Z] \quad (2.11)$$

This value is maximum when the value of  $\alpha$  is equal to zero. This value corresponds to the short circuiting instant of the voltage wave when it is passing through zero. Thus the final expression for the maximum momentary current is obtained as:

$$i_{mm} = 2 [\sqrt{2}V/Z] \quad (2.12)$$

Thus it is observed that the maximum momentary current is twice the maximum value of symmetrical short circuit current. This is referred as the *doubling effect* of the short circuit current during the symmetrical fault on a transmission line.

### Short circuit of an unloaded synchronous machine

#### Short Circuit Reactances

Under steady state short circuit conditions, the armature reaction in synchronous generator produces a demagnetizing effect. This effect can be modeled as a reactance,  $X_a$  in series with the induced emf and the leakage reactance,  $X_l$  of the machine as shown in figure 3. Thus the equivalent reactance is given by:

$$X_d = X_a + X_l \quad (2.13)$$

Where  $X_d$  is called as the direct axis *synchronous reactance* of the synchronous machine. Consider now a sudden three-phase short circuit of the synchronous generator on no-load. The machine experiences a transient in all the 3 phases, finally ending up in steady state conditions.

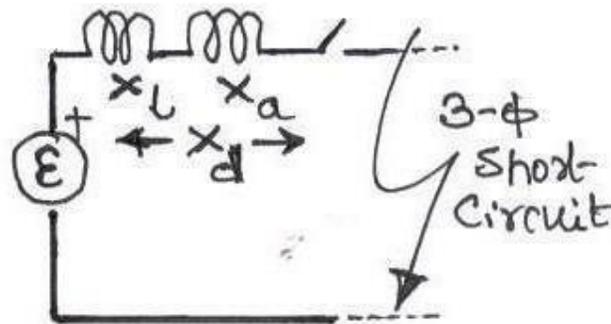
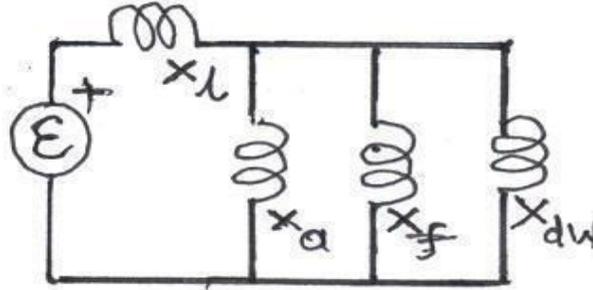


Figure 3. Steady State Short Circuit Model

Immediately after the short circuit, the symmetrical short circuit current is limited only by the leakage reactance of the machine. However, to encounter the demagnetization of the armature short circuit current, current appears in field and damper windings, assisting the rotor field winding to sustain the air-gap flux. Thus during the initial part of the short circuit, there is mutual coupling between stator, rotor and damper windings and hence the corresponding equivalent circuit would be as shown in figure 4. Thus the equivalent reactance is given by:

$$X_d'' = X_l + [1/X_a + 1/X_f + 1/X_{dw}]^{-1} \quad (2.14)$$

Where  $X_d''$  is called as the *sub-transient reactance* of the synchronous machine. Here, the equivalent resistance of the damper winding is more than that of the rotor field winding. Hence, the time constant of the damper field winding is smaller. Thus the damper field effects and the eddy currents disappear after a few cycles.

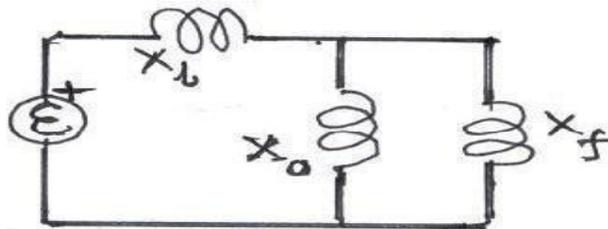


**Figure 4. Model during Sub-transient Period of Short Circuit**

In other words,  $X_{dw}$  gets open circuited from the model of Figure 5 to yield the model as shown in figure 4. Thus the equivalent reactance is given by:

$$X_d' = X_l + [1/X_a + 1/X_f]^{-1} \quad (2.15)$$

Where  $X_d'$  is called as the *transient reactance* of the synchronous machine. Subsequently,  $X_f$  also gets open circuited depending on the field winding time constant and yields back the steady state model of figure 3.



**Figure 5. Model during transient Period of Short Circuit**

Thus the machine offers a time varying reactance during short circuit and this value of reactance varies from initial stage to final one such that:  $X_d > X_d' > X_d''$

### Short Circuit Current Oscillogram

Consider the oscillogram of short circuit current of a synchronous machine upon the occurrence of a fault as shown in figure 6. The symmetrical short circuit current can be divided into three zones: the initial sub transient period, the middle transient period and finally the steady state period. The corresponding reactances,  $X_d''$ ,  $X_d'$  and  $X_d$  respectively, are offered by the synchronous machine during these time periods

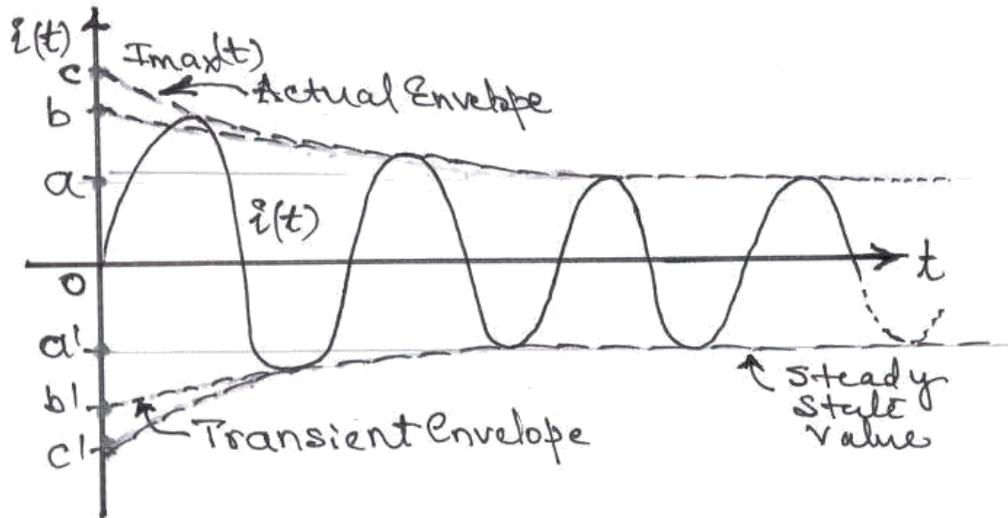


Figure 6. SC current Oscillogram of Armature Current.

The currents and reactances during the three zones of period are related as under in terms of the intercepts on the oscillogram (oa, ob and oc are the y-intercepts as indicated in figure 6):

$$\begin{aligned} \text{RMS value of the steady state current} &= I = [oa/\sqrt{2}] = [E_g/X_d] \\ \text{RMS value of the transient current} &= I' = [ob/\sqrt{2}] = [E_g/X_d'] \\ \text{RMS value of the sub transient current} &= I'' = [oc/\sqrt{2}] = [E_g/X_d''] \end{aligned} \quad (2.16)$$

**short circuit of a loaded machine**

In the analysis of section 2.3 above, it has been assumed that the machine operates at no load prior to the occurrence of the fault. On similar lines, the analysis of the fault occurring on a loaded machine can also be considered.

Figure 7 gives the circuit model of a synchronous generator operating under steady state conditions supplying a load current  $I_L$  to the bus at a terminal voltage  $V_t$ .  $E_g$  is the induced emf under the loaded conditions and  $X_d$  is the direct axis synchronous reactance of the generator.

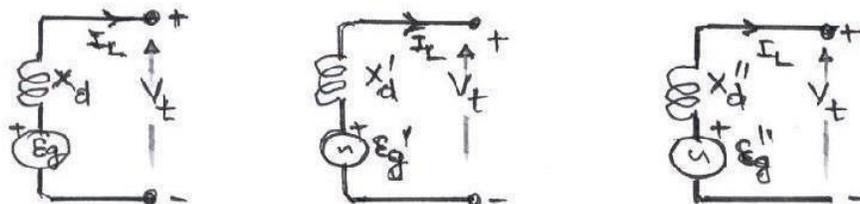


Figure 7. Circuit models for a fault on a loaded machine.

Also shown in figure 7, are the circuit models to be used for short circuit current calculations when a fault occurs at the terminals of the generator, for sub-transient current and transient current values. The induced emf values used in these models are given by the expressions as under:

$$\begin{aligned} E_g &= V_t + j I_L X_d = \text{Voltage behind syn. reactance} \\ E_g' &= V_t + j I_L X_d' = \text{Voltage behind transient reactance} \\ E_g'' &= V_t + j I_L X_d'' = \text{Voltage behind subtr. Reactance} \end{aligned} \quad (2.17)$$

The synchronous motors will also have the terminal emf values and reactances. However, then the current direction is reversed. During short circuit studies, they can be replaced by circuit models similar to those shown in figure 7 above, except that the voltages are given by the relations as under:

$$\begin{aligned} E_m &= V_t - j I_L X_d = \text{Voltage behind syn. reactance} \\ E_m' &= V_t - j I_L X_d' = \text{Voltage behind transient reactance} \\ E_m'' &= V_t - j I_L X_d'' = \text{Voltage behind subtr. Reactance} \end{aligned} \quad (2.18)$$

The circuit models shown above for the synchronous machines are also very useful while dealing with the short circuit of an interconnected system.

### Selection of circuit breaker ratings

For *selection of circuit breakers*, the maximum momentary current is considered corresponding to its maximum possible value. Later, the current to be interrupted is usually taken as symmetrical short circuit current multiplied by an empirical factor in order to account for the DC off-set current. A value of 1.6 is usually selected as the multiplying factor.

Normally, both the generator and motor reactances are used to determine the momentary current flowing on occurrence of a short circuit. The interrupting capacity of a circuit breaker is decided by  $X_d''$  for the generators and  $X_d'$  for the motors.

### Examples

**Problem #1:** A transmission line of inductance 0.1 H and resistance 5  $\Omega$  is suddenly short circuited at  $t = 0$ , at the far end of a transmission line and is supplied by an ac source of voltage  $v = 100 \sin (100\pi t + 15^\circ)$ . Write the expression for the short circuit current,  $i(t)$ . Find the approximate value of the first current maximum for the given values of  $\alpha$  and  $\theta$ . What is this value for  $\alpha=0$ , and  $\theta=90^\circ$ ? What should be the instant of short circuit so that the DC offset current is (i) zero and (ii) maximum?

Solution:

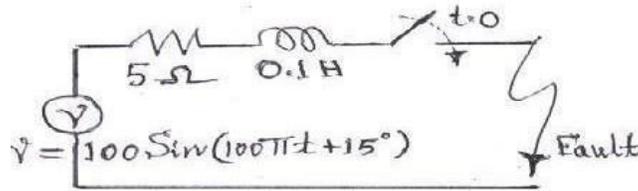


Figure P1.

Consider the expression for voltage applied to the transmission system given by

$$v = V_m \sin(\omega t + \alpha) = 100 \sin(100\pi t + 15^\circ)$$

Thus we get:  $V_m = 100$  volts;  $f = 50$  Hz and  $\alpha = 15^\circ$ .

Consider the impedance of the circuit given by:

$$Z = R + j\omega L = 5 + j(100\pi)(0.1) = 5 + j 31.416 \text{ ohms.}$$

Thus we have:  $Z_{\text{mag}} = 31.8113$  Ohms;  $\theta = 80.957^\circ$  and  $\tau = L/R = 0.1/5 = 0.02$  seconds. The short circuit current is given by:

$$i(t) = [V_m/Z] \sin(\omega t + \alpha - \theta) + [V_m/Z] \sin(\theta - \alpha) e^{-(R/L)t}$$

$$\square [100/31.8113] [\sin(100\pi t + 15^\circ - 80.957^\circ) + \sin(80.957^\circ - 15^\circ) e^{-(t/0.02)}]$$

$$\square 3.1435 \sin(314.16 t - 65.96) + 2.871 e^{-50t}$$

Thus we have:

$$i) i_{\text{imm}} = 3.1435 + 2.871 e^{-50t}$$

where  $t$  is the time instant of maximum of symmetrical short circuit current. This instant occurs at  $(314.16 t - 65.96) = 90^\circ$ ; Solving we get,  $t = 0.00867$  seconds so that  $i_{\text{imm}} = 5$  Amps.

$$ii) i_{\text{imm}} = 2V_m/Z = 6.287 \text{ A; for } \alpha = 0, \text{ and } \theta = 90^\circ \text{ (Also, } i_{\text{imm}} = 2(3.1435) = 6.287 \text{ A)}$$

$$iii) \text{DC offset current} = [V_m/Z] \sin(\theta - \alpha) e^{-(R/L)t}$$

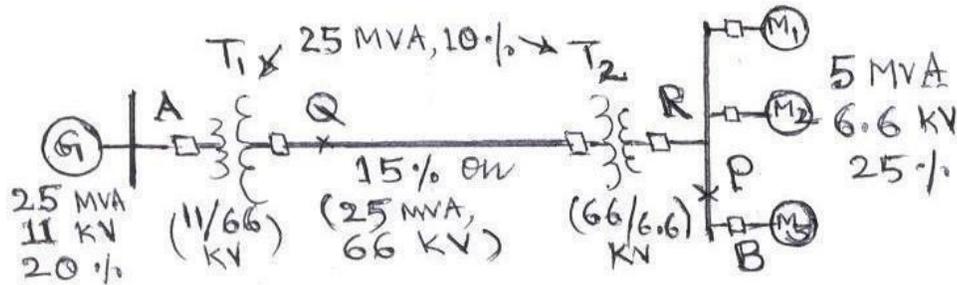
$$= \text{zero, if } (\theta - \alpha) = \text{zero, i.e., } \theta = \alpha, \quad \text{or} \quad \alpha = 80.957^\circ$$

$$= \text{maximum if } (\theta - \alpha) = 90^\circ, \text{ i.e., } \alpha = \theta - 90^\circ, \text{ or} \quad \alpha = -9.043^\circ.$$

**Problem #2:** A 25 MVA, 11 KV, 20% generator is connected through a step-up transformer- T<sub>1</sub> (25 MVA, 11/66 KV, 10%), transmission line (15% reactance on a base of 25 MVA, 66 KV) and step-down transformer-T<sub>2</sub> (25 MVA, 66/6.6 KV, 10%) to a bus that supplies 3 identical motors in parallel (all motors rated: 5 MVA, 6.6 KV, 25%). A circuit breaker-A is used near the primary of the transformer T<sub>1</sub> and breaker-B is used near the motor M<sub>3</sub>. Find the symmetrical currents to be interrupted by circuit breakers A and B for a fault at a point P, near the circuit breaker B.

**Solution:**

Consider the SLD with the data given in the problem statement. The base values are selected as under:



**Figure P2(a)**

*Selection of bases:*

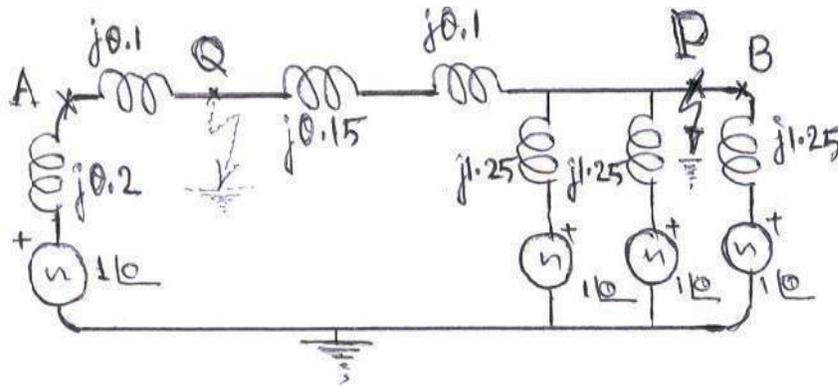
$S_b = 25 \text{ MVA}$  (common);  $V_b = 11 \text{ KV}$  (Gen. circuit)- chosen so that then  $V_b = 66 \text{ KV}$  (line circuit) and  $V_b = 6.6 \text{ KV}$  (Motor circuit).

*Pu values:*

$$X_g = j0.2 \text{ pu}, X_{t1} = X_{t2} = j0.1 \text{ pu}; X_{m1} = X_{m2} = X_{m3} = j0.25(25/5) = j1.25 \text{ pu}; X_{line} = j0.15 \text{ pu}.$$

Since the system is operating at no load, all the voltages before fault are 1 pu.

Considering the pu reactance diagram with the faults at P, we have:



**Figure P2(b)**

Current to be interrupted by circuit breaker A =  $1.0 / j[0.2+0.1+0.15+0.1]$

$$= -j 1.818 \text{ pu} = -j 1.818 (25/[\sqrt{3}(11)]) = -j 1.818 (1.312) \text{ KA} = \mathbf{2.386 \text{ KA}}$$

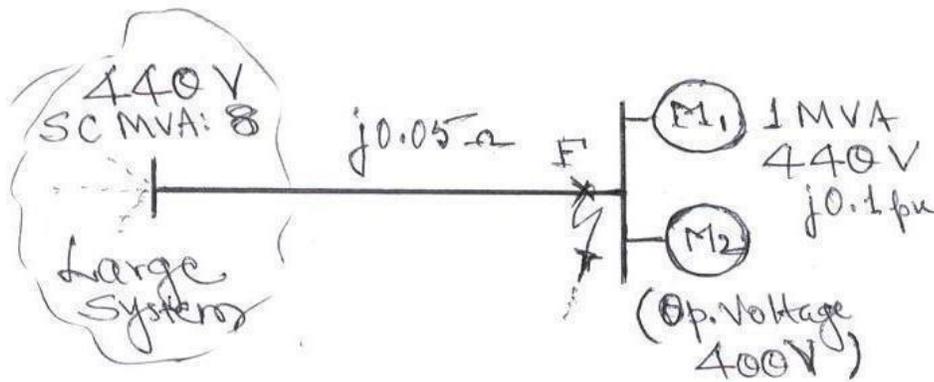
And Current to be interrupted by breaker B =  $1/j1.25 = -j 0.8 \text{ pu}$

$$= -j0.8 (25/[\sqrt{3}(6.6)]) = -j0.8 (2.187) \text{ KA} = \mathbf{1.75 \text{ KA.}}$$

**Problem #3:** Two synchronous motors are connected to a large system bus through a short line. The ratings of the various components are: Motors(each)= 1 MVA, 440 volts, 0.1 pu reactance; line of 0.05 ohm reactance and the short circuit MVA at the bus of the large system is 8 at 440 volts. Calculate the symmetrical short circuit current fed into a three-phase fault at the motor bus when the motors are operating at 400 volts.

**Solution:**

Consider the SLD with the data given in the problem statement. The base values are selected as under:



**Figure P3.**

$S_b = 1 \text{ MVA}$ ;  $V_b = 0.44 \text{ KV}$  (common)- chosen so that  $X_m(\text{each})=j0.1 \text{ pu}$ ,  $E_m = 1.0 \angle 0^\circ$ ,  $X_{\text{line}}=j0.05 (1/0.44^2) = j 0.258 \text{ pu}$  and  $X_{\text{large-system}} = (1/8) = j 0.125 \text{ pu}$ .

Thus the prefault voltage at the motor bus;  $V_t = 0.4/0.44 = 0.909 \angle 0^\circ$ ,

Short circuit current fed to the fault at motor bus ( $I_f = YV$ );

$$I_f = [0.125 + 0.258]^{-1} + 2.0 \} 0.909 = [20.55 \text{ pu}] [1000/(\sqrt{3}(0.4))] \\ = 20.55 (1.312) \text{ KA} = \mathbf{26.966 \text{ KA}}$$

**Problem #4:** A generator-transformer unit is connected to a line through a circuit breaker. The unit ratings are: Gen.: 10 MVA, 6.6 KV,  $X_d'' = 0.1 \text{ pu}$ ,  $X_d' = 0.2 \text{ pu}$  and  $X_d = 0.8 \text{ pu}$ ; and Transformer: 10 MVA, 6.9/33 KV,  $X_1 = 0.08 \text{ pu}$ ; The system is operating on no-load at a line voltage of 30 KV, when a three-phase fault occurs on the line just beyond the circuit breaker. Determine the following:

- Initial symmetrical RMS current in the breaker,
- Maximum possible DC off-set current in the breaker,
- Momentary current rating of the breaker,
- Current to be interrupted by the breaker and the interrupting KVA and
- Sustained short circuit current in the breaker.

**Solution:**

Consider the base values selected as **10 MVA, 6.6 KV** (in the generator circuit) and  $6.6(33/6.9) = \mathbf{31.56 KV}$  (in the transformer circuit). Thus the base current is:

$$I_b = 10 / [\sqrt{3}(31.56)] = \mathbf{0.183 KA}$$

The pu values are:  $X_d'' = 0.1$  pu,  $X_d' = 0.2$  pu and  $X_d = 0.8$  pu; and  $X_{Tr} = 0.08$   
 $(6.9/6.6)^2 = 0.0874$  pu;  $V_t = (30/31.6) = 0.95 \angle 0^\circ$  pu.

Initial symmetrical RMS current =  $0.95 \angle 0^\circ / [0.1 + 0.0874] = 5.069$  pu = **0.9277**

KA; Maximum possible DC off-set current =  $2(0.9277) = \mathbf{1.312 KA}$ ;

Momentary current rating =  $1.6(0.9277) = \mathbf{1.4843 KA}$ ; (assuming 60%

allowance) Current to be interrupted by the breaker (5 Cycles) =  $1.1(0.9277) =$

**1.0205 KA**; Interrupting MVA =  $3(30)(1.0205) = \mathbf{53.03 MVA}$ ;

Sustained short circuit current in the breaker =  $0.95 \angle 0^\circ (0.183) / [0.8 + 0.0874] = \mathbf{0.1959 KA}$ .

### Exercises for Practice

#### PROBLEMS

1. The one line diagram for a radial system network consists of two generators, rated 10 MVA, 15% and 10 MVA, 12.5 % respectively and connected in parallel to a bus bar A at 11 KV. Supply from bus A is fed to bus B (at 33 KV) through a transformer T<sub>1</sub> (rated: 10 MVA, 10%) and OH line (30 KM long). A transformer T<sub>2</sub> (rated: 5 MVA, 8%) is used in between bus B (at 33 KV) and bus C (at 6.6 KV). The length of cable running from the bus C up to the point of fault, F is 3 KM. Determine the current and line voltage at 11 kV bus A under fault conditions, when a fault occurs at the point F, given that  $Z_{cable} = 0.135 + j 0.08$  ohm/ km and  $Z_{OH-line} = 0.27 + j 0.36$  ohm/kM. [Answer: 9.62 kV at the 11 kV bus]

2. A generator (rated: 25MVA, 12. KV, 10%) supplies power to a motor (rated: 20 MVA, 3.8 KV, 10%) through a step-up transformer (rated:25 MVA, 11/33 KV, 8%), transmission line (of reactance 20 ohms) and a step-down transformer (rated:20 MVA, 33/3.3 KV, 10%). Write the pu reactance diagram. The system is loaded such that the motor is drawing 15 MW at 0.9 leading power factor, the motor terminal voltage being 3.1 KV. Find the sub-transient current in the generator and motor for a fault at the generator bus. [Answer:  $I_g'' = 9.337 KA$ ;  $I_m'' = 6.9 KA$ ]

3. A synchronous generator feeds bus 1 and a power network feed bus 2 of a system. Buses 1 and 2 are connected through a transformer and a line. Per unit reactances of the components are: Generator(bus-1):0.25; Transformer:0.12 and Line:0.28. The power network is represented by a generator with an unknown reactance in series. With the generator on no-load and with 1.0 pu voltage at each bus, a three phase fault occurring on bus-1 causes a current of 5 pu to flow into the fault. Determine the equivalent reactance of the power network. [Answer:  $X = 0.6$  pu]

4. A synchronous generator, rated 500 KVA, 440 Volts, 0.1 pu sub-transient reactance is supplying a passive load of 400 KW, at 0.8 power factor (lag). Calculate the initial symmetrical RMS current for a three-phase fault at the generator terminals.

[Answer:  $S_b = 0.5 MVA$ ;  $V_b = 0.44 KV$ ; load =  $0.8 \angle -36.9^\circ$ ;  $I_b = 0.656 KA$ ;  $I_f = 6.97 KA$ ]

**OBJECTIVE TYPE QUESTIONS**

1. When a 1-phase supply is across a 1-phase winding, the nature of the magnetic field produced is
  - a) Constant in magnitude and direction
  - b) Constant in magnitude and rotating at synchronous speed
  - c) Pulsating in nature
  - d) Rotating in nature
2. The damper windings are used in alternators to
  - a) Reduce eddy current loss
  - b) Reduce hunting
  - c) Make rotor dynamically balanced
  - d) Reduce armature reaction
3. The neutral path impedance  $Z_n$  is used in the equivalent sequence network models as
  - a)  $Z_n^2$
  - b)  $Z_n$
  - c)  $3 Z_n$
  - d) An ineffective value
4. An infinite bus-bar should maintain
  - a) Constant frequency and Constant voltage
  - b) Infinite frequency and Infinite voltage
  - c) Constant frequency and Variable voltage
  - d) Variable frequency and Variable voltage
5. Voltages under extra high voltage are
  - a) 1KV & above
  - b) 11KV & above
  - c) 132 KV & above
  - d) 330 KV & above

[Ans.: 1(c), 2(b), 3(c), 4(a), 5(d)]

## SYMMETRICAL COMPONENTS

[CONTENTS: Introduction, The a operator, Power in terms of symmetrical components, Phase shift in Y-transformer banks, Unsymmetrical series impedances, Sequence impedances, Sequence networks, Sequence networks of an unloaded generator, Sequence networks of elements, Sequence networks of power system]

### INTRODUCTION

Power systems are large and complex three-phase systems. In the normal operating conditions, these systems are in balanced condition and hence can be represented as an equivalent single phase system. However, a fault can cause the system to become unbalanced. Specifically, the unsymmetrical faults: open circuit, LG, LL, and LLG faults cause the system to become unsymmetrical. The single-phase equivalent system method of analysis (using SLD and the reactance diagram) cannot be applied to such unsymmetrical systems. Now the question is how to analyze power systems under unsymmetrical conditions? There are two methods available for such an analysis: Kirchhoff's laws method and Symmetrical components method.

The method of symmetrical components developed by C.L. Fortescue in 1918 is a powerful technique for analyzing unbalanced three phase systems. Fortescue defined a linear transformation from phase components to a new set of components called symmetrical components. This transformation represents an unbalanced three-phase system by a set of three balanced three-phase systems. The symmetrical component method is a modeling technique that permits systematic analysis and design of three-phase systems. Decoupling a complex three-phase network into three simpler networks reveals complicated phenomena in more simplistic terms.

Consider a set of three-phase unbalanced voltages designated as  $V_a$ ,  $V_b$ , and  $V_c$ . According to Fortescue theorem, these phase voltages can be resolved into following three sets of components.

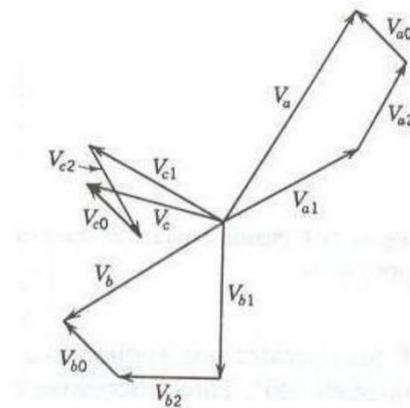
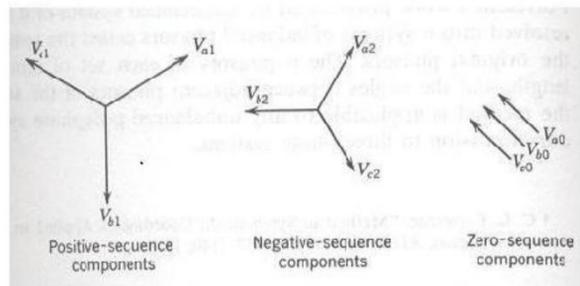
1. Positive-sequence components, consisting of three phasors equal in magnitude, displaced from each other by  $120^\circ$  in phase, and having the same phase sequence as the original phasors, designated as  $V_{a1}$ ,  $V_{b1}$ , and  $V_{c1}$
2. Negative-sequence components, consisting of three phasors equal in magnitude, displaced from each other by  $120^\circ$  in phase, and having the phase sequence opposite to that of the original phasors, designated as  $V_{a2}$ ,  $V_{b2}$ , and  $V_{c2}$
3. Zero-sequence components, consisting of three phasors equal in magnitude, and with zero phase displacement from each other, designated as  $V_{a0}$ ,  $V_{b0}$ , and  $V_{c0}$   
Since each of the original unbalanced phasors is the sum of its components, the original phasors expressed in terms of their components are

$$V_a = V_{a1} + V_{a2} + V_{a0}$$

$$V_b = V_{b1} + V_{b2} + V_{b0}$$

$$V_c = V_{c1} + V_{c2} + V_{c0}$$

The synthesis of a set of three unbalanced phasors from the three sets of symmetrical components is shown in Figure 1.



**Figure 3.1 Graphical addition of symmetrical components To obtain unbalanced phasors.**

### THE OPERATOR ‘a’

The relation between the symmetrical components reveals that the phase displacement among them is either  $120^\circ$  or  $0^\circ$ . Using this relationship, only three independent components is sufficient to determine all the nine components. For this purpose an operator which rotates a given phasor by  $120^\circ$  in the positive direction (counterclockwise) is very useful. The letter ‘a’ is used to designate such a complex operator of unit magnitude with an angle of  $120^\circ$ . It is defined by

$$a = 1\angle 120^\circ = -0.5 + j 0.866 \quad (3.2)$$

If the operator ‘a’ is applied to a phasor twice in succession, the phasor is rotated through  $240^{\circ}$ . Similarly, three successive applications of ‘a’ rotate the phasor through  $360^{\circ}$ .

To reduce the number of unknown quantities, let the symmetrical components of  $V_b$  and  $V_c$  can be expressed as product of some function of the operator a and a component of  $V_a$ . Thus,

$$\begin{aligned} V_{b1} &= a^2 V_{a1} & V_{b2} &= a V_{a2} & V_{b0} &= V_{a0} \\ V_{c1} &= a V_{a1} & V_{c2} &= a^2 V_{a2} & V_{c0} &= V_{a0} \end{aligned}$$

Using these relations the unbalanced phasors can be written as

$$\begin{aligned} V_a &= V_{a0} + V_{a1} + V_{a2} \\ V_b &= V_{a0} + a^2 V_{a1} + a V_{a2} \\ V_c &= V_{a0} + a V_{a1} + a^2 V_{a2} \end{aligned} \tag{3.3}$$

In matrix form,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \tag{3.4}$$

$$\text{Let } V_p = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}; \quad V_s = \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}; \quad A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{3.5}$$

The inverse of A matrix is

$$A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{3.6}$$

With these definitions, the above relations can be written as

$$V_p = A V_s; \quad V_s = A^{-1} V_p \tag{3.7}$$

Thus the symmetrical components of  $V_a$ ,  $V_b$  and  $V_c$  are given by

$$\begin{aligned} V_{a0} &= 1/3 (V_a + V_b + V_c) \\ V_{a1} &= 1/3 (V_a + a V_b + a^2 V_c) \\ V_{a2} &= 1/3 (V_a + a^2 V_b + a V_c) \end{aligned} \tag{3.8}$$

Since the sum of three balanced voltages is zero, the zero-sequence component voltage in a balanced three-phase system is always zero. Further, the sum of line voltages of even an unbalanced three-phase system is zero and hence the corresponding zero-sequence component of line voltages.

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**NUMERICAL EXAMPLES**

**Example 1 :** The line currents in a 3-ph 4 –wire system are  $I_a = 100\angle 30^\circ$ ;  $I_b = 50\angle 300^\circ$ ;  $I_c = 30\angle 180^\circ$ . Find the symmetrical components and the neutral current.

**Solution:**

$$I_{a0} = 1/3(I_a + I_b + I_c) = 27.29 \angle 4.7^\circ A$$

$$I_{a1} = 1/3(I_a + a I_b + a^2 I_c) = 57.98 \angle 43.3^\circ A$$

$$I_{a2} = 1/3(I_a + a^2 I_b + a I_c) = 18.96 \angle 24.9^\circ A$$

$$I_n = I_a + I_b + I_c = 3 I_{a0} = 81.87 \angle 4.7^\circ A$$

**Example 2:** The sequence component voltages of phase voltages of a 3-ph system are:  $V_{a0} = 100 \angle 0^\circ V$ ;  $V_{a1} = 223.6 \angle -26.6^\circ V$ ;  $V_{a2} = 100 \angle 180^\circ V$ . Determine the phase voltages.

**Solution:**

$$V_a = V_{a0} + V_{a1} + V_{a2} = 223.6 \angle -26.6^\circ V$$

$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2} = 213 \angle -99.9^\circ V$$

$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2} = 338.6 \angle 66.2^\circ V$$

**Example 3:** The two seq. components and the corresponding phase voltage of a 3-ph system are  $V_{a0} = 1 \angle -60^\circ V$ ;  $V_{a1} = 2 \angle 0^\circ V$ ; &  $V_a = 3 \angle 0^\circ V$ . Determine the other phase voltages.

**Solution:**

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$V_{a2} = V_a - V_{a0} - V_{a1} = 1 \angle 60^\circ V$$

$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2} = 3 \angle -120^\circ V$$

$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2} = 0 V$$

**Example 4:** Determine the sequence components if  $I_a = 10\angle 60^\circ A$ ;  $I_b = 10\angle -60^\circ A$ ;  $I_c = 10\angle 180^\circ A$ .

**Solution:**

$$I_{a0} = 1/3(I_a + I_b + I_c) = 0 A$$

$$I_{a1} = 1/3(I_a + a I_b + a^2 I_c) = 10\angle 60^\circ A$$

$$I_{a2} = 1/3(I_a + a^2 I_b + a I_c) = 0 A$$

Observation: If the phasors are balanced, two sequence components will be zero.

**Example 5:** Determine the sequence components if  $V_a = 100 \angle 30^\circ V$ ;  $V_b = 100 \angle 150^\circ V$  &  $V_c = 100 \angle -90^\circ V$ .

**Solution:**

$$V_{a0} = 1/3(V_a + V_b + V_c) = 0 V$$

$$V_{a1} = 1/3(V_a + a V_b + a^2 V_c) = 0 V$$

$$V_{a2} = 1/3(V_a + a^2 V_b + a V_c) = 100\angle 30^\circ V$$

Observation: If the phasors are balanced, two sequence components will be zero

**Example 6:** The line b of a 3-ph line feeding a balanced Y-load with neutral grounded is open resulting in line currents:  $I_a = 10\angle 0^\circ$  A &  $I_c = 10\angle 120^\circ$  A. Determine the sequence current components.

**Solution:**

$$\begin{aligned}
 I_b &= 0 \text{ A.} \\
 I_{a0} &= 1/3(I_a + I_b + I_c) = 3.33\angle 60^\circ \text{ A} \\
 I_{a1} &= 1/3(I_a + a I_b + a^2 I_c) = 6.66\angle 0^\circ \text{ A} \\
 I_{a2} &= 1/3(I_a + a^2 I_b + a I_c) = 3.33\angle -60^\circ \text{ A}
 \end{aligned}$$

**Example 7:** One conductor of a 3-ph line feeding a balanced delta-load is open. Assuming that line c is open, if current in line a is  $10\angle 0^\circ$  A, determine the sequence components of the line currents.

**Solution:**

$$\begin{aligned}
 I_c &= 0 \text{ A; } I_a = 10\angle 0^\circ \text{ A. } \quad \text{Ib} = 10\angle 120^\circ \text{ A} \\
 I_{a0} &= 1/3(I_a + I_b + I_c) = 0 \text{ A} \\
 I_{a1} &= 1/3(I_a + a I_b + a^2 I_c) = 5.78\angle -30^\circ \text{ A} \\
 I_{a2} &= 1/3(I_a + a^2 I_b + a I_c) = 5.78\angle 30^\circ \text{ A}
 \end{aligned}$$

Note: The zero-sequence components of line currents of a delta load (3-ph 3-wire) system are zero.

### POWER IN TERMS OF SYMMETRICAL COMPONENTS

The power in a three-phase system can be expressed in terms of symmetrical components of the associated voltages and currents. The power flowing into a three-phase system through three lines a, b and c is

$$S = P + j Q = V_a I_a^* + V_b I_b^* + V_c I_c^* \tag{3.9}$$

where  $V_a$ ,  $V_b$  and  $V_c$  are voltages to neutral at the terminals and  $I_a$ ,  $I_b$ , and  $I_c$  are the currents flowing into the system in the three lines. In matrix form

$$S = \begin{bmatrix} v_a & v_b & v_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Thus

$$S = [A V]^T [AI]^*$$

Using the reversal rule of the matrix algebra

$$S = V^T A^T A^* I^*$$

Noting that  $A^T = A$  and  $a$  and  $a^2$  are conjugates,

$$S = \begin{bmatrix} v_{a0} & v_{a1} & v_{a2} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

or, since  $A^T A^*$  is equal to  $3U$  where  $U$  is  $3 \times 3$  unit matrix

$$S = 3 \begin{bmatrix} v_{a0} & v_{a1} & v_{a2} \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

Thus the complex three-phase power is given by

$$S = V_a I_a^* + V_b I_b^* + V_c I_c^* = 3 V_{a0} I_{a0} + 3 V_{a1} I_{a1} + 3 V_{a2} I_{a2} \quad (3.10)$$

Here,  $3V_{a0}I_{a0}$ ,  $3V_{a1}I_{a1}$  and  $3V_{a2}I_{a2}$  correspond to the three-phase power delivered to the zero-sequence system, positive-sequence system, and negative-sequence system, respectively. Thus, the total three-phase power in the unbalanced system is equal to the sum of the power delivered to the three sequence systems representing the three-phase system.

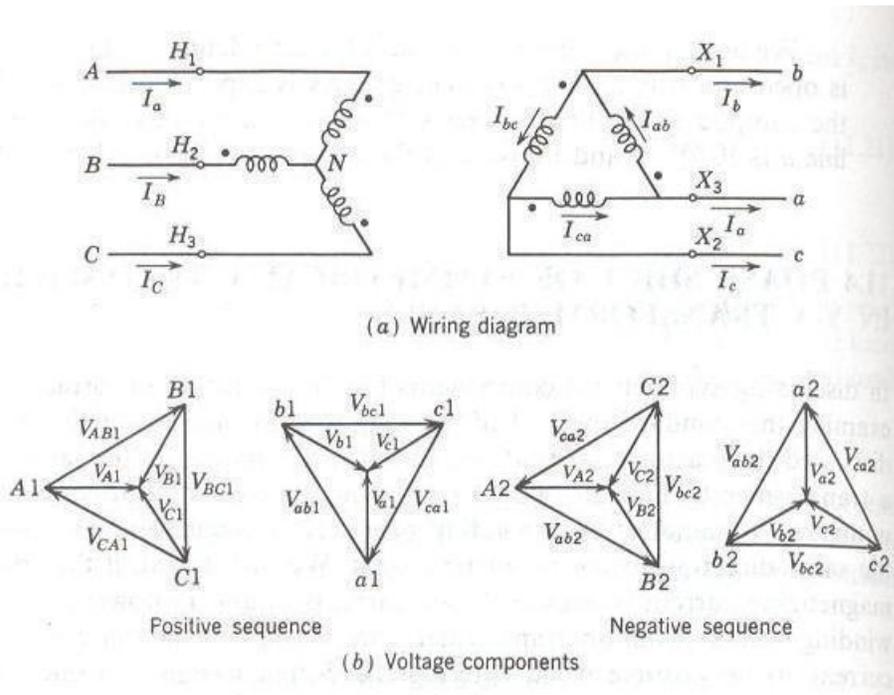
### PHASE SHIFT OF COMPONENTS IN Y- TRANSFORMER BANKS

The dot convention is used to designate the terminals of transformers. The dots are placed at one end of each of the winding on the same iron core of a transformer to indicate that the currents flowing from the dotted terminal to the unmarked terminal of each winding produces an mmf acting in the same direction in the magnetic circuit. In that case, the voltage drops from dotted terminal to unmarked terminal in each side of the windings are in phase.

The HT terminals of three-phase transformers are marked as H1, H2 and H3 and the corresponding LT side terminals are marked X1, X2 and X3. In Y-Y or - transformers, the markings are such that voltages to neutral from terminals H1, H2, and H3 are in phase with the voltages to neutral from terminals X1, X2, and X3, respectively. But, there will be a phase shift (of  $30^0$ ) between the corresponding quantities of the primary and secondary sides of a star-delta (or delta-star) transformer. The standard for connection and designation of transformer banks is as follows:

1. The HT side terminals are marked as H1, H2 and H3 and the corresponding LT side terminals are marked X1, X2 and X3.
2. The phases in the HT side are marked in uppercase letters as A, B, and C. Thus for the sequence abc, A is connected to H1, B to H2 and C to H3. Similarly, the phases in the LT side are marked in lowercase letters as a, b and c.
3. The standard for designating the terminals H1 and X1 on transformer banks requires that the positive-sequence voltage drop from H1 to neutral lead the positive sequence voltage drop from X1 to neutral by  $30^0$  regardless of the type of connection in the HT

and LT sides. Similarly, the voltage drops from H2 to neutral and H3 to neutral lead their corresponding values, X2 to neutral and X3 to neutral by  $30^{\circ}$ .



**Figure 3.2 Wiring diagram and voltage phasors of a Y- transformer With Y connection on HT side.**

Consider a Y- transformer as shown in Figure a. The HT side terminals H1, H2, and H3 are connected to phases A, B, and C, respectively and the phase sequence is ABC. The windings that are drawn in parallel directions are those linked magnetically (by being wound on the same core). In Figure a winding AN is the phase on the Y-side which is linked magnetically with the phase winding bc on the side. For the location of the dots on the windings  $V_{AN}$  is in phase with  $V_{bc}$ . Following the standards for the phase shift, the phasor diagrams for the sequence components of voltages are shown in Figure b. The sequence component of  $V_{AN1}$  is represented as  $V_{A1}$  (leaving subscript 'N' for convenience and all other voltages to neutral are similarly represented). The phasor diagram reveals that  $V_{A1}$  leads  $V_{b1}$  by  $30^{\circ}$ . This will enable to designate the terminal to which b is connected as X1. Inspection of the positive-sequence and negative-sequence phasor diagrams reveals that  $V_{a1}$  leads  $V_{A1}$  by  $90^{\circ}$  and  $V_{a2}$  lags  $V_{A2}$  by  $90^{\circ}$ .

From the dot convention and the current directions assumed in Figure a, the phasor diagram for the sequence components of currents can be drawn as shown in Figure c. Since the direction specified for  $I_A$  in Figure a is away from the dot in the winding and the direction of  $I_{bc}$  is also away from the dot in its winding,  $I_A$  and  $I_{bc}$  are  $180^{\circ}$  out of phase. Hence the phase relation between the Y and currents is as shown in Figure c. From this diagram, it can be seen that  $I_{a1}$  leads  $I_{A1}$  by  $90^{\circ}$  and  $I_{a2}$  lags  $I_{A2}$  by  $90^{\circ}$ . Summarizing these relations between the symmetrical components on the two sides of the transformer gives:

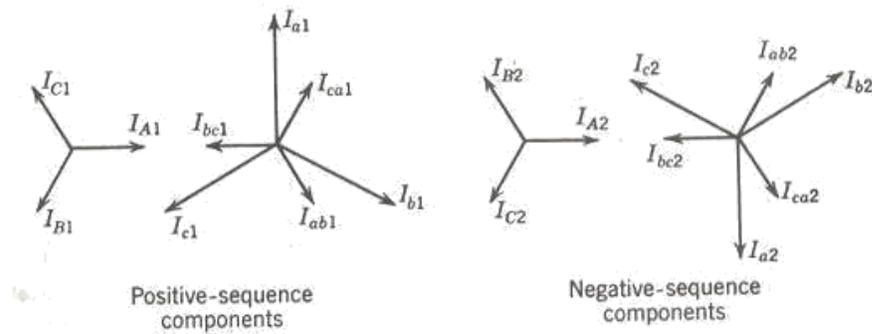


Figure 3.3 Current phasors of Y- transformer with Y connection on HT side.

$$\begin{aligned} V_{a1} &= +j V_{A1} & I_{a1} &= +j I_{A1} \\ V_{a2} &= -j V_{A2} & I_{a1} &= -j I_{A2} \end{aligned} \quad (3.11)$$

Where each voltage and current is expressed in per unit. Although, these relations are obtained for Y- transformer with Y connection in the HT side, they are valid even when the HT side is connected in  $\Delta$  and the LT side in Y.

### NUMERICAL EXAMPLES

**Example 8:** Three identical resistors are Y-connected to the LT Y-side of a delta-star transformer. The voltages at the resistor loads are  $|V_{ab}| = 0.8$  pu.,  $|V_{bc}| = 1.2$  pu., and  $|V_{ca}| = 1.0$  pu. Assume that the neutral of the load is not connected to the neutral of the transformer secondary. Find the line voltages on the HT side of the transformer.

**Solution:**

Assuming an angle of  $180^\circ$  for  $V_{ca}$ , find the angles of other voltages

$$\begin{aligned} V_{ab} &= 0.8 \angle 82.8^\circ \text{ pu} \\ V_{bc} &= 1.2 \angle -41.4^\circ \text{ pu} \\ V_{ca} &= 1.0 \angle 180^\circ \text{ pu} \end{aligned}$$

The symmetrical components of line voltages are

$$\begin{aligned} V_{ab0} &= 1/3 (V_{ab} + V_{bc} + V_{ca}) = 0 \\ V_{ab1} &= 1/3 (V_{ab} + aV_{bc} + a^2V_{ca}) = 0.985 \angle 73.6^\circ \text{ V} \\ V_{ab2} &= 1/3 (V_{ab} + a^2V_{bc} + aV_{ca}) = 0.235 \angle 220.3^\circ \text{ V} \end{aligned}$$

Since  $V_{an1} = V_{ab1} \angle -30^\circ$  and  $V_{an2} = V_{ab2} \angle 30^\circ$

$$\begin{aligned} V_{an1} &= 0.985 \angle 73.6^\circ - 30^\circ \\ &= 0.985 \angle 43.6^\circ \text{ pu (L-L base)} \\ V_{an2} &= 0.235 \angle 220.3^\circ + 30^\circ \\ &= 0.235 \angle 250.3^\circ \text{ pu (L-L base)} \end{aligned}$$

Since each resistor is of  $1.0 \angle 0^\circ$  pu. Impedance,

$$I_{an1} = (V_{an1}/Z) = 0.985 \angle 43.6^\circ \text{ pu.}$$

$$I_{a2} = (V_{a2}/Z) = 0.235 \angle -250.3^\circ \text{ pu.}$$

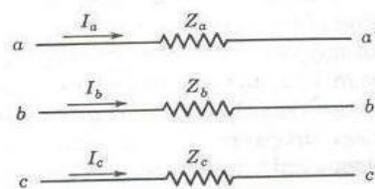
The directions are +ve for currents from supply toward the delta primary and away from the Y-side toward the load. The HT side line to neutral voltages are

$$\begin{aligned} V_{A1} &= -j V_{a1} = 0.985 \angle -46.4^\circ \\ V_{A2} &= +j V_{a2} = 0.235 \angle -19.7^\circ \\ V_A &= V_{A1} + V_{A2} = 1.2 \angle -41.3^\circ \text{ pu.} \\ V_{B1} &= a^2 V_{A1} \quad \text{and} \quad V_{B2} = a V_{A2} \\ V_B &= V_{B1} + V_{B2} = 1 \angle 180^\circ \text{ pu.} \\ V_{C1} &= a V_{A1} \quad \text{and} \quad V_{C2} = a^2 V_{A2} \\ V_C &= V_{C1} + V_{C2} = 0.8 \angle 82.9^\circ \text{ pu.} \end{aligned}$$

The HT side line voltages are

$$\begin{aligned} V_{AB} &= V_A - V_B = 2.06 \angle -22.6^\circ \text{ pu. (L-N base)} \\ &= (1/\sqrt{3}) V_{AB} = 1.19 \angle -22.6^\circ \text{ pu. (L-L base)} \\ V_{BC} &= V_B - V_C = 1.355 \angle 215.8^\circ \text{ pu. (L-N base)} \\ &= (1/\sqrt{3}) V_{BC} = 0.782 \angle 215.8^\circ \text{ pu. (L-L base)} \\ V_{CA} &= V_C - V_A = 1.78 \angle 116.9^\circ \text{ pu. (L-N base)} \\ &= (1/\sqrt{3}) V_{CA} = 1.028 \angle 116.9^\circ \text{ pu. (L-L base)} \end{aligned}$$

### UNSYMMETRICAL IMPEDANCES



**Figure 3.4** Portion of three-phase system representing three unequal series impedances.

Consider the network shown in Figure. Assuming that there is no mutual impedance between the impedances  $Z_a$ ,  $Z_b$ , and  $Z_c$ , the voltage drops  $V_{aa'}$ ,  $v_{bb'}$ , and  $V_{cc'}$  can be expressed in matrix form as

$$\begin{bmatrix} V \\ V \\ V \\ V \end{bmatrix} \begin{bmatrix} Z & 0 & 0 \\ 0 & Z_b & 0 \\ 0 & 0 & Z \\ c \end{bmatrix} \begin{bmatrix} I \\ I \\ I \\ c \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad (3.12)$$

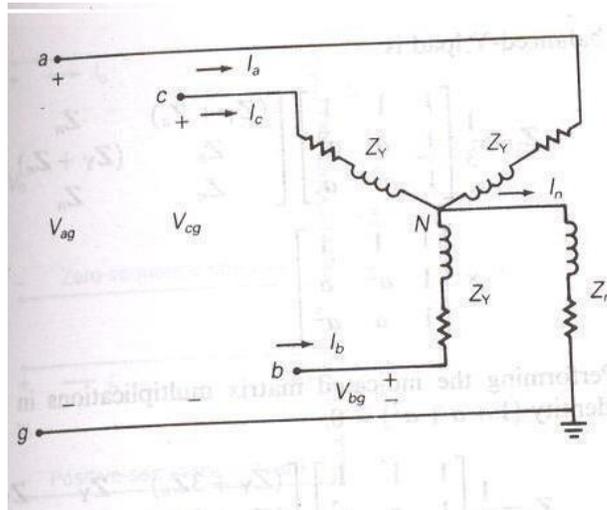
And in terms of symmetrical components of voltage and current as

$$A \begin{bmatrix} V \\ V \\ V \end{bmatrix} \begin{bmatrix} Z_a & 0 & 0 \\ 0 & Z_b & 0 \\ 0 & 0 & Z_c \end{bmatrix} A \begin{bmatrix} I \\ I \\ I \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \quad (3.13)$$

If the three impedances are equal ( i.e., if  $Z_a = Z_b = Z_c$ ), Eq reduces to

$$V_{aa'1} = Z_a I_{a1}; \quad V_{aa'2} = Z_a I_{a2}; \quad V_{aa'0} = Z_a I_{a0} \quad (3.14)$$

Thus, the symmetrical components of unbalanced currents flowing in balanced series impedances (or in a balanced Y load) produce voltage drops of like sequence only. However, if the impedances are unequal or if there exists mutual coupling, then voltage drop of any one sequence is dependent on the currents of all the sequences.



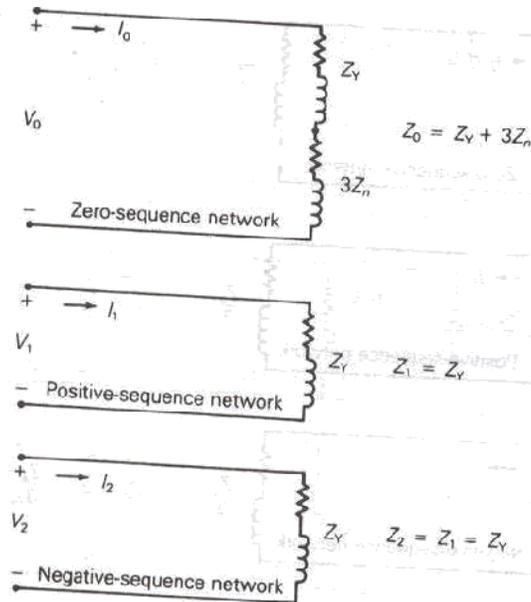


Figure 3.5 Sequence impedances of a Y-connected load.

### NUMERICAL EXAMPLES

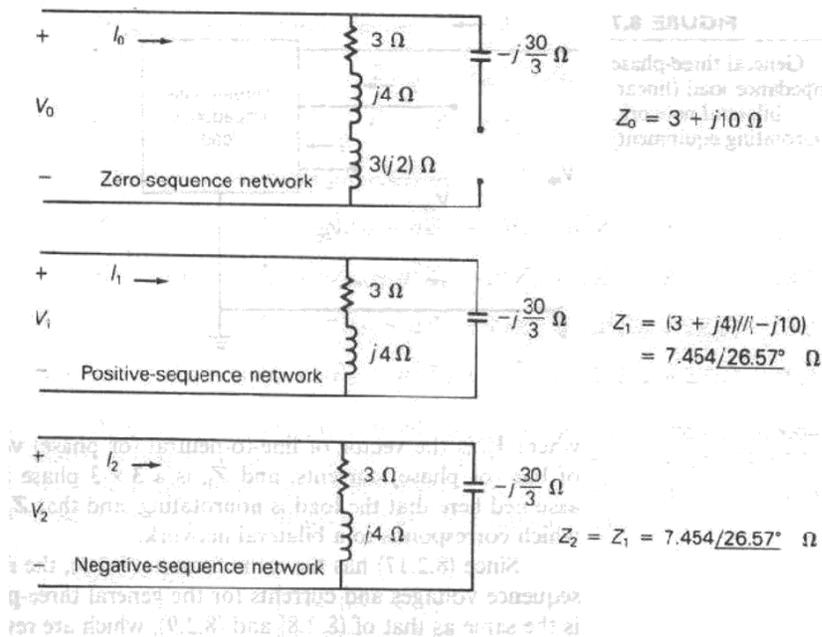
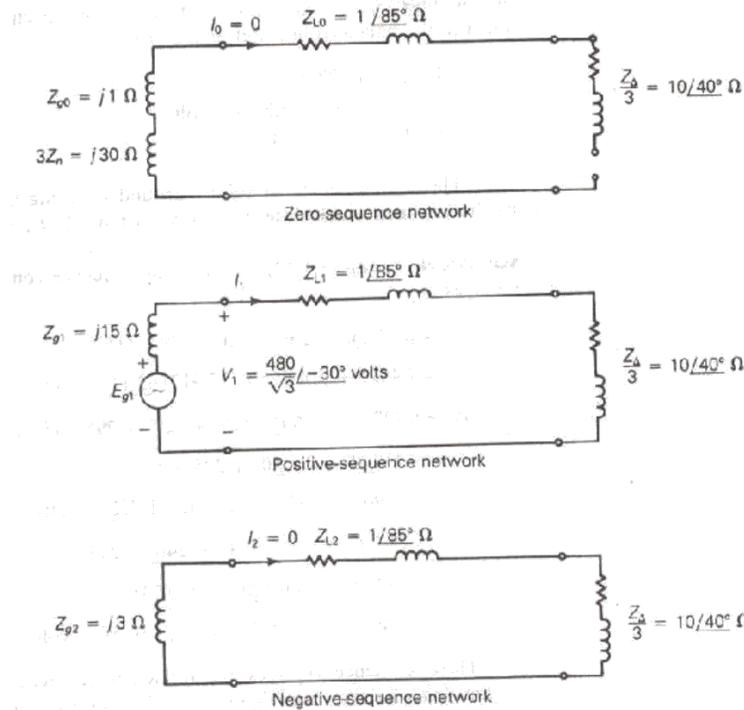
**Example 9:** A Y-connected source with phase voltages  $V_{ag} = 277\angle 0^\circ$ ,  $V_{bg} = 260\angle -120^\circ$  and  $V_{cg} = 295\angle 115^\circ$  is applied to a balanced load of  $30\angle 40^\circ \Omega$ /phase through a line of impedance  $1\angle 85^\circ \Omega$ . The neutral of the source is solidly grounded. Draw the sequence networks of the system and find source currents.

**Solution:**

$$\begin{aligned} V_{a0} &= 15.91\angle 62.110 \text{ V} \\ V_{a1} &= 277.1\angle -1.70 \text{ V} \\ V_{a2} &= 9.22\angle 216.70 \text{ V} \\ Y \text{ eq. of load} &= 10\angle 400 \text{ } \Omega/\text{phase} \\ Z_{\text{line}} &= 1\angle 850 \text{ } \Omega. \\ Z_{\text{neutral}} &= 0 \end{aligned}$$

$$\begin{aligned} I_{a0} &= 0\angle 00 \text{ A} \\ I_{a1} &= 25.82\angle -45.60 \text{ A} \\ I_{a2} &= 0.86\angle 172.80 \text{ A} \end{aligned}$$

$$\begin{aligned} I_a &= 25.15\angle -46.80 \text{ A} \\ I_b &= 25.71\angle 196.40 \text{ A} \\ I_c &= 26.62\angle 73.80 \text{ A} \end{aligned}$$



### SEQUENCE IMPEDANCES AND SEQUENCE NETWORKS

The impedance of a circuit to positive-sequence currents alone is called the impedance to positive-sequence current or simply *positive-sequence impedance*, which is generally denoted as  $Z_1$ . Similarly, the impedance of a circuit to negative-sequence currents alone is called the impedance to negative-sequence current or simply *negative-sequence*

impedance, which is generally denoted as  $Z_2$ . The impedance of a circuit to zero-sequence currents alone is called the impedance to zero-sequence current or simply *zero-sequence impedance*, which is generally denoted as  $Z_0$ . In the analysis of an unsymmetrical fault on a symmetrical system, the symmetrical components of the unbalanced currents that are flowing are determined. Since in a balanced system, the components currents of one sequence cause voltage drops of like sequence only and are independent of currents of other sequences, currents of any one sequence may be considered to flow in an independent network composed of the generated voltages, if any, and impedances to the current of that sequence only.

The single-phase equivalent circuit consisting of the impedances to currents of any one sequence only is called the *sequence network* of that particular sequence. Thus, the sequence network corresponding to positive-sequence current is called the *positive-sequence network*. Similarly, the sequence network corresponding to negative-sequence current is called *negative-sequence network*, and that corresponding to zero-sequence current is called *zero-sequence network*. The sequence networks are interconnected in a particular way to represent various unsymmetrical fault conditions. Therefore, to calculate the effect of a fault by the method of symmetrical components, it is required to determine the sequence networks.

### SEQUENCE NETWORKS OF UNLOADED GENERATOR

Consider an unloaded generator which is grounded through a reactor as shown in Figure. When a fault occurs, unbalanced currents depending on the type of fault will flow through the lines. These currents can be resolved into their symmetrical components. To draw the sequence networks of this generator, the component voltages/currents, component impedances are to be determined. The generated voltages are of positive-sequence only as the generators are designed to supply balanced three-phase voltages. Hence, positive-sequence network is composed of an emf in series with the positive-sequence impedance. The generated emf in this network is the no-load terminal voltage to neutral, which is also equal to the transient and subtransient voltages as the generator is not loaded. The reactance in this network is the subtransient, transient, or synchronous reactance, depending on the condition of study.

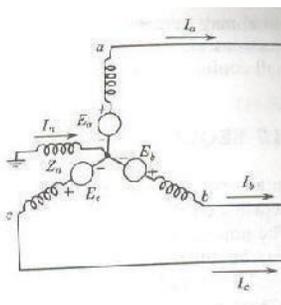


Figure 3.6 Circuit of an unloaded generator grounded through reactance.

The negative- and zero-sequence networks are composed of only the respective sequence impedances as there is no corresponding sequence emf. The reference bus for the positive- and negative-sequence networks is the neutral of the generator.

The current flowing in the impedance  $Z_n$  between neutral and ground is  $3I_{a0}$  as shown in Fig. 3.6. Thus the zero-sequence voltage drop from point a to the ground, is given by:  $(-I_{a0}Z_{g0} - 3I_{a0}Z_n)$ , where  $Z_{g0}$  is the zero-sequence impedance of the generator. Thus the zero-sequence network, which is single-phase equivalent circuit assumed to carry only one phase, must have an zero-sequence impedance of  $Z_0 = (Z_{g0} + 3Z_n)$ .

From the sequence networks, the voltage drops from point a to reference bus (or ground) are given by

$$\begin{aligned} V_{a1} &= E_a - I_{a1}Z_1 \\ V_{a2} &= -I_{a2}Z_2 \\ V_{a0} &= -I_{a0}Z_0 \end{aligned} \tag{3.15}$$

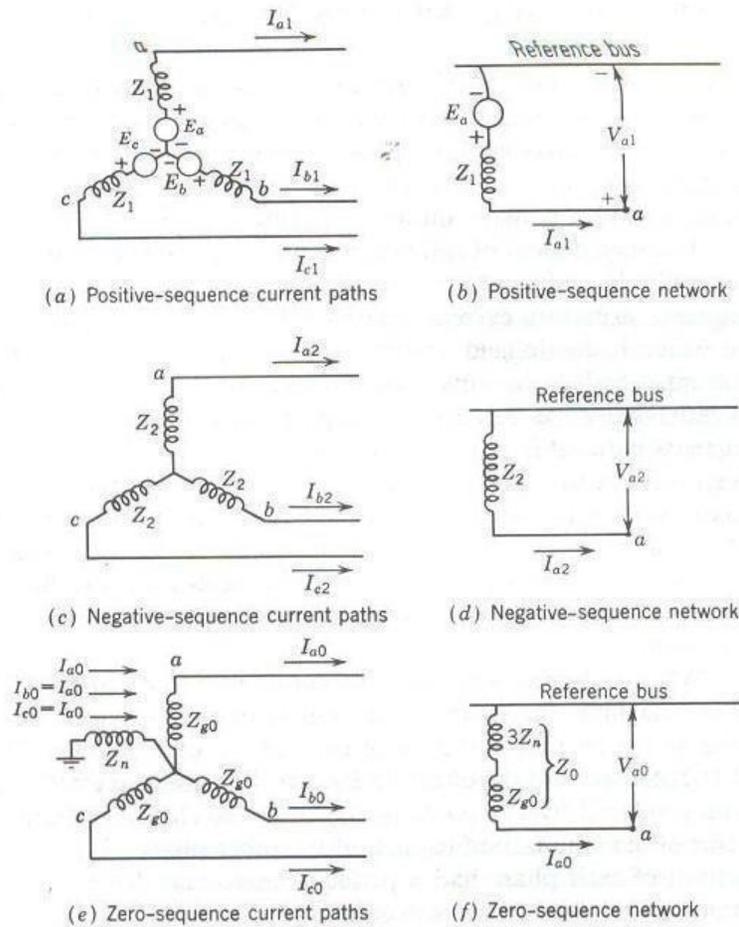


Figure 3.7 Sequence current paths in a generator and The corresponding sequence networks.

Eq. 3.15 applicable to any unloaded generator are valid for loaded generator under steady state conditions. These relations are also applicable for transient or subtransient conditions of a loaded generator if  $E_g'$  or  $E_g''$  is substituted for  $E_a$ .

### SEQUENCE IMPEDANCE OF CIRCUIT ELEMENTS

For obtaining the sequence networks, the component voltages/ currents and the component impedances of all the elements of the network are to be determined. The usual elements of a power system are: passive loads, rotating machines (generators/ motors), transmission lines and transformers. The positive- and negative-sequence impedances of linear, symmetrical, static circuits are identical (because the impedance of such circuits is independent of phase order provided the applied voltages are balanced).

The sequence impedances of rotating machines will generally differ from one another. This is due to the different conditions that exists when the sequence currents flows. The flux due to negative-sequence currents rotates at double the speed of rotor while that the positive-sequence currents is stationary with respect to the rotor. The resultant flux due to zero-sequence currents is ideally zero as these flux components adds up to zero, and hence the zero-sequence reactance is only due to the leakage flux. Thus, the zero-sequence impedance of these machines is smaller than positive- and negative-sequence impedances.

The positive- and negative-sequence impedances of a transmission line are identical, while the zero-sequence impedance differs from these. The positive- and negative-sequence impedances are identical as the transposed transmission lines are balanced linear circuits. The zero-sequence impedance is higher due to magnetic field set up by the zero-sequence currents is very different from that of the positive- or negative-sequence currents ( because of no phase difference). The zero-sequence reactance is generally 2 to 3.5 times greater than the positive- sequence reactance. It is customary to take all the sequence impedances of a transformer to be identical, although the zero-sequence impedance slightly differs with respect to the other two.

### SEQUENCE NETWORKS OF POWER SYSTEMS

In the method of symmetrical components, to calculate the effect of a fault on a power system, the sequence networks are developed corresponding to the fault condition. These networks are then interconnected depending on the type of fault. The resulting network is then analyzed to find the fault current and other parameters.

**Positive- and Negative-Sequence Networks:** The positive-sequence network is obtained by determining all the positive-sequence voltages and positive-sequence impedances of individual elements, and connecting them according to the SLD. All the generated emfs are positive-sequence voltages. Hence all the per unit reactance/impedance diagrams obtained in the earlier chapters are positive-sequence networks. The negative-sequence generated emfs are not present. Hence, the negative-sequence network for a power system is obtained by omitting all the generated emfs (short circuiting emf sources) and

replacing all impedances by negative-sequence impedances from the positive-sequence networks.

Since all the neutral points of a symmetrical three-phase system are at the same potential when balanced currents are flowing, the neutral of a symmetrical three-phase system is the logical reference point. It is therefore taken as the reference bus for the positive- and negative-sequence networks. Impedances connected between the neutral of the machine and ground is not a part of either the positive- or negative- sequence networks because neither positive- nor negative-sequence currents can flow in such impedances.

**Zero-Sequence Networks:** The zero-sequence components are the same both in magnitude and in phase. Thus, it is equivalent to a single-phase system and hence, zero-sequence currents will flow only if a return path exists. The reference point for this network is the ground (Since zero-sequence currents are flowing, the ground is not necessarily at the same point at all points and the reference bus of zero-sequence network does not represent a ground of uniform potential. The return path is conductor of zero impedance, which is the reference bus of the zero-sequence network.).

If a circuit is Y-connected, with no connection from the neutral to ground or to another neutral point in the circuit, no zero-sequence currents can flow, and hence the impedance to zero-sequence current is infinite. This is represented by an open circuit between the neutral of the Y-connected circuit and the reference bus, as shown in Fig. 3.8a. If the neutral of the Y-connected circuit is grounded through zero impedance, a zero-impedance path (short circuit) is connected between the neutral point and the reference bus, as shown in Fig. 3.8b. If an impedance  $Z_n$  is connected between the neutral and the ground of a Y-connected circuit, an impedance of  $3Z_n$  must be connected between the neutral and the reference bus (because, all the three zero-sequence currents ( $3I_{a0}$ ) flows through this impedance to cause a voltage drop of  $3I_{a0} Z_0$ ), as shown in Fig. 3.8c.

Δ-connected circuit can provide no return path; its impedance to zero-sequence line currents is therefore infinite. Thus, the zero-sequence network is open at the Δ-connected circuit, as shown in Fig.3.9 However zero-sequence currents can circulate inside the Δ-connected circuit.

The zero-sequence equivalent circuits of three-phase transformers deserve special attention. The different possible combinations of the primary and the secondary windings in Y and alter the zero-sequence network. The five possible connections of two-winding transformers and their equivalent zero-sequence networks are shown in Fig.3.10. The networks are drawn remembering that there will be no primary current when there is no secondary current, neglecting the no-load component. The arrows on the connection diagram show the possible paths for the zero-sequence current. Absence of an arrow indicates that the connection is such that zero-sequence currents cannot flow. The letters P and Q identify the corresponding points on the connection diagram and equivalent circuit:

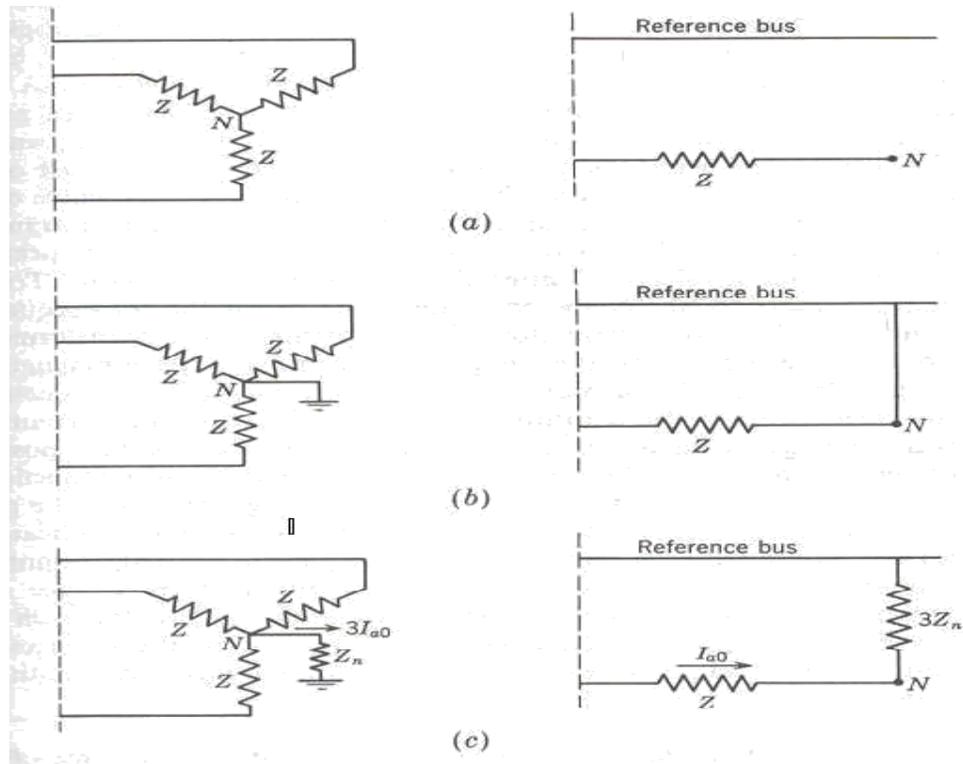


Figure 3.8 Zero-sequence equivalent networks of Y-connected load

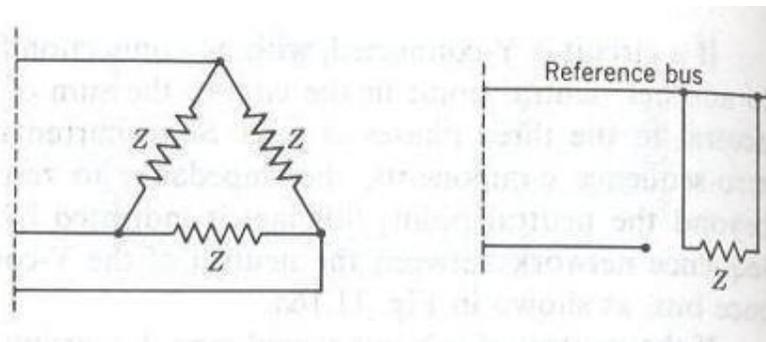


Figure 3.9 Zero-sequence equivalent networks of  $\Delta$ -connected load

1. *Case 1: Y-Y Bank with one neutral grounded:* If either one of the neutrals of a Y-Y bank is ungrounded, zero-sequence current cannot flow in either winding ( as the absence of a path through one winding prevents current in the other). An open circuit exists for zero-sequence current between two parts of the system connected by the transformer bank.
2. *Case 2: Y-Y Bank with both neutral grounded:* In this case, a path through transformer exists for the zero-sequence current. Hence zero-sequence current can flow in both sides of the transformer provided there is complete outside closed path for it to flow. Hence the points on the two sides of the transformer are connected by the zero-sequence impedance of the transformer.

SYMBOLS	CONNECTION DIAGRAMS	ZERO-SEQUENCE EQUIVALENT CIRCUITS

Figure 3.10 Zero-sequence equivalent networks of three-phase transformer banks for various combinations.

3. *Case 3: Y- Bank with grounded Y:* In this case, there is path for zero-sequence current to ground through the Y as the corresponding induced current can circulate in the . The equivalent circuit must provide for a path from lines on the Y side through zero-sequence impedance of the transformer to the reference bus. However, an open circuit must exist between line and the reference bus on the side. If there is an impedance  $Z_n$  between neutral and ground, then the zero-sequence impedance must include  $3Z_n$  along with zero-sequence impedance of the transformer.

4. *Case 4: Y-Bank with ungrounded Y:* In this case, there is no path for zero-sequence current. The zero-sequence impedance is infinite and is shown by an open circuit.
5. *Case 5: - Bank:* In this case, there is no return path for zero-sequence current. The zero-sequence current cannot flow in lines although it can circulate in the windings.
6. The zero-sequence equivalent circuits determined for the individual parts separately are connected according to the SLD to form the complete zero-sequence network.

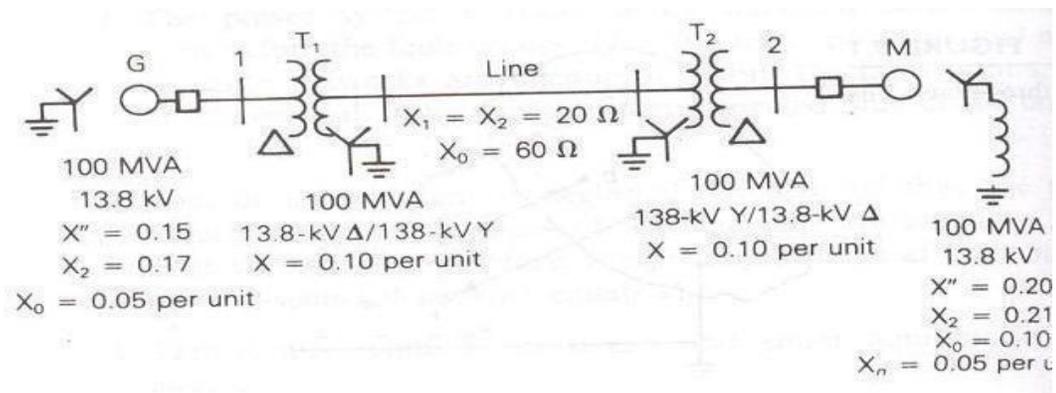
**Procedure to draw the sequence networks**

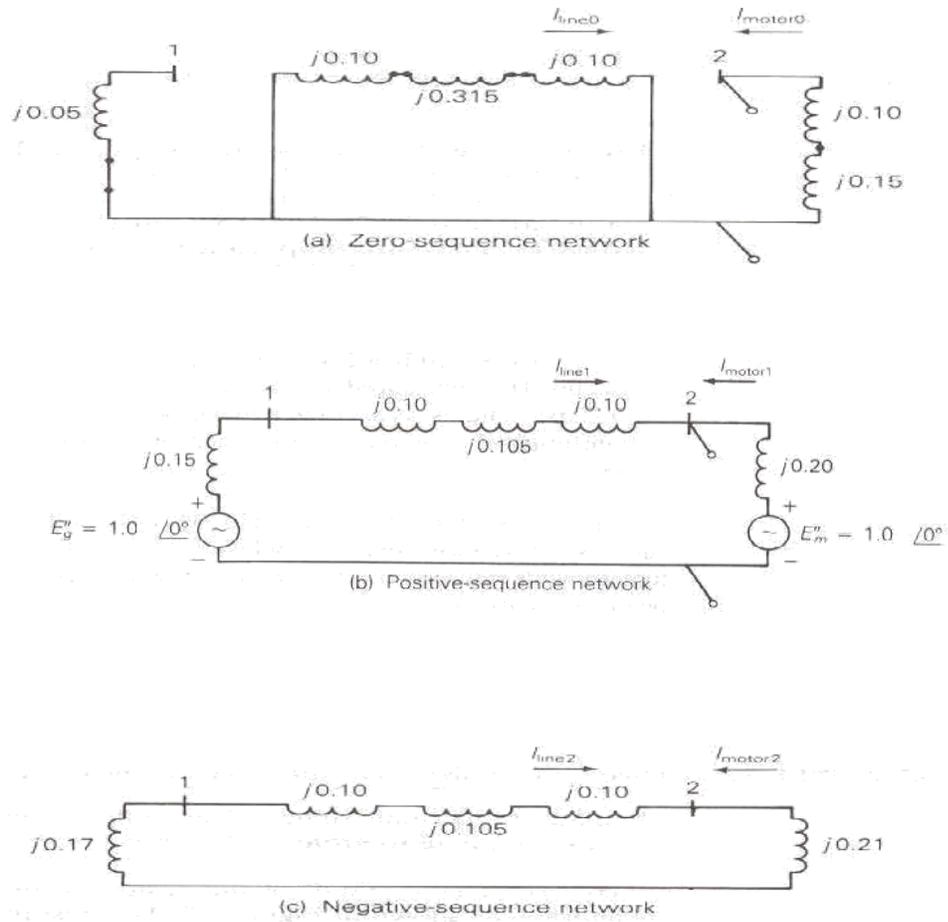
The sequence networks are three separate networks which are the single-phase equivalent of the corresponding symmetrical sequence systems. These networks can be drawn as follows:

1. For the given condition (steady state, transient, or subtransient), draw the reactance diagram (selecting proper base values and converting all the per unit values to the selected base, if necessary). This will correspond to the positive-sequence network.
2. Determine the per unit negative-sequence impedances of all elements (if the values of negative sequence is not given to any element, it can approximately be taken as equal to the positive-sequence impedance). Draw the negative-sequence network by replacing all emf sources by short circuit and all impedances by corresponding negative-sequence impedances in the positive-sequence network.
3. Determine the per unit zero-sequence impedances of all the elements and draw the zero-sequence network corresponding to the grounding conditions of different elements.

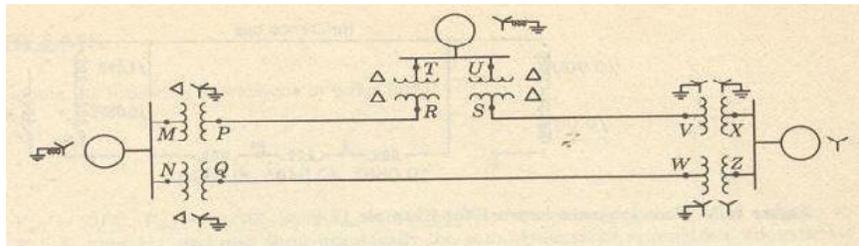
**NUMERICAL EXAMPLES**

**Example 10:** For the power system shown in the SLD, draw the sequence networks.





**EXERCISE PROBLEM:** For the power system shown in the SLD, draw the sequence networks.



## CHAPTER 4: UNSYMMETRICAL FAULTS

[CONTENTS: Preamble, L-G, L-L, L-L-G and 3-phase faults on an unloaded alternator without and with fault impedance, faults on a power system without and with fault impedance, open conductor faults in power systems, examples]

### PREAMBLE

The unsymmetrical faults will have faulty parameters at random. They can be analyzed by using the symmetrical components. The standard types of unsymmetrical faults considered for analysis include the following (in the order of their severity):

- Line-to-Ground (L-G) Fault
- Line-to-Line (L-L) Fault
- Double Line-to-Ground (L-L-G) Fault and
- Three-Phase-to-Ground (LLL-G) Fault.

Further the neutrals of various equipment may be grounded or isolated, the faults can occur at any general point F of the given system, the faults can be through a fault impedance, etc. Of the various types of faults as above, the 3- $\phi$  fault involving the ground is the most severe one. Here the analysis is considered in two stages as under:

(i) Fault at the terminals of a Conventional (Unloaded) Generator and (ii) Faults at any point F, of a given Electric Power System (EPS).

Consider now the symmetrical component relational equations derived from the three sequence networks corresponding to a given unsymmetrical system as a function of sequence impedances and the positive sequence voltage source in the form as under:

$$\begin{aligned} V_{a0} &= -I_{a0}Z_0 \\ V_{a1} &= E_a - I_{a1}Z_1 \\ V_{a2} &= -I_{a2}Z_2 \end{aligned} \quad (4.1)$$

These equations are referred as the *sequence equations*. In matrix Form the sequence equations can be considered as:

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \quad (4.2)$$

This equation is used along with the equations i.e., conditions under fault (c.u.f.), derived to describe the fault under consideration, to determine the sequence current  $I_{a1}$  and hence the fault current  $I_f$ , in terms of  $E_a$  and the sequence impedances,  $Z_1$ ,  $Z_2$  and  $Z_0$ . Thus during unsymmetrical fault analysis of any given type of fault, two sets of equations as follows are considered for solving them simultaneously to get the required fault parameters:

- Equations for the conditions under fault (c.u.f.)

- Equations for the sequence components (sequence equations) as per (4.2) above. **SINGLE LINE TO GROUND FAULT ON A CONVENTIONAL (UNLOADED) GENERATOR**

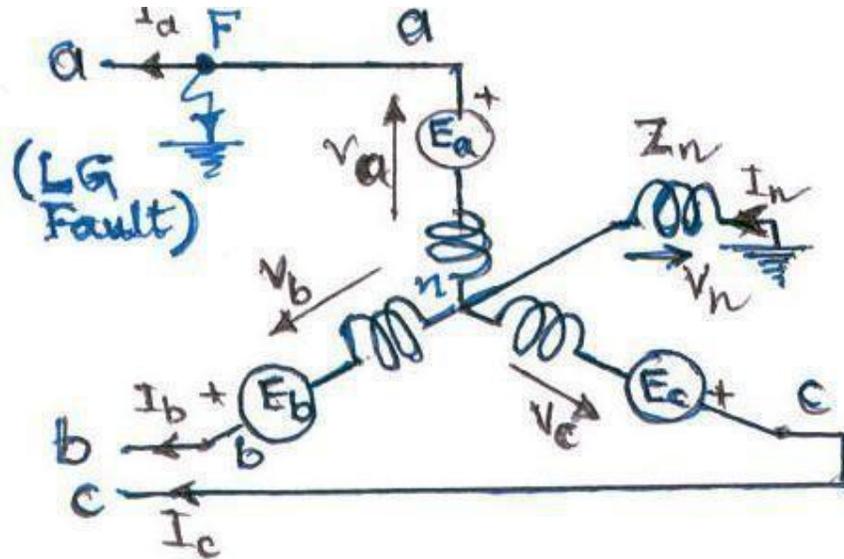


Figure 4.1 LG Fault on a Conventional Generator

A conventional generator is one that produces only the balanced voltages. Let  $E_a$ , and  $E_c$  be the internally generated voltages and  $Z_n$  be the neutral impedance. The fault is assumed to be on the phase 'a' as shown in figure 4.1. Consider now the conditions under fault as under:

**c.u.f.:**

$$I_b = 0; I_c = 0; \text{ and } V_a = 0. \tag{4.3}$$

Now consider the symmetrical components of the current  $I_a$  with  $I_b=I_c=0$ , given by:

$$\begin{vmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{vmatrix} = (1/3) \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \begin{vmatrix} I_a \\ 0 \\ 0 \end{vmatrix} \tag{4.4}$$

Solving (4.4) we get,

$$I_{a1} = I_{a2} = I_{a0} = (I_a/3) \tag{4.5}$$

Further, using equation (4.5) in (4.2), we get,

$$\begin{vmatrix} V_{a0} \\ V_{a1} \end{vmatrix} = \begin{vmatrix} 0 \\ E_a \end{vmatrix} - \begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \end{vmatrix} \begin{vmatrix} I_{a1} \\ I_{a1} \end{vmatrix}$$

$$V_{a2} = 0 = 0 + Z_2 I_{a1} \quad (4.6)$$

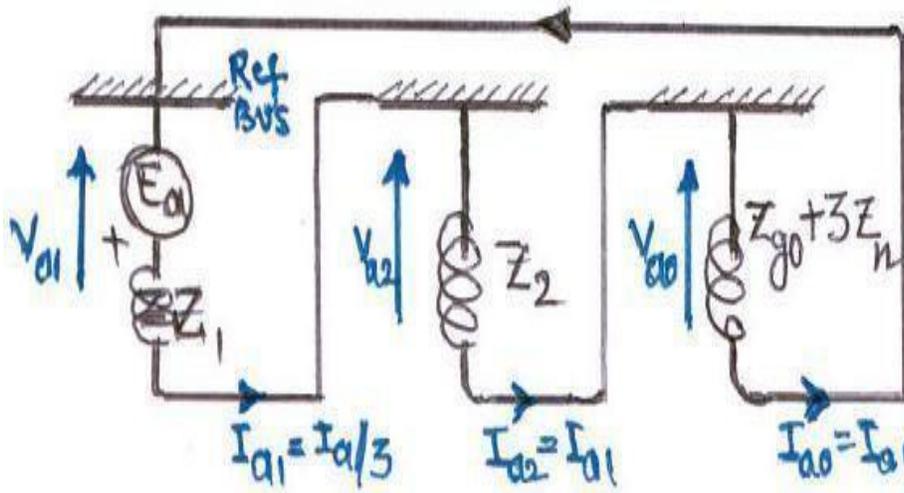
Pre-multiplying equation (4.6) throughout by  $[1 \ 1 \ 1]$ , we get,

$$V_{a1} + V_{a2} + V_{a0} = -I_{a1}Z_0 + E_a - I_{a1}Z_1 - I_{a2}Z_2$$

i.e.,  $V_a = E_a - I_{a1}(Z_1 + Z_2 + Z_0) = 0$ ,

Or in other words,

$$I_{a1} = [E_a / (Z_1 + Z_2 + Z_0)] \quad (4.7)$$



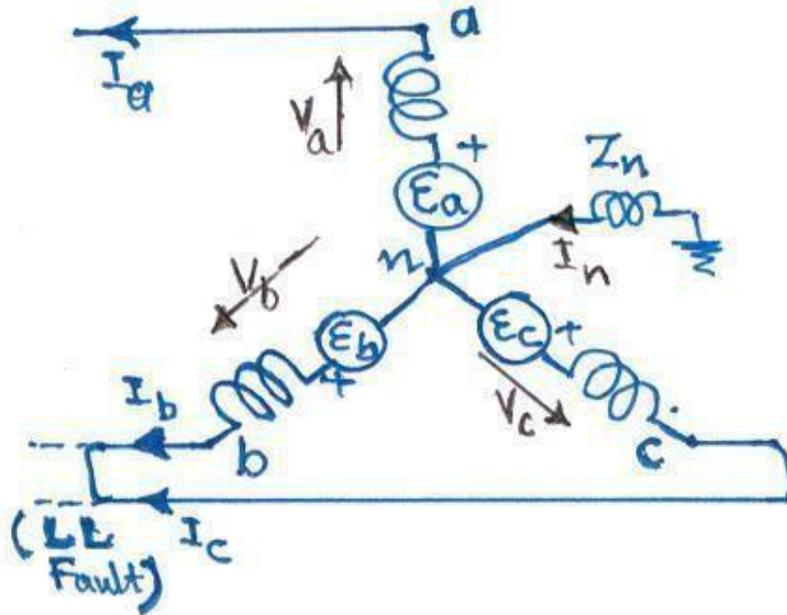
**Figure 4.2 Connection of sequence networks for LG Fault on phase a of a Conventional Generator**

The equation (4.7) derived as above implies that the three sequence networks are connected in series to simulate a LG fault, as shown in figure 4.2. Further we have the following relations satisfied under the fault conditions:

1.  $I_{a1} = I_{a2} = I_{a0} = (I_a/3) = [E_a / (Z_1 + Z_2 + Z_0)]$
2. Fault current  $I_f = I_a = 3I_{a1} = [3E_a / (Z_1 + Z_2 + Z_0)]$
3.  $V_{a1} = E_a - I_{a1}Z_1 = E_a(Z_2 + Z_0) / (Z_1 + Z_2 + Z_0)$
4.  $V_{a2} = -E_a Z_2 / (Z_1 + Z_2 + Z_0)$
5.  $V_{a0} = -E_a Z_0 / (Z_1 + Z_2 + Z_0)$
6. Fault phase voltage  $V_a = 0$ ,
7. Sound phase voltages  $V_b = a^2 V_{a1} + a V_{a2} + V_{a0}$ ;  $V_c = a V_{a1} + a^2 V_{a2} + V_{a0}$
8. Fault phase power:  $V_a I_a^* = 0$ , Sound phase powers:  $V_b I_b^* = 0$ , and  $V_c I_c^* = 0$ ,
9. If  $Z_n = 0$ , then  $Z_0 = Z_{g0}$ ,

10. If  $Z_n = \infty$ , then  $Z_0 = \infty$ , i.e., the zero sequence network is open so that then,  $I_f = I_a = 0$ .

**LINE TO LINE FAULT ON A CONVENTIONAL GENERATOR**



**Figure 4.3 LL Fault on a Conventional Generator**

Consider a line to line fault between phase ‘b’ and phase ‘c’ as shown in figure 4.3, at the terminals of a conventional generator, whose neutral is grounded through a reactance. Consider now the conditions under fault as under:

**c.u.f.:**

$$I_a = 0; I_b = -I_c; \text{ and } V_b = V_c \tag{4.8}$$

Now consider the symmetrical components of the voltage  $V_a$  with  $V_b = V_c$ , given by:

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix} \tag{4.9}$$

Solving (4.4) we get,

$$V_{a1} = V_{a2} \tag{4.10}$$

Further, consider the symmetrical components of current  $I_a$  with  $I_b = -I_c$ , and  $I_a = 0$ ; given by:

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix} \tag{4.11}$$

Solving (4.11) we get,

$$I_{a0} = 0; \text{ and } I_{a2} = -I_{a1} \quad (4.12)$$

Using equation (4.10) and (4.12) in (4.2), and since  $V_{a0} = 0$  ( $I_{a0}$  being 0), we get,

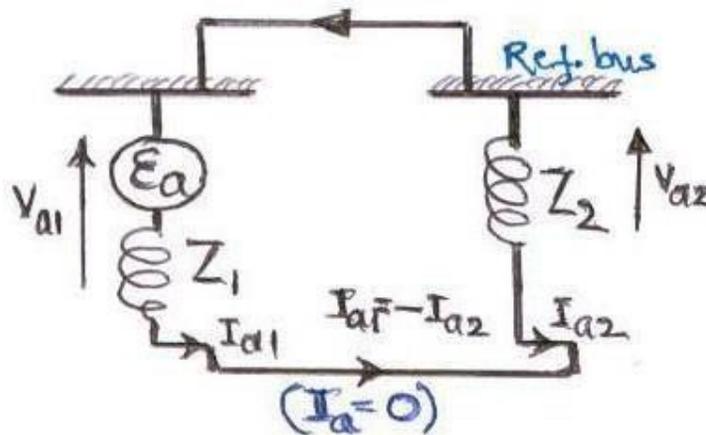
$$\begin{bmatrix} 0 \\ V_{a1} \\ V_{a1} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} 0 \\ I_{a1} \\ -I_{a1} \end{bmatrix} \quad (4.13)$$

Pre-multiplying equation (4.13) throughout by  $[0 \ 1 \ -1]$ , we get,

$$V_{a1} - V_{a1} = E_a - I_{a1}Z_1 - I_{a1}Z_2 = 0$$

Or in other words,

$$I_{a1} = [E_a / (Z_1 + Z_2)] \quad (4.14)$$



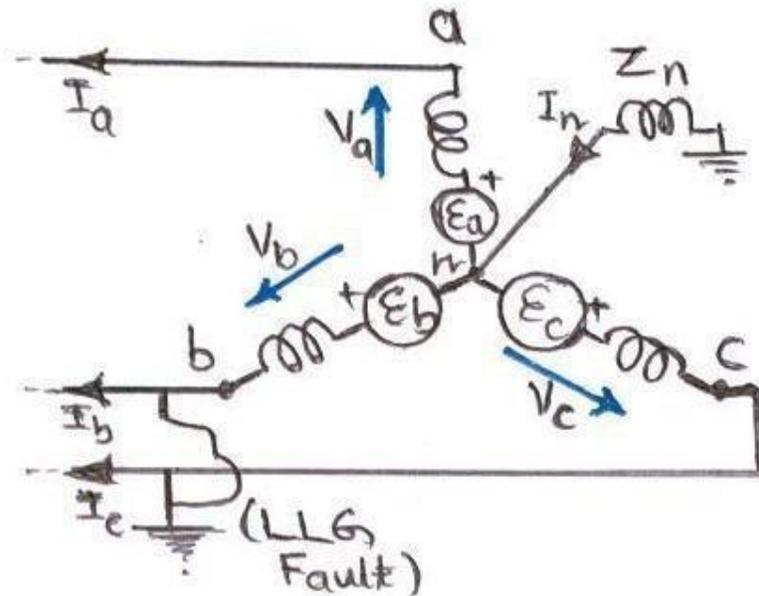
**Figure 4.4 Connection of sequence networks for LL Fault on phases b & c of a Conventional Generator**

The equation (4.14) derived as above implies that the three sequence networks are connected such that the zero sequence network is absent and only the positive and negative sequence networks are connected in series-opposition to simulate the LL fault, as shown in figure 4.4. Further we have the following relations satisfied under the fault conditions:

1.  $I_{a1} = -I_{a2} = [E_a / (Z_1 + Z_2)]$  and  $I_{a0} = 0$ ,
2. Fault current  $I_f = I_b = -I_c = [\sqrt{3}E_a / (Z_1 + Z_2)]$  (since  $I_b = (a^2 - a)I_{a1} = \sqrt{3}I_{a1}$ )
3.  $V_{a1} = E_a - I_{a1}Z_1 = E_a Z_2 / (Z_1 + Z_2)$
4.  $V_{a2} = V_{a1} = E_a Z_1 / (Z_1 + Z_2)$
5.  $V_{a0} = 0$ ,
6. Fault phase voltages;  $V_b = V_c = aV_{a1} + a^2V_{a2} + V_{a0} = (a + a^2)V_{a1} = -V_{a1}$
7. Sound phase voltage:  $V_a = V_{a1} + V_{a2} + V_{a0} = 2V_{a1}$ ;
8. Fault phase powers are  $V_{fb} I_{fb}$  and  $V_{fc} I_{fc}$ ;
9. Sound phase power:  $V_{fa} I_{fa}^* = 0$ ,

10. Since  $I_{a0}=0$ , the presence of absence of neutral impedance does not make any difference in the analysis.

**DOUBLE LINE TO GROUND FAULT ON A CONVENTIONAL GENERATOR**



**Figure 4.5 LLG Fault on a Conventional Generator**

Consider a double-line to ground fault at the terminals of a conventional unloaded generator, whose neutral is grounded through a reactance, between phase ‘b’ and phase ‘c’ as shown in figure 4.5, Consider now the conditions under fault as under:

**c.u.f.:**

$$I_a = 0 \text{ and } V_b = V_c = 0 \tag{4.15}$$

Now consider the symmetrical components of the voltage with  $V_b=V_c=0$ , given by:

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix} \tag{4.16}$$

Solving (4) we get,

$$V_{a1} = V_{a2} = V_{a0} = V_a/3 \tag{4.17}$$

Consider now the sequence equations (4.2) as under,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \tag{4.18}$$

Pre-multiplying equation (4.18) throughout by

$$Z^{-1} = \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix} \quad (4.19)$$

We get,

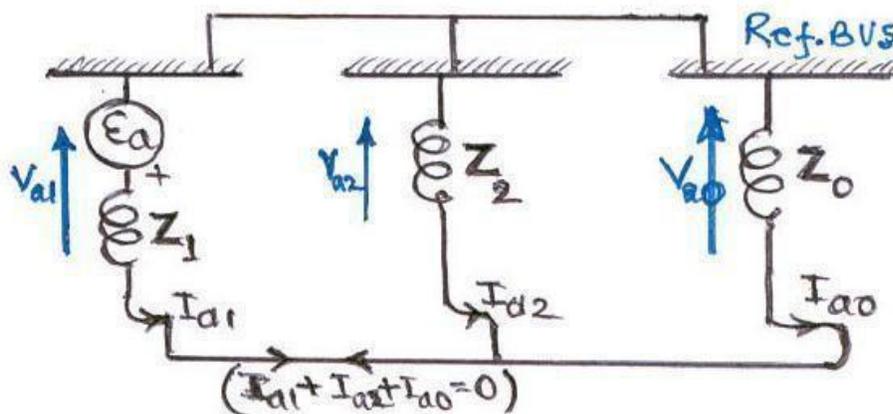
$$Z^{-1} \begin{bmatrix} V_{a1} \\ V_{a1} \\ V_{a1} \end{bmatrix} = Z^{-1} \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - Z^{-1} \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \quad (4.20)$$

Using the identity:  $V_{a1} = (E_a - I_{a1}Z_1)$  in equation (4.19), pre-multiplying throughout by  $[1 \ 1 \ 1]$  and finally adding, we get,

$$\begin{aligned} E_a/Z_0 - I_{a1}(Z_1/Z_0) + (E_a/Z_1) - I_{a1} + E_a/Z_2 - I_{a1}(Z_1/Z_2) &= (E_a/Z_1) - (I_{a0} + I_{a1} + I_{a2}) \\ &= (E_a/Z_1) - I_a = (E_a/Z_1) \end{aligned} \quad (4.21)$$

Since  $I_a = 0$ , solving the equation (4.21), we get,

$$I_{a1} = \left\{ \frac{E_a}{Z_1 + Z_2 Z_0 / (Z_2 + Z_0)} \right\} \quad (4.22)$$



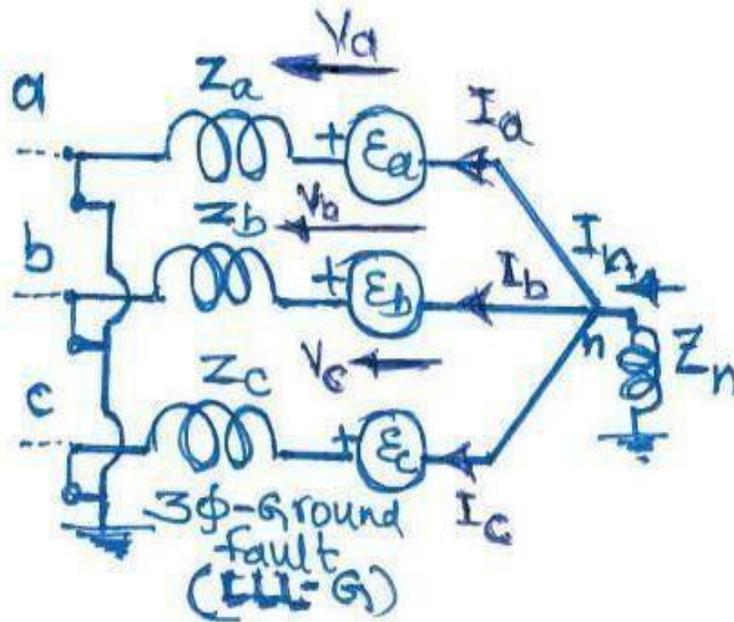
**Figure 4.6 Connection of sequence networks for LLG Fault on phases b and c of a Conventional Generator**

The equation (4.22) derived as above implies that, to simulate the LLG fault, the three sequence networks are connected such that the positive network is connected in series with the parallel combination of the negative and zero sequence networks, as shown in figure 4.6. Further we have the following relations satisfied under the fault conditions:

1.  $I_{a1} = \left\{ \frac{E_a}{Z_1 + Z_2 Z_0 / (Z_2 + Z_0)} \right\}$ ;  $I_{a2} = -I_{a1} Z_0 / (Z_2 + Z_0)$  and  $I_{a0} = -I_{a1} Z_2 / (Z_2 + Z_0)$ ,
2. Fault current  $I_f$ :  $I_{a0} = (1/3)(I_a + I_b + I_c) = (1/3)(I_b + I_c) = I_f/3$ , Hence  $I_f = 3I_{a0}$
3.  $I_a = 0$ ,  $V_b = V_c = 0$  and hence  $V_{a1} = V_{a2} = V_{a0} = V_a/3$
4. Fault phase voltages;  $V_b = V_c = 0$
5. Sound phase voltage;  $V_a = V_{a1} + V_{a2} + V_{a0} = 3V_{a1}$ ;
6. Fault phase powers are  $V_b I_b^* = 0$ , and  $V_c I_c^* = 0$ , since  $V_b = V_c = 0$

7. Healthy phase power:  $V_a I_a^* = 0$ , since  $I_a=0$
8. If  $Z_0=\infty$ , (i.e., the ground is isolated), then  $I_{a0}=0$ , and hence the result is the same as that of the LL fault [with  $Z_0=\infty$ , equation (4.22) yields equation(4.14)].

**THREE PHASE TO GROUND FAULT ON A CONVENTIONAL GENERATOR**



**Figure 4.7 Three phase ground Fault on a Conventional Generator**

Consider a three phase to ground (LLLG) fault at the terminals of a conventional unloaded generator, whose neutral is grounded through a reactance, between all its three phases a, b and c, as shown in figure 4.7, Consider now the conditions under fault as under:

**c.u.f.:**

$$V_a = V_b = V_c = 0, I_a + I_b + I_c = 0 \tag{4.23}$$

Now consider the symmetrical components of the voltage with  $V_a=V_b=V_c=0$ , given by:

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.24}$$

Solving (4.24) we get,

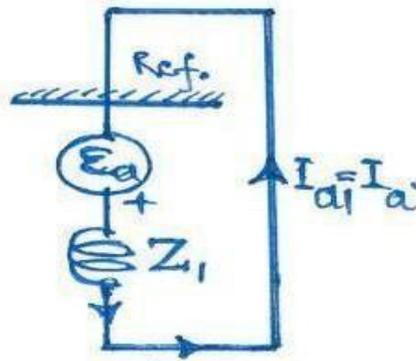
$$V_{a1} = V_{a2} = V_{a0} = 0 \quad (4.25)$$

Thus we have

$$V_{a1} = E_{a1} - I_{a1}Z_1 \quad (4.26)$$

So that after solving for  $I_{a1}$  we, get,

$$I_{a1} = [ E_a / Z_1 ] \quad (4.27)$$



**Figure 4.8 Connection of sequence networks for 3-phase ground Fault on phases b and c of a Conventional Generator**

The equation (4.26) derived as above implies that, to simulate the 3-phase ground fault, the three sequence networks are connected such that the negative and zero sequence networks are absent and only the positive sequence network is present, as shown in figure 4.8. Further the fault current,  $I_f$  in case of a 3-phase ground fault is given by

$$I_f = I_{a1} = I_a = (E_a / Z_1) \quad (4.28)$$

It is to be noted that the presence of a neutral connection without or with a neutral impedance,  $Z_n$  will not alter the simulated conditions in case of a three phase to ground fault.

### UNSYMMETRICAL FAULTS ON POWER SYSTEMS

In all the analysis so far, only the fault at the terminals of an unloaded generator have been considered. However, faults can also occur at any part of the system and hence the power system fault at any general point is also quite important. The analysis of unsymmetrical fault on power systems is done in a similar way as that followed thus far for the case of a fault at the terminals of a generator. Here, instead of the sequence impedances of the generator, each and every element is to be replaced by their corresponding sequence impedances and the fault is analyzed by suitably connecting them together to arrive at the Thevenin equivalent impedance if that given sequence. Also, the internal voltage of the generators of the equivalent circuit for the positive

sequence network is now  $V_f$  (and not  $E_a$ ), the pre-fault voltage to neutral at the point of fault (PoF) (ref. Figure 4.9).

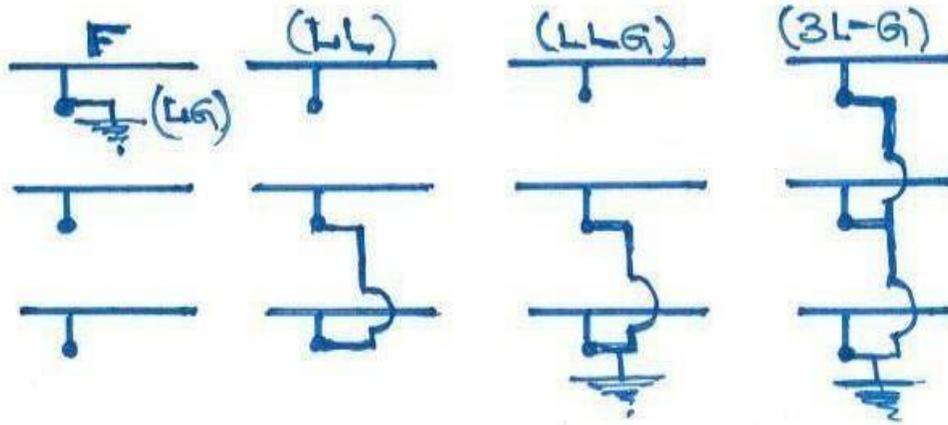


Figure 4.9 Unsymmetrical faults in Power Systems

Thus, for all the cases of unsymmetrical fault analysis considered above, the sequence equations are to be changed as under so as to account for these changes:

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ V_f \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \quad (4.29)$$

(i) *LG Fault at any point F of a given Power system*

Let phase 'a' be on fault at F so that then, the c.u.f. would be:

$$I_b = 0; I_c = 0; \text{ and } V_a = 0.$$

Hence the derived conditions under fault would be:

$$I_{a1} = I_{a2} = I_{a0} = (I_a/3)$$

$$I_{a1} = [V_f / (Z_1 + Z_2 + Z_0)] \text{ and}$$

$$I_f = 3I_{a1} \quad (4.30)$$

(ii) *LL Fault at any point F of a given Power system*

Let phases 'b' and 'c' be on fault at F so that then, the c.u.f. would be:

$$I_a = 0; I_b = -I_c; \text{ and } V_b = V_c$$

Hence the derived conditions under fault would be:

$$V_{a1} = V_{a2}; I_{a0} = 0; I_{a2} = -I_{a1}$$

$$I_{a1} = [V_f / (Z_1 + Z_2)] \text{ and}$$

$$I_f = I_b = -I_c = [\sqrt{3} V_f / (Z_1 + Z_2)] \quad (4.31)$$

(ii) *LLG Fault at any point F of a given Power system*

Let phases 'b' and 'c' be on fault at F so that then, the c.u.f. would be:

$$I_a = 0 \text{ and } V_b = V_c = 0$$

Hence the derived conditions under fault would be:

$$V_{a1} = V_{a2} = V_{a0} = (V_a/3)$$



$$I_{a1} + I_{a2} + I_{a0} = 0 \text{ and}$$

$$V_{aa1}' = V_{aa2}' = V_{aa0}' = (V_{aa}'/3) \tag{4.34}$$

These relations suggest a parallel combination of the three sequence networks as shown in fig. 4.11.

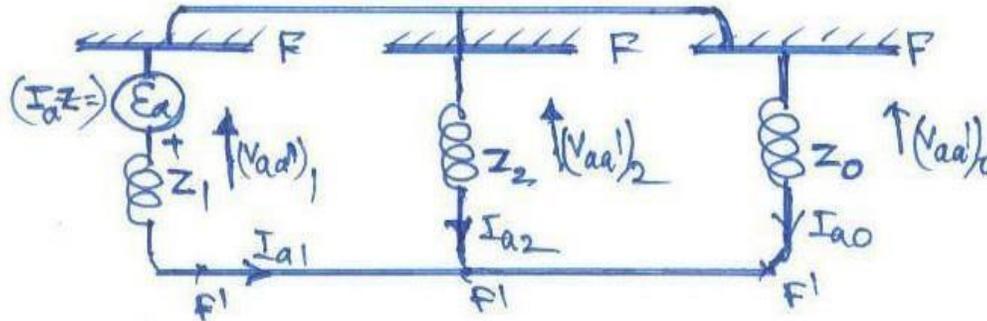


Figure 4.11 Sequence network connection for 1-conductor open fault

It is observed that a single conductor fault is similar to a LLG fault at the fault point F of the system considered.

(ii) *Two Conductor Open Fault*: consider the phases 'b' and 'c' under open condition so that then the conditions under fault are:

$$I_b = I_c = 0; V_{aa}' = 0$$

The derived conditions are:

$$I_{a1} = I_{a2} = I_{a0} = I_a/3 \text{ and}$$

$$V_{aa1}' = V_{aa2}' = V_{aa0}' = 0 \tag{4.35}$$

These relations suggest a series combination of the three sequence networks as shown in fig. 4.12. It is observed that a double conductor fault is similar to a LG fault at the fault point F of the system considered.

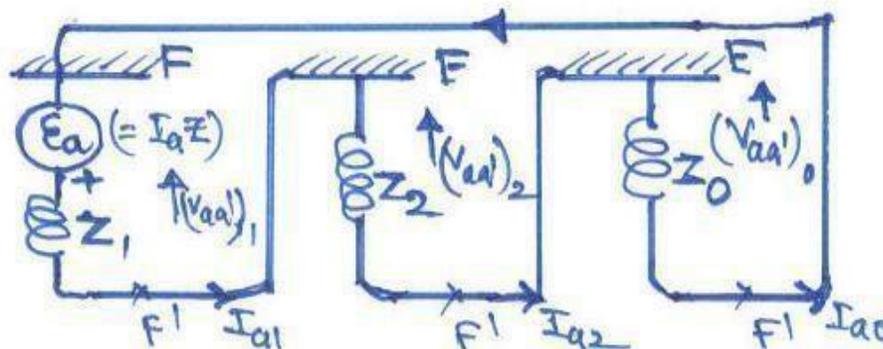


Figure 4.12 Sequence network connection for 2-conductor open fault.

(iii) *Three Conductor Open Fault*: consider all the three phases a, b and c, of a 3-phase system conductors be open. The conditions under fault are:

$$I_a + I_b + I_c = 0$$

The derived conditions are:

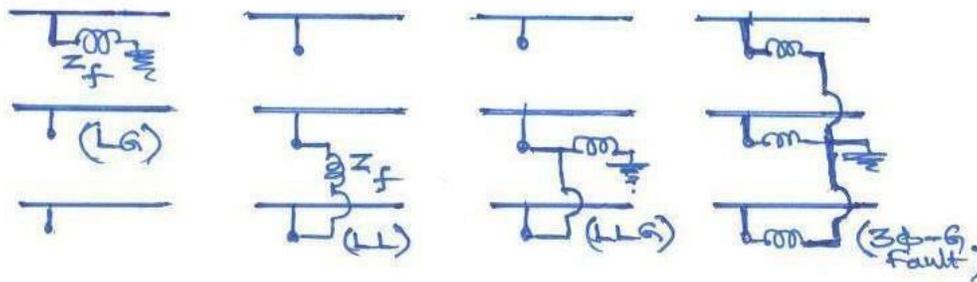
$$I_{a1} = I_{a2} = I_{a0} = 0 \text{ and}$$

$$V_{a0} = V_{a2} = 0 \text{ and } V_{a1} = V_f \tag{4.36}$$

These relations imply that the sequence networks are all open circuited. Hence, in a strict analytical sense, this is not a fault at all!

### FAULTS THROUGH IMPEDANCE

All the faults considered so far have comprised of a direct short circuit from one or two lines to ground. The effect of impedance in the fault is found out by deriving equations similar to those for faults through zero valued neutral impedance. The connections of the hypothetical stubs for consideration of faults through fault impedance  $Z_f$  are as shown in figure 4.13.



Fig

Figure 4.13 Stubs Connections for faults through fault impedance  $Z_f$ .

(i) *LG Fault at any point F of a given Power system through  $Z_f$*

Let phase 'a' be on fault at F through  $Z_f$ , so that then, the c.u.f. would be:

$$I_b = 0; I_c = 0; \text{ and } V_a = 0.$$

Hence the derived conditions under fault would be:

$$I_{a1} = I_{a2} = I_{a0} = (I_a/3)$$

$$I_{a1} = [V_f / (Z_1 + Z_2 + Z_0 + 3Z_f)] \text{ and}$$

$$I_f = 3I_{a1} \tag{4.37}$$

(ii) *LL Fault at any point F of a given Power system through  $Z_f$*

Let phases 'b' and 'c' be on fault at F through  $Z_f$ , so that then, the c.u.f. would be:

$$I_a = 0; I_b = -I_c; \text{ and } V_b = V_c$$

Hence the derived conditions under fault would be:

$$V_{a1} = V_{a2}; I_{a0} = 0; I_{a2} = -I_{a1}$$

$$I_{a1} = [V_f / (Z_1 + Z_2 + Z_f)] \text{ and}$$

$$I_f = I_b = -I_c = [\sqrt{3} V_f / (Z_1 + Z_2 + Z_f)] \tag{4.38}$$

(iii) *LLG Fault at any point F of a given Power system through  $Z_f$*

Let phases 'b' and 'c' be on fault at F through  $Z_f$ , so that then, the c.u.f. would be:

$$I_a = 0 \text{ and } V_b = V_c = 0$$

Hence the derived conditions under fault would be:

$$V_{a1} = V_{a2} = V_{a0} = (V_a/3)$$

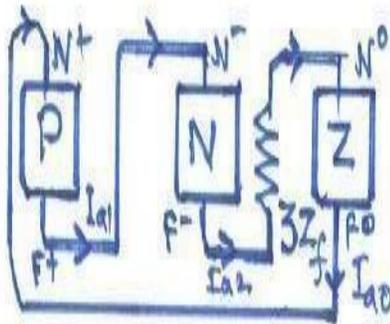
$$I_{a1} = \{V_f / [Z_1 + Z_2(Z_0 + 3Z_f)/(Z_2 + Z_0 + 3Z_f)]\}$$

$$I_{a2} = -I_{a1}(Z_0 + 3Z_f)/(Z_2 + Z_0 + 3Z_f); I_{a0} = -I_{a1}Z_2/(Z_2 + (Z_0 + 3Z_f)) \text{ and}$$

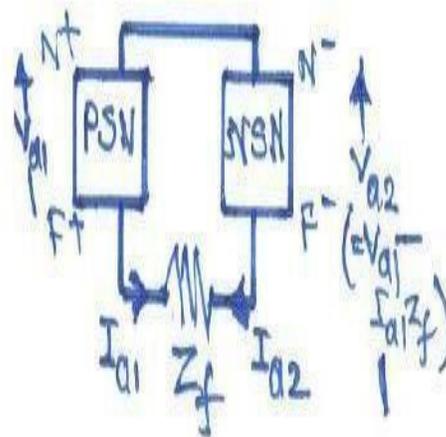
$$I_f = 3I_{a0} \tag{4.39}$$

(iv) Three Phase Fault at any point F of a given Power system through  $Z_f$

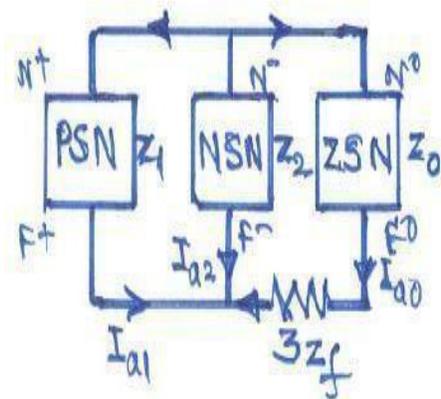
Let all the 3 phases a, b and c be on fault at F, through  $Z_f$  so that the c.u.f. would be:  $V_a = I_a Z_f$ ; Hence the derived conditions under fault would be:  $I_{a1} = [V_f / (Z_1 + Z_f)]$ ; The connections of the sequence networks for all the above types of faults through  $Z_f$  are as shown in figure 4.14.



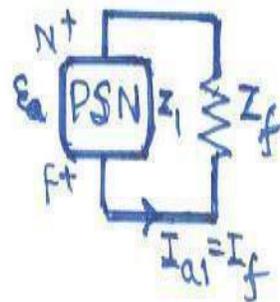
LG Fault



LL Fault



LLG Fault



3-Ph. Fault

Figure 4.15 Sequence network connections for faults through impedance

**EXAMPLES**

**Example-1:** A three phase generator with constant terminal voltages gives the following currents when under fault: 1400 A for a line-to-line fault and 2200 A for a line-to-ground fault. If the positive sequence generated voltage to neutral is 2 ohms, find the reactances of the negative and zero sequence currents.

Solution: Case a) Consider the conditions w.r.t. the LL fault:

$$I_{a1} = [E_{a1}/(Z_1 + Z_2)]$$

$$I_f = I_b = -I_c = \sqrt{3} I_{a1}$$

$$= \sqrt{3} E_{a1} / (Z_1 + Z_2) \text{ or}$$

$$(Z_1 + Z_2) = \sqrt{3} E_{a1} / I_f$$

$$\text{i.e., } 2 + Z_2 = \sqrt{3} [2000/1400]$$

Solving, we get,  $Z_2 = 0.474$  ohms.

Case b) Consider the conditions w.r.t. a LG fault:

$$I_{a1} = [E_{a1}/(Z_1 + Z_2 + Z_0)]$$

$$I_f = 3 I_{a1}$$

$$= 3 E_{a1} / (Z_1 + Z_2 + Z_0) \text{ or}$$

$$(Z_1 + Z_2 + Z_0) = 3 E_{a1} / I_f$$

$$\text{i.e., } 2 + 0.474 + Z_0 = 3 [2000/2200]$$

Solving, we get,  $Z_0 = 0.253$  ohms.

**Example-2:** A dead fault occurs on one conductor of a 3-conductor cable supplied by a 10 MVA alternator with earthed neutral. The alternator has +ve, -ve and 0-sequence components of impedances per phase respectively as:  $(0.5+j4.7)$ ,  $(0.2+j0.6)$  and  $(j0.43)$  ohms. The corresponding LN values for the cable up to the point of fault are:  $(0.36+j0.25)$ ,  $(0.36+j0.25)$  and  $(2.9+j0.95)$  ohms respectively. If the generator voltage at no load ( $E_{a1}$ ) is 6600 volts between the lines, determine the (i) Fault current, (ii) Sequence components of currents in lines and (iii) Voltages of healthy phases.

Solution: There is LG fault on any one of the conductors. Consider the LG fault to be on conductor in phase a. Thus the fault current is given by:

$$\text{(i) Fault current: } I_f = 3I_{a0} = [3E_a/(Z_1 + Z_2 + Z_0)]$$

$$= 3(6600/\sqrt{3}) / (4.32+j7.18)$$

$$= 1364.24 \angle 58.97^\circ$$

(ii) sequence components of line currents:

$$I_{a1} = I_{a2} = I_{a0} = I_a/3 = I_f/3 = 454.75 \angle 58.97^\circ.$$

(iii) Sound phase voltages:

$$V_{a1} = E_a - I_{a1}Z_1 = E_a(Z_2+Z_0)/(Z_1+Z_2+Z_0) = 1871.83 \angle -$$

$$26.17^\circ, V_{a2} = -E_aZ_2/(Z_1+Z_2+Z_0) = 462.91 \angle 177.6^\circ,$$

$$V_{a0} = -E_aZ_0/(Z_1+Z_2+Z_0) = 1460.54 \angle 146.5^\circ,$$

Thus,

$$\text{Sound phase voltages } V_b = a^2V_{a1} + aV_{a2} + V_{a0} = 2638.73 \angle -165.8^\circ$$

$$\text{Volts, And } V_c = aV_{a1} + a^2V_{a2} + V_{a0} = 3236.35 \angle 110.8^\circ \text{ Volts.}$$

**Example-3:** A generator rated 11 kV, 20 MVA has reactances of  $X_1=15\%$ ,  $X_2=10\%$  and  $X_0=20\%$ . Find the reactances in ohms that are required to limit the fault current to 2 p.u. when a line to ground fault occurs. Repeat the analysis for a LLG fault also for a fault current of 2 pu.

Solution: **Case a:** Consider the fault current expression for LG fault given by:

$$I_f = 3 I_{a0}$$

$$\text{i.e., } 2.0 = 3E_a / j[X_1+X_2+X_0]$$

$$= 3(1.0 \angle 0^\circ) / j[0.15+0.1+0.2+3X_n]$$

Solving we get

$$3X_n = 2.1 \text{ pu}$$

$$= 2.1 (Z_b) \text{ ohms} = 2.1 (11^2/20) = 2.1(6.05)$$

$$= 12.715 \text{ ohms.}$$

Thus  $X_n = 4.235 \text{ ohms.}$

**Case b:** Consider the fault current expression for LLG fault given

$$\text{by: } I_f = 3I_{a0} = 3 \{ -I_{a1}X_2/(X_2 + X_0+3X_n) \} = 2.0,$$

$$\text{where, } I_{a1} = \{ E_a / [X_1+X_2(X_0+3X_n)/(X_2+X_0+3X_n)] \}$$

Substituting and solving for  $X_n$  we get,

$$X_n = 0.078 \text{ pu}$$

$$= 0.47$$

$$\text{ohms.}$$

**Example-4:** A three phase 50 MVA, 11 kV generator is subjected to the various faults and the currents so obtained in each fault are: 2000 A for a three phase fault; 1800 A for a line-to-line fault and 2200 A for a line-to-ground fault. Find the sequence impedances of the generator.

Solution: *Case a)* Consider the conditions w.r.t. the three phase fault:

$$I_f = I_a = I_{a1} = E_{a1}/Z_1$$

$$\text{i.e., } 2000 = 11000 / (\sqrt{3}Z_1)$$

Solving, we get,  $Z_1 = 3.18 \text{ ohms}$  (1.3 pu, with  $Z_b = (11^2/50) = 2.42 \text{ ohms}$ ).

Case b) Consider the conditions w.r.t. the LL fault:

$$I_{a1} = [E_{a1}/(Z_1 + Z_2)]$$

$$I_f = I_b = -I_c = \sqrt{3} I_{a1}$$

$$= \sqrt{3} E_{a1} / (Z_1 + Z_2) \text{ or}$$

$$(Z_1 + Z_2) = \sqrt{3} E_{a1} / I_f$$

$$\text{i.e., } 3.18 + Z_2 = \sqrt{3} (11000/\sqrt{3})/1800$$

Solving, we get,  $Z_2 = 2.936 \text{ ohms} = 1.213 \text{ pu.}$

Case c) Consider the conditions w.r.t. a LG fault:

$$I_{a1} = [E_{a1}/(Z_1 + Z_2 + Z_0)]$$

$$I_f = 3 I_{a1}$$

$$= 3 E_{a1} / (Z_1 + Z_2 + Z_0) \text{ or}$$

$$(Z_1 + Z_2 + Z_0) = 3 E_{a1} / I_f$$

$$\text{i.e., } 3.18 + 2.936 + Z_0 = 3 (11000/\sqrt{3})/ 2200$$

Solving, we get,  $Z_0 = 2.55 \text{ ohms} = 1.054 \text{ pu.}$

## **CHAPTER 5**

### **POWER SYSTEM STABILITY**

#### **INTRODUCTION**

Power system stability of modern large inter-connected systems is a major problem for secure operation of the system. Recent major black-outs across the globe caused by system instability, even in very sophisticated and secure systems, illustrate the problems facing secure operation of power systems. Earlier, stability was defined as the ability of a system to return to normal or stable operation after having been subjected to some form of disturbance. This fundamentally refers to the ability of the system to remain in synchronism. However, modern power systems operate under complex interconnections, controls and extremely stressed conditions. Further, with increased automation and use of electronic equipment, the quality of power has gained utmost importance, shifting focus on to concepts of voltage stability, frequency stability, inter-area oscillations etc.

The IEEE/CIGRE Joint Task Force on stability terms and conditions have proposed the following definition in 2004: “*Power System stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded, so that practically the entire system remains intact*”.

The Power System is an extremely non-linear and dynamic system, with operating parameters continuously varying. Stability is hence, a function of the initial operating condition and the nature of the disturbance. Power systems are continually subjected to small disturbances in the form of load changes. The system must be in a position to be able to adjust to the changing conditions and operate satisfactorily. The system must also withstand large disturbances, which may even cause structural changes due to isolation of some faulted elements.

A power system may be stable for a particular (large) disturbance and unstable for another disturbance. It is impossible to design a system which is stable under all

disturbances. The power system is generally designed to be stable under those disturbances which have a high degree of occurrence. The response to a disturbance is extremely complex and involves practically all the equipment of the power system. For example, a short circuit leading to a line isolation by circuit breakers will cause variations in the power flows, network bus voltages and generators rotor speeds. The voltage variations will actuate the voltage regulators in the system and generator speed variations will actuate the prime mover governors; voltage and frequency variations will affect the system loads. In stable systems, practically all generators and loads remain connected, even though parts of the system may be isolated to preserve bulk operations. On the other hand, an unstable system condition could lead to cascading outages and a shutdown of a major portion of the power system.

### **CLASSIFICATION OF POWER SYSTEM STABILITY**

The high complexity of stability problems has led to a meaningful classification of the power system stability into various categories. The classification takes into account the main system variable in which instability can be observed, the size of the disturbance and the time span to be considered for assessing stability.

### **ROTOR ANGLE STABILITY**

Rotor angle stability refers to the ability of the synchronous machines of an interconnected power system to remain in synchronism after being subjected to a disturbance. Instability results in some generators accelerating (decelerating) and losing synchronism with other generators. Rotor angle stability depends on the ability of each synchronous machine to maintain equilibrium between electromagnetic torque and mechanical torque. Under steady state, there is equilibrium between the input mechanical torque and output electromagnetic torque of each generator, and its speed remains a constant. Under a disturbance, this equilibrium is upset and the generators accelerate/decelerate according to the mechanics of a rotating body. Rotor angle stability is further categorized as follows:

**Small single (or small disturbance) rotor angle stability:** It is the ability of the power system to maintain synchronism under small disturbances. In this case, the system equation can be linearized around the initial operating point and the stability depends only on the operating point and not on the disturbance. Instability may result in

- (i) A non oscillatory or a periodic increase of rotor angle
- (ii) Increasing amplitude of rotor oscillations due to insufficient damping.

The first form of instability is largely eliminated by modern fast acting voltage regulators and the second form of instability is more common. The time frame of small signal stability is of the order of 10-20 seconds after a disturbance.

**Large-signal rotor angle stability or transient stability:** This refers to the ability of the power system to maintain synchronism under large disturbances, such as short circuit, line outages etc. The system response involves large excursions of the generator rotor angles. Transient stability depends on both the initial operating point and the disturbance parameters like location, type, magnitude etc. Instability is normally in the form of a periodic angular separation. The time frame of interest is 3-5 seconds after disturbance.

The term dynamic stability was earlier used to denote the steady-state stability in the presence of automatic controls (especially excitation controls) as opposed to manual controls. Since all generators are equipped with automatic controllers today, dynamic stability has lost relevance and the Task Force has recommended against its usage.

## **VOLTAGE STABILITY**

Voltage stability refers to the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance. It depends on the ability of the system to maintain equilibrium between load demand and load supply. Instability results in a progressive fall or rise of voltages of some buses, which could lead to loss of load in an area or tripping of transmission lines, leading to cascading outages. This may eventually lead to loss of synchronism of some generators.

The cause of voltage instability is usually the loads. A run-down situation causing voltage instability occurs when load dynamics attempt to restore power consumption beyond the capability of the transmission network. Voltage stability is also threatened when a disturbance increases the reactive power demand beyond the sustainable capacity of the available reactive power resources. Voltage stability is categorized into the following sub-categories:

**Small – disturbance voltage stability:** It refers to the system's ability to maintain steady voltages when subjected to small perturbations such as incremental changes in load. This is primarily influenced by the load characteristics and the controls at a given point of time.

**Large disturbance voltage stability:** It refers to the systems ability to maintain steady voltages following large disturbances; It requires computation of the non-linear response of the power system to include interaction between various devices like motors, transformer tap changers and field current limiters. Short term voltage stability involves dynamics of fast acting load components and period of interest is in the order of several seconds. Long term voltage stability involves slower acting equipment like tap-changing transformers and generator current limiters. Instability is due to loss of long-term equilibrium.

### **FREQUENCY STABILITY**

Frequency stability refers to the ability of a power system to maintain steady frequency following a severe disturbance, causing considerable imbalance between generation and load. Instability occurs in the form of sustained frequency swings leading to tripping of generating units or loads. During frequency swings, devices such as under frequency load shedding, generator controls and protection equipment get activated in a few seconds. However, devices such as prime mover energy supply systems and load voltage regulators respond after a few minutes. Hence, frequency stability could be a short-term or a long-term phenomenon.

**MECHANICS OF ROTATORY MOTION**

Since a synchronous machine is a rotating body, the laws of mechanics of rotating bodies are applicable to it. In rotation we first define the fundamental quantities. The angle  $\theta_m$  is defined, with respect to a circular arc with its center at the vertex of the angle, as the ratio of the arc length  $s$  to radius  $r$ .

$$\theta_m = \frac{s}{r} \quad (5.1)$$

The unit is radian. Angular velocity  $\omega_m$  is defined as

$$\omega_m = \frac{d\theta_m}{dt} \quad (5.2)$$

and angular acceleration as

$$\alpha = \frac{d\omega_m}{dt} = \frac{d^2\theta_m}{dt^2} \quad (5.3)$$

The torque on a body due to a tangential force  $F$  at a distance  $r$  from axis of rotation is given by

$$T = r F \quad (5.4)$$

The total torque is the summation of infinitesimal forces, given by

$$T = \int r dF \quad (5.5)$$

The unit of torque is N-m. When torque is applied to a body, the body experiences angular acceleration. Each particle experiences a tangential acceleration  $a = r\alpha$ , where  $r$  is the distance of the particle from axis of rotation. The tangential force required to accelerate a particle of mass  $dm$  is

$$dF = a dm = r \alpha dm \quad (5.6)$$

The torque required for the particle is

$$dT = r dF = r^2 \alpha dm \quad (5.7)$$

and that required for the whole body is given by

$$T = \alpha \int r^2 dm = I \alpha \quad (5.8)$$

Here,

$$I = \int r^2 dm \quad (5.9)$$

It is called the moment of inertia of the body. The unit is  $\text{Kg} - \text{m}^2$ . If the torque is assumed to be the result of a number of tangential forces  $F$ , which act at different points of the body

$$T = \sum r F$$

Now each force acts through a distance,  $ds = r d\theta_m$  and the work done is  $\sum F \cdot ds$  i.e.,

$$dW = \sum F r d\theta_m = d\theta_m T \quad (5.10)$$

$$W = \int T d\theta_m \quad (5.11)$$

and  $T = \frac{dW}{d\theta_m}$

Thus the unit of torque may also be Joule per radian. The power is defined as rate of doing work. Using (5.11)

$$P = \frac{dW}{dt} = \frac{T d\theta_m}{dt} = T \omega_m \quad (5.12)$$

The angular momentum M is defined as

$$M = I \omega_m \quad (5.13)$$

And the kinetic energy is given by

$$KE = \frac{1}{2} I \omega_m^2 = \frac{1}{2} M \omega_m \quad (5.14)$$

From (5.14) we can see that the unit of M has to be J-sec/rad.

#### SWING EQUATION:

The laws of rotation developed in section.3 are applicable to the synchronous machine.

From(.5.8)

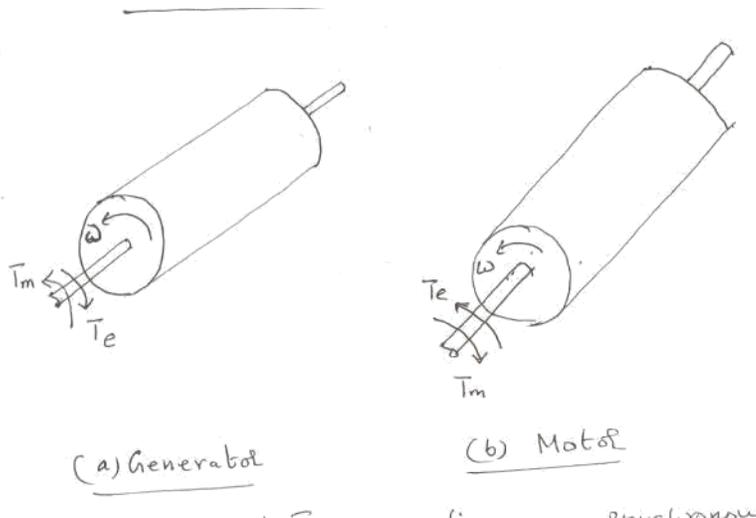
$$I\alpha = T$$

or  $\frac{I d^2\theta_m}{dt^2} = T \quad (5.15)$

Here T is the net torque of all torques acting on the machine, which includes the shaft torque (due to prime mover of a generator or load on a motor), torque due to rotational losses (friction, windage and core loss) and electromagnetic torque.

Let  $T_m$  = shaft torque or mechanical torque corrected for rotational losses  
 $T_e$  = Electromagnetic or electrical torque

For a generator  $T_m$  tends to accelerate the rotor in positive direction of rotation as shown in Fig 5.1. It also shows the corresponding torque for a motor with respect to the direction of rotation



(a) Generator

(b) Motor

**Fig. 5.1 Torque acting on a synchronous machine**

The accelerating torque for a generator is given by:

$$T^a = T_m - T_e \quad (5.16)$$

Under steady-state operation of the generator,  $T_m$  is equal to  $T_e$  and the accelerating torque is zero. There is no acceleration or deceleration of the rotor masses and the machines run at a constant synchronous speed. In the stability analysis in the following sections,  $T_m$  is assumed to be a constant since the prime movers (steam turbines or hydro turbines) do not act during the short time period in which rotor dynamics are of interest in the stability studies.

Now (5.15) has to be solved to determine  $\theta_m$  as a function of time. Since  $\theta_m$  is measured with respect to a stationary reference axis on the stator, it is the measure of the absolute rotor angle and increases continuously with time even at constant synchronous speed. Since machine acceleration /deceleration is always measured relative to synchronous speed, the rotor angle is measured with respect to a synchronously rotating reference axis. Let

5.17)

where  $\omega_{sm}$  is the synchronous speed in mechanical rad/s and  $\delta_m$  isplacement in mechanical radians. Taking the derivative of (5.17) we get

(5.18)

$$\frac{d\delta_m}{dt} = \frac{d\theta_m}{dt} \cdot \omega_{sm}$$

$$\frac{d^2\delta_m}{dt^2} = \frac{d^2\theta_m}{dt^2}$$

Substituting (5.18) in (5.15) we get

≍

$$I \frac{d^2\delta_m}{dt^2} = T_a - T_m - T_e \quad \text{N-m} \quad (5.19)$$

Multiplying by  $\omega_m$  on both sides we get

$$\omega_m I \frac{d^2\delta_m}{dt^2} = \omega_m (T_a - T_m - T_e) \quad \text{N-m} \quad (5.20)$$

From (5.12) and (5.13), we can write

$$M \frac{d^2\delta_m}{dt^2} = P_m - P_e \quad \text{W} \quad (5.21)$$

where M is the angular momentum, also called inertia constant,  $P_m$  is shaft power input less rotational losses,  $P_e$  is electrical power output corrected for losses and  $P_a$  is the acceleration power. M depends on the angular velocity  $\omega_m$ , and hence is strictly not a constant, because  $\omega_m$  deviates from the synchronous speed during and after a disturbance. However, under stable conditions  $\omega_m$  does not vary considerably and M can be treated as a constant. (21) is called the “Swing equation”. The constant M depends on the rating of the machine and varies widely with the size and type of the machine. Another constant called H constant (also referred to as inertia constant) is defined as

$$H = \frac{\text{stored kinetic energy in mega joules}}{\text{Machine rating in MVA}} \quad \text{MJ / MVA} \quad (5.22)$$

H falls within a narrow range and typical values are given in Table 5.1. If the rating of the machine is G MVA, from (5.22) the stored kinetic energy is GH Mega Joules. From (5.14)

$$GH = \frac{1}{2} M \omega_{sm}^2 \quad \text{MJ} \quad (5.23)$$

or

$$M = \frac{2GH}{\omega_{sm}^2} \quad \text{MJ-s/mech rad} \quad (5.24)$$

The swing equation (5.21) is written as

$$\frac{2H}{\omega_{sm}} \frac{d^2 \delta}{dt^2} = \frac{P_m - P_e}{G} \quad (5.25)$$

In (5.25)  $\delta$  is expressed in mechanical radians and  $\omega_{sm}$  in mechanical radians per second (the subscript  $m$  indicates mechanical units). If  $\delta$  and  $\omega$  have consistent units (both are mechanical or electrical units) (5.25) can be written as

$$\frac{2H}{\omega_s} \frac{d^2 \delta}{dt^2} = P_a - P_{me} \quad \text{pu} \quad (5.26)$$

Here  $\omega_s$  is the synchronous speed in electrical rad/s ( $\omega_s = \frac{p}{2} \omega_{sm}$ ) and  $P_a$  is

acceleration power in per unit on same base as H. For a system with an electrical frequency  $f$  Hz, (5.26) becomes

$$\frac{H}{\pi f} \frac{d^2 \delta}{dt^2} = P_a = P - P_{me} \quad \text{pu} \quad (5.27)$$

when  $\delta$  is in electrical radians and

$$\frac{H}{180 f} \frac{d^2 \delta}{dt^2} = P_a = P - P_{me} \quad \text{pu} \quad (5.28)$$

when  $\delta$  is in electrical degrees. Equations (5.27) and (5.28) also represent the swing equation. It can be seen that the swing equation is a second order differential equation which can be written as two first order differential equations:

$$\frac{2H}{\omega_s} \frac{d \omega}{dt} = P - P_{me} \quad \text{pu} \quad (5.29)$$

$$\frac{d \delta}{dt} = \omega - \omega_s \quad (5.30)$$

In which  $\omega$ ,  $\omega_s$  and  $\delta$  are in electrical units. In deriving the swing equation, damping has been neglected.

s

**Table 5.1 H constants of synchronous machines**

Type of machine	H (MJ/MVA)
Turbine generator condensing 1800 rpm	9 – 6
3600 rpm	7 – 4
Non condensing 3600rpm	4 – 3
Water wheel generator	
Slow speed < 200 rpm	2 – 3
High speed > 200 rpm	2 – 4
Synchronous condenser	1.25
Large	} 25% less for hydrogen cooled 1.0 }
Small	
Synchronous motor with load varying from 1.0 to 5.0	2.0

In defining the inertia constant H, the MVA base used is the rating of the machine. In a multi machine system, swing equation has to be solved for each machine, in which case, a common MVA base for the system has to be chosen. The constant H of each machine must be consistent with the system base.

Let  $G_{mach} = \text{Machine MVA rating (base)}$

$G_{system} = \text{System MVA base}$

In (5.25), H is computed on the machine rating  $G = G_{mach}$

Multiplying (5.25) by  $\frac{G_{mach}}{G_{system}}$  on both sides we get

$$\left( \frac{G_{mach}}{G_{system}} \right) \frac{2H_{system} d^2 \delta}{\omega_{sm}} = \frac{P_m - P_e}{G_{mach}} \left( \frac{G_{mach}}{G_{system}} \right) \quad (5.31)$$

$$\frac{2H_{system}}{\omega_{sm}} d^2 \delta_m = P_m - P_e \quad \text{pu (on system base)}$$

where  $H_{system} = H_{mach} \frac{G_{mach}}{G_{system}}$  (5.32)

In the stability analysis of a multi machine system, computation is considerably reduced if the number of swing equations to be solved is reduced. Machines within a plant normally swing together after a disturbance. Such machines are called coherent machines and can be replaced by a single equivalent machine, whose dynamics reflects the dynamics of the plant. The concept is best understood by considering a two machine system.

**SWING EQUATION OF TWO COHERENT MACHINES**

The swing equations for two machines on a common system base are:

$$\frac{2H_1}{\omega_s} \frac{d^2 \delta_1}{dt^2} = P_{m1} - P_{e1} \tag{5.33}$$

$$\frac{2H_2}{\omega_s} \frac{d^2 \delta_2}{dt^2} = P_{m2} - P_{e2} \tag{5.34}$$

Now  $\delta_1 = \delta_2 = \delta$  (since they swing together). Adding (5.33) and (5.34) we get

$$\frac{2H_{eq}}{\omega_s} \frac{d^2 \delta}{dt^2} = P_m - P_e \tag{5.35}$$

Where  $H_{eq} = H_1 + H_2$

$$P_m = P_{m1} + P_{m2}$$

$$P_e = P_{e1} + P_{e2}$$

The relation (5.35) represents the dynamics of the single equivalent machine.

**SWING EQUATION OF TWO NON – COHERENT MACHINES**

For any two non – coherent machines also (5.33) and (5.34) are valid. Subtracting

(5.34) from (33) we get

$$\frac{2}{\omega_s} \frac{d^2 \delta_1}{dt^2} - \frac{2}{\omega_s} \frac{d^2 \delta_2}{dt^2} = \frac{P_{m1} - P_{e1}}{H_1} - \frac{P_{m2} - P_{e2}}{H_2} \tag{5.36}$$

Multiplying both sides by  $\frac{H_1 H_2}{H_1 + H_2}$  we get

$$H_1 + H_2$$

$$\frac{2(H_1 H_2)}{\omega_s^2} \frac{d^2(\delta_1 - \delta_2)}{dt^2} = P_{m12} - P_{e12} \quad (5.37)$$

i.e.  $\delta$   
 where  $\delta_{12} = \delta_1 - \delta_2$ , the relative angle of the two machines

$$H_{12} = \frac{H_1 H_2}{H_1 + H_2}$$

$$P_{m12} = \frac{P_{m1} H_2 - P_{m2} H_1}{H_1 + H_2}$$

$$P_{e12} = \frac{P_{e1} H_2 - P_{e2} H_1}{H_1 + H_2}$$

From (5.37) it is obvious that the swing of a machine is associated with dynamics of other machines in the system. To be stable, the angular differences between all the machines must decrease after the disturbance. In many cases, when the system loses stability, the machines split into two coherent groups, swinging against each other. Each coherent group of machines can be replaced by a single equivalent machine and the relative swing of the two equivalent machines solved using an equation similar to (5.37), from which stability can be assessed.

The acceleration power is given by  $P_a = P_m - P_e$ . Hence, under the condition that  $P_m$  is a constant, an accelerating machine should have a power characteristic, which would increase  $P_e$  as  $\delta$  increases.

This would reduce  $P_a$  and hence the acceleration and help in maintaining stability. If on the other hand,  $P_e$  decreases when  $\delta$  increases,  $P_a$  would further increase which is

$$\frac{\partial P}{\partial \delta}$$

detrimental to stability. Therefore,  $\partial \delta$  must be positive for a stable system. Thus the power-angle relationship plays a crucial role in stability.

**POWER-ANGLE EQUATION:**

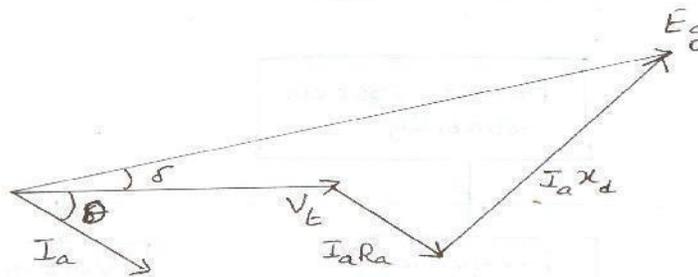
In solving the swing equation, certain assumptions are normally made

- (i) Mechanical power input  $P_m$  is a constant during the period of interest, immediately after the disturbance
- (ii) Rotor speed changes are insignificant.
- (iii) Effect of voltage regulating loop during the transient is neglected i.e the excitation is assumed to be a constant.

As discussed in section 4, the power-angle relationship plays a vital role in the solution of the swing equation.

**POWER-ANGLE EQUATION FOR A NON-SALIENT POLE**

**MACHINE:** The simplest model for the synchronous generator is that of a constant voltage behind an impedance. This model is called the classical model and can be used for cylindrical rotor (non-salient pole) machines. Practically all high-speed turbo alternators are of cylindrical rotor construction, where the physical air gap around the periphery of the rotor is uniform. This type of generator has approximately equal magnetic reluctance, regardless of the angular position of the rotor, with respect to the armature mmf. The phasor diagram of the voltages and currents at constant speed and excitation is shown in Fig. 5.2.



**Fig 5.2 Phasor diagram of a non-salient pole synchronous generator**

$E_g$  = Generator internal emf.

$V_t$  = Terminal voltage

$\theta$  = Power factor angle

$I_a$  = Armature current  
 $R_a$  = Armature resistance  
 $x_d$  = Direct axis reactance

The power output of the generator is given by the real part of  $E_g I_a^*$ .

$$I_a = \frac{E_g \angle \delta - V_t \angle 0^\circ}{R_a + jx_d} \tag{5.38}$$

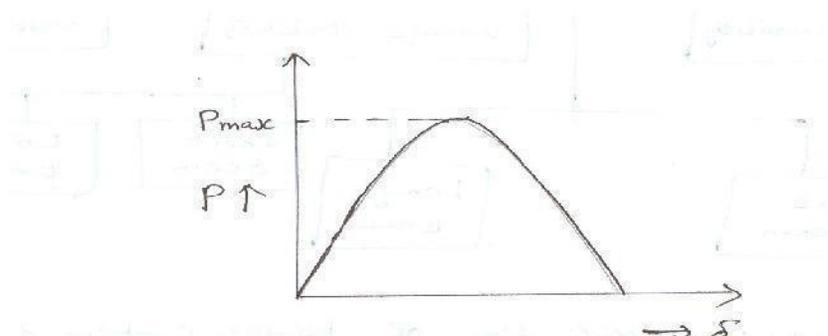
Neglecting  $R_a$ , 
$$I_a = \frac{E_g \angle \delta - V_t \angle 0^\circ}{jx_d}$$

$$P = \text{Re} \left\{ \left( E_g \angle \delta \right) \left( \frac{E_g \angle 90^\circ - \delta - V_t \angle 90^\circ}{x_d} \right)^* \right\}$$

$$= \frac{E_g^2 \cos 90^\circ}{x_d} - \frac{E_g V_t \cos (90^\circ + \delta)}{x_d}$$

$$= \frac{E_g V_t \sin \delta}{x_d} \tag{5.39}$$

(Note- **R** stands for real part of). The graphical representation of (9.39) is called the power angle curve and it is as shown in Fig 5.3.

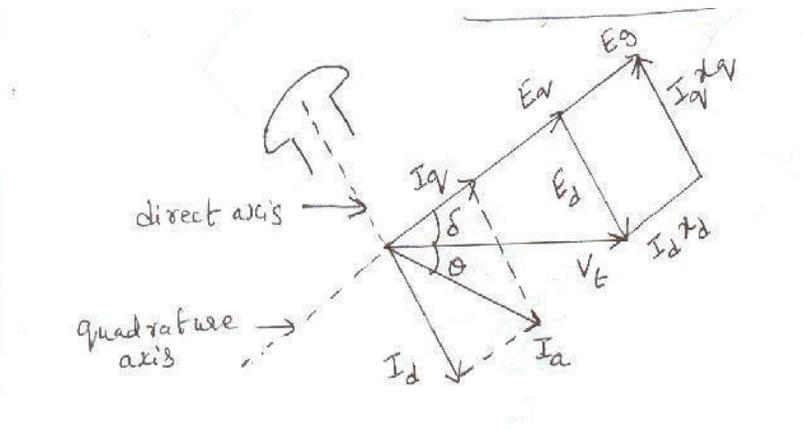


**Fig 5.3 Power angle curve of a non – salient pole machine**

The maximum power that can be transferred for a particular excitation is given by  $\frac{E_g V_t}{x_d}$  at  $\delta = 90^\circ$ .

**POWER ANGLE EQUATION FOR A SALIENT POLE MACHINE:**

Here because of the salient poles, the reluctance of the magnetic circuit in which flows the flux produced by an armature mmf in line with the quadrature axis is higher than that of the magnetic circuit in which flows the flux produced by the armature mmf in line with the direct axis. These two components of armature mmf are proportional to the corresponding components of armature current. The component of armature current producing an mmf acting in line with direct axis is called the direct component,  $I_d$ . The component of armature current producing an mmf acting in line with the quadrature axis is called the quadrature axis component,  $I_q$ . The phasor diagram is shown in Fig 5, with same terminology as Fig 5.4 and  $R_a$  neglected.



**Fig 5.4 Phasor diagram of a salient pole machine**

Power output  $P = V_t I_a \cos \theta$

$$= E_d I_d + E_q I_q \tag{5.40}$$

From Fig 5.4,

$$E_d = V_t \sin \delta; \quad E_q = V_t \cos \delta$$

$$I_d = \frac{E_g - E_q}{x_d} = I_a \sin(\delta + \theta)$$

$$I_q = \frac{E_d}{x_q} = I_a \cos(\delta + \theta) \tag{5.41}$$

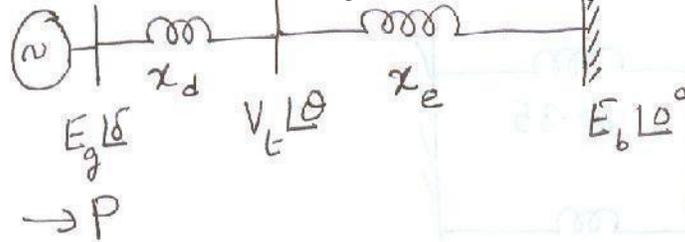
Substituting (5.41) in (5.40), we obtain

$$P = \frac{E_g V_t \sin \delta}{x_d} + \frac{V_t^2 (x_d - x_q) \sin 2\delta}{2 x_d x_q} \tag{5.42}$$

the relation (5.42) gives the steady state power angle relationship for a salient pole machine. The second term does not depend on the excitation and is called the reluctance power component. This component makes the maximum power greater than in the classical model. However, the angle at which the maximum power occurs is less than  $90^\circ$ .

**POWER ANGLE RELATIONSHIP IN A SMIB SYSTEM:**

Without loss of generality, many important conclusions on stability can be arrived at by considering the simple case of a Single Machine Infinite Bus (SMIB), where a generator supplies power to an infinite bus. The concept of an infinite bus arises from the fact that if we connect a generator to a much larger power system, it is reasonable to assume that the voltage and frequency of the larger system will not be affected by control of the generator parameters. Hence, the external system can be approximated by an infinite bus, which is equivalent to an ideal voltage source, whose voltage and frequency are constant. The one line diagram is shown in Fig 7.



**Fig. 5.5 SMIB System**

In Fig. 5.5, the infinite bus voltage is taken as reference and  $\delta$  is the angle between  $E_g$  and  $E_b$ . The generator is assumed to be connected to the infinite bus through a lossless line of reactance  $x_e$ . The power transferred (using classical model) is given by

$$P = \frac{E_g E_b}{x_d + x_e} \sin \delta \tag{5.43}$$

and using salient pole model,

$$P = \frac{E_g E_b}{x_d + x_e} \sin \delta + \frac{E_b^2 (x_d - x_q)}{2(x_d + x_e)} \sin 2\delta \tag{5.44}$$

An important measure of performance is the *steady state stability limit*, which is defined as *the maximum power that can be transmitted in steady state without loss of synchronism*, to the receiving end. If transient analysis is required, respective transient quantities namely  $E_g$ ,  $x_d$  and  $x_q$  are used in (5.43) and (5.44) to calculate the power output.

### **TRANSIENT STABILITY**

Transient stability is the ability of the system to remain stable under large disturbances like short circuits, line outages, generation or load loss etc. The evaluation of the transient stability is required offline for planning, design etc. and online for load management, emergency control and security assessment. Transient stability analysis deals with actual solution of the nonlinear differential equations describing the dynamics of the machines and their controls and interfacing it with the algebraic equations describing the interconnections through the transmission network.

Since the disturbance is large, linearized analysis of the swing equation (which describes the rotor dynamics) is not possible. Further, the fault may cause structural changes in the network, because of which the power angle curve prior to fault, during the fault and post fault may be different (See example 9.8). Due to these reasons, a general stability criteria for transient stability cannot be established, as was done in the case of steady state stability (namely  $P_s > 0$ ). Stability can be established, for a given fault, by actual solution of the swing equation. The time taken for the fault to be cleared (by the circuit breakers) is called the *clearing time*. If the fault is cleared fast enough, the probability of the system remaining stable after the clearance is more. If the fault persists for a longer time, likelihood of instability is increased.

*Critical clearing time* is the maximum time available for clearing the fault, before the system loses stability. Modern circuit breakers are equipped with auto reclosure facility, wherein the breaker automatically recloses after two sequential openings. If the fault still persists, the breakers open permanently. Since most faults are transient, the first reclosure

is in general successful. Hence, transient stability has been greatly enhanced by auto closure breakers.

Some common assumptions made during transient stability studies are as follows:

1. Transmission line and synchronous machine resistances are neglected. Since resistance introduces a damping term in the swing equation, this gives pessimistic results.
2. Effect of damper windings is neglected which again gives pessimistic results.
3. Variations in rotor speed are neglected.
4. Mechanical input to the generator is assumed constant. The governor control loop is neglected. This also leads to pessimistic results.
5. The generator is modeled as a constant voltage source behind a transient reactance, neglecting the voltage regulator action.
6. Loads are modeled as constant admittances and absorbed into the bus admittance matrix.

The above assumptions, vastly simplify the equations. A digital computer program for transient stability analysis can easily include more detailed generator models and effect of controls, the discussion of which is beyond the scope of present treatment. Studies on the transient stability of an SMIB system, can shed light on some important aspects of stability of larger systems. One of the important methods for studying the transient stability of an SMIB system is the application of equal-area criterion.

### **5. 8 EQUAL- AREA CRITERION**

Transient stability assessment of an SMIB system is possible without resorting to actual solution of the swing equation, by a method known as equal–area criterion. In a SMIB system, if the system is unstable after a fault is cleared,  $\delta(t)$  increases indefinitely with time, till the machine loses synchronism. In contrast, in a stable system,  $\delta(t)$  reaches a maximum and then starts reducing as shown in Fig.5.6.

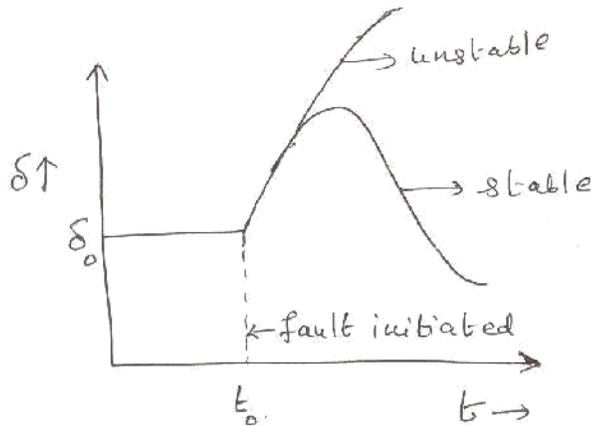


Fig.5.6 Swing Curve ( $\delta$  VS  $t$ ) for stable and unstable system

Mathematically stated,

$$\frac{d\delta(t)}{dt} = 0$$

some time after the fault is cleared in a stable system and  $\frac{d\delta}{dt} > 0$ , for a long time after

the fault is cleared in an unstable system.

Consider the swing equation (21)

$$\frac{M}{P_a} \frac{d^2 \delta}{dt^2} = P_m - P_e$$

$$\frac{d^2 \delta}{dt^2} = \frac{P}{M}$$

Multiplying both sides by  $2 \frac{d\delta}{dt}$ , we get

$$2 \frac{d\delta}{dt} \frac{d^2 \delta}{dt^2} = 2 \frac{d\delta}{dt} \frac{P}{M}$$

This may be written as

$$\left[ \left( \frac{d\delta}{dt} \right)^2 \right] = \frac{2P}{M} \delta + C$$

Integrating both sides we get

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M} \int_{\delta_0}^{\delta} P_a d\delta$$

$$\text{or } \frac{d\delta}{dt} = \sqrt{\frac{2}{M} \int_{\delta_0}^{\delta} P_a d\delta} \tag{5.45}$$

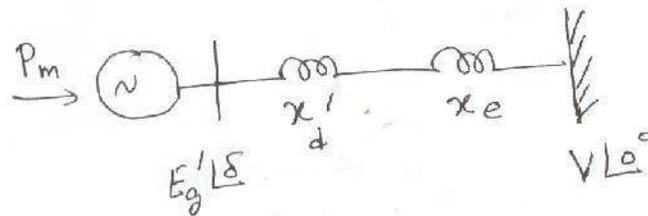
For stability  $\frac{d\delta}{dt} = 0$ , some time after fault is cleared. This means

$$\int_{\delta_0}^{\delta} P_a d\delta = 0 \tag{5.46}$$

The integral gives the area under the  $P_a - \delta$  curve. The condition for stability can be, thus stated as follows: A SMIB system is stable if the area under the  $P_a - \delta$  curve, becomes zero at some value of  $\delta$ . This means that the accelerating (positive) area under  $P_a - \delta$  curve, must equal the decelerating (negative) area under  $P_a - \delta$  curve. Application of equal area criterion for several disturbances is discussed next.

**SUDDEN CHANGE IN MECHANICAL INPUT**

Consider the SMIB system shown in Fig. 5.7.



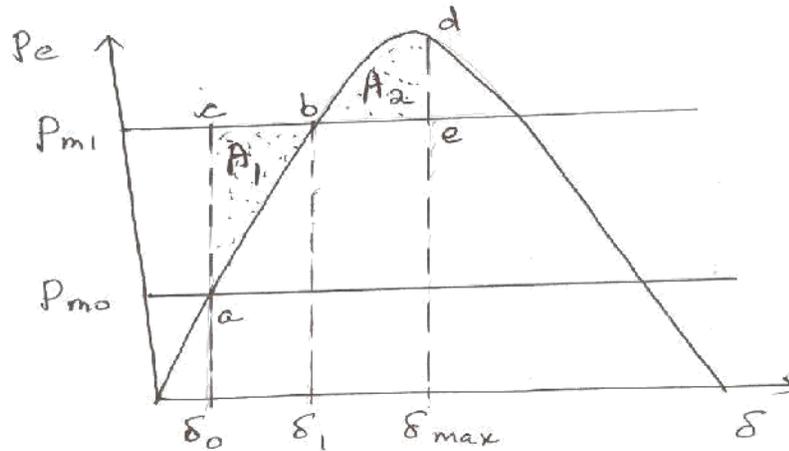
**Fig.5.7 SMIB System**

The electrical power transferred is given by

$$P_e = P_{\max} \sin \delta$$

$$P_{\max} = \frac{E_g V}{x_d' + x_e}$$

Under steady state  $P_m = P_e$ . Let the machine be initially operating at a steady state angle  $\delta_0$ , at synchronous speed  $\omega_s$ , with a mechanical input  $P_{m0}$ , as shown in Fig.5.8 ( point a).



**Fig.5.8 Equal area criterion–sudden change in mechanical input**

If there is a sudden step increase in input power to  $P_{m1}$  the accelerating power is positive (since  $P_{m1} > P_{m0}$ ) and power angle  $\delta$  increases. With increase in  $\delta$ , the electrical power  $P_e$  increases, the accelerating power decreases, till at  $\delta = \delta_1$ , the electrical power matches the new input  $P_{m1}$ . The area  $A_1$ , during acceleration is given by

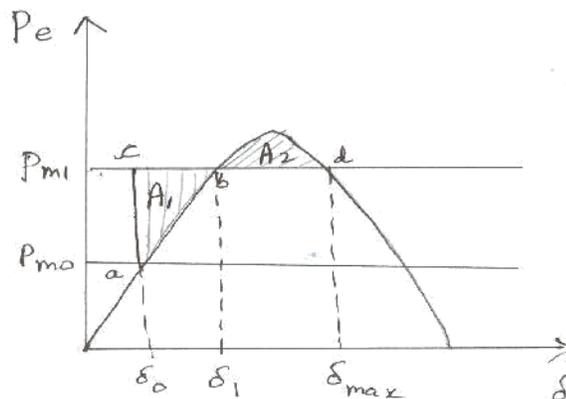
$$A_1 = \int_{\delta_0}^{\delta_1} (P_e - P_{m1}) d\delta$$

$$= P_{m1} (\delta_1 - \delta_0) - P_{max} (\cos \delta_0 - \cos \delta_1) \quad (5.47)$$

At  $b$ , even though the accelerating power is zero, the rotor is running above synchronous speed. Hence,  $\delta$  and  $P_e$  increase beyond  $b$ , wherein  $P_e < P_{m1}$  and the rotor is subjected to deceleration. The rotor decelerates and speed starts dropping, till at point  $d$ , the machine reaches synchronous speed and  $\delta = \delta_{max}$ . The area  $A_2$ , during deceleration is given by

$$A_2 = \int_{\delta_1}^{\delta_{max}} (P_{m1} - P_e) d\delta = P_{m1} (\delta_{max} - \delta_1) - P_{max} (\cos \delta_{max} - \cos \delta_1) \quad (5.48)$$

By equal area criterion  $A_1 = A_2$ . The rotor would then oscillate between  $\delta_0$  and  $\delta_{max}$  at its natural frequency. However, damping forces will reduce subsequent swings and the machine finally settles down to the new steady state value  $\delta_1$  (at point  $b$ ). Stability can be maintained only if area  $A_2$  at least equal to  $A_1$ , can be located above  $P_{m1}$ . The limiting case is shown in Fig.5.9, where  $A_2$  is just equal to  $A_1$ .



**Fig.5.9 Maximum increase in mechanical power**

Here  $\delta_{max}$  is at the intersection of  $P_e$  and  $P_{m1}$ . If the machine does not reach synchronous speed at  $d$ , then beyond  $d$ ,  $P_e$  decreases with increase in  $\delta$ , causing  $\delta$  to increase indefinitely. Applying equal area criterion to Fig.5.9 we get

$$A_1 = A_2.$$

From (5.47) and (5.48) we get

$$P_{m1} (\delta_{max} - \delta_0) = P_{max} (\cos \delta_0 - \cos \delta_{max})$$

Substituting  $P_{m1} = P_{max} \sin \delta_{max}$ , we get

$$(\delta_{max} - \delta_0) \sin \delta_{max} + \cos \delta_{max} = \cos \delta_0 \tag{5.49}$$

Equation (5.49) is a non-linear equation in  $\delta_{max}$  and can be solved by trial and error or by using any numerical method for solution of non-linear algebraic equation (like Newton-Raphson, bisection etc). From solution of  $\delta_{max}$ ,  $P_{m1}$  can be calculated.  $P_{m1} - P_{m0}$  will give the maximum possible increase in mechanical input before the machine loses stability.

### NUMERICAL EXAMPLES

**Example 1:** A 50Hz, 4 pole turbo alternator rated 150 MVA, 11 kV has an inertia constant of 9 MJ / MVA. Find the (a) stored energy at synchronous speed (b) the rotor acceleration if the input mechanical power is raised to 100 MW when the electrical load is 75 MW, (c) the speed at the end of 10 cycles if acceleration is assumed constant at the initial value.

**Solution:**

(a) Stored energy = GH = 150 × 9 = 1350 MJ

(b) P<sub>a</sub> = P<sub>m</sub> - P<sub>e</sub> = 100 - 75 = 25 MW

$$M = \frac{GH}{180f} = \frac{1350}{180 \times 50} = 0.15 \text{ MJ - s / } ^\circ\text{e}$$

$$0.15 \frac{d^2 \delta}{dt^2} = 25$$

$$\text{Acceleration } \alpha = \frac{d^2 \delta}{dt^2} = \frac{25}{0.15} = 166.6 \text{ } ^\circ\text{e/s}^2$$

$$= 166.6 \times \frac{2}{P} \text{ } ^\circ\text{m/s}^2$$

$$= 166.6 \times \frac{2}{P} \times \frac{1}{360} \text{ rps /s}$$

$$= 166.6 \times \frac{2}{P} \times \frac{1}{360} \times 60 \text{ rpm/s}$$

$$= 13.88 \text{ rpm/s}$$

\* Note °e = electrical degree; °m = mechanical degree; P=number of poles.

(c) 10 cycles =  $\frac{10}{50} = 0.2 \text{ s}$

$$N_s = \text{Synchronous speed} = \frac{120 \times 50}{\text{rpm } 4} = 1500$$

Rotor speed at end of 10 cycles = N<sub>S</sub> + α × 0.2 = 1500 + 13.88 × 0.2 = 1502.776 rpm.

**Example 2:** Two 50 Hz generating units operate in parallel within the same plant, with the following ratings: Unit 1: 500 MVA, 0.8 pf, 13.2 kV, 3600 rpm: H = 4 MJ/MVA; Unit 2: 1000 MVA, 0.9 pf, 13.8 kV, 1800 rpm: H = 5 MJ/MVA. Calculate the equivalent H constant on a base of 100 MVA.

**Solution:**

$$H_{1 \text{ system}} = H_{1 \text{ mach}} \times \frac{G}{G^{\text{system}}} = 4 \times \frac{500}{100} = 20 \text{ MJ/MVA}$$

$$H_{2 \text{ system}} = H_{2 \text{ mach}} \times \frac{G}{G^{\text{system}}} = 5 \times \frac{1000}{100} = 50 \text{ MJ/MVA}$$

$$H_{eq} = H_1 + H_2 = 20 + 50 = 70 \text{ MJ/MVA}$$

This is the equivalent inertia constant on a base of 100 MVA and can be used when the two machines swing coherently.

**Example 3:** Obtain the power angle relationship and the generator internal emf for (i) classical model (ii) salient pole model with following data:  $x_d = 1.0 \text{ pu} : x_q = 0.6 \text{ pu} : V_t = 1.0 \text{ pu} : I_a = 1.0 \text{ pu}$  at upf

**Solution:**

(i) **Classical model:** The phasor diagram is shown in Fig P3.

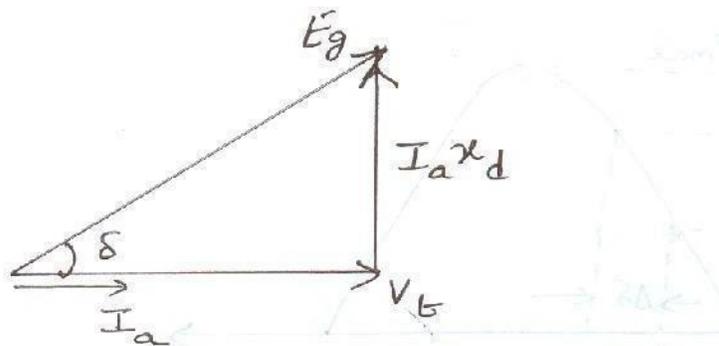


Fig.P3 Example 3, case(i)

$$|E_g| = \sqrt{V_t^2 + (I_a x_d)^2} = \sqrt{(1.0)^2 + (1.0 \times 1.0)^2} = 1.414$$

$$\delta = \tan^{-1} \frac{I_a x_d}{V_t} = \tan^{-1} \frac{1.0}{1.0} = 45^\circ$$

$$\therefore E_g = 1.414 \angle 45^\circ .$$

If the excitation is held constant so that  $|E_g| = 1.414$ , then power output

$$P = \frac{1.414 \times 1.0 \sin \delta}{\sin \delta 1.0} = 1.414$$

(ii) **Salient pole:** From Fig (5), we get using (41a) to

$$\begin{aligned} (41d) E_g &= E_q + I_d x_d = V_t \cos \delta + I_d x_d \\ &= V_t \cos \delta + I_a \sin \delta x_d \end{aligned}$$

(\*  $\theta = 0^\circ$ , since we are considering upf)

Substituting given values we get

$$E_g = \cos \delta + \sin \delta.$$

Again from Fig (9.5) we have

$$E_d = V_t \sin \delta = I_q x_q$$

$$\therefore V_t \sin \delta - I_q x_q = 0$$

$$V_t \sin \delta - I_a \cos \delta x_q = 0$$

Substituting the given values we get

$$0 = \sin \delta - 0.6 \cos \delta$$

We thus have two simultaneous equations.

$$E_g = \cos \delta + \sin \delta$$

$$0 = \sin \delta - 0.6 \cos \delta$$

Solving we get  $\delta = 30.96^\circ$

$$E_g = 1.372 \text{ pu}$$

If the excitation is held constant, then from (42)

$$P = 1.372 \sin \delta + 0.333 \sin 2\delta$$

**Example 4:** Determine the steady state stability limit of the system shown in Fig 8, if  $V_t = 1.0 \text{ pu}$  and the reactances are in pu.

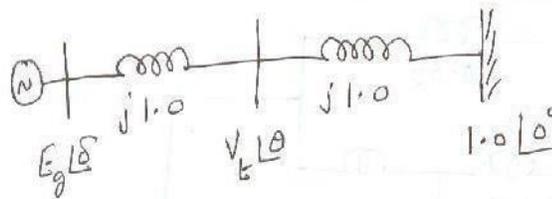


Fig. P4 Example 4

**Solution:**

$$\text{Current } I = \frac{V_t \angle \theta - 1.0 \angle 0^\circ}{j1.0} = \frac{1.0 \angle \theta - 1.0 \angle 0^\circ}{j1.0}$$

$$E_g \angle \delta = V_t \angle \theta + j1.0 (I)$$

$$\begin{aligned}
 &= 1 \frac{\angle\theta + j1.0(1.0\angle\theta - 1.0\angle 0^\circ)}{j1.0} \\
 &= \cos\theta + j\sin\theta + \cos\theta + j\sin\theta - 1.0 \\
 &= 2\cos\theta - 1 + j2\sin\theta
 \end{aligned}$$

When maximum power is transferred  $\delta = 90^\circ$ ; which means real part of  $E = 0$

$$\therefore 2\cos\theta - 1 = 0$$

$$\theta = \cos^{-1} 0.5 = 60^\circ$$

$$|E_g| = 2 \times \sin 60^\circ = 1.732$$

$$E_g = 1.732 \angle 90^\circ \text{ (for maximum power)}$$

$$\text{Steady state stability limit} = \frac{1.732 \times 1.0}{\text{pu } 1.0 + 1.0} = 0.866$$

**Example 5:** A 50 Hz synchronous generator having an internal voltage 1.2 pu,  $H = 5.2$  MJ/MVA and a reactance of 0.4 pu is connected to an infinite bus through a double circuit line, each line of reactance 0.35 pu. The generator is delivering 0.8 pu power and the infinite bus voltage is 1.0 pu. Determine: maximum power transfer, Steady state operating angle, and Natural frequency of oscillation if damping is neglected.

**Solution:** The one line diagram is shown in Fig P5.

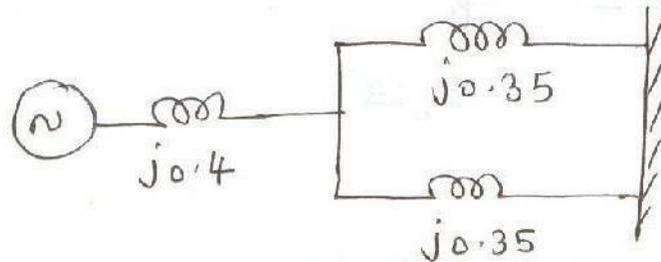


Fig . P5 Example 6

$$(a) X = 0.4 + 0.35 = 0.575 \text{ pu}$$

$$2P_{\max} = \frac{E_g \overline{E_b}}{X} = \frac{1.2 \times 1.0}{0.575} = 2.087 \text{ pu}$$

(b)  $P_e = P_{\max} \sin \delta_o$

$$\therefore \delta_o = \sin^{-1} \frac{P_e}{P_{\max}} = \sin^{-1} \left( \frac{0.8}{2.087} \right) = 22.54^\circ$$

(c)  $P_s = P_{\max} \cos \delta_o = 2.087 \cos (22.54^\circ)$   
 $= 1.927 \text{ MW (pu)/ elec rad.}$

$$M \text{ (pu)} = \frac{H}{\Pi f} = \frac{5.2}{\Pi \times 50} = 0.0331 \text{ s}^2 / \text{elec rad}$$

Without damping  $s = \pm j \sqrt{\frac{P_s}{M}} = \pm j \sqrt{\frac{1.927}{0.0331}}$   
 $= \pm j 7.63 \text{ rad/sec} = 1.21 \text{ Hz}$

Natural frequency of oscillation  $\omega_n = 1.21 \text{ Hz.}$

**Example 6:** In example .6, if the damping is 0.14 and there is a minor disturbance of  $\Delta \delta = 0.15 \text{ rad}$  from the initial operating point, determine: (a)  $\omega_n$  (b)  $\xi$  (c)  $\omega_d$  (d) setting time and (e) expression for  $\delta$ .

**Solution:**

(a)  $\omega_n = \sqrt{\frac{P_s}{M}} = \sqrt{\frac{1.927}{0.0331}} = 7.63 \text{ rad/sec} = 1.21 \text{ Hz}$

(b)  $\xi = \frac{D}{2 \sqrt{M P_s}} = \frac{0.14}{2 \sqrt{0.0331 \times 1.927}} = 0.277$

(c)  $\omega_d = \omega_n \sqrt{1 - \xi^2} = 7.63 \sqrt{1 - (0.277)^2} = 7.33 \text{ rad/sec} = 1.16 \text{ Hz}$

(d) Setting time  $= 4\tau = 4 \cdot \frac{1}{\xi \omega_n} = 4 \times \frac{1}{0.277 \times 7.63} = 1.892 \text{ s}$

(e)  $\Delta \delta_o = 0.15 \text{ rad} = 8.59^\circ$

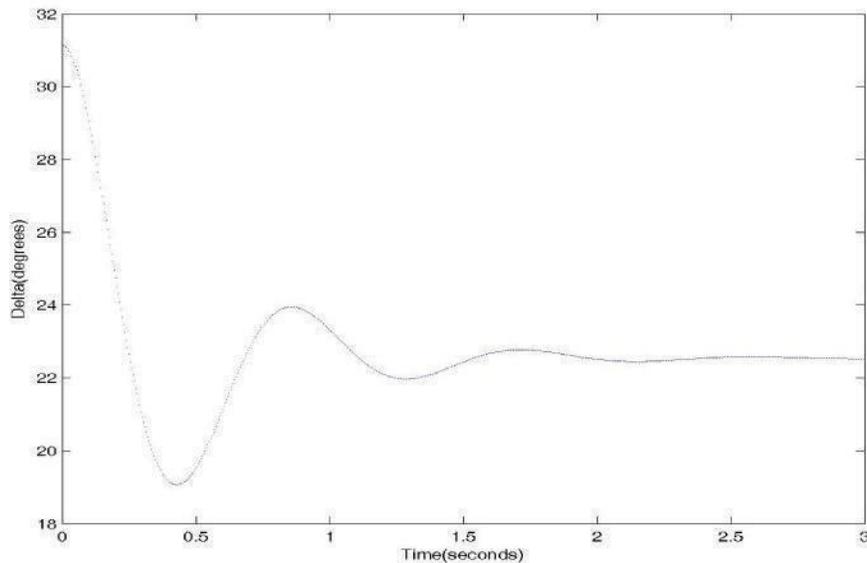
$$\theta = \cos^{-1} \xi = \cos^{-1} 0.277 = 73.9^\circ$$

$$\delta = \delta_o + \frac{\Delta \delta_o}{\sqrt{1 - \xi^2}} e^{-\xi \omega_n t} \sin(\omega_d t + \theta)$$

$$= 22.54^\circ + \frac{8.59}{\sqrt{1 - 0.277^2}} e_{-0.277 \times 7.63 t} \sin(7.33 t + 73.9^\circ)$$

$$= 22.54^\circ + 8.94 e^{-2.11t} \sin(7.33t + 73.9^\circ)$$

The variation of delta with respect to time is shown below. It can be observed that the angle reaches the steady state value of  $22.54^\circ$  after the initial transient. It should be noted that the magnitudes of the swings decrease in a stable system with damping.



**Fig.P6 Swing Curve for example 7**

**Example 7:** In example 6, find the power angle relationship

- (i) For the given network
- (ii) If a short circuit occurs in the middle of a line
- (iii) If fault is cleared by line outage

Assume the generator to be supplying 1.0 pu power initially.

**Solution:**

- (i) From example 6,  $P_{\max} = 2.087$ ,  $P_e = 2.087 \sin \delta$ .
- (ii) If a short circuit occurs in the middle of the line, the network equivalent can be drawn as shown in Fig. 12a.

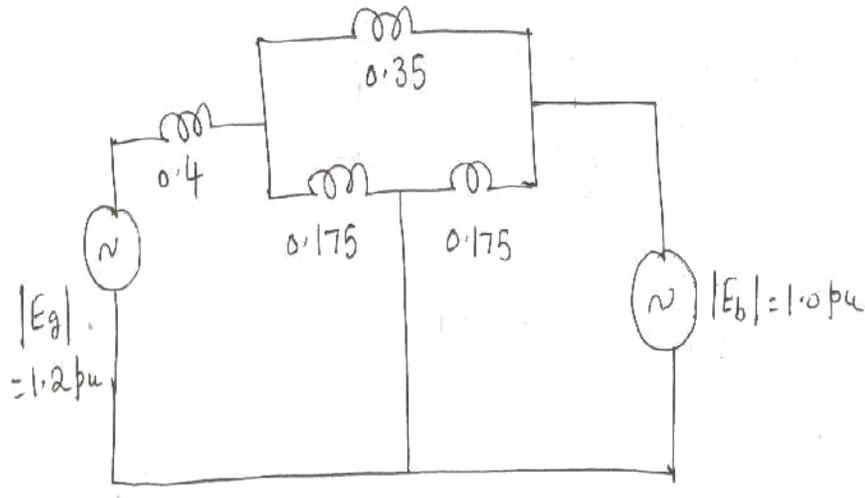


Fig.P7a Short circuit in middle of line

The network is reduced by converting the delta to star and again the resulting star to delta as shown in Fig P7a, P7b and P7c.

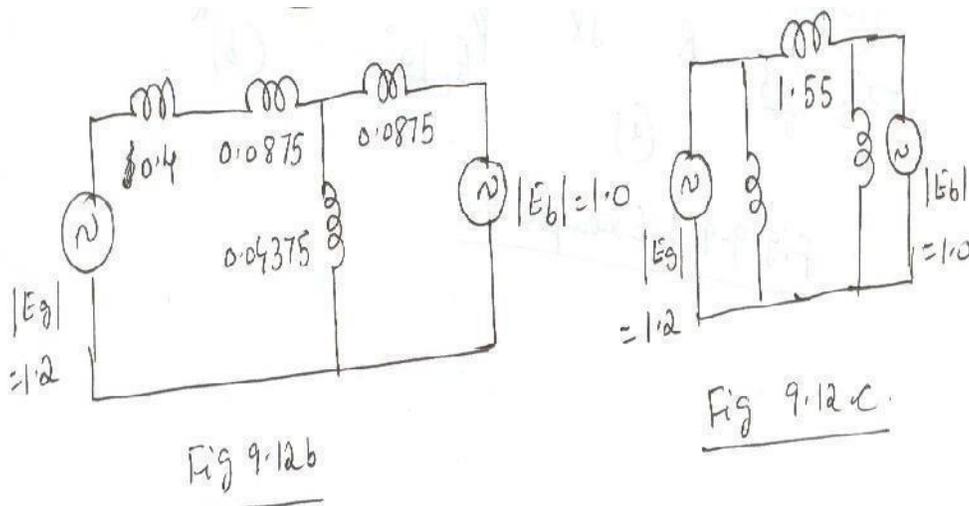


Fig.P7b

Fig.P7c

The transfer reactance is 1.55 pu. Hence,

$$P_{\max} = \frac{1.2 \times 1.0}{1.55} = 0.744 ; P_e = 0.744 \sin \delta$$

(iii) When there is a line outage

$$X = 0.4 + 0.35 = 0.75$$

$$P_{\max} = \frac{1.2 \times 1.0}{0.75} = 1.6$$

$$P_e = 1.6 \sin \delta$$

**Example 8:** A generator supplies active power of 1.0 pu to an infinite bus, through a lossless line of reactance  $x_e = 0.6$  pu. The reactance of the generator and the connecting transformer is 0.3 pu. The transient internal voltage of the generator is 1.12 pu and infinite bus voltage is 1.0 pu. Find the maximum increase in mechanical power that will not cause instability.

**Solution:**

$$P_{\max} = \frac{1.12 \times 1.0}{0.9} = 1.244$$

$$P_{m0} = P_{e0} = 1.0 = P_{\max} \sin \delta_0 = 1.244 \sin \delta_0$$

$$\therefore \delta_0 = \sin^{-1} \frac{1.0}{1.244} = 53.47^\circ = 0.933 \text{ rad.}$$

The above can be solved by N-R method since it is of the form  $f(\delta_{\max}) = K$ . Applying N-R method, at any iteration 'r', we get

$$\delta_{\max}^{(r)} = \frac{K - f(\delta_{\max}^{(r)})}{\frac{df}{d\delta_{\max}^{(r)}}}$$

$$\frac{df}{d\delta_{\max}^{(r)}} = \left( \frac{\delta_{\max}^{(r)}}{x} - \delta_0 \right) \cos \delta_{\max}^{(r)}$$

(This is the derivative evaluated at a value of  $\delta = \delta_{\max}^{(r)}$ )  $\delta_{\max}^{(r+1)} = \delta_{\max}^{(r)} + \Delta \delta_{\max}^{(r)}$

Starting from an initial guess of  $\delta_{\max}$  between  $\frac{\pi}{2}$  to  $\pi$ , the above equations are solved

iteratively till  $\delta_{\max}^{(r)} \leq \epsilon$ . Here  $K = \cos \delta_0 = 0.595$ . The computations are shown in table

P8, starting from an initial guess  $\delta_{\max}^{(1)} = 1.745$  rad.

Table P8

Interaction r	$\delta_{\max}^{(r)}$	$\frac{df}{d\delta_{\max}^{(r)}}$	$f(\delta_{\max}^{(r)})$	$\Delta \delta_{\max}^{(r)}$	$\delta_{\max}^{(r+1)}$
1	1.745	-0.1407	0.626	0.22	1.965
2	1.965	-0.396	0.568	-0.068	1.897
3	1.897	-0.309	0.592	-0.0097	1.887
4	1.887	-0.2963	0.596	-0.0033	1.883

Since  $\Delta \delta_{\max}^{(r)}$  is sufficiently small, we can take

$$\delta_{\max} = 1.883 \text{ rad} = 107.88^\circ$$

$$\delta_1 = 180 - \delta_{\max} = 72.1^\circ$$

$$P_{m1} = P_{\max} \sin \delta_{\max} = 1.183$$

Maximum step increase permissible is  $P_{m1} - P_{m0} = 1.183 - 1.0 = 0.183 \text{ pu}$

**Example 9:** Transform a two machine system to an equivalent SMIB system and show how equal area criterion is applicable to it.

**Solution:** Consider the two machine system show in Fig.P9.

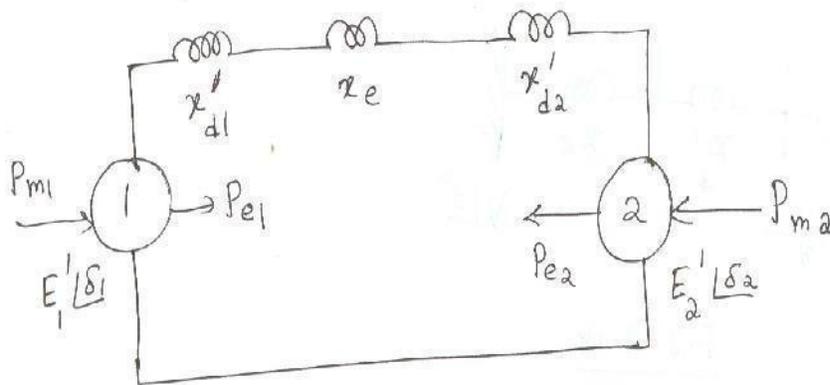


Fig.P9 Two machine system under steady state (neglecting losses)

$$P_{m1} = -P_{m2} = P_m ; P_{e1} = -P_{e2} = P_e$$

The swing equations are

$$\frac{d^2 \delta_1}{dt^2} = \frac{P_{m1} - P_{e1}}{M_1} = \frac{P_m - P_e}{M_1}$$

$$\frac{d^2 \delta_2}{dt^2} = \frac{P_{m2} - P_{e2}}{M_2} = \frac{P_e - P_m}{M_2}$$

Simplifying, we get

$$\frac{d^2 (\delta_1 - \delta_2)}{dt^2} = \frac{M_1 + M_2}{M_1 M_2} (P_m - P_e)$$

$$\text{or } M_{eq} \frac{d^2 \delta}{dt^2} = P_m - P_e$$

$$\text{where } M_{eq} = \frac{M_1 M_2}{M_1 + M_2}$$

$$\delta = \delta_1 - \delta_2$$

$$P_e = \frac{E'_1 E'_2}{x'_{d1} + x_e + x'_{d2}} \sin \delta$$

This relation is identical to that of an SMIB system in form and can be used to determine the relative swing ( $\delta_1 - \delta_2$ ) between the two machines to assess the stability.