LECTURE NOTES

ON

ELECTRICAL CIRCUITS-II

2019 – 2020

II B. Tech I Semester (JNTUA-R15)

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JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY ANANTAPUR

B. Tech II - I sem (E.E.E)

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(15A02301) ELECTRICAL CIRCUITS- II

OBJECTIVES:

To make the students learn about:

- How to determine the transient response of R-L, R-C, R-L-C series circuits for D.C. and A.C. excitations
- The analysis of three phase balanced and unbalanced circuits
- How to measure active and reactive power in three phase circuits
- Applications of Fourier transforms to electrical circuits excited by non-sinusoidal sources
- Study of Network topology, Analysis of Electrical Networks, Duality and Dual Networks
- Different types of filters and equalizers

UNIT- I TRANSIENT RESPONSE ANALYSIS

D.C Transient Analysis: Transient Response of R-L, R-C, R-L-C Series Circuits for D.C Excitation-Initial Conditions-Solution Method Using Differential Equations and Laplace Transforms, Response of R-L & R-C Networks to Pulse Excitation.

A.C Transient Analysis: Transient Response of R-L, R-C, R-L-C Series Circuits for Sinusoidal Excitations-Initial Conditions-Solution Method Using Differential Equations and Laplace Transforms

UNIT- II THREE PHASE A.C CIRCUITS

Phase Sequence- Star and Delta Connection-Relation between Line and Phase Voltages and Currents in Balanced Systems-Analysis of Balanced and unbalanced Three Phase Circuits- Measurement of Active and Reactive Power in Balanced and Unbalanced Three Phase Systems. Loop Method- Application of Millman's Theorem- Star Delta Transformation Technique – for balanced and unbalanced circuits, Measurement of Active and reactive Power.

UNIT- III FOURIER TRANSFORMS

Fourier Theorem- Trigonometric Form and Exponential Form of Fourier Series – Conditions of Symmetry- Line Spectra and Phase Angle Spectra- Analysis of Electrical Circuits excited by Non Sinusoidal sources of Periodic Waveforms. Fourier Integrals and Fourier Transforms – Properties of Fourier Transforms and Application to Electrical Circuits.

UNIT- IV NETWORK TOPOLOGY

Definitions – Graph – Tree, Basic Cut set and Basic Tie set Matrices for Planar Networks – Loop and Nodal Methods of Analysis of Networks with Dependent & Independent Voltage and Current Sources – Duality & Dual Networks. Nodal Analysis, Mesh Analysis, Super Node and Super Mesh for D.C Excitations.

UNIT - V FILTER DESIGN & CIRCUIT SIMULATION

Filters – Low Pass – High Pass and Band Pass – RC, RL filters– derived filters and composite filters design.

Circuit simulation – Description of Circuit elements, nodes, and sources, Input and Output variables – Modeling of the above elements – DC analysis.

OUTCOMES:

After completing the course, the student should be able to do the following:

- Determine the transient response of R-L, R-C, R-L-C circuits for D.C. and A.C. excitations
- Analyze three phase balanced and unbalanced circuits and determine line voltages, line currents, phase voltages and phase currents
- Measure active and reactive power consumed by a given three phase circuit
- Apply Fourier transforms to electrical circuits excited by non-sinusoidal sources
- Analysis of electrical networks, duality and dual networks
- Design different types of filters
- Simulate D.C. Circuits

TEXT BOOKS:

1. Electrical Circuit Theory and Technology, John Bird, ELSEVIER, 4th Edition, 2010.
2. Network Analysis, M.E Van Valkenburg, Pearson Education, 3rd Edition, 2015.

REFERENCES:

1. Circuit Theory (Analysis & Synthesis), A. Chakrabarti, Dhanpat Rai & Co., 6th Edition, 2008.
2. Electric Circuits by N.Sreenivasulu, REEM Publications Pvt. Ltd., 2012
3. Engineering circuit analysis by William Hayt, Jack E. Kemmerly and Steven M. Durbin, Mc Graw Hill Education (India) Pvt. Ltd., 6th Edition, 2013.

Syllabus:

UNIT- I

TRANSIENT ANALYSIS

- D.C Transient Response of Series R-L circuit Using Differential Equation and Laplace Transforms
- Transient response of Series RC Circuit Using Differential Equation and Laplace Transforms
- Transient response of Series R-L- C Circuits Using Differential Equation and Laplace Transforms
- Response of RL to Pulse Excitation
- Response of R-C Networks to Pulse Excitation.
- Transient Response of R-L Series Circuits for Sinusoidal Excitations Using Differential Equation and Laplace Transforms
- Transient Response of R-C Series Circuits for Sinusoidal Excitations Using Differential Equation and Laplace Transforms
- Transient Response R-L-C Series Circuits for Sinusoidal Excitations Using Differential Equation and Laplace Transforms

UNIT-I: TRANSIENT ANALYSIS

If a circuit is switched from one condition to another either by a change in the applied voltage or change in a circuit parameter, there exists a transitional period during which the branch currents and voltage drops change from their former values to new ones. After transition period, the circuit becomes steady.

The transient disturbances in the electrical circuits are disturbances caused by sudden switching off and on or short circuit of the circuit and sudden change in the applied voltage. The current developed in the circuit due to this disturbance is called the “transient current”. The “resultant current” in the circuit is the steady state current with a transient current superimposed. The transient currents are found to be associated with the changes in stored energy in capacitors and inductors. Hence in a purely resistive circuit no transient current is developed since there is no stored energy in a resistor.

Single energy and double energy transients:

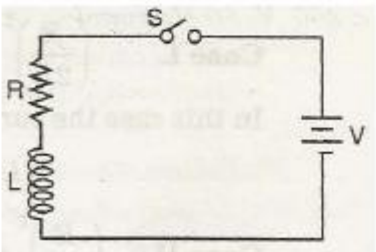
• Single energy transient is the transient disturbance where only one form of energy, either electromagnetic or electrostatic is involved e.g., transient disturbance in a circuit consisting of resistor and inductor i.e., R-L circuit or a circuit consisting of resistor and capacitor i. e., R-C circuit.

• Double energy transient is the transient disturbance where both electromagnetic and electrostatic energies are involved e.g., transient disturbance in a circuit consisting of resistor, inductor and capacitor i.e., R-L-C circuit.

S. No	Steady State Response	Transient Response
1	Amplitude will not change.	Amplitude may change.
2	Frequency will not change.	Frequency may change.
3	Constant voltage and current with time.	Change from one steady state to another.
4	Algebraic equations are used.	Integro-differential equations are used

D.C Transient Response of Series R-L circuit Using Differential Equation and Laplace Transforms

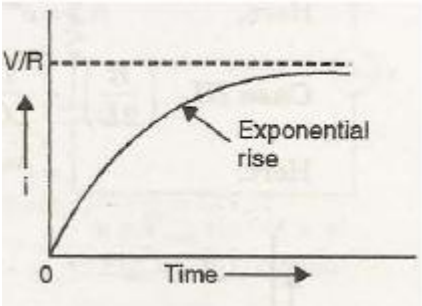
In the R-L circuit shown in Fig. below



$i = V / R\,[1 - e^{-(R/L)t}]$

The plot of i (exponential rise equation) versus time is shown in Fig below

The time constant (λ) for the above function is the time at which the exponent of e is unity. Thus in this case time constant (λ) is L/R. At one time constant, the value of i will be



i = (1- e-1) = 1-0.368 = 0.632

At this time current will be 63.2% of its final value.

The voltage across inductance,

$V_L = L\ di / dt = Ve^{-(R/L)t}$

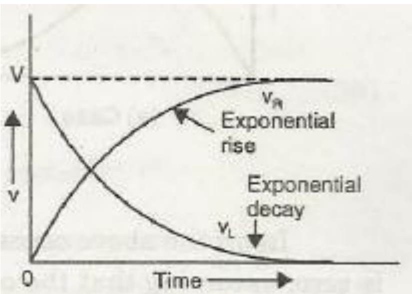
and voltage across resistor,

$V_R = V[1- e^{-(R/L)t}]$

The exponential rise of resistor voltage and exponential decay of inductor voltage are shown in Fig below.

Also, $V_R + V_L = V[1- e^{-(R/L)t}]+ Ve^{-R/L)t} = V$

Power in the circuit elements is given by



$P_R = V / R\ [\ 1 – 2e^{-(R/Lt)t} + e^{-2(R/L)t}$

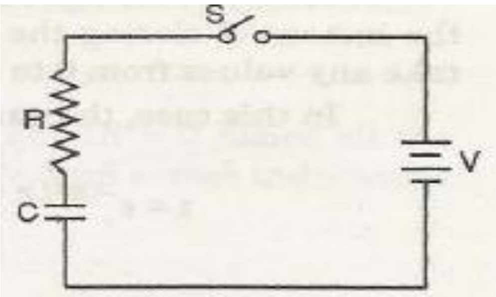
$P_L = V/ R\ [\ e^{-(R/L)t} – e^{-2(R/L)t}$

Total power,

$P = P_R + P_L = V/ R\ [1\text{-e} –(R / L)t$

(ii) D.C Transient Response of Series R-C circuit Using Differential Equation and Laplace Transforms

In the R-C circuit shown in Fig.



$i = V / R\ e^{-t/RC}$

Transients voltages across R and C are given by

$u_R = Ve^{-t/RC}$

$u_c = V(1 - e^{-t/RC)}$

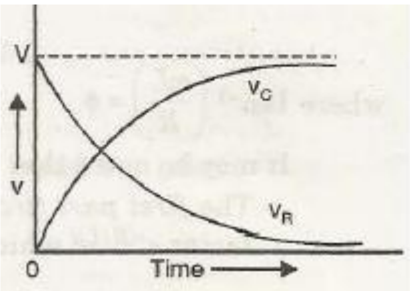
Also, the power in circuit elements is given by

$P_R = V_2 / R\ e^{-2t/RC}$

$P_L = V_2 / V_2 / R\ (e^{-t/RC - e\ 2t/RC})$

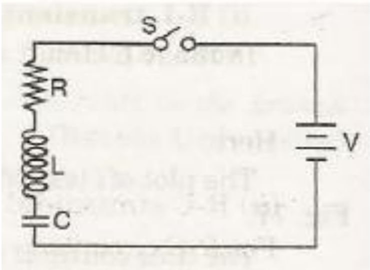
(iii) D.C Transient Response of Series R-C circuit Using Differential Equation and Laplace Transforms

For R -L-C circuit shown in Fig. below he following integro differential equation can be written as follows :



While solving for i, the following three cases are considered

Case I. $(R / 2L)^{2} > 1 / LC$



In this case the current is given by

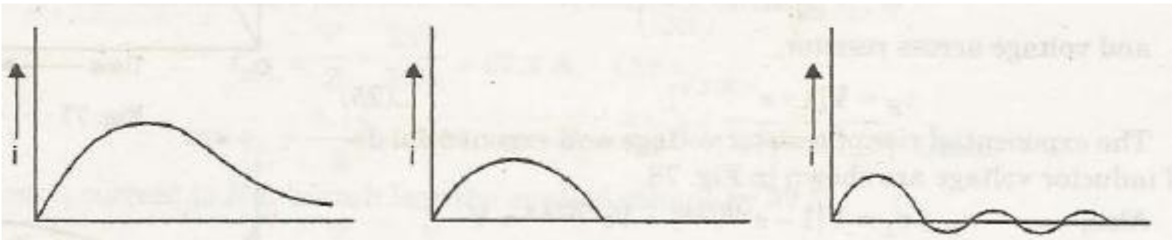
$i = C1e^{at} C1(e^{\beta t} + C2e^{-\beta t})$

Case II. $(R / 2L)^2 = 1 / LC$

Here , $i = e\ at\ (C1 + C2\ t)$

Case III. $(R / 2L)^2 < 1 / LC$

Here , $i = e^{at} (C_1 \cos \beta t + C_2 \sin \beta t)$



(a) Case I (b) Case II (c) Case III

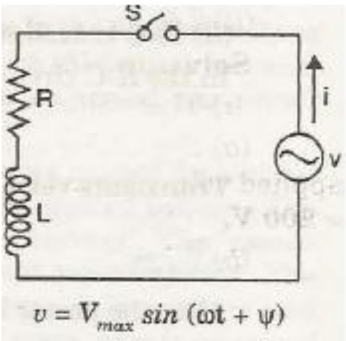
In all the above cases the current contains the factor e^{at} and since ex = – R/2L the final value is zero, assuming that the complimentary function decays in a relatively short time. Fig shows the value of i for initial values zero and initial slope positive.

AC TRANSIENTS

(i) Transient Response of R-L Series Circuits for Sinusoidal Excitations

Here the voltage function could be at any point in the period at the instant of closing the switch and therefore the phase angle can take any values from 0 to 2 π rad /sec.

In this case, the current (i) is given by



$i = e^{-(R / L)t}$ [$-V_{max} / \sqrt{R2 + w2\ L2}$ sin (ψ – tan $^{-1}$ wL /R)]

+ $V_{max} / \sqrt{R2 + w2\ L}$ sin (wt + ψ – tan $^{-1}$ wL /R)

where tan -1 (wL / R) =

It may be noted that:

The first part (transient component of current, it, of the above equation contains the

Factor $e^{-(R / L)t}$ which has a value of zero in a relatively short time.

The second part of the above equation is the steady current (is) which lags the applied

voltage by $\tan^{-1}$ wL / R

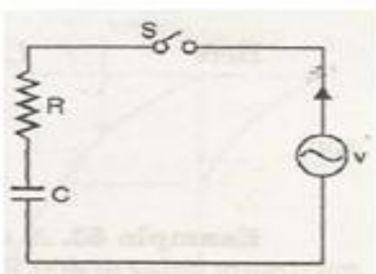
Here , V max / R2 + w2 L2 = I max , tan -1 (wL/R)

(ii) Transient Response of R-C Series Circuits for Sinusoidal Excitations

For R-C circuit shown in Fig. 84 the basic equation is :

Ri + 1 / C ∫ vdt = V max sin (wt + ψ)

Here the current i is given by,



$v = V_{max} \sin(\omega t + \psi)$

$$i = e^{-(t/RC)}\left[\frac{V_{max}}{R}\sin\psi - \frac{V_{max}}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\sin\left(\psi + \tan^{-1}\frac{1}{\omega CR}\right)\right] + \frac{V_{max}}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\sin\left(\omega t + \psi + \tan^{-1}\frac{1}{\omega CR}\right)$$

It may be noted that :

-The first part of the above equation is the transient with decay factor e-t/ RC

-The second part is the steady current which leads the applied voltage by tan-1 1 /wCR.

Problem: A series circuit has R = 10 n and L = 0.1 H. A direct voltage of200 Vis suddenly applied to it. Calculate the following:

(i) The voltage drop across the inductance at the instance of switching on and tLt 0. 01 second,

(ii) The flux linkages at these instants.

Solution. Given: R = 10 n; L = 0.1 H; V = 200 volts

(i) The voltage drop across the inductance:

(a) Switching instant. At the instant of switching on, i = 0, so that i R = 0 hence all the

applied voltage must drop across the inductance only. Therefore, voltage drop across inductance

= 200 V. (Ans.)

(b) When t = 0.01 second

i = V / R [1-e –(R / L)y]

= 200 / 10 [1- e –(10 / 0.1) x 0.01] = 20 (1-e -1) = 12.64 A.

and $iR = 12.64 \times 10 = 126.4$ V.

The voltage drop across the inductance = $\sqrt{(200)2 - (126.4)2} = 155$ v.

Now , $L = N\phi / i$ or $N\phi = Li$

Flux linkages $(N\phi) = Li = 0.1 \times 12.64 = 1.264$ Wb-turns. (Ans.)

Problem: A choke has a resistance of 50 Q and inductance of 1. 0 H. It is supplied with an A. C. voltage given by 141 sin 314t. Find the expression for transient component of the current flowing through the choke after the voltage is suddenly switched on.

Solution. Given: R = 50 Q; L = 1.0 H; e = 141 sin 314t

Expression for transient component of the current :

The equation of the transient component of the current is given by:

$it = I\ m_{ax} \sin \phi\ e -(R / L)t$

Here $I_{max} - V_{max} / Z = V_{max} / \sqrt{R^2 + w^2 L^2} = 141 / \sqrt{(50)^2 + (3.14)^2 (1.0)^2} = 1.443$ A

and $\phi = \tan^{-1} (wL / R) = \tan^{-1} (314 \times 1.0 / 500 = 80.95$

$i = 0.443 \sin 80.95\ e^{-(50 / 1.0)\ t} = 0.437\ e^{-50t}$

Problem: A series circuit has R = 10 Q and L = 0.1 H. A 50 Hz sinusoidal voltage of maximum value of 400 Vis applied across this circuit. Find an expression for the value of current at any instant after the voltage is applied, assuming that the voltage is zero at the instant of application. Calculate its value 0.02 second after switching on.

Solution. Given: R = 10 Q; L = 0.1 H ;f= 50 Hz; $V_{max} = 400$ V; t = 0.02 s.

The current consists of transient component and steady-state component. The equation of

the resultant current is given by :

$i = e - (R / L)t\ [\ -V\ max / \sqrt{R^2 + w^2L^2} \sin (\psi - \tan^{-1} wL / R)$

$+ V\ max / \sqrt{R^2 + w^2L^2} \sin (wt + \psi - \tan^{-1} wL / R)$

where $V\ max / \sqrt{R2 + w2\ L2} = I\ max$, $\tan -1\ (wL / R) = \phi$

Here, $\psi = 0$ as per given condition, then

$$i = e^{-(R/L)t}\left[-\frac{V_{max}}{\sqrt{R^2+\omega^2L^2}}\sin\left(-\tan^{-1}\frac{\omega L}{R}\right)\right] + \frac{V_{max}}{\sqrt{R^2+\omega^2L^2}}\sin\left(\omega t-\tan^{-1}\frac{\omega L}{R}\right)$$

Now, $V_{max} / R^2 + w^2L^2$ = I max = 400 / √ 102 + (314)2 (0.1)2 = 12.14 A

Tan -1 wL / R = ϕ = tan -1 (314 x 0.1 / 10) = 73.3 = 1.262 rad

Substituting the value in eqn. (i), we get

i = e-(10/0.1) x 0.02 [- 12.14 sin (- 72.3°)] + 12.14 sin (314 x 0.02 – 1.262)

= e-2 [12.14 sin (72 .3°)] + 12.14 sin (5.018)

(rad)

= 0.1353(12.14 x 0.9527) + 12.14 sin (287.5°)

(deg)

= 1.56 – 11.58 = – 10.02 A. (Ans.)

i = 6 –(10 / 0.1) x [-12.14 sin (-72.3)] + 12.14 sin (314 x 0.02 -1.262)

= e -2 [12.14 x 0.9527 x 0.9527) + 12.14 sin (287.5)

= 1.56 – 11.58 = – 10.02 A.

Objective Questions and Answers

S.No	Objective Questions
1.	In an R-L circuit connected to an A.C supply, the magnitude of transient current primarily depends on the ----------- [ ] a**) Instant in the voltage cycle at which circuit is closed** b) Impedance of circuit c) Frequency of voltage d) Peak value of steady state current
2	Time constant of series R-L circuit is -------- [ ] a) R/L b) RL c) **L/R** d) None
3	The order of the series R-L-C circuit is -------- [ ] a) 1 b) **2** c) 3 d) 0

4	When the series R-L-C circuit is over damped------------ a) $(R^2/4L^2)=(1/LC)$ b) $(R^2/4L)<(1/C)$ c) **$(R^2/4L)>(1/C)$** d) $(R^2/4C^2)=(1/LC$	[ ]
5	Time constant is defined as the time taken to reach ----- % of its initial value a) **36.7 %** b) 63.4% c) 37.6% d) 64.3%	[ ]
6	Analysing the circuit at t=0 means --------- a) **Transient analysis** b) steady state analysis c) both a & b d) Nome	[ ]
7	In transients Inductor will acts as ------------- a) **open circuit** b) short circuit c) Neither a nor b d) None	[ ]
8	Time constant is defined as the time taken to reach ----- % of its final value a) **36.7 %** b) 63.4% c) 37.6% d) 64.3%	[ ]
9	An R-C series circuit is excited by a DC source. After its switching on ------- a) The voltage across R and C are equal b) The voltage across R and C are equal c) The voltage across R and C are equal d) **The sum of the voltage across R and C is always equal to Vs**	[ ]
10	In transients capacitor will acts as ------------- a) open circuit b) **short circuit** c) Neither a nor b d) None	[ ]
11	Time constant of series R-C circuit is -------- a) **RC** b) R/C c) 1/CR d) C/R	[ ]
12	Transient behavior occurs in any circuit when (a)Sudden changes of applied voltage (b) Voltage source is shorted (c)The circuit is connected or Disconnected **(d)All of the above**	[ ]
13	Inductor does not allow sudden changes **(a)in currents** (b)in voltages (c)in voltages (d)none of the above	[ ]
14	capacitor does not allow sudden changes (a)in currents **(b)in voltages** (c) in both currents and voltages (d)in neither of two	[ ]
15.	The Transient Response occurs (a)only resistive circuits (b)only capacitive circuits (c)only inductive circuits **(d)both b and c**	[ ]

Note: Need to prepare 15 to 20 Objective Question and Answers from each unit. Highlight the correct answer with bold.

TWO MARKS QUESTIONS AND ANSWERS

TRANSIENT ANALYSIS

1. What is transient state?

If a network contains energy storage elements, with change in excitation, the current and voltages change from one state to other state. The behavior of the voltage (or) current when it is changed from one state to another state is called transient state.

2. What is natural response?

If a circuit containing storage elements which are independent of sources, the response depends upon the nature of the circuit, it is called natural response.

3.Write down the equation of current for an Rc circuit when it is supplied by a Dc source ?

Sol: the equation of current for an Rc circuit when it is supplied by a Dc source is

$i(t)=V/R.e^{-t/RC}$

4. What is the time constant of an Rc circuit exited by Dc source?

Sol: The constant of an Rc circuit excited by a Dc source is Rc.

$T=Rc$

5. When an inductor is connected in a circuit , what is the state of inductor under stedy state condition ? Draw the equivalent circuit?

Sol: When an inductror is connected in a circuit,the inductor act as short circuit at steady state .

6. When an capacitor is connected in a circuit what is the status of capacitor under steady state condition? Draw thw equivalent circuit?

Sol: when an capacitor is connected in a circuit, the capacitor act as open circuit at steady state.

7. When an inductor is connected in a circuit , what is the status of inductor under transient state condition? Draw the equivalent circuit?

Sol: when an inductor is connected in a circuit, the inductor act as open ciruit at transient state.

8. When an capacitor is connected in a circuit, what is the status of capacitor under transient state condition? Draw the equivalent circuit ?

Sol: when a capacitor is connected in a circuit, the capacitor act as a short circuit.

9.Define transient circuit ?

Sol: the state which exists between two steady states is called transient state.

10. A Dc voltage of 100 v applied to a series RL circuit with R= 25Ω what will be the current in the circuit at twice the time constant?

sol: Given V= 100v ; R=25Ω

In RL circuit

$i(t) = v/R\ (1- e^{-R/L\ t})$

$i(t) = V/R(1-e^{-t/T})$

$i(t) = 100/25\ (1-e^{-t/T})$

$i(t) = 4(1-e^{-t/T})$

At twice the time constant

$i(t) = 4(1-e^{-2T/T})$

$= 4(1-e^{-2}) = 4(1-0.1353)$

$= 4(0.9757)$

` $= 3.8928$ A

The current in the RL circuit at twice the time constant is i(t) = 3.8928A

11. A DC source is 1V suddenly applied to a series RLC circuit with R=2Ω , L=1H

C=1/2 F . sketch the current response i9t) in the circuit ?

<u>Sol:</u> By applying kvkl foor this circuit

$1-2\,i(t) - 1\,di(t)/dt - 1/1/2 \int i(t)\,dt = 0$

Differentiate with respect to t on both sides

$0-2\,di(t)/dt - 1\,d^2i(t)/dt - 2\,i(t) = 0$

$d^2\,i(t)/dt^2 + 2\,d\,i(t)/dt + 2\,i(t) = 0$

$D_1, D_2 = -2 \pm \sqrt{(2)^2 - 4(1)(2)}\ /2(1)$

$-2 \pm \sqrt{4-8} / 2$

$= -2 \pm \sqrt{-4} /2$

$= -2 \pm 2i /2$

$D_1, D_2 = -1 \pm i$

Here the roots are imaginary

So, the graph is under damped

12. A unit pulse of width 1 sec as shown is applied to an RL series circuit with R=1Ω and L=1H sketch the current response i(t) in the circuit ?

<u>Sol:</u> Given R= 1Ω L= 1 H , t = 1 sec , v= 1v

in RL circuit

$i(t) = V/R\,(1-e^{-R/L}\,t)$

$= 1/1\,(1- e^{-1/1t})$

$i(t) = (1-e^{-t})$

13 Distinguish between steady state and transient response of an electric circuit.

SL. No	STEADY STATE RESPONSE	TRANSIENT RESPONSE
1	Amplitude will not change.	Amplitude may change.
2	Frequency will not change.	Frequency may change.
3	Constant voltage and current with time.	Change from one steady state to another.
4	Algebraic equations are used.	Integro-differential equations are used

14. What are the causes of transient behavior occurring in a circuit?

The causes are

a) It may be due to the sudden change of applied voltage.

b) When the voltage source is shorted.

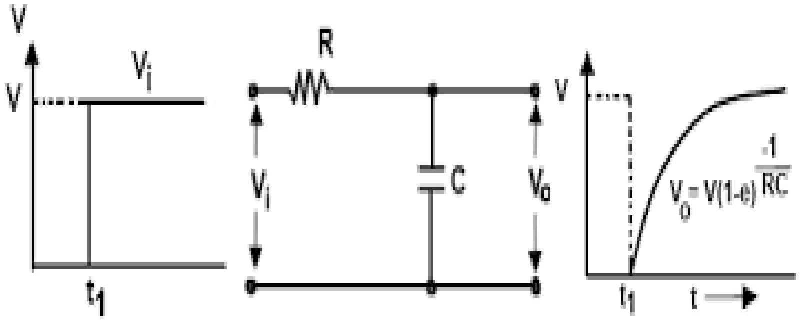
c) When a circuit is connected or disconnected and

d) Due to storage elements in the circuit.

15.What is the Laplace transform?

The Laplace transform is an integral transform perhaps second only to the Fourier transform in its utility in solving physical problems. The Laplace transform is particularly useful in solving linear ordinary differential equations such as those arising in the analysis of electronic circuits.

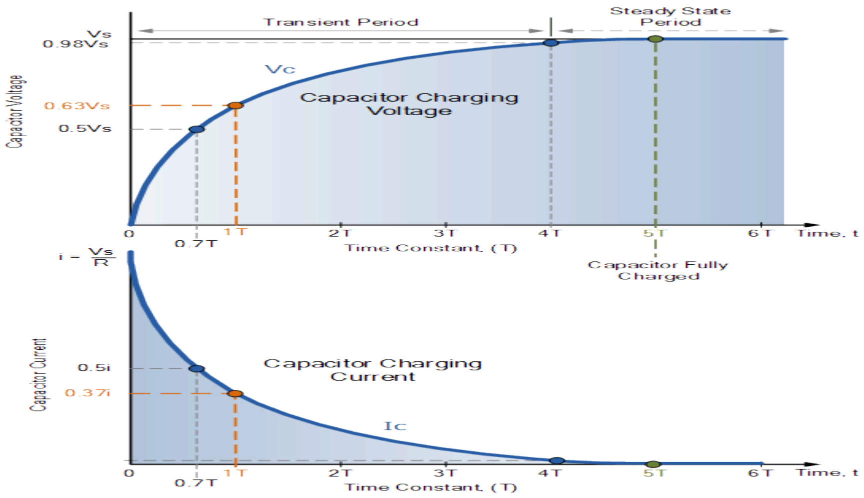
16.Draw the transient response of RC circuit for step input?



17. What is meant by free response?

Free response is due to the internal energy stored in the network. It depends upon the type of elements, their size etc. This response is independent of the source. This response dies gradually, i.e., it approaches zero as time becomes infinity. Free response is also known as natural (or) transient response.

18. Draw the time response of voltage and current in a series RC circuit.

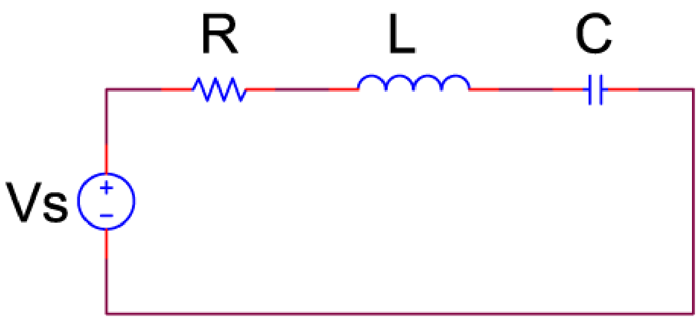


19.Distinguish between free and forced response.

When a circuit contains storage elements which are independent of the sources, the response depends upon the nature of circuit. This response is called natural (or) free response.

The storage elements deliver the energy to the resistances. So the response changes with time, gets saturated after some time. It is referred to as the transient response. When we consider sources acting on a circuit, the response depends on the nature of such sources. This response is called forced response

20.Write equation for voltage in a RLC series circuit.



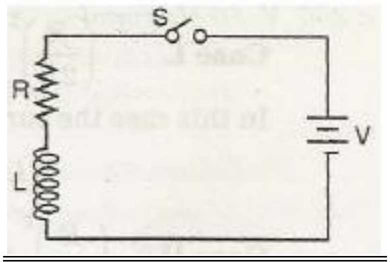
Consider a series RLC circuit with resistance denoted as R, inductor as L, capacitor as C. applying KVL for the above closed loop, we will get the voltage equation v(t).

$$\mathbf{V(t) = Ri(t) + Li(t)^{I} + \frac{1}{C}\int i\,dt}$$

Descriptive Questions and Answers

1. Analyze the D.C response for RL circuit?

Sol:



At t=0 when the switch is closed ,

By KVL,

L di(t)/dt + Ri(t) = v

Consider , series RL circuit , where inductor is un charged,

di(t) /dt +R/L i(t)= V/L → ①

the sol;ution is ,

$i(t) = e^{-R/L} \int V/L\, e^{Rt/L}\, dt + c\, e^{-Rt/L}$

$=V/L\, e^{-Rt/L}[e^{Rt/L} /R/L] + c\, e^{-Rt} / L$

$V/L \times L/R\, e^{-Rt/L}(e^{-Rt/L}) = ce^{-Rt/L}$

$i(t) = V/R\ (1) = ce^{-Rt/L}$

$i(t)=V/R = c\, e^{-Rt/L}$ → ②

Initial conditions :-

When t=0, inductor act as open circuit ,

No current flows through inductor

i.e., At t=0, i(t)=0

substitute initial conditions in equation →②

$0 = V/R = c\, e^{0}$

$C = -V/R$

Equation →2 → $i(t0 = -V/R\ [e^{-RT/L}] = V/R$

The complete solutions , $i(t) = V/R\ [1 - e^{-Rt/L}]$

$i(t) = V/R\ [1 - e^{-t/T}]$

voltage across resistor $(V_R) = Ri(t)(T = L/R)$

$V_R = R.V/R\ [1 - e^{-Rt/L}]$

$V_R = V[1 - e^{-Rt/L}]$

Voltage across inductor $(V_L) = L\, di(t)/dt$

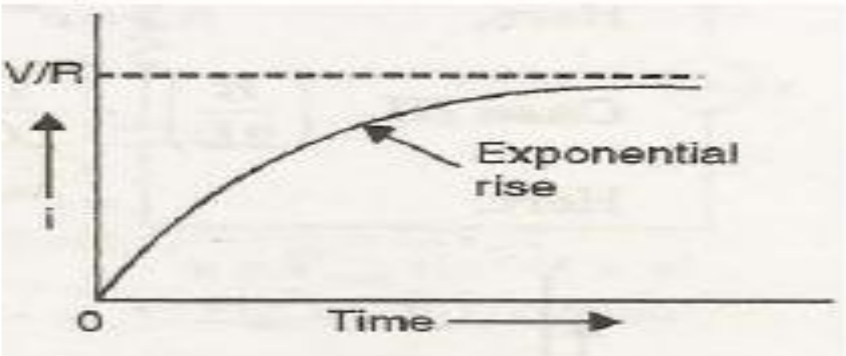
$L.d/dt\ [V/R\ (1 - e^{-Rt/L}]$

$L.\ V/R\ d/dt\ (1 - e - Rt/L)$

$VL/R(0 - e^{-Rt/L(-R/L)})$

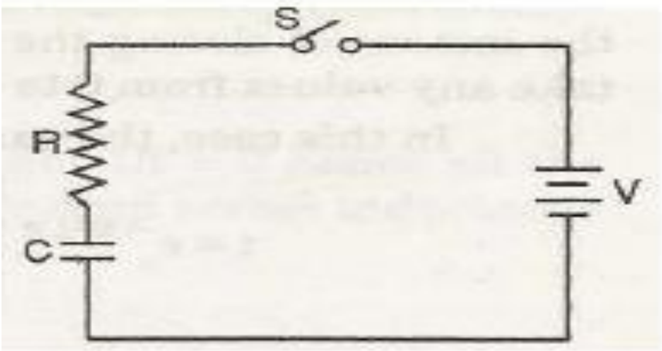
$VL/R\ .\ \ R/L\ e^{-Rt/L}$

$V_L = Ve^{-Rt/L}$



2. Analyse D.C response for RC circuit ?

Sol:-



initially capacitor is uncharged

At t=0, the switch S is closed ,

Let, i(t) Current be flowing throught the circuit,

$1/c \int i(t)\, dt + Ri(t) = V$

Differentiate W.r .to 't' on both sides

$1/c\, i(t) + R\, di(t)/dt = 0$

$1/RC\, i(t) + di(t)/dt = 0 \quad \rightarrow 1$

The solution is ,

$i(t0 = e^{-t/RC} \int e^{t/RC} (0)\, dt + ce^{-t/RC}$

$i(t) = ce^{-t/Rc} \quad \rightarrow 2$

initial conditions :-

initially the capacitor act as short circuit,

At t=0,

$I(t) = V/r$

At t=0, $i(t0 = V/R$

Equation $\rightarrow 2 \rightarrow V/R \pm C$ (substitute initial conditions in equation 2)

$I(t) = V/R\, e^{-t/RC}$

The complete solution is,

$I(t) = V/R\ e^{-t/T}$

Where , T= time constant =Rc

Voltage across resistor (V_R)= Ri(t)

$= R.V/R\ e^{-t/T}$

$V_R = Ve^{-t/T}$

Voltage across capacitor (V_c) =1/c $\int i(t)\,dt$

$1/c\int V/R\ e^{-t/RC\ dt} + c$

$V/RC\ [e^{-t/RC}/-1/RC]$

$V_c = -V/RC \times RC\ e^{-t/RC} + c$

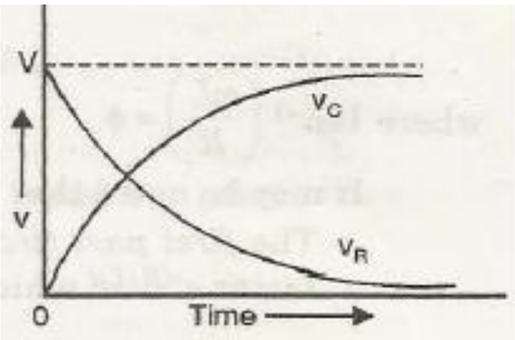
$V_C = -Ve^{-t/T} + c$

Initial conditions at t=0, $V_c = 0$

0= -v=c

C=V

$V_c = V\ (1-e^{-t/t})$



3. In figure below the switch is closed at position (i) at t=0 . at t=0 .5 milli sec the switch is moved to position (ii) find the expression for the current in both the conditions and sketch the transient

Sol;- case -1:-

When the switch is in postion (1)

The circuit can be modified as,

By applying KVL,

$50\ i(t) + 0.5\ di(t)/dt = 10$

$0.5\ di(t9/dt + 50/0.5\ i(t) = 10/0.5$

$di(t)/dt + 100\ i(t) = 20 \rightarrow (1)$

the solution is ,

$i(t) = e^{-100t} \int 20\ e^{+100t}\ dt + ce^{-100t}$

$i(t) = 20\ e^{-100t} (e^{100t}/100) + ce^{-100t}$

$i(t) = 1/5 + ce^{-100t}$

$i(t) = 0.2 + ce^{-100t} \rightarrow (2)$

Initial conditions :-

At t=0, i(t) =0

Equation $\rightarrow (2) \rightarrow 0 = 0.2 + c$

C= -0.2

$I(t) = 0.2 (1 - e^{-100t}) \rightarrow (3)$

<u>Case 2 :-</u>

When switch is in positon (2) , the circuit can be modified as,

By applying KVL,

$5 = 50\ i(t) + 0.5\ di(t)/dt$

$di(t)/dt + 100\ i(t0 = 10 \rightarrow (4)$

the solution is,

$i(t) = e^{-100t} \int 10 e^{100t} dt + ce^{-100t}$

$i(t) = 10\ e^{-100\ t} (e^{100t}/100) + ce^{-100t}$

$i(t) = 0.1 + ce^{-100t} \rightarrow (5)$

initial conditions :-

current through position (2) at t=0

= current passing through position at t= 0.5 milli sec

$i(t0 = 0.2\ (1-e^{-100(0.5\times}10^{-3)})$

$=0.2(1-e^{-0.5\times}10^{-1)}$

$= 0.2\ (1- e^{-0.05})$

$=0.2\ (0.0487)$

$i(t)=9.74\times10^{-3}$

<u>At position (2) :-</u>

At $t=0$, $i(t) = 9.7 \times10^{-3}$

Equation →5

` $9.74\times10^{-3}=0.1 +ce^{0}$

$= c= 9.74\times10^{-3}-0.1$

$=c=0.0097$

The complete solution , $i(t)= 0.1 – 0.0097e^{-100t}$

$I(0) = 0.1$

The graph between currents in positon (1) & (2)

4. The switch in circuit shown below is moved from position 1 to 2 at t=0 . find the expressions for V_c &V_R for t>0 and also derive the formula used.

<u>Sol:-</u> At t=0, switch is moved from positon 1 to 2 ,

By KVL,

$-5000\ i(t) + 50- 1/10^{-6}(\int i(t)dt)=0$

Differentiate w.r.to i(t) on both sides

$-5000\ di(t)/dt + 0- 1/10^{-6}i(t) =0$

$10^{6}/5000\ i(t) + di(t)/dt =0$

$di(t)/dt + 200\ i(t) =0$ →1

the solution is

$i(t)=ce^{-200t}$→2

<u>initial conditions :-</u>

when switch is at position (10)

the voltage across capacitor is 100v

At t=0,i(t)=0

When switch is moved to position (20

At that instant $V_c(0^+)$=100V

The circuit is ,

By KVL,

-50-5000i(t) -100=0

-5000 i(t) =150

I(t) = 0.03

At t=0. Equation →②

-0.03=C

C=0.03

$i(t)=0.03\ e^{-200t}$

voltage across resistor (V_R)=Ri(t)

$=5000\times(-3/100\ e^{-200t})$

$=-150\ e^{-200t}$

The voltage across resistor $(V_c) = 1/c \int i(t)\, dt$

$=1/10^{-6}\int -0.03e^{-200t}dt$

$= -0.03/10^{-6}(e^{-200t}/-200) +c_1$

$V_c = 150\ e^{-200t}+c1$→③

<u>Initial conditions:-</u>

At t=0,$V_c(0)$=100v

Equation (3)

$100=150e^{-200(0)}+c_1$

C_1=100-150

C_1=-50

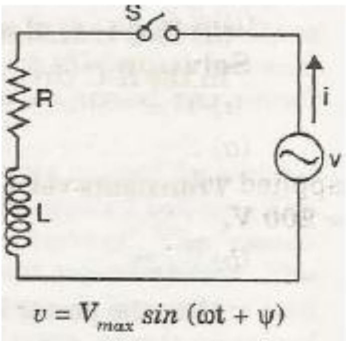
$V_c =150\ e^{-200t}-50$

5. Obtain the R-L and RC Response of Sinusoidal Excitation?

(i) Transient Response of R-L Series Circuits for Sinusoidal Excitations

Here the voltage function could be at any point in the period at the instant of closing the switch and therefore the phase angle ϕ can take any values from 0 to 2 π rad /sec.

In this case, the current (i) is given by



i = e –(R / L)t [-V $_{max}$ / √ R2 + w2 L2 sin (ψ – tan $^{-1}$ wL /R)]

+ V $_{max}$ / √ R2 + w2 L sin (wt + ψ – tan $^{-1}$ wL /R)

where tan -1 (wL / R) = ϕ

It may be noted that:

The first part (transient component of current, it, of the above equation contains the

factor $e^{-(R / L)t}$ which has a value of zero in a relatively short time.

The second part of the above equation is the steady current (is) which lags the applied

voltage by tan-1 wL / R

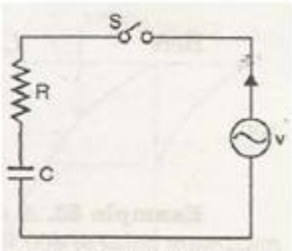
Here , V max / R2 + w2 L2 = I max , tan -1 (wL/R) = ϕ

(ii) Transient Response of R-C Series Circuits for Sinusoidal Excitations

For R-C circuit shown in Fig. 84 the basic equation is :

Ri + 1 / C ∫ vdt = V max sin (wt + ψ)

Here the current i is given by,



$v = V_{max} \sin(\omega t + \psi)$

$$i = e^{-(t/RC)}\left[\frac{V_{max}}{R}\sin\psi - \frac{V_{max}}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\sin\left(\psi + \tan^{-1}\frac{1}{\omega CR}\right)\right] + \frac{V_{max}}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\sin\left(\omega t + \psi + \tan^{-1}\frac{1}{\omega CR}\right)$$

It may be noted that :

-The first part of the above equation is the transient with decay factor e-t/ RC

-The second part is the steady current which leads the applied voltage by tan-1 1 /wCR

6. A series circuit has R = 10 n and L = 0.1 H. A direct voltage of200 Vis suddenly applied to it. Calculate the following :

(i) The voltage drop across the inductance at the instance of switching on and tLt 0. 01 second,

(ii) The flux linkages at these instants.

Solution. Given: R = 10 n; L = 0.1 H; V = 200 volts

(i) The voltage drop across the inductance :

(a) Switching instant. At the instant of switching on, i = 0, so that i R = 0 hence all the

applied voltage must drop across the inductance only. Therefore, voltage drop across inductance

= 200 V. (Ans.)

(b) When t = 0.01 second

i = V / R [1-e –(R / L)y]

= 200 / 10 [1- e –(10 / 0.1) x 0.01] = 20 (1-e -1) = 12.64 A.

and i R = 12.64 x 10 = 126.4 V.

The voltage drop across the inductance = √ (200)2 – (126 .4)2 = 155 v.

Now , L = Nϕ / i or Nϕ = Li

Flux linkages (Nϕ) = Li = 0.1 x 12.64 = 1.264 Wb-turns. (Ans.)

7. A choke has a resistance of 50 Q and inductance of 1. 0 H. It is supplied with an A. C. voltage given by 141 sin 314t. Find the expression for transient component of the current flowing through the choke after the voltage is suddenly switched on.

Solution. Given: R = 50 Q; L = 1.0 H; e = 141 sin 314t

Expression for transient component of the current :

The equation of the transient component of the current is given by:

it = I m_{ax} sin ϕ e –(R / L)t

Here $I_{max} – V_{max} / Z = V_{max} / \sqrt{} R^2 + w^2 L^2 = 141 / \sqrt{} (50)^2 + (3.14)^2 (1.0)^2 = 1.443$ A

and $\phi = \tan^{-1} (wL / R) = \tan^{-1}$ (314 x 1.0 / 500 = 80.95

i = 0.443 sin 80.95 $e^{-(50 / 1.0)\, t}$ = 0.437 e – 50t

8. A series circuit has R = 10 Q and L = 0.1 H. A 50 Hz sinusoidal voltage of maximum value of 400 Vis applied across this circuit. Find an expression for the value of current at any instant after the voltage is applied, assuming that the voltage is zero at the instant of application. Calculate its value 0.02 second after switching on.

Solution. Given: R = 10 Q; L = 0.1 H ;f= 50 Hz; V_{max} = 400 V; t = 0.02 s.

The current consists of transient component and steady-state component. The equation of

the resultant current is given by :

i = e – (R / L)t [-V max / $\sqrt{} R^2 + w^2L^2 \sin (\psi – \tan^{-1} wL / R)$

+ V max / $\sqrt{} R^2 + w^2L^2 \sin (wt + \psi – \tan^{-1} wL / R)$

where V max / √ R2+ w2 L2 = I max , tan -1 (wL / R) = ϕ

Here, ψ = 0 as per given condition, then

$$i = e^{-(R/L)t}\left[-\frac{V_{max}}{\sqrt{R^2+\omega^2L^2}}\sin\left(-\tan^{-1}\frac{\omega L}{R}\right)\right] + \frac{V_{max}}{\sqrt{R^2+\omega^2L^2}}\sin\left(\omega t-\tan^{-1}\frac{\omega L}{R}\right)$$

Now, V_{max} / $R^2 + w^2L^2$ = I max = 400 / √ 102 + (314)2 (0.1)2 = 12.14 A

Tan -1 wL / R = ϕ = tan -1 (314 x 0.1 / 10) = 73.3 = 1.262 rad

Substituting the value in eqn. (i), we get

i = e-(10/0.1) x 0.02 [- 12.14 sin (- 72.3°)] + 12.14 sin (314 x 0.02 – 1.262)

= e-2 [12.14 sin (72 .3°)] + 12.14 sin (5.018)(rad)

= 0.1353(12.14 x 0.9527) + 12.14 sin (287.5°)(deg)

= 1.56 – 11.58 = – 10.02 A. (Ans.)

i = 6 –(10 / 0.1) x [-12.14 sin (-72.3)] + 12.14 sin (314 x 0.02 -1.262)

= e -2 [12.14 x 0.9527 x 0.9527) + 12.14 sin (287.5)

= 1.56 – 11.58 = – 10.02 A.

UNIT – II

3 PHASE AC CIRCUITS

- Phase Sequence-
- Star and Delta Connection-
- Relation Between Line and Phase Voltages and Currents in Balanced Systems-Analysis of Balanced Three Phase Circuits-
- Measurement of Active and Reactive Power in Balanced and Unbalanced Three Phase Systems.
- Analysis of Three Phase Unbalanced Circuits-
- Loop Method- Application of Millmanns Theorem

INTRODUCTION:

Generation, transmission and heavy-power utilisation of A.C. electric energy almost invariably involve a type of system or circuit called a polyphase system or polyphase circuit. In such a system, each voltage source consists of a group of voltages having relative magnitudes and phase angles. Thus, am-phase system will employ voltage sources which, conventionally, consist of m voltages substantially equal in magnitude and successively displaced by a phase angle of 360° I m.

A 3-phase system will employ voltage sources which, conventionally, consist of three voltages substantially equal in magnitude and displaced by phase angles of 120°. Because it possesses definite economic and operating advantages, the 3-phase system is by far the most common, and consequently emphasis is placed on 3-phase circuits.

The advantages of polyphase systems over single-phase systems are:

1. A polyphase transmission line requires less conductor material than a single-phase line for transmitting the same amount power at the same voltage.
2. For a given frame size a polyphase machine gives a higher output than a single-phase machine For example, output of a 3-phase motor is 1.5 times the output of single-phase motor of same size
3. Polyphase motors have a uniform torque where most of the single-phase motors have a pulsating torque.

4. Polyphase induction motors are self-starting and are more efficient. On the other hand single phase induction motors are not self-starting and are less efficient.

5. Per unit of output, the polyphase machine is very much cheaper.
6. Power factor of a single-phase motor is lower than that of polyphase motor of the same
7. Rotating field can be set up by passing polyphase current through stationary coils.
8. Parallel operation of polyphase alternators is simple as compared to that of single-phase alternators because of pulsating reaction in single-phase alternator.

It has been found that the above advantages are best realised in the case of three-phase systems. Consequently, the electric power is generated and transmitted in the form of three-phase system.

Phase Sequence

By phase sequence is meant the order in which the three phases attain their peak or maximum,

In the generation of three-phase e.m.fs. in Fig. 2 clockwise rotation of the field system in Fig. 1 was assumed. This assumption made the e.m.f. of phase '**m**' lag behind that of '**l**' by 120^0 and in a similar way, made that of '**n**' lag behind that of '**m**' by 120° (or that of **l** by 240°). Hence, the order in which the e.m.fs. of phases **l, m** and **n** attain their maximum value is **lmn.** It is called the *phase order* or phase sequence **l** → **m** → **n**. If now the rotation offield structure of Fig. 1 is reversed *i.e.* made counter-clockwise, then the order in which three phases would attain their corresponding maximum voltages would also be reversed. The phase sequencewould become **l** → **n** → **m**. This means that e.m.f. of phase **'n'** would now lag behind that of phase '**l**' by 120° instead of 240° as in the previous case.

The phase sequence of the voltages applied to a load, in general, is determined by the order in which the 3-phase lines are connected. The *phase sequence can be reversed* by *interchanging any pair of lines* . (In the case of an induction motor, reversal of sequence results in the reversed direction of motor rotation).

- The three-phases may be *numbered* **l, m, n** or 1, 2, 3 or they may be given three *colours* (as is customary).

The colours used commercially are *red, yellow* (or sometimes white) and *blue.* In this case sequence is RYE.

Evidently in any three-phase system, there two possible sequences, in which three coils or phase voltages may pass through their maximum value *i.e.*, red → yellow → blue (RYE) or red → blue → yellow (RBY).

By convention:

RYE ….. taken as *positive.*

RBY ….. taken as *negative.*

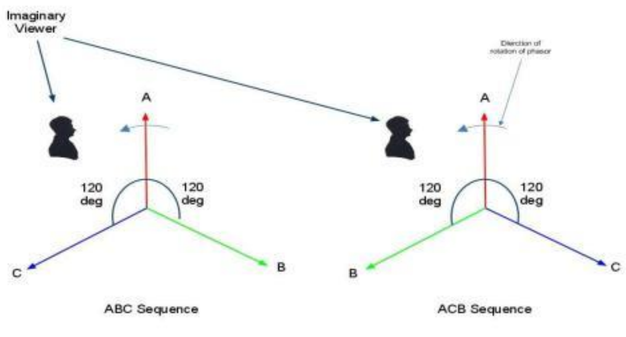


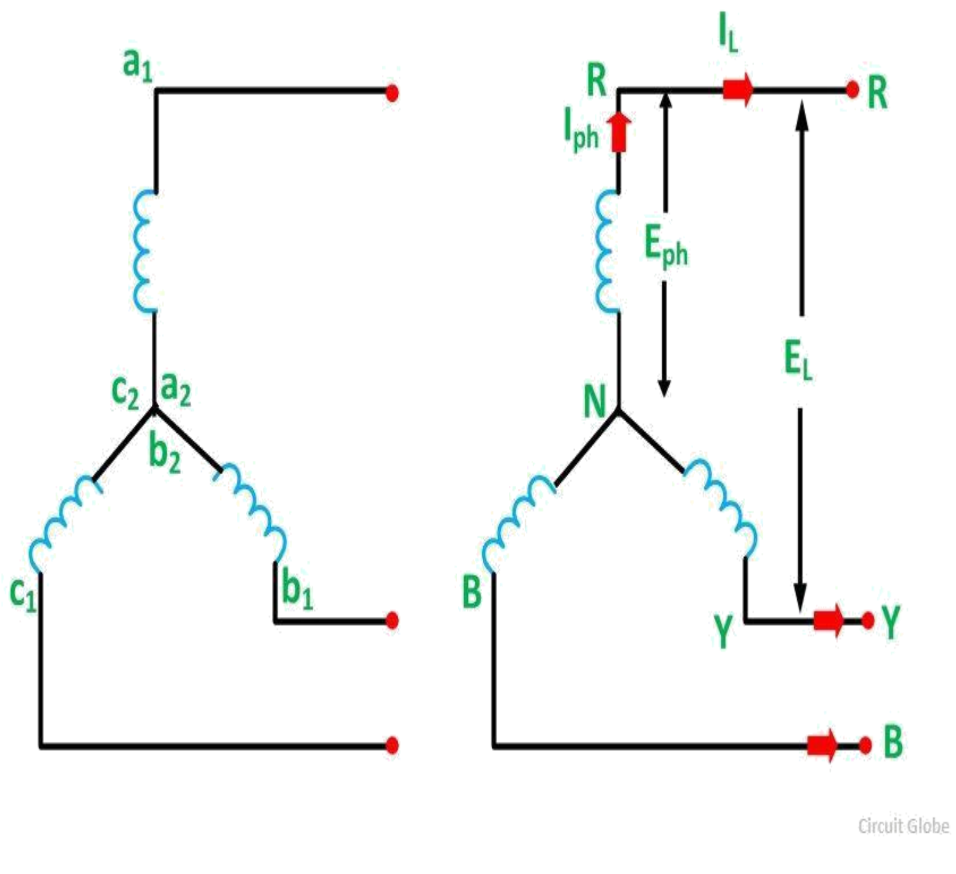
Fig A: ABC and ACB phase sequence, Anti clock wise Rotation

Inter connection of phases:

The three phases can be inter connected either in star (Y)or in delta (Δ).These connections result in a compact and a relatively economical system as the number of conductors gets reduced by 33% for a three phase 4 - wire star system and by 50% for 3phase 3 - wire star or delta systems when compared to independent connection of phases.

Star connection:

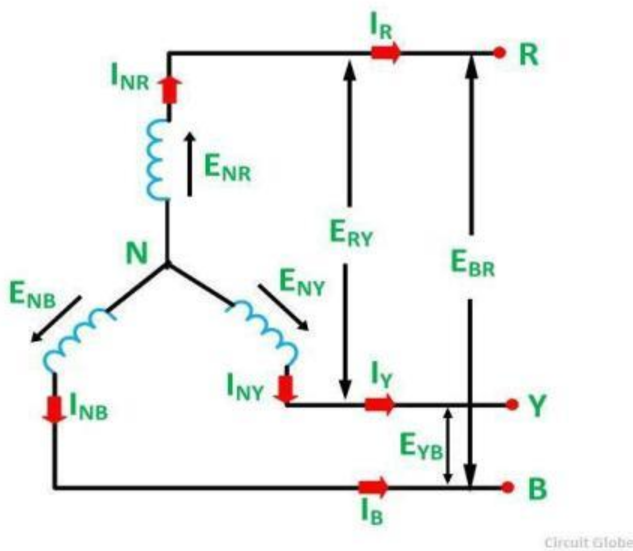
In the **Star Connection**, the similar ends (either start or finish) of the three windings are connected to a common point called star or neutral point. The three line conductors run from the remaining three free terminals called **line conductors**. The wires are carried to the external circuit, giving three phase, three wire star connected systems. However, sometimes a fourth wire is carried from the star point to the external circuit, called **neutral wire**, forming three phase, four wire star connected systems. The star connection is shown in the diagram below.



Considering the above figure, the finish terminals a_2, b_2, and c_2 of the three windings are connected to form a star or neutral point. The three conductors named as R, Y and B run from the remaining three free

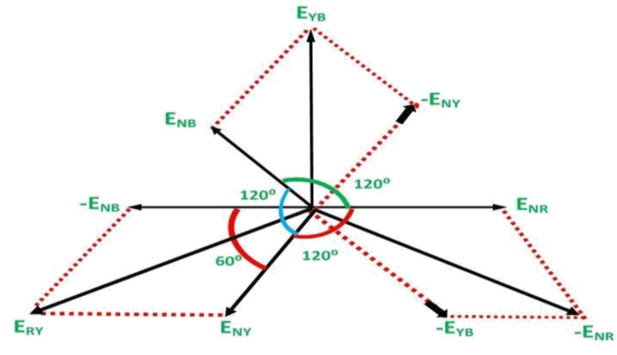
terminals as shown in the above figure. The current flowing through each phase is called **Phase current I_{ph}**, and the current flowing through each line conductor is called **Line Current I_L**. Similarly, the voltage across each phase is called **Phase Voltage E_{ph}**, and the voltage across two line conductors is known as the **Line Voltage E_L. Relation Between Phase Voltage and Line Voltage in Star Connection:**

The Star connection is shown in the figure below.



As the system is balanced, a balanced system means that in all the three phases, i.e., R, Y and B, the equal amount of current flows through them. Therefore, the three voltages E_{NR}, E_{NY} and E_{NB} are equal in magnitude but displaced from one another by 120 degrees electrical.

The **Phasor Diagram** of Star Connection is shown below.



$$E_{YB} = E_{NB} - E_{NY} \quad \text{or} \quad E_L = \sqrt{3}\, E_{ph} \quad \text{and}$$

$$E_{BR} = E_{NR} - E_{NB} \quad \text{or} \quad E_L = \sqrt{3}\, E_{ph}$$

The arrowheads on the emfs and current indicate direction and not their actual direction at any instant. Now,

$$E_{NR} = E_{NY} = E_{NB} = E_{ph} \text{ (in magnitude)}$$

There are two phase voltages between any two lines.

Tracing the loop NRYN

$$\overline{E_{NR}} + \overline{E_{RY}} - \overline{E_{NY}} = 0 \quad \text{or}$$

$$\overline{E_{RY}} = \overline{E_{NY}} - \overline{E_{NR}} \text{ (vector difference)}$$

To find the vector sum of ENY and –ENR, we have to reverse the vector ENR and add it with ENY as shown in the phasor diagram above.

Therefore,

$$E_{RY} = \sqrt{E_{NY}^2 + E_{NR}^2 + 2E_{NY}E_{NR}\cos 60^\circ} \quad \text{or}$$

$$E_L = \sqrt{E_{ph}^2 + E_{ph}^2 + 2E_{ph}E_{ph} \times 0.5} \quad \text{or}$$

$$E_L = \sqrt{3E_{ph}^2} = \sqrt{3}\, E_{ph} \text{ (in magnitude)}$$

Similarly,

Hence, in Star Connections Line voltage is root 3 times of phase voltage.

$$\text{Line voltage} = \sqrt{3} \times \text{Phase voltage}$$

$V_{RY}=V_{NR}-V_{NY}= V_{rms}\angle 0- V_{rms}\angle -120$

$=V_{rms}[(\cos 0+j\sin 0)-[\cos(-120)+J\sin(-120)]]$

$$= V_{rms}\left[1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right] = V_{rms}\left[\frac{3 + j\sqrt{3}}{2}\right] = \sqrt{3}\,V_{rms}\left[\frac{\sqrt{3} + j1}{2}\right]$$

$$= \sqrt{3}\,V_{rms}\left[1\angle 30^o\right]$$

$$= \sqrt{3}\,V_{rms}\angle 30^o$$

$I_R = I_{NR}$

$I_Y = I_{NY}$ and

$I_B = I_{NB}$

Similarly

$V_{YB} = \sqrt{3}\, V_{rms} \angle -90^o$

$= \sqrt{3}\, V_{rms} \angle 150^o$

V_{BR}

If we compare the line-to-neutral voltages with the line-to-line voltages,
we find the following relationships,

Line-to-neutral voltages	Line-to-line voltages
$V_{an} = V_{rms} \angle 0^o$	$V_{ab} = \sqrt{3}\, V_{rms} \angle 30^o$
$V_{bn} = V_{rms} \angle -120^o$	$V_{bc} = \sqrt{3}\, V_{rms} \angle -90^o$
$V_{cn} = V_{rms} \angle 120^o$	$V_{ca} = \sqrt{3}\, V_{rms} \angle 150^o$

Line-to-line voltages in terms of line-to-neutral voltages:

$$V_{ab} = \sqrt{3}\, V_{an} e^{j30^o}$$

$$V_{bc} = \sqrt{3}\, V_{bn} e^{j30^o}$$

$$V_{ca} = \sqrt{3}\, V_{cn} e^{j30^o}$$

Relation Between Phase Current and Line Current in Star Connection

The same current flows through phase winding as well as in the line conductor as it is connected in serie with the phase winding.

Where the phase current will be

$$I_{NR} = I_{NY} = I_{NB} = I_{ph}$$

The line current will be

$$I_R = I_Y = I_B = I_L$$

Hence, in a 3 Phase system of Star Connections, the Line Current is equal to Phase Current.

Power in Star Connection

In a three phase AC circuit, the total True or Active power is the sum of the three phase power. Or the sum of the all three phase powers is the Total Active or True Power.

Hence, total active or true power in a three phase AC

system; Total True or Active Power = 3 Phase Power Or

$P = 3 \times V_{PH} \times I_{PH} \times \text{Cos}\Phi$ ….. Eq … (1)

Good to Know: Where Cos Φ = Power factor = the phase angle between Phase Voltage and Phase Current and not between Line current and line voltage.

We know that the values of Phase Current and Phase Voltage in Star Connection;

$I_L = I_{PH}$

$V_{PH} = V_L / \sqrt{3}$ ….. (From $V_L = \sqrt{3}\, V_{PH}$)

Putting these values in power eq……. (1)

$P = 3 \; x(V_L/\sqrt{3}) \; xI_L \; x \; \text{Cos}\Phi$ ………. $(V_{PH} = V_L/\sqrt{3})$

$P = \sqrt{3} \; x\sqrt{3}x \; (V_L/\sqrt{3}) \; x \; I_L x\text{Cos}\Phi$ ……. $\{3 = \sqrt{3}x\sqrt{3}\}$

$P = \sqrt{3} \times V_L \times I_L \times \text{Cos}\Phi$ Hence proved;

Power in Star Connection,

$P = 3 \times V_{PH} \times I_{PH} \times \text{Cos}\Phi$ or

$P = \sqrt{3} \times V_L \times I_L \times \text{Cos}\Phi$

Similarly,

Total Reactive Power = $Q = \sqrt{3} \times V_L \times I_L \times \sin\Phi$

Good to know: Reactive Power of Inductive coil is taken as Positive (+) and that of a Capacitor as Negative (-).

Also, the total apparent power of the three phases

Total Apparent Power = $S = \sqrt{3} \times V_L \times I_L$

Or, $S = \sqrt{(P^2 + Q^2)}$

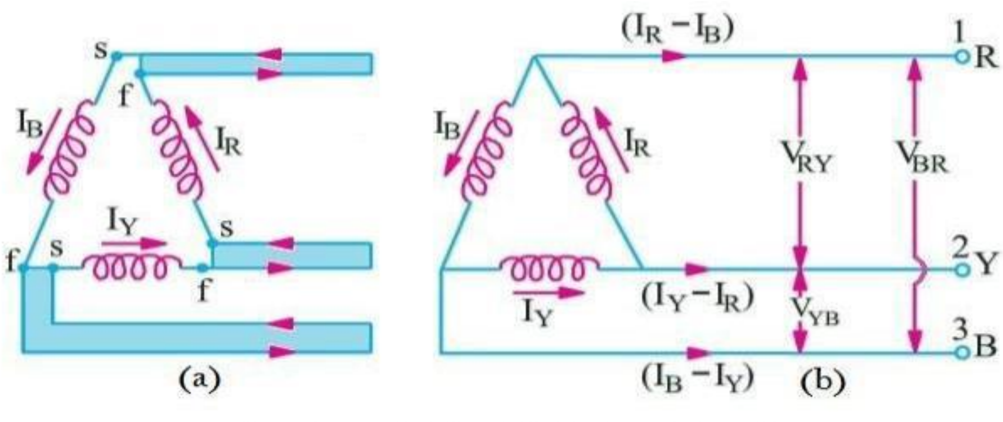
<u>Delta Connection In a 3 Phase System :</u>

In this system of interconnection, the starting ends of the three phases or coils are connected to the finishing ends of the coil. Or the starting end of the first coil is connected to the finishing end of the second coil and so on (for all three coils) and it looks like a closed mesh or circuit as shown in fig (1).

In more clear words, all three coils are connected in series to form a close mesh or circuit. Three wires are taken out from three junctions and the all outgoing currents from junction assumed to be positive.

In Delta connection, the three windings interconnection looks like a short circuit, but this is not true, if the system is balanced, then the value of the algebraic sum of all voltages around the mesh is zero.

When a terminal is open, then there is no chance of flowing currents with basic frequency around the closed mesh.



Delta Connection (Δ): 3 Phase Power, Voltage & Current Values

1. Line Voltages and Phase Voltages in Delta Connection

It is seen from fig 2 that there is only one phase winding between two terminals (i.e. there is one phase winding between two wires). Therefore, in Delta Connection, the voltage between (any pair of) two lines is equal to the phase voltage of the phase winding which is connected between two lines. Since the phase sequence is R → Y → B, therefore, the direction of voltage from R phase towards Y phase is positive (+), and the voltage of R phase is leading by 120°from Y phase voltage. Likewise, the voltage of Y phase is leading by 120° from the phase voltage of B and its direction is positive from Y towards B. If the line voltage between;

Line 1 and Line 2 = V_{RY} ; Line 2 and Line 3 = V_{YB} : ; Line 3 and Line 1 = V_{BR}
Then, we see that V_{RY} leads V_{YB} by 120° and V_{YB} leads V_{BR} by 120°.

Let"s suppose,

$V_{RY} = V_{YB} = V_{BR} = V_L$ …………… (Line Voltage)

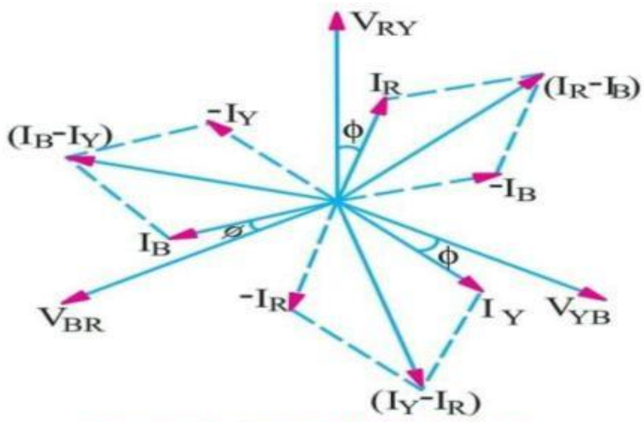
Then $V_L = V_{PH}$

I.e. in Delta connection, the Line Voltage is equal to the Phase Voltage.

2. Line Currents and Phase Currents in Delta Connection

It will be noted from the below (fig-2) that the total current of each Line is equal to the vector difference between two phase currents flowing through that line. i.e.;

- Current in Line 1= $I_1 = I_R – I_B$
- Current in Line 2 =$I_2 = I_Y – I_R$
- Current in Line 3 =$I_3 = I_B – I_Y$



Line & Phase Current and Line & Phase Voltage in Delta (Δ) Connection

The current of Line 1 can be found by determining the vector difference between I_R and I_B and we can do that by increasing the I_B Vector in reverse, so that, I_R and I_B makes a parallelogram. The diagonal of that parallelogram shows the vector difference of I_R and I_B which is equal to Current in Line 1= I_1. Moreover, by reversing the vector of I_B, it may indicate as ($-I_B$), therefore, the angle between I_R and $-I_B$ (I_B, when reversed = $-I_B$) is 60°. If,

$I_R = I_Y = I_B = I_{PH}$ …. The phase currents

Then;

The current flowing in Line 1 would be;

I_L or $I_L = \sqrt{(I_R^2+I_Y^2 +2.I_R*I_Y \text{ C0S } 60^o)}$

$$I_L = \sqrt{3I_{ph}^2} = \sqrt{3}I_{ph}$$

i.e. In Delta Connection, The Line current is √3 times of Phase Current

Similarly, we can find the reaming two Line currents as same as above. i.e.,

$I_2 = I_Y – I_R$ … Vector Difference = $\sqrt{3}\ I_{PH}$

$I_3 = I_B – I_Y$ … Vector difference = $\sqrt{3}\ I_{PH}$

As, all the Line current are equal in magnitude i.e.

$I_1 = I_2 = I_3 = I_L$

Hence

$I_L = \sqrt{3}\, I_{PH}$

It is seen from the fig above that;

- The Line Currents are 120° apart from each other
- Line currents are lagging by 30° from their corresponding Phase Currents
- The angle Φ between line currents and respective line voltages is (30°+Φ), i.e. each line current is lagging by (30°+Φ) from the corresponding line voltage.

3. Power in Delta Connection

We know that the power of each phase

Power / Phase = $V_{PH} \times I_{PH} \times \cos\Phi$

And the total power of three phases;

Total Power = $P = 3 \times V_{PH} \times I_{PH} \times \cos\Phi$ ….. (1)

We know that the values of Phase Current and Phase Voltage in Delta
Connection; $I_{PH} = I_L / /\sqrt{3}$ ….. (From $I_L = \sqrt{3}\, I_{PH}$)

$V_{PH} = V_L$

Putting these values in power eq……. (1)

$P = 3 \times V_L \times (I_L/\sqrt{3}) \times \cos\Phi$ …… ($I_{PH} = I_L / /\sqrt{3}$)

$P = \sqrt{3} \times \sqrt{3} \times V_L \times (I_L/\sqrt{3}) \times \cos\Phi$ …{ $3 = \sqrt{3} \times \sqrt{3}$ }

$P = \sqrt{3} \times V_L \times I_L \times \cos\Phi$ …

Hence proved;

Power in Delta Connection,

$P = 3 \times V_{PH} \times I_{PH} \times \cos\Phi$ …. or

$P = \sqrt{3} \times V_L \times I_L \times \cos\Phi$

Analysis of Balanced Three Phase Circuits:

It is always better to solve the balanced three phase circuits on per phase basis. When the three phase supply voltage is given without reference to the line or phase value, then it is the line voltage which is taken into consideration.

The following steps are given below to solve the balanced three phase circuits.

Step 1 – First of all draw the circuit diagram.

Step 2 – Determine $X_{LP} = X_L/\text{phase} = 2\pi f_L$.

Step 3 – Determine $X_{CP} = X_C/\text{phase} = 1/2\pi f_C$.

Step 4 – Determine $X_P = X/\text{ phase} = X_L – X_C$

Step 5 – Determine $Z_P = Z/\text{phase} = \sqrt{R^2{}_P + X^2{}_P}$

Step 6 – Determine $\cos\phi = R_P/Z_P$; the power factor is lagging when $X_{LP} > X_{CP}$ and it is leading when $X_{CP} > X_{LP}$.

Step 7 – Determine V phase.

For star connection $V_P = V_L/\sqrt{3}$ and for delta connection $V_P = V_L$

Step 8 – Determine $I_P = V_P/Z_P$.

Step 9 – Now, determine the line current I_L.

For star connection $I_L = I_P$ and for delta connection $I_L = \sqrt{3}\ I_P$

Step 10 – Determine the Active, Reactive and Apparent power.

THREE-PHASE CONNECTIONS:

The sources and loads in a three-phase system can each be connected in either a *wye* (Y) or *delta* ( ) configuration. Note that the wye connections are line-to-neutral while the delta connections are line-to-line with no neutral. Also note the convention on the node designations (lowercase letters at the source connections and uppercase letters at the load connections).

Both the three phase source and the three phase load can be connected either Wye or DELTA.

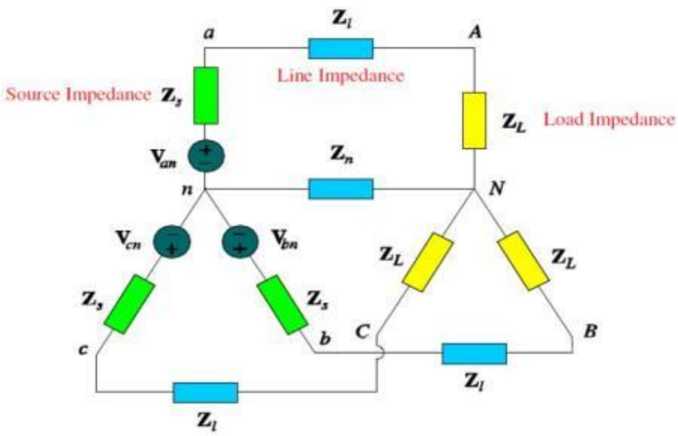
We have 4 possible connection types.

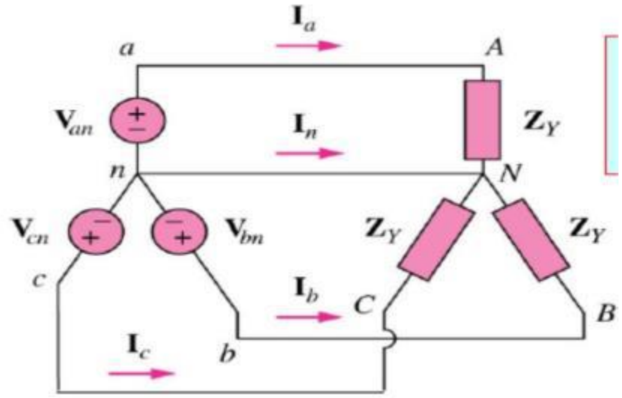
1. Y-Y connection
2. Y- connection
3. - Y connection
4. - connection

Balanced connected load is more common & Y connected sources are more common.

BALANCED WYE-WYE CONNECTION

The balanced three-phase wye-wye connection is shown below. Note that the line impedance for each of the individual phases in included in the circuit. The line impedances are assumed to be equal for all three phases. The *line currents* (***I****aA*, ***I****bB* and ***I****cC*) are designated according to the source/load node naming convention. The *source* current, *line* current, and *load* current are all one in the same current for a given phase in a wye-wye connection. Wye source Wye load Assuming a positive phase sequence, the application of Kirchoff‟s voltage law around each phase gives where ***Z****total*





Line Currents

$$I_a = \frac{V_{rms}}{|Z_{total}|}\angle(-\theta_Z)$$

$$I_b = \frac{V_{rms}}{|Z_{total}|}\angle(-120^o-\theta_Z)$$

$$I_c = \frac{V_{rms}}{|Z_{total}|}\angle(120^o-\theta_Z)$$

Line-to-neutral voltages

$$V_{an} = V_{rms}\angle 0^o$$

$$V_{bn} = V_{rms}\angle -120^o$$

$$V_{cn} = V_{rms}\angle 120^o$$

Assuming a positive phase sequence, the application of Kirchoff‟s voltage law around each phase gives

$V_{an} = V_{rms}\llcorner 0^O = I_a(Z_l+Z_L) = I_a\,Z_{total} = I_a\ |Z_{total}|\llcorner\theta_Z$

$V_{bn} = V_{rms}\llcorner -120^O = I_b(Z_l+Z_L) = I_b\,Z_{total} = I_b\ |Z_{total}|\llcorner\theta_Z$

$V_{cn} = V_{rms}\llcorner 120^O = I_c(Z_l+Z_L) = I_c\,Z_{total} = I_c\ |Z_{total}|\llcorner\theta_Z$

Line current I_n add up to zero.
Neutral current is zero:

$$I_n = -(I_a + I_b + I_c) = 0$$

➢ Magnitude of line voltages is √3 times the magnitude of phase voltages. $V_L = \sqrt{3}\ V_p$

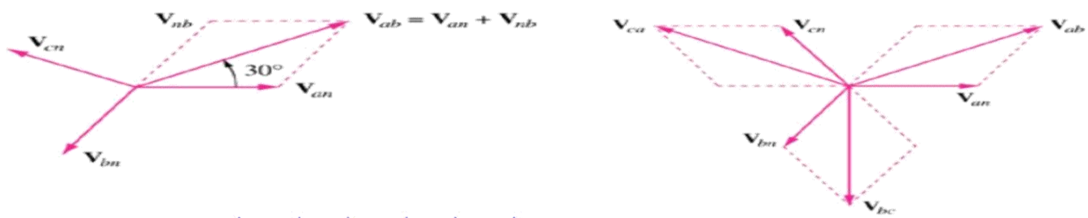
$$V_{an} = V_p\angle 0°, \quad V_{bn} = V_p\angle -120°, \quad V_{cn} = V_p\angle +120°$$

$$V_{ab} = V_{an} + V_{nb} = V_{an} - V_{bn} = \sqrt{3}V_p\angle 30°$$

$$V_{bc} = V_{bn} - V_{cn} = \sqrt{3}V_p\angle -90°$$

$$V_{ca} = V_{cn} - V_{an} = V_{an} + V_{bn} = \sqrt{3}V_p\angle -210°$$

➢ Phasor diagram of phase and line voltages

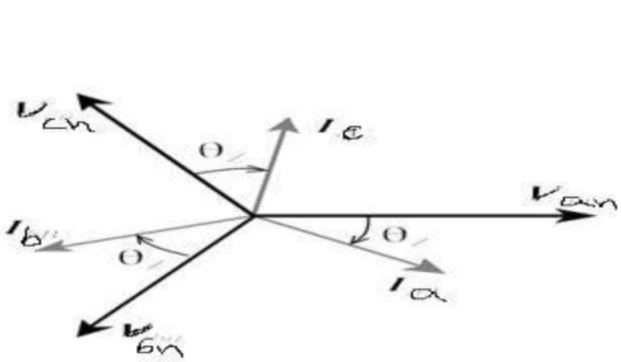


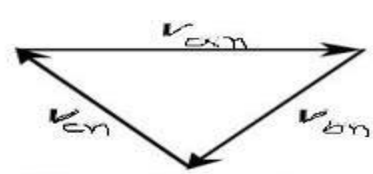
$$V_L = |V_{ab}| = |V_{bc}| = |V_{ca}|$$
$$= \sqrt{3}\,|V_{an}| = \sqrt{3}\,|V_{bn}| = \sqrt{3}\,|V_{cn}| = \sqrt{3}V_p$$

$$V_p = |V_{an}| = |V_{bn}| = |V_{cn}|$$

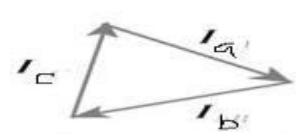
Where ***Z****total* is the total impedance in each phase and θZ is the phase angle associated with the total phase impedance. The preceding equations can be solved for the line currents.

Note that the line current magnitudes are equal and each line current lags the respective line-to-neutral voltage by the impedance phase angle 2*Z*. Thus, the balanced voltages yield balanced currents. The phasor diagram for the line currents and the line-to-neutral voltages is shown below. If we lay the line-to-neutral voltage phasors end to end, they form a closed triangle (the same property is true for the line currents). The closed triangle shows that the sum of these phasors is zero.





$$V_{an} + V_{bn} + V_{cn} = 0$$



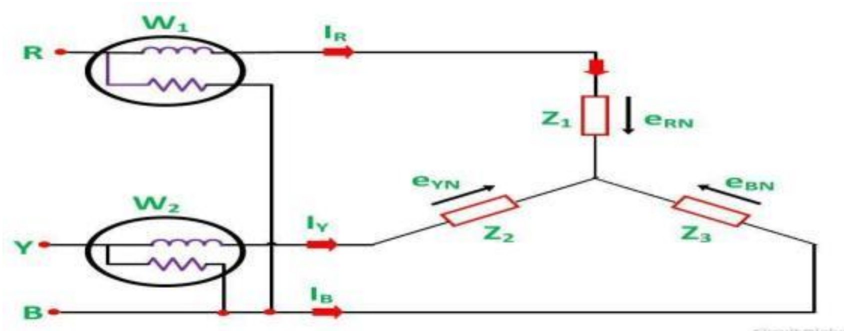
$$I_a + I_b + I_c = 0 = I_n$$

The fact that the line currents sum to zero in the balanced wye-wye connection shows that the neutral current *In* is zero in this balanced system. Thus, the impedance of the neutral is immaterial to the performance of the circuit under balanced conditions. However, any imbalance in the system (loads, line impedances, source variations, etc.) will produce a non-zero neutral current.In any balanced three-phase system (balanced voltages, balanced line and load impedances), the resulting currents are balanced. Thus, there is no real need to analyze all three phases. We may analyze one phase to determine its current, and infer the currents in the other phases based on a simple balanced phase shift (120o phase difference between any two line currents). This technique is known as the *per phase analysis*

Two Wattmeter Method of Power Measurement:

Two Wattmeter Method can be employed to measure the power in a 3 phase, 3 wire star or delta connected balanced or unbalanced load. In Two wattmeter method the current coils of the wattmeter are connected with any two lines, say R and Y and the potential coil of each wattmeter is joined across the same line, the third line i.e. B as shown below in the figure

Measurement of Power by Two Wattmeter Method in Star Connection:



The instantaneous current through the coil of the Wattmeter, W_1 is given by the equation

$$W_1 = i_R = i_1 - i_3$$

Instantaneous voltage measured by the Wattmeter, W_1 will be

$$W_1 = e_{RB}$$

Therefore, the instantaneous power measured by the Wattmeter, W_1 will be given as

$$W_1 = e_{RB}\,(i_1 - i_3) \;\ldots\ldots\ldots (3)$$

The instantaneous current through the current coil of the Wattmeter, W_2 is given as

$$W_2 = i_Y = i_2 - i_1$$

The instantaneous potential difference across the potential coil of Wattmeter, W_2 is

$$W_2 = e_{YB}$$

Therefore, the instantaneous power measured by Wattmeter, W_2 will be

$$W_2 = e_{YB}(i_2 - i_1) \ldots\ldots\ldots (4)$$

Hence, to obtain the total power measured by the Two Wattmeter the two equations, i.e. equation (3) and (4) has to be added.

$$W_1 + W_2 = e_{RB}(i_1 - i_3) + e_{YB}(i_2 - i_1)$$

$$W_1 + W_2 = i_1 e_{RB} + i_1 e_{YB} - i_3 e_{RB} - i_1 e_{YB}$$

$$W_1 + W_2 = i_2 e_{YB} + i_3 e_{BR} - i_1(e_{YB} + e_{BR}) \qquad (\text{i.e.} -e_{RB} = e_{RB})$$

$$W_1 + W_2 = i_1 e_{RY} + i_2 e_{YB} + i_3 e_{BR} \qquad (\text{i.e } e_{RY} + e_{YB} + e_{BR} = 0)$$

$$W_1 + W_2 = P$$

Where, P is the total power absorbed in the three loads at any instant.

The power measured by the Two Wattmeter at any instant is the instantaneous power absorbed by the three loads connected in three phases. In fact, this power is the average power drawn by the load since the Wattmeter reads the average power because of the inertia of their moving system.

The circuit diagram for two wattmeter method of measurement of three phase real power is as shown in the figure 4.7. The current coil of the wattmeters W1 and W2 are inserted respectively in R and Y phases. The potential coils of the two wattmeters are joined together to phase B, the third phase. Thus, the voltage applied to the voltage coil of the meter, W1 is $V_{RB} = V_R - V_B$, while the voltage applied to the voltage coil of the meter, W2 is $V_{YB}=V_Y-V_B$, where, V_R, V_B and V_C are the phase voltage values of lines R, Y and B respectively, as illustrated by the phasor diagram of figure 4.8. Thus, the reading of the two wattmeters can be obtained based on the phasor diagram of figure 4.8, as follows:

$$W_1 = I_R V_{RB}$$
$$= I_L V_L \cos(30 - \emptyset)$$
$$W_2 = I_Y V_{YB}$$
$$= I_L V_L \cos(30 + \emptyset)$$

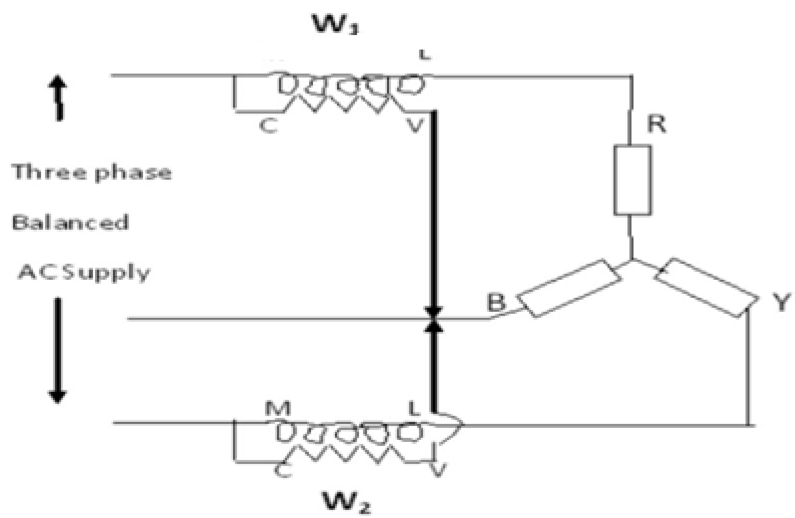
Hence, $W_1+W_2 = \sqrt{3}\, V_L I_L \cos\emptyset = P_{3ph}$

And $W_1-W_2 = V_L I_L \sin\emptyset$

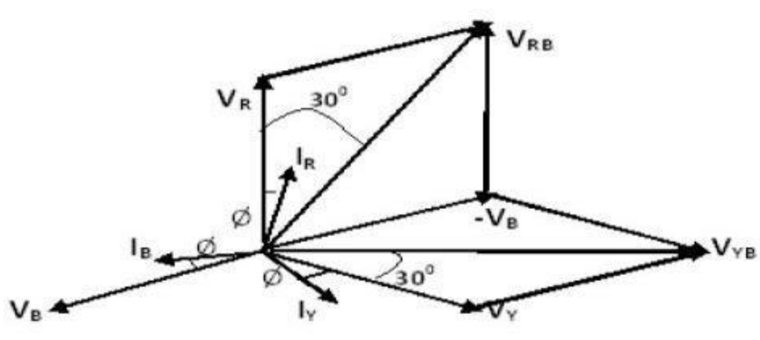
So that then,

$$\mathbf{Tan\,\emptyset = \sqrt{3}\,[W_1-W_2]/[W_1+W_2]}$$

Where Ø is the lagging PF angle of the load. It is to be noted that the equations get exchanged if the load is considered to be of leading PF.



Two wattmeter method of 3-phase power measurement



Phasor diagram for real power measurements

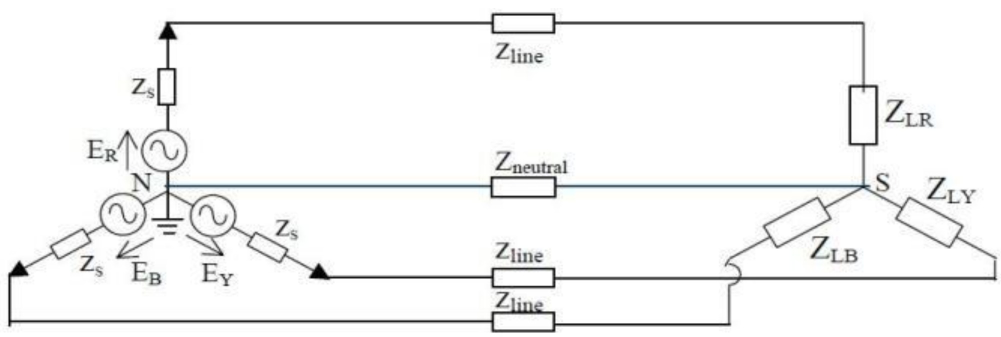
The readings of the two watt meters used for real power measurements in three phase circuits as above vary with the load power factor as described in the table below

PF angle	PF	W_1	W_2	$W_{3ph}=W_1+W_2$	Remarks
φ (lag)	$\cos\varphi$	$V_LI_L\cos(30-\varphi)$	$V_LI_L\cos(30+\varphi)$	$\sqrt{3}V_LI_L\cos\varphi$	Gen. Case (always $W_1 \geq W_2$)
0^0	UPF	$\sqrt{3}/2\ V_LI_L$	$\sqrt{3}/2\ V_LI_L$	$2W_1$ or $2W_2$	$W_1=W_2$
30^0	0.866	V_LI_L	$V_LI_L/2$	$1.5W_1$ or $3W_2$	$W_2=W_1/2$
60^0	0.5	$\sqrt{3}/2\ V_LI_L$	ZERO	W_1 alone	W_2 reads zero
$>60^0$	<0.5	W_1	W_2 reads negative	$W_1+(-W_2)$	For taking readings, the PC or CC connection of W_2 should be reversed) (LPF case)

Unbalanced three phase systems

An unbalanced three phase system is one which is not perfectly balanced. It may be caused by the supply being unbalanced, or more usually the load being unbalanced or both. In such a case, knowledge of the currents or voltages in one phase does not tell us the currents or voltages in the other phases. Thus all phase quantities must be independently determined. Let us consider some of the common unbalanced situations to see how this may be done.

a) Star connected supply feeding a star connected load



(i) If $Z_{neutral}$ is considered zero, each individual phase current can be independently determined from the supply voltage in that phase and the impedance of that phase.

$$I_{LR} = \frac{E_R}{z_s + z_{line} + Z_{LR}}, \; I_{LY} = \frac{E_Y}{z_s + z_{line} + Z_{LY}}, \; I_{LB} = \frac{EB}{z_s + z_{line} + Z_{LB}}$$

Then the load voltages etc can be determined.

(ii) If there is a neutral impedance, then using **Millmann's theorem**, we will first have to determine the voltage of the star point of the load with respect to the supply neutral.

$$V_{SN} = \frac{\sum Y.V}{Y} = \frac{\frac{1}{z_s + z_{line} + Z_{LR}}.E_R + \frac{1}{z_s + z_{line} + Z_{LY}}.E_Y + \frac{1}{z_s + z_{line} + Z_{LB}}.E_B + \frac{1}{z_{neutral}}.0}{\frac{1}{z_s + z_{line} + Z_{LR}} + \frac{1}{z_s + z_{line} + Z_{LY}} + \frac{1}{z_s + z_{line} + Z_{LB}} + \frac{1}{z_{neutral}}}$$

from which V_{SN} is known.

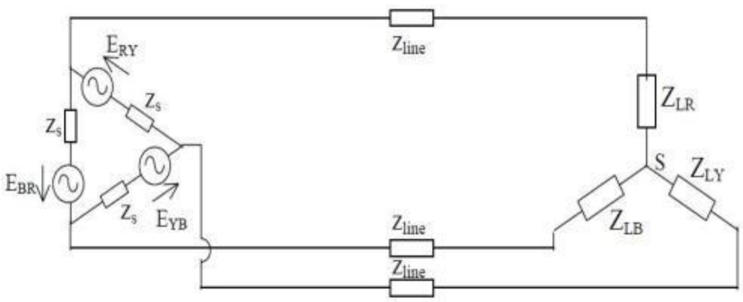
Thus the load currents can be determined from

$$I_{LR} = \frac{E_R - V_{SN}}{z_s + z_{line} + Z_{LR}}, \; I_{LY} = \frac{E_Y - V_{SN}}{z_s + z_{line} + Z_{LY}}, \; I_{LB} = \frac{E_B - V_{SN}}{z_s + z_{line} + Z_{LB}}$$

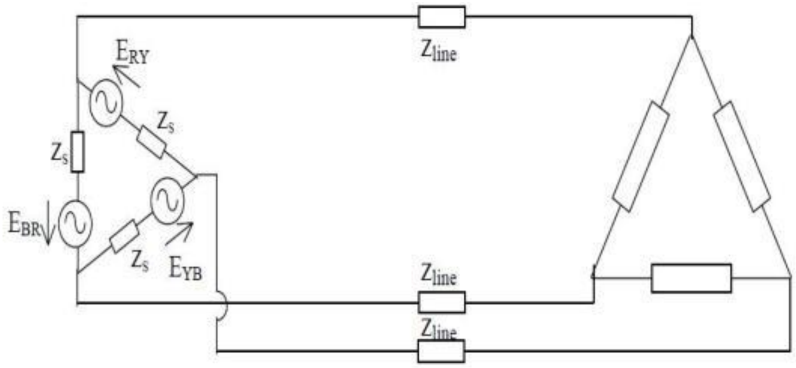
Hence the remaining quantities can be determined.

iii) If the system is a 3-wire system, rather than a 4-wire system, the analysis is the same as if $z_{neutral}$ were (i.e. $1/z_{neutral} = 0$). Thus again Millmann‟s theorem is used to determine V_{SN} and the load currents are then determined

(b) Delta connected supply feeding a star connected load: If the supply was connected, not in star but in delta, which is not the case in practice, then we would have to write the Kirchoff‟s current law for the loops and solve as a normal circuit problem.

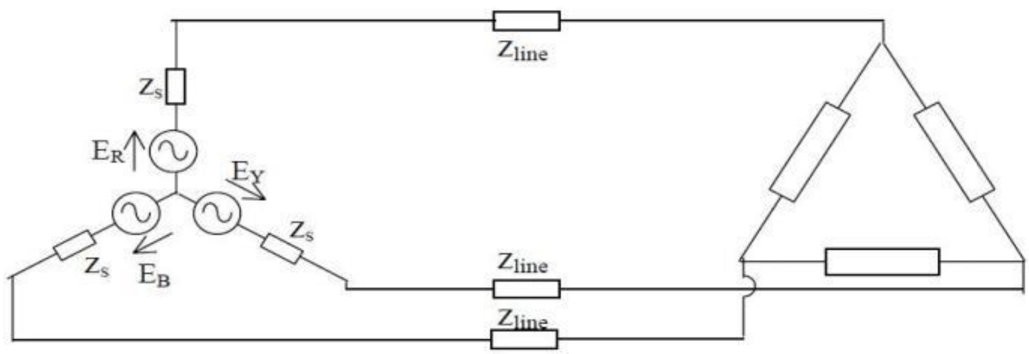


(c) Delta connected supply feeding a delta connected load:



When a delta connected supply feeds a delta connected load, which is not usual, then the line voltages are known so that the currents inside the delta can be obtained directly from Ohm’s Law. The line currents can then be obtained by phasor summing of the currents inside the delta. The remaining variables are then obtained directly

(d) Star connected supply feeding a delta connected load

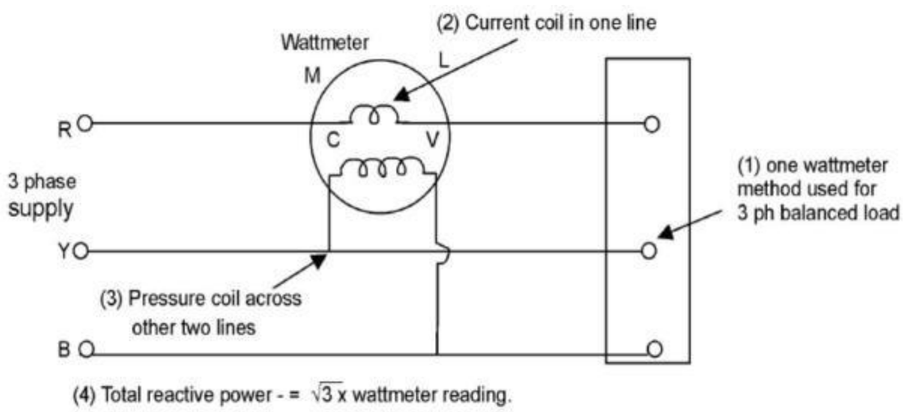


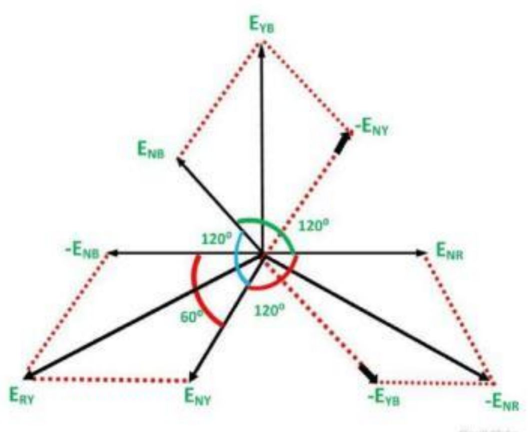
When a star connected supply feeds a delta connected load, then from the phase voltages the line voltages are known so that the currents inside the delta can be obtained directly from Ohm‟s Law. The line currents can then be obtained by phasor summing of the currents inside the delta. The remaining variables are then obtained directly.

Thus basically, any unbalanced system can be calculated using the basic network theorems.

Measurement of Reactive Power in Balanced and Unbalanced Three Phase Systems:

One wattmeter method for measurement of reactive power is for 3 phase balanced load only. The current coil of the wattmeter is connected in one of the lines. The pressure coil is connected across two lines. The reactive power is Ö3 times the wattmeter reading.





$$W_1 = I_R V_{RB}$$
$$= I_L V_L \cos(90 - \phi)$$
$$W = I_L V_L \sin\phi$$
$$KVAR = \sqrt{3}\,W$$

Example 1.*A star-connected, 6000 V, 3-phase alternator is supplying 4000 kW at factor of 0.8. Calculate the active and reactive components of the current in each phase.*

Solution. Line voltage, E_L = 6000 V

Power supplied, P = 4000 kW

Power factor, cos = 0.8

Active and reactive components of current:

We know that, $P = E_L I_L$ cos0

4000 x 1000 = 6000 I_L. 0.8

Active component = I_{ph} cos0 = 481 * 0.8 = **384.8 A. (Ans.)**

Reactive component = I_{ph} sin0 = 481 * 0.6 = **288.6 A. (Ans.)**

Example 2. *In a 3-phase, 4-wire system, two phases have currents of 20 A and* 12 *Air. power factors of 0.8 and 0.6 respectively, while the third phase is open-circuited. Calculate the in the neutral and draw the vector diagram.*

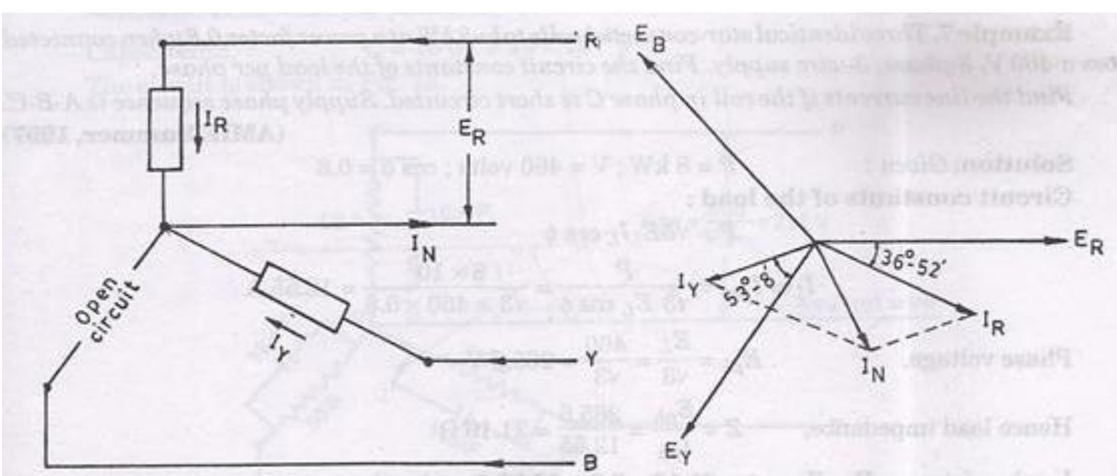
Solution. Refer Figs. 10 and 11.

cos $_1$ = 0.8

$_1$ = cos^{-1} 0.8 = 36° 52′ (lag)

cos $_2$ = 0.6

$_2$ = cos^{-1} 0.6 = 53° 8′ (lag)



Let E_R be the reference vector.

Then $I_R = 20\ -36^\circ\ 52' = 20\ (0.8 - j0.6) = (16 - j12)$

and $I_Y = 12\ -173^\circ\ 8' = 12\ (-1 - j0.12) = (-12 - j1.44)$

The current through the neutral

$$I_N = I_R + l_Y$$

$= (16 - j12) + (-12 - j1.44) = 4 - j13.44 = 14\ -73^\circ\ 24'$

Hence, **current in the neutral = 14 – 73° 24′. (Ans.)**

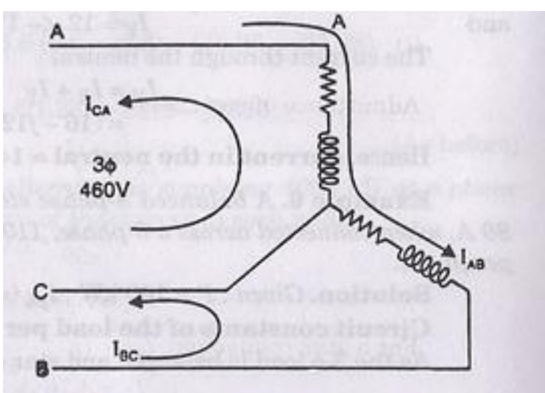
Example 3. *Three identical star-connected coils take* 8 *k W at a power factor 0.8 when connected across a 460 V, 3-phase, 3-wire supply. Find the circuit constants of the load per phase.*

Find the line currents if the coil in phase C *is short circuited. Supply phase sequence is A-B-C.*

Solution. *Given:* $P = 8$ kW ; $V = 460$ volts; $\cos\theta = 0.8$

Circuit constants of the load:

Load resistance, **R** = Z cos θ = 21.16 × 0.8 = **16.93** Ω. **(Ans.)**
Load reactance, **X** = Z sin θ = 21.16 × 0.6 = **12.7 Ω. (Ans.)**
Line currents if the coil in phase C is short circuited:



The connection arrangement is shown in above figure when coil C is shorted.

Given that phase sequence is A-B-C.
Hence, $E_{AB} = 460\ 0^\circ$

$E_{BC} = 460\ -120^\circ$

$E_{CA} = 460\ +120^\circ$

Also, Z = 21.6 *36.8°*

Line current in line A $= I_{AB} - I_{CA}$

= 10.87 36.8° – 21.74 *83.2°*

= 10.87(0.8 – *j0.6)* – 21.74(0.118 + *j0.993)*

= 8.70 – *j6.52* – 2.56 – *j21.59* = **(6.14 – j28.11)A. (Ans.)**

Line current in line B $= I_{BC} - I_{AB}$

= 21.74 -*156.8°* -10.87 – *36.8°*

= 21.74(- 0.92 – *j0.394)* – (8.70 – *j6.52)*

= – 20.0 – *j8.56* – 8.70 + *j6.52* = **(- 28.70 – j2.04)A. (Ans.)**

Line current in line C $= I_{CA} - I_{BC}$

= (- 2.56 – *j21.59)* – (- 20 – *j8.56)* = – 2.56 – *j21.59* + 20 +8.56
= (17.44 – j13.03)A. (Ans.)

Example 4. *In a star-connected load each phase consists of a resistance" of 50 Ω in parallel with a capacitor of capacitance 16 µF. When it is connected to 400 V, 3-phase, 5 Hz supply, calculate:*

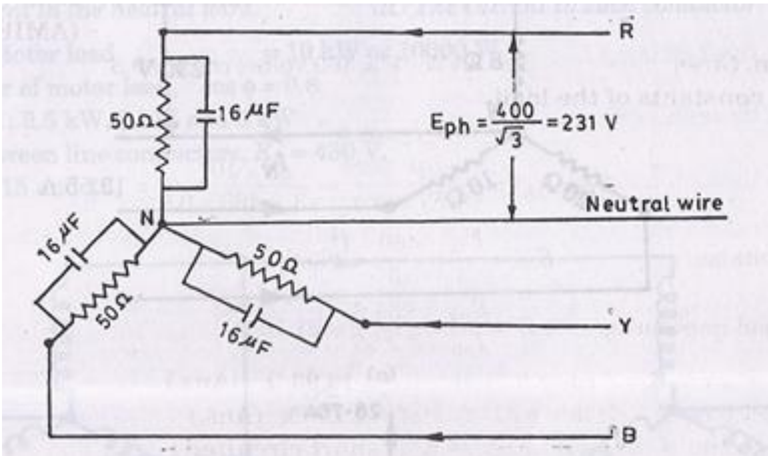
(i) The line current *(ii) The power factor,*

(iii) The power absorbed, and *(iv) The total kVA*

Solution. Resistance per phase, $R = 50\ \Omega$

Capacitance, $C = 16\ \mu F = 16 \times 10^{-6}$ F

The circuit is shown in Fig.



Admittance of each phase,

$l_{ph} = E_{ph}.Y_{ph}$ = 231 (0.02 + *j0.005)* = 4.62 + *j1.155* = 4.76 *14°*

.. *h* =4.76A.. *h* =4.76A

Hence, line current = 4.76 A.

(ii) Power factor
(iii) Now
(Ans.)

For a star connection, $I_{ph} = I_L$

I_L = 4.76 A

Hence, **line current = 4.76 A. (Ans.)**

(ii) **Power factor** = cos 14° = **0.97 (leading). (Ans.)**
(iii) Now E_{ph} = (231 + *j0); l_{ph}* = (4.62 + *j1.155)*
P_{YA} = (231+ *j0)* (4.62 – *j1.155)*

= 231 × 4.62 – *j1.155* × 231 = 1067.22 – *j266.8*

= 1100 *14°* ***(per phase)***

Total power absorbed = 3 × 1067.22 = 3201.66 W = **3.201 kW. (Ans.)**

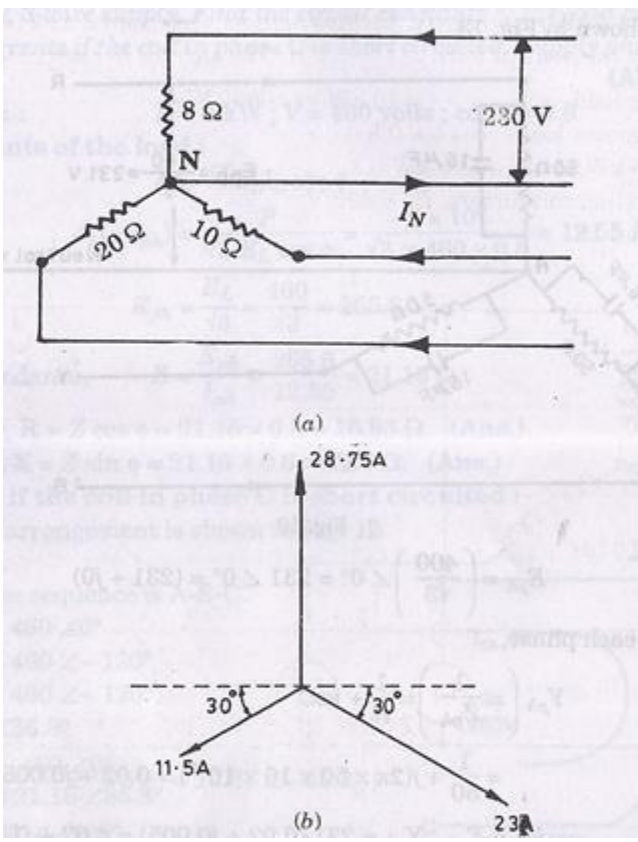
(iv) **Total volt-amperes** = 3 × 1100 = 3300 VA = **3.3 kVA. (Ans.)**

Example5. *A 3-phase, star-connected system with 230 V between each phase and neutral has resistance of* 8, *10 and 20 Ω respectively in three phases, calculate:*

(i) The current flowing in each phase, *(ii) The neutral current, and*

(iii) The total power absorbed.

Solution. Refer Figs. *(a)* and *(b).*



Phase voltage, E_{ph} = 230 V

ii) The above currents are mutually displaced by 120°. The neutral current I_N is the vector sum of these three currents.

I_N can be found by splitting up these three-phase currents into their X-components and Y-components and then by combining them together.

ΣX-components = 23 cos 30° – 11.5 cos 30° =11.5 cos 30° = 9.96 A

ΣY-components = 28.75 – 23 sin 30° -11.5 sin 30° = 28.75 – 34.5 sin 30° = 11.5 A

Neutral current, I_N **= 15.21 A. (Ans.)**

(iii) **Total power absorbed,**

Example 6. *In a 3-phase, 4-wire system, there is a balanced 3-phase motor load taking 10 kW at a power factor of 0.8 lagging while lamps connected between phase conductors and the neutral are taking* 2.5 *kW,* 2 *kW and* 5 *kW respectively. Voltage between line conductors is 430 V. Calculate:*

(i) The current in the neutral wire. *(ii) The current in each conductor.*

Solution. Motor load = 10 kW or 10000 W

Power factor of motor load, cos θ = 0.8

Lamp loads: 2.5 kW, 2 kW and 5 kW

Voltage between line conductors, E_L = 430 V.

Refer Figs. 15, and 16.

Let us first find the current in the neutral wire due to lamp loads L_1 (2.5 kW), L_2 (2 kW) and L_3 (5 kW) respectively.

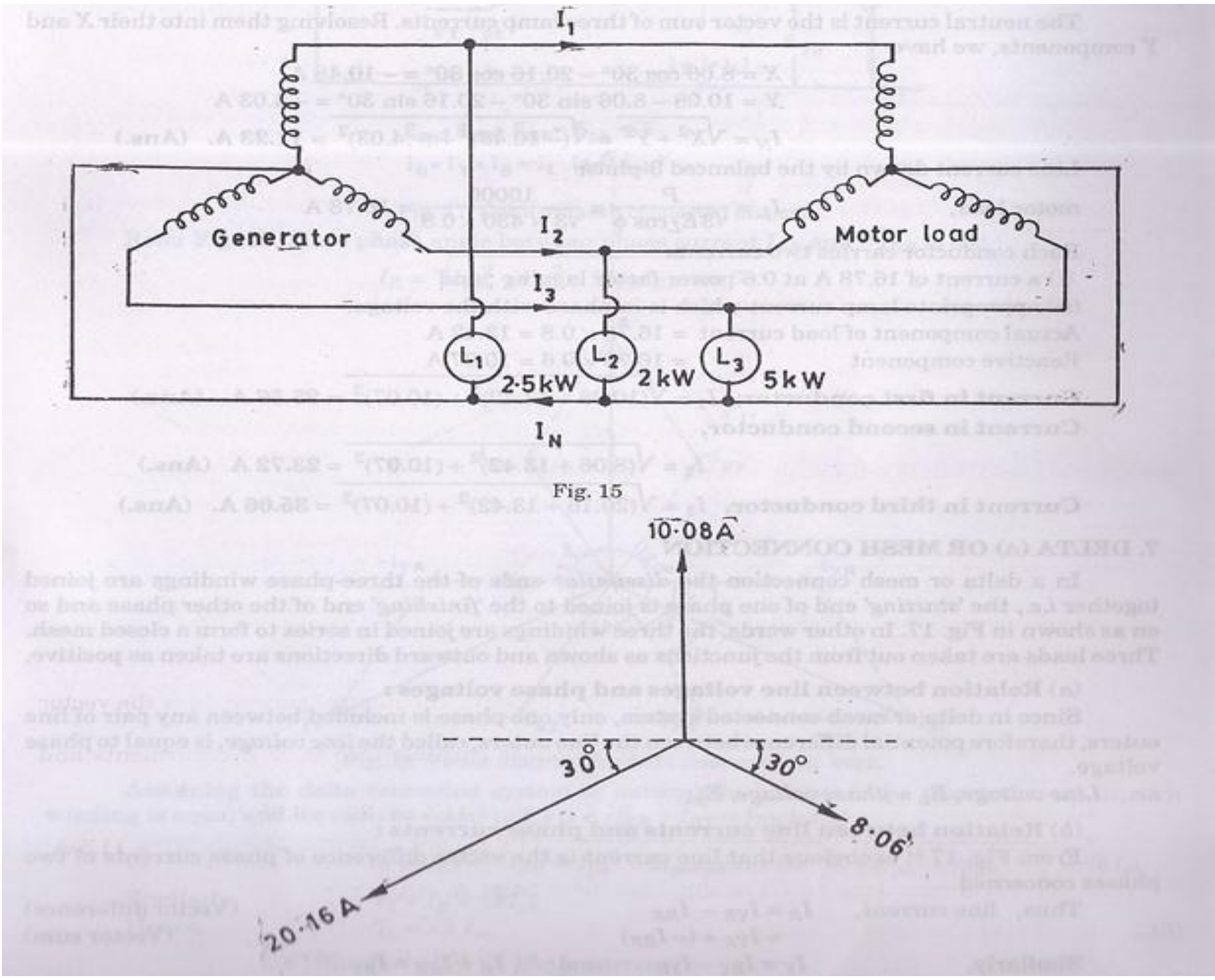


Fig. 15

The neutral current is the vector sum of three lamp currents. Resolving them into their X and *Y* components, we have

X = 8.06 cos 30° – 20.16 cos 30° = -10.48 A

Y = 10.08 – 8.06 sin 30° – 20.16 sin 30° = – 4.03 A

Line current drawn by the balanced 3-phase

Each conductor carries-two currents:

(i) a current of 16.78 A at'O.8 power factor lagging; and

(ii) appropriate lamp current which is in phase with the voltage.

Actual component of load current = 16.78 × 0.8 = 13.42 A

Reactive component = 16.78 × 0.6 = 10.07 A

Example7. *Three identical coils connected in delta across 400 V, 50 Hz, 3-phase supply take a line current of* 15 *A at a power factor 0.8 lagging. Calculate:*

(i) The phase current, and (ii) The impedance, resistance and inductance of each winding.

Solution. Line voltage, $E_L = 400$ V

Line current, $I_L = 15$ A

Power factor, $\cos = 0.8$ lagging

I_{ph} ; Z_{ph} ; R_{ph} ; L :

Phase voltage, $E_{ph} = E_L = 400$ V

(i) Phase current, $I_{ph} = I_L/\sqrt{3} = 15/\sqrt{3} = 8.66$A. (Ans.)

(ii) Impedance of each phase, $Z_{ph} = E_{ph}/I_{ph} = 400/8.66 = 46.19\ \Omega$.(Ans.)

Resistance of each phase, $R_{ph} = Z_{ph} \cos\Phi = 46.19 \times 0.8 = 36.95\ \Omega$.(Ans.)

Reactance of each phase, $X_{ph} = Z_{ph} \sin\Phi = 46.19 \sqrt{(1-\cos^2 \Phi)}$

$= 46.19 \sqrt{(1-(0.8)^2)} = 27.71\ \Omega$

$\therefore$Inductance, $L = X_{ph}/2\pi f = 27.71/(2\pi \times 50) = 0.088$ H.(Ans.)

Example 8. *A 220 V; 3-phase voltage is applied to a balanced delta-connected 3-phase load of phase impedance* (6 + *j*8).

(i) Find the phasor current in each line.

(ii) What is the power consumed per phase?

(iii) What is the phasor sum of the three line currents? What does it have this value?

Solution.

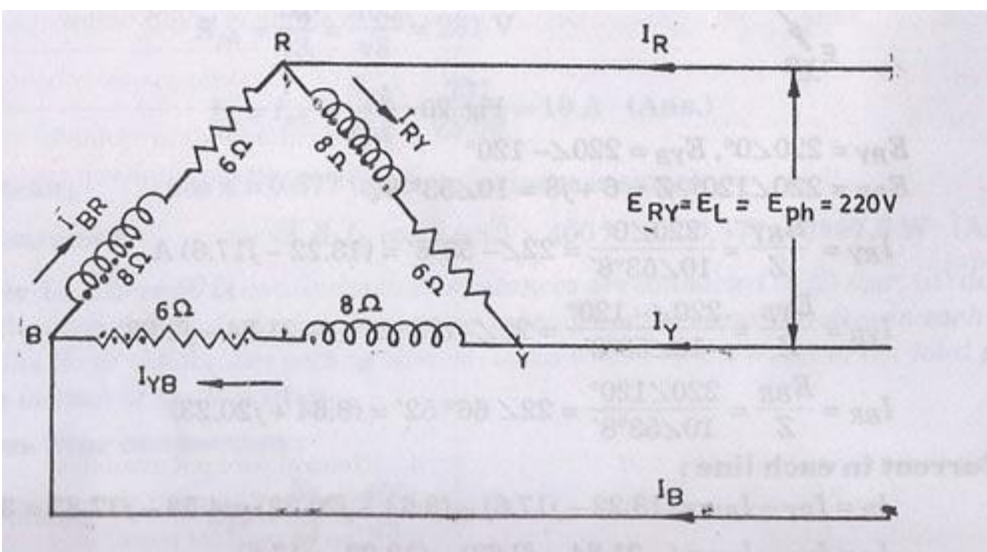


Fig. 19

Resistance per phase, $R_{ph} = 6\ \Omega$

Reactance per phase, $X_{ph} = 8\ \Omega$

$E_L = E_{ph} = 220$ V

Impedance per phase, $Z_{ph} = \sqrt{R_{ph}^2 + X_{ph}^2} = \sqrt{6^2 + 8^2} = 10\ \Omega$

(i) Phase current, $I_{ph}=E_{ph}/Z_{ph}$ = 220/10=22 A

∴ **Line current**, $I_L=\sqrt{3} \times 22=$**38.1 A (Ans.)**

(ii) Power consumed per phase,

$P_{ph}=I_{ph}^2 \times R_{ph}= 22^2 \times 6=$**2204 W. (Ans)**

(iii) Phasor sum would be zero because the three currents are equal in magnitudes and have a mutual phase difference of 120°.

Solution by Symbolic Notation. Let ERY is taken as a reference vector (Fig, 20).

$E_{RY}=220\angle 0^o, E_{YB}=220\angle -120^o$

$E_{BR}=220\angle 120^o, Z=6+j8=10\angle 53^o 8'$

$I_{RY}=E_{RY}/Z= (220\angle 0^o)/(10\angle 53^o 8')= 22\angle -53^o 8', =(13-j17.6)$ A

$I_{YB}=E_{YB}/Z= (220\angle -120^o)/(10\angle 53^o 8')= 22\angle -173^o 8', =(-21.84-j2.63)$

$I_{BR}=E_{BR}/Z= (220\angle 120^o)/(10\angle 53^o 8')= 22\angle 66^o 52', =(8.64+j20.23)$

(i) Current in each line:

$I_R=I_{RY}- I_{BR}=(13.22-j17.6)- (8.64+j20.23)=4.58-j37.83 = 38.1 \angle 83.1°$Ans.

$I_Y=I_{YB}- I_{RY}=(- 21.84 -j2.63) – (13.22 – j17.6)$

$=21.84 – j2.63 – 13.22 +j17.60 = – 35.06 + j14.97 =38.12\angle 156.8^o$.(Ans.)

$I_B=I_{BR}- I_{YB}=(8.64+j20.23)-(-21.84+j2.63)=8.64+j20.23+21.84+j2.63$

$=30.48+j22.86=38.1\angle 36.8^o$.(Ans.)

(ii) Power consumed per phase:

Using conjugate of voltage, we get for R-phase

PVA = ERY.IRY = (220 – j0)(13.22 – j17.6) = (2908.4 – j3872) volt ampere

True power per phase = 2.908 kW. (Ans.)

(iii) Phase sum of the three line currents

=IR+IY+IB

= (4.58 – j37.83) + (- 35.06 + j14.96) + (30.48 + j22.86) = 0

Hence, the phasor sum of three line currents drawn by a ‘balanced load’ is zero. (Ans.)

S.No.	Objective Questions
1	In delta connection ------------- ( ) **a) $V_L=V_{Ph}$** b) $I_L= L_{Ph}$ c) both a & b d) None
2	In star connection------------- ( ) a) $V_L=V_{Ph}$ **b) $I_L= L_{Ph}$** c) both a & b d) None
3	In two wattmeter method if two meters reads same value then power factor ( ) a) 0 b) 0.5 **c) 1** d) none
4	In two wattmeter method if one meter reads zero value then phase angle ( ) a) 0^0 b)90^0 c)180^0 **d) none**
5	In a balanced 3phase star connected load the total power dissipated is 1000W. If the same impedances are connected in delta across the same supply, the total power dissipated is [ ] a) 9000W b) $\frac{1000}{3}W$ c) **1000W** d) 3000W
6	In a $3-\phi$ unbalanced, 4 – wire Star connected system, the currents in the neutral wire is given by [ ] a) Zero b) 3 times the current in individual phases **c) The vector sum of the currents in the three lines** d) None
7	A 3 phase 220V, balanced supply is applied to a balanced 3 phase Delta connected load of impedance (6+j8) Ω/ph . The current through load impedance is [ ] a) 22A lags by 53.2^0 b) 22A leads by 53^0

	c) $\frac{22}{\sqrt{3}}$ **A lags by 53.2^0** d) $\frac{22}{\sqrt{3}}$ A lags by 36.8^0
8	In a balanced 3-phase star connected load, the power consumed /ph is 500W. The P.F. of the load is 0.8 lag. The total reactive power taken by the load is [ ] **a) 1875 VAR** b) 375 VAR c) 625 VAR d) 1125 VAR
9	A balanced 220V, 3 phase supply is given to a balanced 3 phase delta connected resistance of $10\Omega / ph$. The line current taken by the load is [ ] a) $\sqrt{2}\times 22$A b) $\sqrt{3}\times 22$A c) 22v **d)** $\frac{22}{\sqrt{3}}A$
10	A balanced 3 phase 200V supply is given to a balanced delta connected 3 phase load and takes total reactive power of 1200 VAR at a line current of $4\sqrt{3}$ Amps. The power factor of the load is [ ] a) 0.707 b) 0.5 c) 0.866 **d) unity**
11	A balanced star connected load is supplied from a balanced 3phase 400V system. The current in each phase is 30 Amps and is 30^0 behind the phase voltage. The total power taken by the load is [ ] **a) 18 KW** b) 9 KW c) 6 KW d) 12 KW
12	1A balance star connected load has a capacitance of $3\mu F/ph$. The Equivalent delta connected load has a capacitance of [ ] a) $1\mu F/ph$ **b)** $9\mu F/ph$ **c)** $3\mu F/ph$ d) $6\mu F/ph$
13	In a $3-\phi$ balanced Star connected system, the phase relation between the line voltages and their respective phase voltages is given by [ ] **a) The line voltages lead their respective phase voltages by 30^0** b) The phase voltages lead their respective line voltages by 30^0 c) The line voltages and their respective phase voltages are in phase d) None

14	In a $3-\phi$ balanced Delta connected system, the phase relation between the line currents and their respective phase currents is given by [ ] **a) The line currents lag behind their respective phase currents by 30^0** b) The phase currents lag behind their respective line currents by 30^0 c) The line currents and their respective phase currents are in phase d) None
15	Three inductors of 10mH each are connected in Delta. The inductance of an equivalent star connected load is [ ] a) 20mH b) 30mH c) 5mH **d) 10/3mH**

2 Marks questions and answers

1. Give short notes on resistor.

It is a property of a substance which opposes the flow of electrons. It is denoted by R and its unit is Ohm (Ω)

2. Distinguish between a Branch and a node of a circuit.

A pair of network which connects the various points of the network is called branch

A point at which two or more elements are joined together is called node.

3. Distinguish between a mesh and a loop of a circuit.

A mesh is a loop that does not contain other loops. All meshes are loop, but all loops are not meshes.

A loop is any closed path of branches

4. Define line voltage and phase voltage?

The voltage across one phase and neutral is called line voltage & the voltage between two lines is called phase voltage

5. Write down the formula for a star connected network is converted into a delta network?

RA=(R1 R2)/(R1 +R2+ R3)

RB=(R1 R3)/(R1 +R2+ R3)

RC=(R2 R3)/(R1 +R2+ R3)

6. Write down the formula for a delta connected network is converted into a star network?

R1=(RARB+RBRC+RCRA)/RC

R2=(RARB+RBRC+RCRA)/RB

R3=(RARB+RBRC+RCRA)/RA

7. Define line currents and phase currents?

• The currents flowing in the lines are called as line currents

• The currents flowing through phase are called phase currents

8. Give the phase value & Line value of a star connected system.

$$V_L = \sqrt{3} V_{ph}$$

9. Give the phase value and line value of a delta connected system.

$$I_L = \sqrt{3} I_{ph}$$

10. What is the power equation for a star connected system?

$$P = \sqrt{3} I_L V_L \cos\Phi \text{ W}$$

11. What is the power equation for a delta connected system?

$$P=\sqrt{3}I_{L}V_{L}\cos\Phi \text{ W}$$

12. What is meant by Real power?

Real power means the useful power transfer from source to load. Unit is watts.

13. What is meant by apparent power?

Apparent power is the product of voltage and current and it is not true power. Unit is VA

14. What is reactive power?

If we consider the circuit as purely inductive the output power is reactive power. Its unit is VAR

15. Define Instrument.

Instrument is defined as a device for determining the value or magnitude of a quantity or variable.

16. Mention the two main differences between an ammeter and a voltmeter.

S.No.	Ammeter	Voltmeter
1	It is a current measuring device	It is a voltage measuring device
2	It is always connected series with circuit	Always connected parallel with circuit
3	The resistance is very small	The resistance is very high

17. Give short notes on resistor.

It is a property of a substance which opposes the flow of electrons. It is denoted by R and its unit is Ohm (Ω)

18. State the disadvantage of wattmeter?

i) Even a small error in measurement of voltages causes serious errors

ii) Supply voltage higher than normal voltage is required.

19. A three phase 500 v motor load has p.f 0.4 .Two wattmeter's connected to measure the input. They show the input to be 30 kw .Find the reading of each instrument?

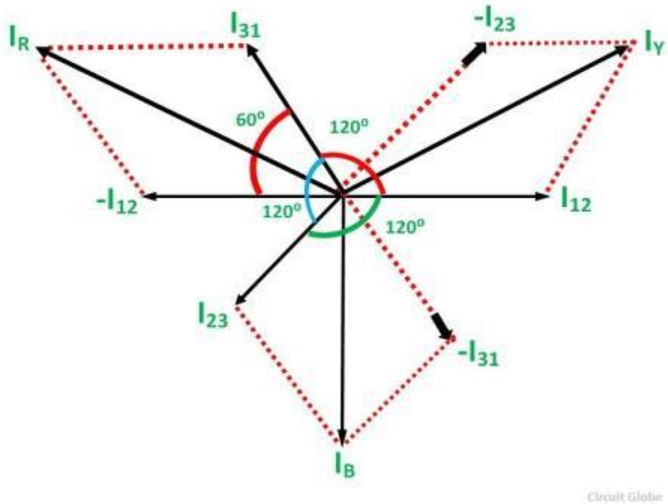
P1+P2=30 kw P1-P2=39.7 kw

P1=34.85 kw P2=-4.85 kw

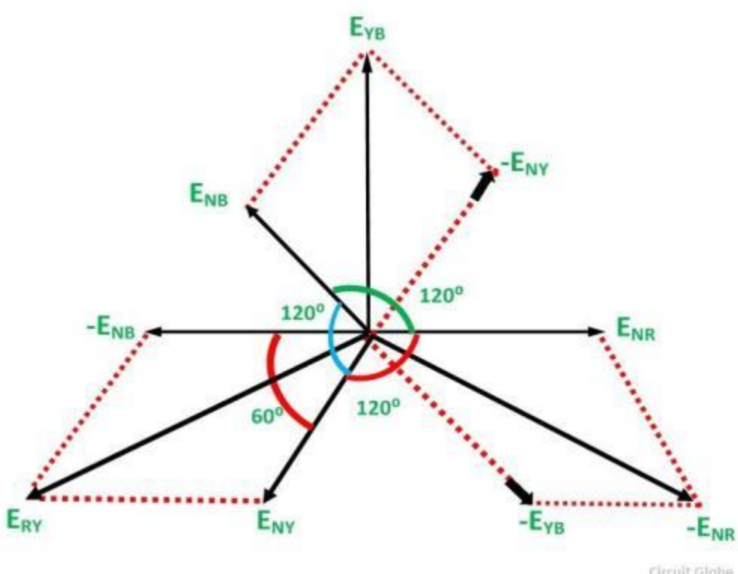
20.Name the methods used for power measurement in three phase circuits.

(i) Three wattmeter method (ii) Two Wattmeter method (iii) single wattmeter method

21. Draw phasor diagram of currents in 3-phase delta connected system.



22.Draw phasor diagram of voltages in 3-phase star connected system.



23. What are the advantages of 3-phase system over other systems?

The advantages of three phase system are

1. Three phase systems requires less conducting material for a given amount of power to transfer than using individual single phase systems
2. Three phase motors are self starting whereas single phase motors are not self starting motors
3. Power factor of single phase motors is poor than compared to three phase motors.

24. Calculate the reactance of a coil of inductance 0.32H when it is connected to a 50 Hz supply.

Given data

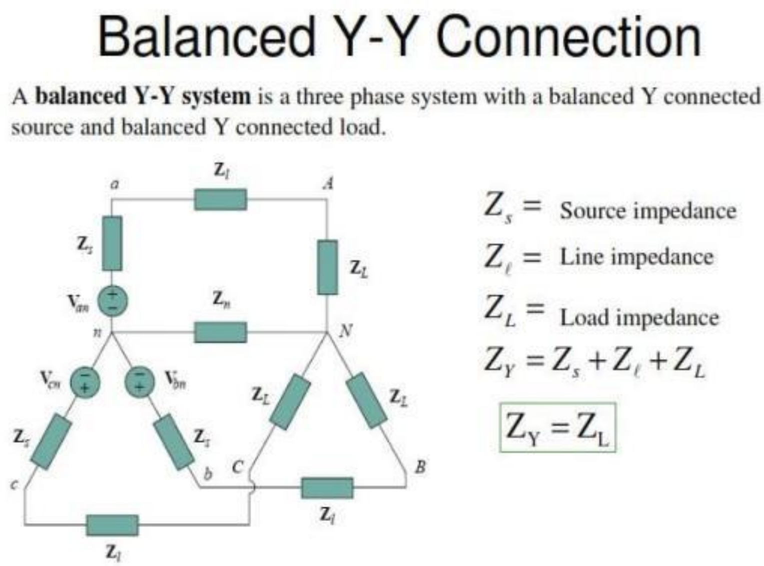
Inductance L=0.32H, frequency f=50Hz

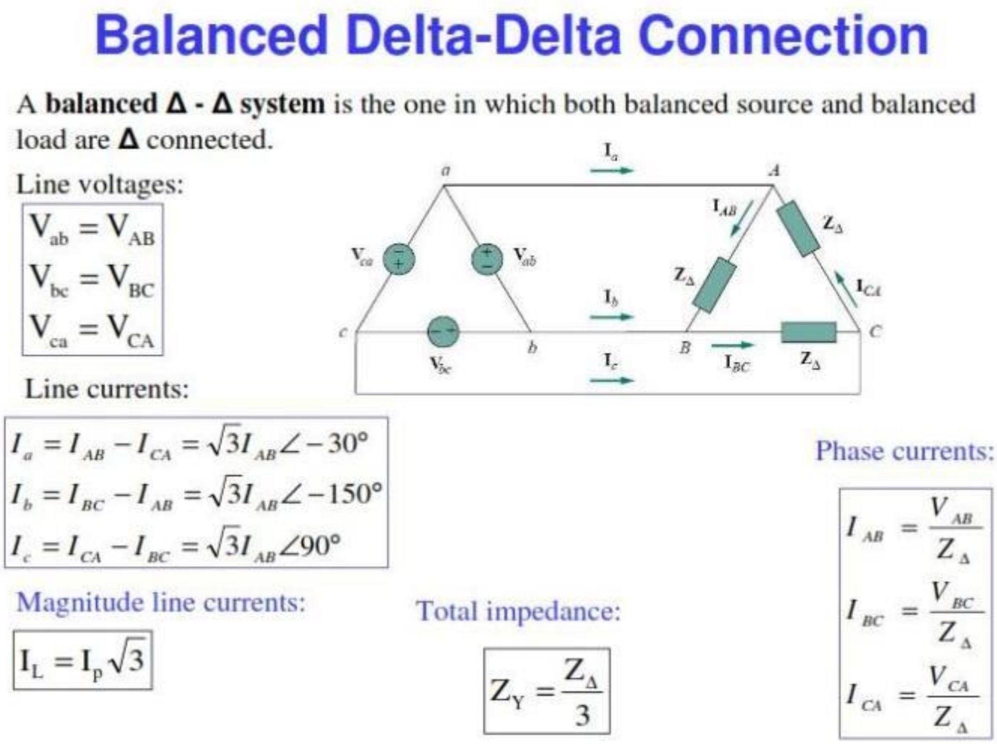
Reactance of the coil =?

Reactance of the coil $X_L = 2\pi f L$ =100.53Ω

Descriptive Questions

1. Explain Balanced Y-Y Connection and balanced Delta –Delta Connection?





2. The comparison between star and delta connected systems is given below:

Star connected system	*Delta connected system*
1. *Similar ends* are joined together.	1. *Dissimilar ends* are joined.
1. Phase voltage = line voltage	1. *2.* Phase voltage = line voltage *(i.e., $E_{ph} = E_L$).*
1. *3.* Phase current = line current *(i.e., $I_{ph} = I_L$).*	1. *3.* Phase current == = × line current
1. Possible to carry neutral to the load.	1. Neutral wire not available.
1. Provides 3-phase 4-wire arrangement.	1. Provides 3-phase 3-wire arrangement.

1. Can be used for lighting as well as power load.	1. Can be used for power loads only,
1. Neutral wire of a star connected alternator can be connected to earth, so relays and protective devices can be provided in the star connected alternators for safety.	7. Not possible. Delta connected system is mostly used in transformer for running of small low voltage 3-phase motors and *best suited for rotary converters.*

3..*Three equal impedances each having a resistance of* 25 Ω *and reactance of 40* Ω *are connected in star to a 400 V, 3-phase, 50 Hz system. Calculate:*

(i) The line current *(ii) Power factor, and*

(iii) Power consumed.

Solution. Resistance per phase, R_{ph}= 25 Ω

Reactance per phase, X_{ph}= 40 Ω

Line voltage, E_L= 400 V

Line current, I_L.:

Power factor, cos

Power consumed, *P* :

Refer Fig. 7.

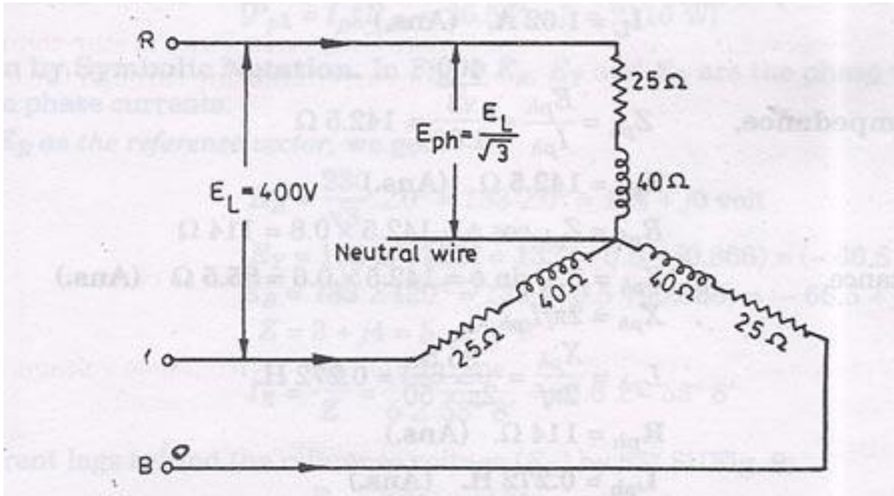


Fig. 7

Impedance per phase, $Z_{ph} = \sqrt{R_{ph^2} + X_{ph^2}}$

$$Z_{ph} = \sqrt{25^2 + 40^2 = 47.17}\ \Omega$$

Phase voltage, $E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231$ V

Phase current, $I_{ph} = \frac{E_{ph}}{Z_{ph}} = \frac{231}{47.17} = 4.9$ A (app.)

(i) **Line current,** I_L = phase current, I_{ph}

I_L = 4.9A. (Ans.)

(ii) **Power factor,** $\cos\phi = \frac{R_{ph}}{Z_{ph}} = \frac{95}{47.17} = \mathbf{0.53}$ **(lag.) (Ans.)**

(iii) **Power consumed,** $P = \sqrt{3}E_L I_L \cos\phi = \sqrt{3} \times 25 = 1800$ W].

[or $P = 3I_{ph}^2R_{ph} = 3 \times 4.9^2 \times 25 = 1800$ W]

4. *Three identical coils are connected in star to a 400 V (line voltage), 3-phase A.C. supply and each coil takes 300* W. *If the power factor is 0.8 (lagging). Calculate:*

(i) The line current, *(ii) Impedance, and*

(iii) Resistance and inductance of each coil.

Solution. Line voltage, $E_L = 400$ V

Power taken by each coil, $P_{ph} = 300$ W

Power factor, $\cos = 0.8$ (lagging)

I_L ; Z ; Z_{ph} ; L_{ph} :

Phase voltage, $$E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{400}{\sqrt{3}}\,V$$

Also $$P_{ph} = E_{ph} I_{ph} \cos\phi$$

$$300 = \frac{400}{\sqrt{3}} \times I_{ph} \times 0.8$$

∴ $$I_{ph} = \frac{300 \times \sqrt{3}}{400 \times 0.8} = 1.62\,A.$$

i) Line current I_L = phase current, I_{ph}

$$\mathbf{I_L = 1.62\ A.}\quad \textbf{(Ans.)}$$

ii) Coil Impedance $$Z_{ph} = \frac{E_{ph}}{I_{ph}} = \frac{\frac{400}{\sqrt{3}}}{\sqrt{3}} V$$

$\therefore$ $$\mathbf{Z_{ph} = 142.5\ \Omega}\quad \textbf{(Ans.)}$$

iii) $$R_{ph} = Z_{ph}\cos\phi = 142.5 \times 0.8 = 114\ \Omega$$

Coil reactance, $$X_{ph} = Z_{ph}\sin\phi = 142.5 \times 0.6 = \mathbf{85.5}\ \Omega \quad \textbf{(Ans.)}$$

But $$X_{ph} = 2\pi f L_{ph}$$

$\therefore$ $$L_{ph} = \frac{X_{ph}}{2\pi f} = \frac{85.5}{2\pi \times 50} = 0.272\ H.$$

Hence $$\mathbf{R_{ph} = 114\ \Omega}\quad \textbf{(Ans.)}$$

and $$\mathbf{L_{ph} = 0.272\ H}\quad \textbf{(Ans.)}$$

5. *In a 3-phase, 3-wire system with star-connected load the impedance of each phase is* (3 + *j4)* Ω*. If the line voltage is 230 V, calculate:*

(i) The line current, and *(ii) The power absorbed by each phase.*

Solution. Line voltage, $E_L = 230$ V

Resistance per phase, $R_{ph} = 3\ \Omega$

Reactance per phase, $X_{ph} = 4$ Q

I_L; P_{ph}:

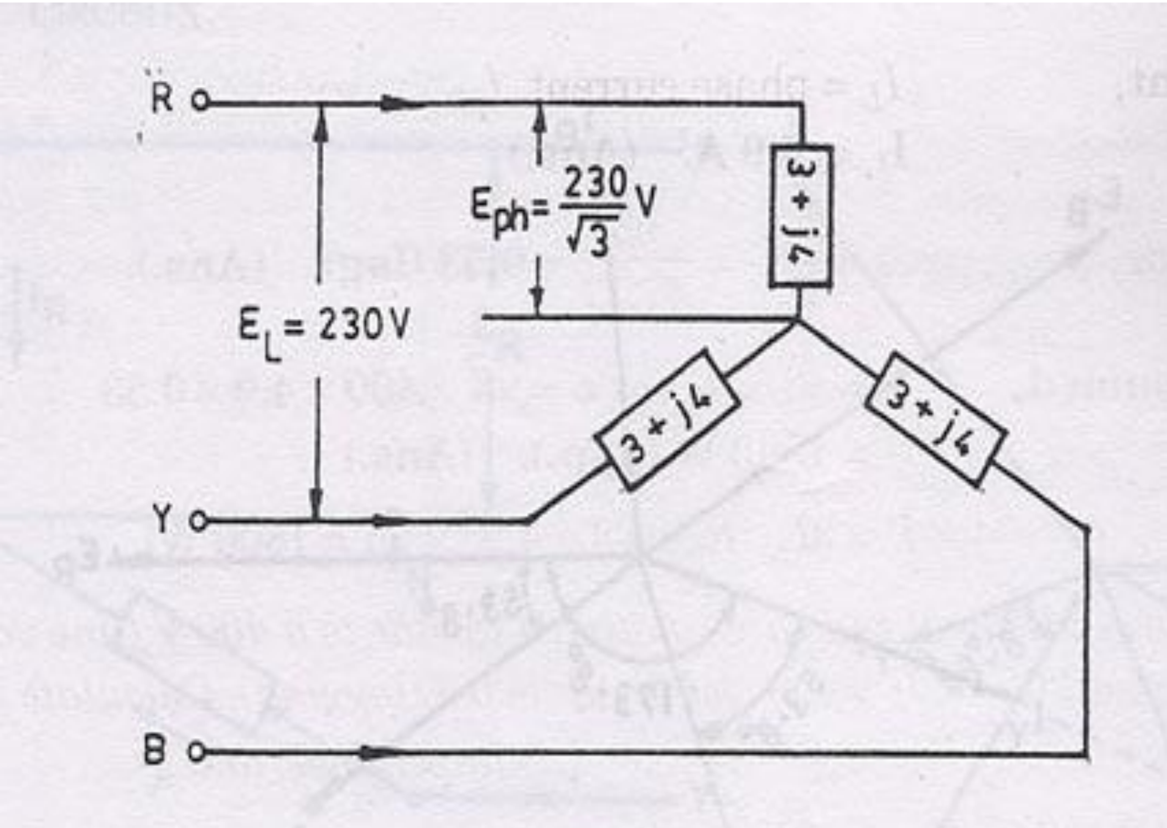


Fig. 8

Phase voltage, $E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} V$

Impedance per phase, $Z_{ph} = \sqrt{R_{ph^2} + X_{ph^2}} = \sqrt{3^2 + 4^2} = 5\ \Omega$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{3}{5} = 0.6$

Phase current, $I_{ph} = \frac{E_{ph}}{Z_{ph}} = \frac{230}{\sqrt{3} \times 5} = 26.56\ A$

Line current, $\mathbf{I_L = I_{ph} = 26.56\ A}$ **(Ans.)**

Power absorbed by each phase,

$$\mathbf{P_{ph}} = E_{ph} I_{ph} \cos\phi = \frac{230}{\sqrt{3}} = 26.56 \times 0.6 = \mathbf{2116\ W}\ (\boldsymbol{Ans.})$$

Solution by symbolic Notation. In Fig 9 E_R, E_Y and E_B are the phase voltages whereas I_R, I_Y and I_B are phase currents.

Taking E_R as the reference vector, we get

This current lags behind the reference voltage (E_R) by 8' (Fig.9)

It lags the reference i.e., E_R by 8' which amounts to lagging behind its phase voltage E_Y by 8'

This current leads E_R by which is the same as *lagging* behind its phase voltage by .

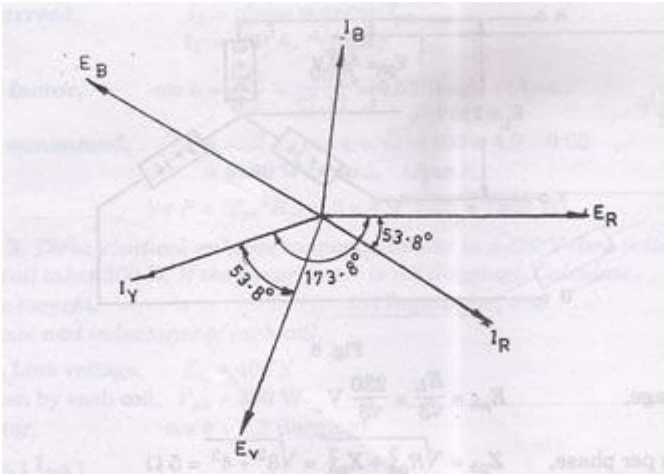


Fig. 9

Let us consider R-phase for calculation of power

$E_R = (133 + j0)$; $I_R = 26.6\,(0.6 - j0.8) = (15.96 - j21.28)$

Using method of conjugates, we get

$P_{VA} = (133 - jO)\,(15.96 - j21.28) = 2116 - j2830$

Real power absorbed/phase = 2116 W

6. A delta-connected balanced 3-phase load is supplied from a 3-phase, 400 V supply. The line current is 30 A and the power taken by the load is 12 k W. Find:
(i) Impedance in each branch; and
(ii) The line current, power factor and power consumed if the same load is connected is star.

Solution. Delta-connection:
$E_{ph} = E_L = 400$ V
$I_L=30$A
$\therefore I_{ph}=I_L/\sqrt{3}= 30/\sqrt{3}=17.32$ A

(i) Impedance per phase
$Z_{ph}=E_{ph}/I_{ph} = 400/17.32=23.09\ \Omega$. (Ans)
Now $P=\sqrt{3}\, E_L I_L \cos\Phi$
$12000=\sqrt{3} \times 400\times 30\times \cos\Phi$
or $\cos\Phi$ (power factor)$=12000/(\sqrt{3} \times 400\times 30)=0.577$
(ii) Star-connection
$E_{ph}=E_L/\sqrt{3}= 400/\sqrt{3}=231$ V
$I_L=I_{ph}\ E_{ph}/Z_{ph} = 231/23.09=10$ A (Ans.)
Power factor, $\cos\phi = 0.577$ (since impedance is same)
Power consumed $= \sqrt{3}\, E_L I_L \cos\Phi=\sqrt{3}\times 400\times 10\times 0.577=3997.6$W (Ans)

7. Three 50 n non-inductive resistances are connected in (i) star, (ii) delta across a 400 V, 50 Hz., 3-phase mains. Calculate the power taken from the supply system in each case. In the event of one of the three resistances getting opened, what would be the value of the total power taken from the mains in each of the two cases.

Solution. Star connection:
Phase voltage,
Delta connection:
Phase voltage, $E_{ph}=E_L/\sqrt{3}=400/\sqrt{3}=231$ V
Phase current, $I_{ph}=E_{ph}/R_{ph}=231/50=4.62$ A
Power consumed, $P = 3I_{ph}^2 R_{ph} = 3 \times 4.62^2 \times 50 =$ **3200 W.(Ans.)**
[or $P = \sqrt{3}\, E_L I_L \cos\Phi = \sqrt{3} \times 400 \times 4.62 \times 1 = 3200$ W.] **Delta connection :**
Phase voltage, $E_{ph}=E_L=400$ V
Phase current, $I_{ph}=E_{ph}/R_{ph}=400/50=8$ A
Power consumed, $P = 3I_{ph}^2 R_{ph} = 3 \times 8^2 \times 50 =$ **9600 W.(Ans.)**
When one of the resistances is disconnected:
(i) Star connection. Refer Fig. 21.

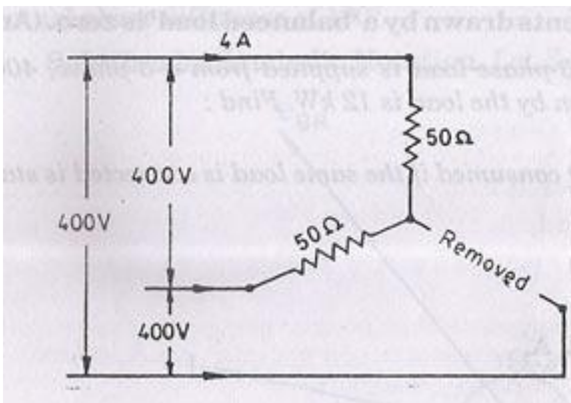


Fig. 21

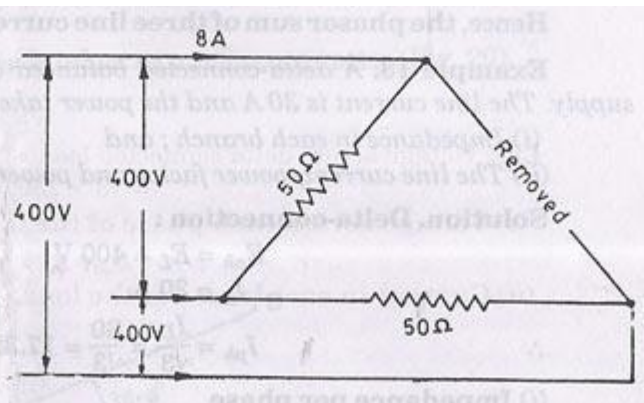


Fig. 22

When one of the resistances is disconnected, the circuit is no longer 3-phase but converted into single-phase circuit, having two resistances each of 50 ohm connected in series across supply of 400V.
Hence line current, $I_L=E_L/2R_{ph}=400/(2\times50)=4$ A
Power consumed, P = 42 (50 + 50) = **1600 W. (Ans.)**
[or $P = VI \cos\Phi = 400\times4\times1 = 1600$ W].
(ii) Delta connection. Refer Fig. 22.
Potential difference across each resistance, EL = 400 V

Current in each resistance =400/50=8 A
Power consumed in both resistances = 2 × 82 × 50 = **6400 W. (Ans.)**
[or $P = 2\times E_{ph} I_{ph} \cos\Phi = 2\times400\times8\times1 = 6400$ W].

8. *A 440 V, 50 Hz, 3-phase supply has delta-connected load having 50 Ω between R and y,* 159 *mH between Y and Band* 15.9 µF *between B and R. Find;*

(i) The line current for the sequence RYB.

(ii) The value of star-connected balanced resistors for the same power.

Solution. Phase voltage *(E_{ph})* = line voltage *(E_L)* = 440 V
The potential difference across three-phases are given by
E_{RY} = 440(1 + *j0)* = *4400°* V

E_{YB} = 440(– 0.5 – *j0.866)* or 440 – 120° V
E_{BR} = 440(- 0.5 + *j0.866)* or 440 120° V

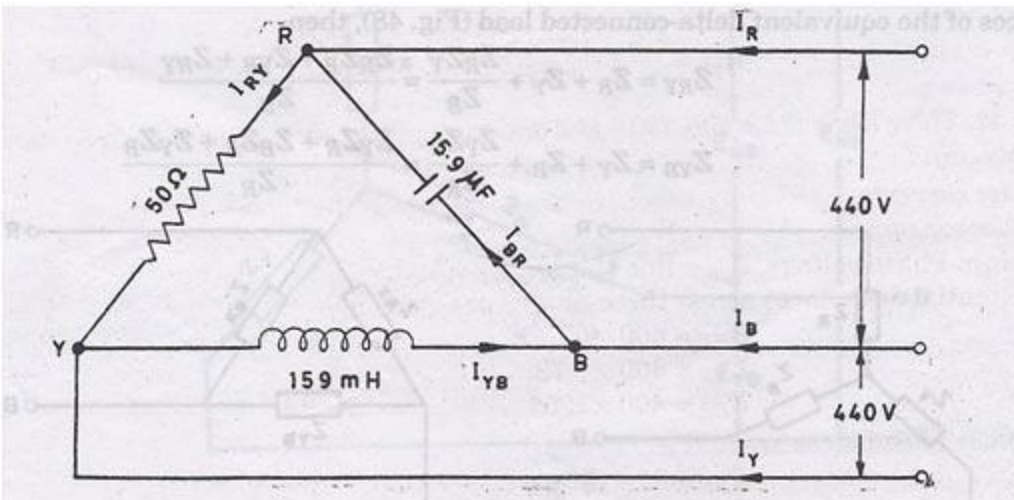


Fig. 50

Impedance , Z_{RY} = (50 + *j0)* = 50 0° Ω

Impedance , Z_{YB} = (0 + *j2rπfL)* = (0 + *j2π* × 50 × 159 × 10^{-3})
= (0 + *j50)* or 50 90° Ω

$$Impedance \quad Z_{BR} = \left(0 - \frac{j}{2\pi fC}\right)$$

$$= \left(0 - \frac{1}{2\pi \times 50 \times 15.9 \times 10^{-6}}\right) = (0 - j200) = 200\angle -90°$$

(i) ***Phase currents :***

$$Phase\ current, \quad I_{RY} = \frac{E_{RY}}{Z_{RY}} = \frac{440\angle 0°}{50\angle 0°} = 8.1(1 + j0)A$$

$$Phase\ current, \quad I_{YB} = \frac{E_{YB}}{Z_{YB}} = \frac{440\angle -120°}{50\angle 0°} = 8.8\angle -120°$$

$$Phase\ current, \quad I_{BR} = \frac{E_{BR}}{Z_{BR}} = \frac{440\angle 120°}{200\angle -90°} = 2.2\angle 210° = 2.2(-0.866 - j0.5)A.$$

Line currents:

Line current $I_R = I_{RY}\text{-}I_{BR} = 8.8(1 + j0) - 2.2(-0.866 - j0.5)$

$= 8.8 + j0 + 1.905 + j1.1 = 10.7 + j1.1 =$ **$10.75\ \angle 5.8°$A. (Ans.)**

Line current $I_Y = I_{YB} - I_{RY} = 8.8(-0.866 + j0.5) - 8.8(1 + j0)$

$= -7.62 + j4.4 - 8.8 - j0$

$= -16.42 + j4.4 =$ **$16.99\ \angle 164.9°$ A. (Ans.)**

Line current $I_B = I_{BR} = I_{YB} = 2.2(-0.866 - j0.5) - 8.8(-0.866 + j0.5)$

$= -1.905 - j1.1 + 7.62 - j4.4 = 5.715 - j5.5$

$= 7.93\ \angle -43.9°$ A. (Ans.)

(ii) **Power supplied:**

$P = I_{RY}^2 \times R = 8.8^2 \times 50 = 3872$ W

Let the resistance per phase be R_{ph} which when connected in star across 440 V, 3-phase supply take power of 38 W.

$$Line\ current, \quad I_L = I_{ph} = \frac{E_L/\sqrt{3}}{R_{ph}} = \frac{440/\sqrt{3}}{R_{ph}}$$

$$Power\ supplied, \quad P = \sqrt{3}E_L I_L$$

$$\therefore \qquad 3872 = \sqrt{3} \times 440 \times \frac{440/\sqrt{3}}{R_{ph}}$$

$$\therefore \qquad \boldsymbol{R_{ph}} = \frac{\sqrt{3} \times 440 \times 440/\sqrt{3}}{3872} = \frac{440 \times 440}{3872} = 50\ \Omega \quad (Ans.)$$

9. *A 440 V, 50 Hz, 3-phase supply has delta-connected load having 50 Ω between R and y, 159 mH between Y and Band 15.9 µF between B and R. Find;*

(i) The line current for the sequence RYB.

(ii) The value of star-connected balanced resistors for the same power.

Solution.

Phase voltage(E_{ph}) = line voltage, (E_L) = 400 V
The potential differences across three-phases are given by:

E_{RY} = 400 0°
E_{YB} = 400 – 120°
E_{BR} = 400 120°

The phase impedances are:

Z_{RY} = 31 + j59 = 66.6 62.3°
Z_{YB} = 30 – j40 = 50 – 53.1°
Z_{BR} = 80 + j60 = 100 36.9°

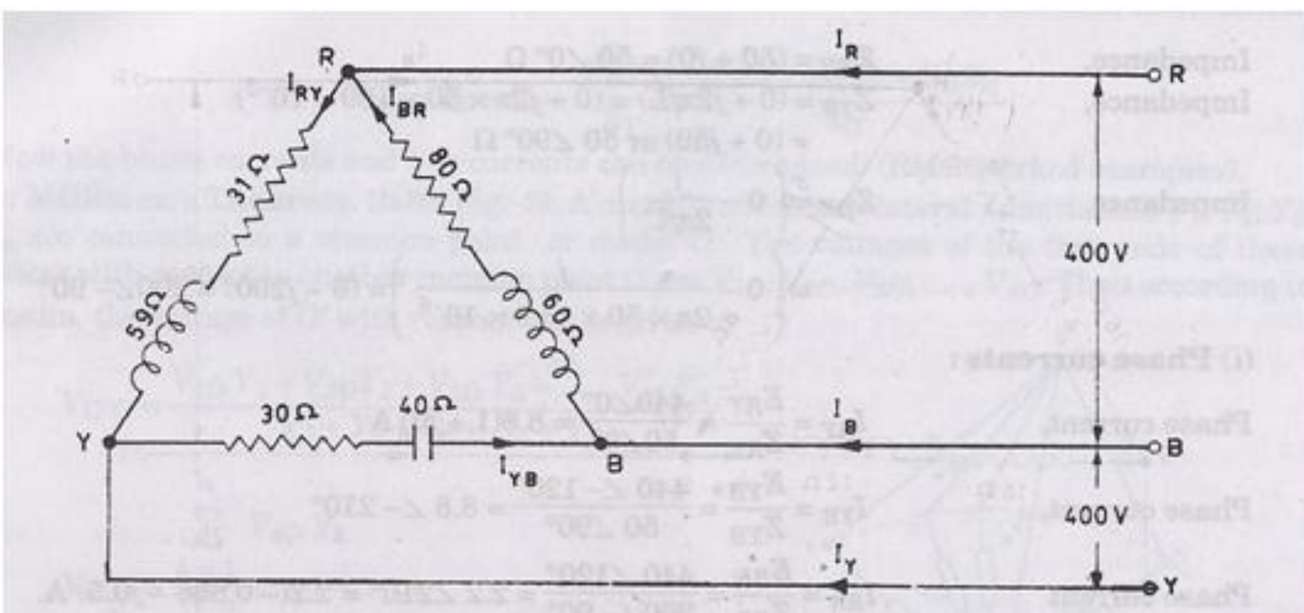


Fig. 51

Phase currents:

$$I_{RY} = \frac{E_{RY}}{Z_{RY}} = \frac{400\angle 0^0}{66.6\angle 62.3^0} = \mathbf{6\angle -62.3^0\ A.\quad Ans.}$$

$$I_{YB} = \frac{E_{YB}}{Z_{YB}} = \frac{400\angle -120^0}{50\angle -53.1^0} = \mathbf{8\angle -66.9^0\ A.\quad Ans.}$$

$$I_{BR} = \frac{E_{BR}}{Z_{BR}} = \frac{400\angle 120^0}{100\angle 36.9^0} = 4\angle\mathbf{83.1\ A.\quad Ans.}$$

Line currents:

$\mathbf{I_R} = I_{RY} - I_{BR} = 6\angle -62.3° - 4\angle 83.1°$

$= 6(0.465 - j0.885) - 4(0.12 + j0.993)$

$= 2.79 - j5.31 - 0.48 - j3.97 = 2.31 - j\,9.28 =$ **9.56 ∠- 76° A. (Ans.)**

$\mathbf{I_Y} = I_{YB} - I_{RY} = 8\angle -66.9° - 6\angle -62.3°$

$= 8(0.392 - j0.92) - 6(0.465 - j0.885)$

$= (3.136 - j7.36) - (2.79 - j5.31)$

$= 0.346 - j2.05 =$ **2.07 ∠- 80.4° A. (Ans.)**

$\mathbf{I_B} = I_{BR} - I_{YB} = 4\angle 83.1° - 8\angle -66.9°$

$= 4(0.12 + j0.993) - 8(0.392 - j0.92)$

$= 0.48 + j3.97 - 3.136 + j7.36$

$= -2.656 + j11.33 =$ **11.637 ∠103.2° A. (Ans.)**

Total power is the sum of power in different phases

$P = E_{RY} \times I_{RY} + E_{YB} \times I_{YB} + E_{BR} \times I_{BR}$

$= 400\angle 0° \times 6\angle -62.3° + 400\angle -120° \times 8\angle -66.9°$

$+ 400\angle 120° \times 4\angle 83.1°$

$= 400 \times 6\cos(-62.3°) + 400 \times 8\cos(53.1°) + 400 \times 4\cos(-36.9°)$

$= 1115.6 + 1921.3 + 1279.5 = 4316.4$ W

$OR\quad P = I_{RY}^2 R_{RY} + I_{YB}^2 \times R_{YB} + I_{BR}^2 \times R_{BR}$

$= 6^2 \times 31 + 8^2 \times 30 + 42 \times 80 = 1116 + 1920 + 1280 = 4316$ W

Hence, **total power = 4316.4 W. (Ans.)**

]UNIT- III FOURIER TRANSFORMS

Fourier Theorem- Trigonometric Form and Exponential Form of Fourier Series – Conditions of Symmetry- Line Spectra and Phase Angle Spectra- Analysis of Electrical Circuits to Non Sinusoidal Periodic Waveforms. Fourier Integrals and Fourier Transforms – Properties of Fourier Transforms and Application to Electrical Circuits.

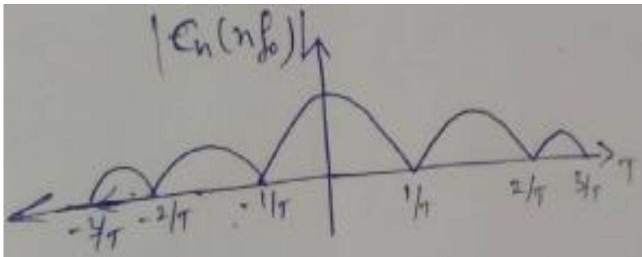
Fourier Transform:- Fourier transform is a transformation technique which transforms signals from the continuous-time domain to corresponding frequency domain and vice versa, and which applies for both periodic as well as aperiodic signals.

Let $x(t)$ be a non-periodic function and $x_T(t)$ be periodic with period T.

$$x(t) = \underset{T \to \infty}{Lt}\; x_T(t) \quad \text{— F.S}$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \quad \text{— F.T}$$

$$x(t) \longleftrightarrow X(\omega)$$



Properties of Fourier Transform :-

The Fourier transform has a number of important properties

(1) Linear Property :- The linearity property states that the Fourier transform of a weighted sum of two signals is equal to weighted sum of their individual Fourier Transform. $x_1(t) \overset{FT}{\longleftrightarrow} X_1(\omega)$ & $x_2(t) \overset{FT}{\longleftrightarrow} X_2(\omega)$

$$a x_1(t) + b x_2(t) \overset{FT}{\longleftrightarrow} a X_1(\omega) + b X_2(\omega)$$

② Time Shifting Property! – The time shifting property states that if a signal $x(t)$ is shifted by t_0 sec, the spectrum is modified by a linear phase shift of slope $-\omega t_0$

$$x(t) \xleftrightarrow{FT} X(\omega)$$
$$x(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0} X(\omega)$$

Proof!– $F[x(t-t_0)] = \int_{-\infty}^{\infty} x(t-t_0) e^{-j\omega t} dt$

$t - t_0 = P$, $t = P + t_0$ and $dt = dP$

$$F[x(t-t_0)] = \int_{-\infty}^{\infty} x(P) e^{-j\omega(P+t_0)} dP$$
$$= e^{-j\omega t_0} \int_{-\infty}^{\infty} x(P) e^{-j\omega P} dP$$
$$= e^{-j\omega t_0} X(\omega)$$

$$x(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0} X(\omega)$$
$$x(t+t_0) \xleftrightarrow{FT} e^{j\omega t_0} X(\omega)$$

Frequency Shifting Property!– Frequency shifting property states that the multiplication of a time domain signal $x(t)$ by $e^{-j\omega t_0}$ results in frequency spectrum shifted b

$$x(t) \xleftrightarrow{FT} X(\omega)$$
$$e^{j\omega_0 t} x(t) \xleftrightarrow{FT} X(\omega - \omega_0)$$

Proof!–
$$F\left[e^{j\omega_0 t} x(t)\right] = \int_{-\infty}^{\infty} e^{j\omega_0 t} x(t) e^{-j\omega t} dt$$
$$= \int_{-\infty}^{\infty} x(t) e^{-j(\omega-\omega_0)t} dt = X(\omega-$$

$$e^{j\omega_0 t} x(t) \xleftrightarrow{FT} X(\omega - \omega_0)$$
$$e^{-j\omega_0 t} x(t) \xleftrightarrow{FT} X(\omega + \omega_0)$$

(4) Time Reversal Property :- The time reversal property states that

if $x(t) \xleftrightarrow{FT} X(\omega)$

$x(-t) \xleftrightarrow{FT} X(-\omega)$

Proof :- $F[x(-t)] = \int_{-\infty}^{\infty} x(-t) e^{-j\omega t} dt$

Replacing t by $-t$, the RHS of the above expression

$$F[x(-t)] = \int_{-\infty}^{\infty} x(t) e^{j\omega t} dt = \int_{-\infty}^{\infty} x(t) e^{-j(-\omega t)} dt$$

$$= X(-\omega)$$

$$x(-t) \xleftrightarrow{FT} X(-\omega)$$

(5) Time scaling property :- Let $x(at)$ is a compressed version of $x(t)$ when $a>1$ (or) expanded version of $x(t)$ when $a<1$

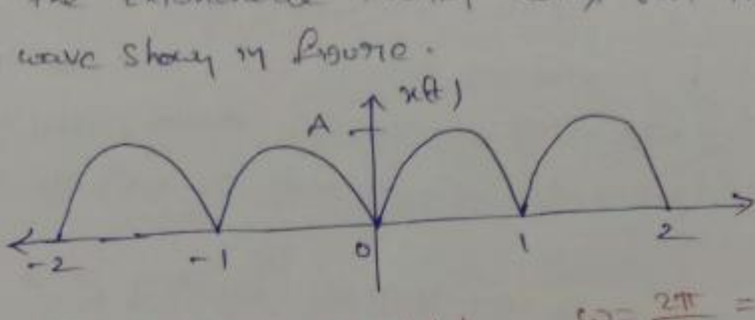
$$x(t) \xleftrightarrow{FT} X(\omega)$$

$$x(at) \xleftrightarrow{FT} \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

(6) Time Integration Property :- The time integration property states that the integration of a function $x(t)$ in time Domain is equivalent to the Division of it's Fourier transform by $j\omega$ $\quad x(t) \xleftrightarrow{FT} X(\omega)$

$$\int_{-\infty}^{t} x(\tau) d\tau \xleftrightarrow{FT} \frac{1}{j\omega} X(j\omega), \text{ if } X(0)=0$$

* Find the Exponential Fourier series for the rectified sine wave shown in figure.



Sol:- $x(t) = A\sin\omega t \quad 0 \le t \le 1, \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

$x(t) = A\sin\pi t \quad 0 \le t \le 1$

Period of rectified wave $T = 1$ (sine)

$t_0 = 0, \quad t_0 + T = 1$

Fundamental freq $\omega = \frac{2\pi}{T} = 2\pi$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jwnt} = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi nt}$$

$$C_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\,dt = \frac{1}{1}\int_0^1 A\sin\pi t\,dt$$

$$= A\left(-\frac{\cos t\pi}{\pi}\right]_0^1 = \frac{2A}{\pi}$$

$$\boxed{C_0 = \frac{2A}{\pi}}$$

$$C_n = \frac{1}{T}\int_0^1 x(t)\,e^{-jn\omega t}\,dt = \int_0^1 A\sin\pi t\,e^{-j2\pi nt}\,dt$$

$$= A\int_0^1\left[\frac{e^{j\pi t} - e^{-j\pi t}}{2j}\right]e^{-j2n\pi t}\,dt$$

$$C_n = \frac{A}{2j}\left[\left(\frac{e^{j\pi(1-2n)t}}{j\pi(1-2n)}\right)_0^1 - \left(\frac{e^{-j\pi(1+2n)t}}{-j\pi(1+2n)}\right)_0^1\right]$$

$$\boxed{C_n = \frac{2A}{\pi(1-4n^2)}}$$

The Exponential Fourier series $x(t) = \frac{2A}{\pi} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{2A}{\pi(1-4n^2)}\,e^{j2n\pi t}$

Trigonometric form of fourier Series

A sinusoidal signal, $x(t) = A \sin \omega_0 t$ is a periodic signal with period $T = 2\pi/\omega_0$. Also, the sum of two sinusiodals is periodic provided that their frequencies are integral multiples of a fundmental frequency ω_0. We can show that a signal $x(t)$, a sum of sine and cosine functions whose frequencies are integral multiples of ω_0, is a periodic signal.

The signal $x(t)$ be

$$x(t) = a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + \cdots + a_k \cos k\omega_0 t$$
$$+ b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + \cdots + b_k \sin k\omega_0 t$$

$$x(t) = a_0 + \sum_{n=1}^{k} a_n \cos \omega_0 n t + b_n \sin \omega_0 n t$$

for the signal $x(t)$ be periodic, it must satisfy the condition $x(t) = x(t+T)$ for all t.

$$x(t+T) = a_0 + \sum_{n=1}^{k} a_n \cos \omega_0 n(t+T) + b_n \sin \omega_0 n(t+T)$$

$$= a_0 + \sum_{n=1}^{k} a_n \cos \omega_0 n\left(t + \frac{2\pi}{\omega_0}\right) + b_n \sin \omega_0 n\left(t + \frac{2\pi}{\omega_0}\right)$$

$$= a_0 + \sum_{n=1}^{k} a_n \cos(\omega_0 n t + 2n\pi) + b_n \sin(\omega_0 n t + 2n\pi)$$

$$x(t+T) = a_0 + \sum_{n=1}^{k} (a_n \cos \omega_0 n t + b_n \sin \omega_0 n t)$$

$$\boxed{x(t+T) = x(t)}$$

Explain the exponential form of representation of a Fourier series:

In the representation of signals over a certain interval of time in terms of the linear combination of orthogonal functions, if the orthogonal functions are exponential functions, then it is called exponential functions.

The exponential Fourier series is the most widely used form of fourier series. The function $x(t)$ is expressed as a weighted sum of the complex exponential functions.

$$x(t) \text{ (or) } f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \left(\frac{e^{j\omega t n} + e^{-j\omega n t}}{2} \right) + b_n \left(\frac{e^{j\omega n t} - e^{-j\omega n t}}{2j} \right) \right]$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \left(\frac{e^{j2\pi n t} + e^{-j2\pi n t}}{2} \right) - j b_n \left(\frac{e^{j2\pi n t} - e^{-j2\pi n t}}{2} \right) \right]$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left[\frac{e^{j2\pi n t}}{2} (a_n - j b_n) + \frac{e^{-j2\pi n t}}{2} (a_n + j b_n) \right]$$

Let $F_n = \frac{1}{2}(a_n - j b_n)$, $F_{-n} = \frac{1}{2}(a_n + j b_n)$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left[F_n e^{j2\pi n t} + F_{-n} e^{-j2\pi n t} \right]$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} F_n e^{j2\pi n t} + \sum_{n=1}^{\infty} F_{-n} e^{-j2\pi n t}$$

$$x(t) = \sum_{n=0}^{\infty} F_n e^{j2\pi n t} + \sum_{n=-1}^{-\infty} F_n e^{j2\pi n t}$$

$$C_0 = \frac{1}{T} \int_{t_0}^{t_0+T} x(t)\,dt$$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} F_n e^{j2\pi n t}\,dt}$$

$$C_n = \frac{1}{T} \int_{t_0}^{t_0+T} x(t) e^{-j\omega n t}\,dt$$

Derive Polar Fourier Series form the Exponential F.S, $D_n = 2|C_n|$

The exponential Fourier series of function

$$x(t) = \sum_{-\infty}^{\infty} C_n e^{jn\omega t}$$

$$C_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\,dt, \quad C_n = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\, e^{-jn\omega_0 t}\,dt$$

General form

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n e^{jn\omega t} + \sum_{n=-\infty} C_n e^{-jn\omega t}$$

Exponential Fourier series co-efficients

$$C_0 = a_0, \quad C_n = \frac{1}{2}(a_n - jb_n), \quad C_{-n} = \frac{1}{2}(a_n + jb_n)$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \frac{1}{2}(a_n - jb_n) e^{jn\omega t} + \sum_{n=1}^{\infty} \frac{1}{2}(a_n + jb_n) e^{-jn\omega t}$$

$$x(t) = a_0 + \frac{1}{2}\left[\sum_{n=1}^{\infty} a_n e^{jn\omega t} + \sum_{n=1}^{\infty} a_n e^{-jn\omega t} + j\sum_{n=1}^{\infty} b_n e^{-jn\omega t} - j\sum_{n=1}^{\infty} b_n e^{jn\omega t}\right]$$

$$x(t) = a_0 + \frac{1}{2}\left[\sum_{n=1}^{\infty} a_n (e^{jn\omega t} + e^{-jn\omega t}) + \sum_{n=1}^{\infty} b_n j(e^{-jn\omega t} - e^{jn\omega t})\right]$$

$$x(t) = a_0 + \frac{1}{2}\left[\sum_{n=1}^{\infty} a_n (2\cos n\omega t) + \sum_{n=1}^{\infty} b_n j(-2j\sin n\omega t)\right]$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$A_n = D_n = \sqrt{a_n^2 + b_n^2}$$

$$= \sqrt{(C_n + C_{-n})^2 + [j(C_n - C_{-n})]^2}$$

$$= \sqrt{C_n^2 + C_{-n}^2 + 2C_n C_{-n} - (C_n^2 + C_{-n}^2 - 2C_n C_{-n})}$$

$$= \sqrt{4C_n C_{-n}}$$

$$\boxed{A_n = 2|C_n|} \quad (\text{or}) \quad \boxed{D_n = 2|C_n|}$$

If the Periodic signal x(t) has some type of symmetry then some of the trigonometric Fourier Series coefficients may become zero and calculation of the coefficients become simple.

There are following four types of symmetry x(t)

(1) Even symmetry
(2) odd symmetry
(3) Half wave symmetry
(4) Quarter wave symmetry

Even symmetry:- A function x(t) is said to have even (or) mirror symmetry, if

$$x(t) = x(-t)$$

x(t) has even symmetry then $b_n = 0$, and a_0 and a_n are to be evaluated.

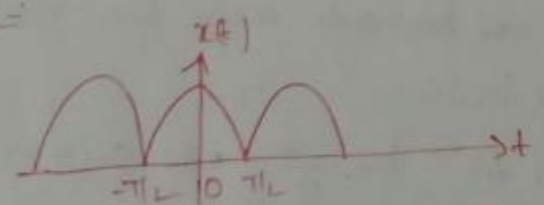
$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt, \quad a_n = \frac{2}{T}\int_0^T x(t)\cos n\omega t$$

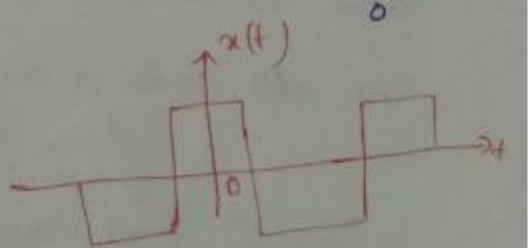
(or)

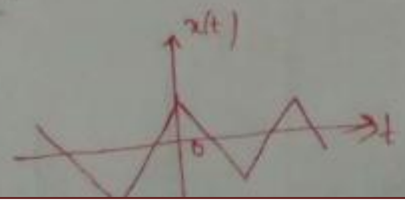
$$a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos n\omega t$$

$$b_n = 2/T\int_0^T x(t)\sin n\omega t = 0$$

Ex:-







Even functions

odd symmetry:- A function $x(t)$ is said to be odd (or) rotation

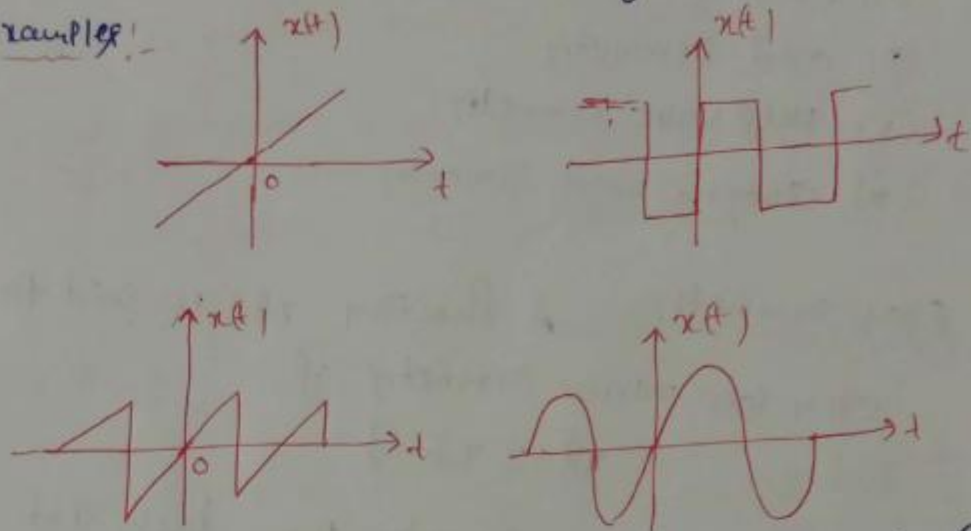
$$x(t) = -x(-t)$$

Then $a_0 = 0$ and $a_n = 0$. only 'b_n' is to be evaluated

$x(t)$ is an odd function then, $x_e(t) = 0$ & $x(t) = x_o(t)$

$$a_0 = 0,\quad a_n = 0,\quad b_n = \frac{4}{T}\int_0^{T/2} x(t)\sin n\omega_0 t\, dt$$

Examples:-



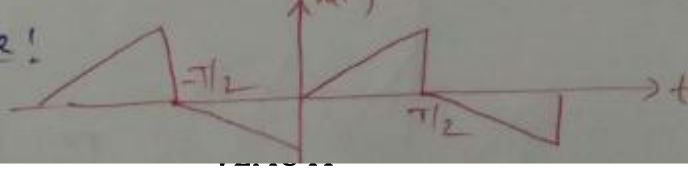
Half wave symmetry:- A periodic signal $x(t)$ which satisfies the condition $x(t) = -x(t \pm T/2)$ is said to have half wave symmetry. This function is neither purely odd nor purely even. For such functions, $a_0 = 0$

As $x(t)$ contains only odd harmonic terms, when n is even

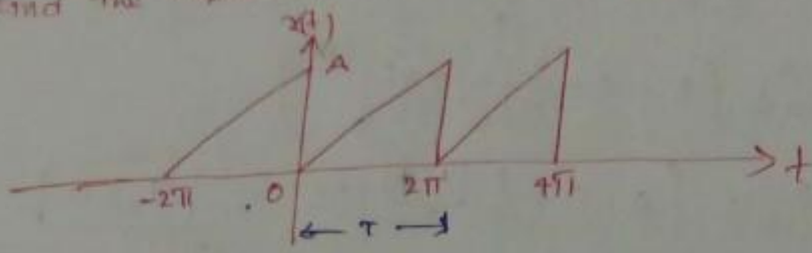
$$a_n = b_n = 0$$

$$a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos n\omega_0 t\, dt,\quad b_n = \frac{4}{T}\int_0^{T/2} x(t)\sin n\omega_0 t\, dt$$

Example!



* Find the cosine Fourier series for the waveform



Sol:-

Time Period $T = 2\pi$, $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$

$t_0 = 0$, $t_0 + T = 2\pi$,

$$x(t) = \frac{A}{2\pi} t, \quad 0 \le t \le 2\pi$$

Trigonometric series

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

The co-efficients $a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\, dt$

$$a_0 = \frac{1}{T}\int_0^T x(t)\, dt = \frac{1}{2\pi}\int_0^{2\pi} \frac{A}{2\pi} t\, dt$$

$$a_0 = \frac{A}{4\pi^2}\int_0^{2\pi} t\, dt = \frac{A}{4\pi^2}\left[\frac{t^2}{2}\right]_0^{2\pi}$$

$$a_0 = \frac{A\, 4\pi^2}{2(4\pi^2)} = A/2$$

$$\boxed{a_0 = A/2}$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cos n\omega_0 t\, dt = \frac{2}{2\pi}\int_0^{2\pi}\left(\frac{A}{2\pi}\right)t \cos nt\, dt$$

$$a_n = \frac{2A}{4\pi^2}\int_0^{2\pi} t \cos nt\, dt$$

$$a_n = \frac{2A}{(2\pi)^2}\left\{\left[\frac{t \sin nt}{n}\right]_0^{2\pi} - \int_0^{2\pi}\frac{\sin nt}{n}\,dt\right\}$$

$$a_n = \frac{2A}{(2\pi)^2}\left[0 - 0 - \frac{1}{n}\left[-\frac{\cos nt}{n}\right]_0^{2\pi}\right]$$

$$a_n = \frac{2A}{(2\pi)^2}\left[0 - 0 + \frac{1}{n}\frac{(1-1)}{n}\right] = 0 \quad . \quad \boxed{a_n = 0}$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\sin n\omega_0 t\,dt = \frac{2}{2\pi}\int_0^{2\pi}\frac{A}{2\pi}t\sin nt\,dt$$

$$b_n = \frac{2A}{(2\pi)^2}\int_0^{2\pi} t\sin nt\,dt = \frac{2A}{(2\pi)^2}\left\{\left[\frac{t(-\cos nt)}{n}\right]_0^{2\pi} - \int_0^{2\pi}\left(-\frac{\cos nt}{n}\right)dt\right\}$$

$$b_n = \frac{2A}{(2\pi)^2}\left[\left[\frac{t(-\cos nt)}{n}\right]_0^{2\pi} + \frac{1}{n}\left(\frac{\sin nt}{n}\right)_0^{2\pi}\right]$$

$$b_n = \frac{2A}{(2\pi)^2}\left[-\frac{2\pi(1)}{n} - 0 + \frac{1}{n}\frac{(0-0)}{n}\right]$$

$$b_n = \frac{2A}{(2\pi)^2}\left[-\frac{2\pi}{n}\right] = -\frac{4\pi A}{4\pi^2 n}$$

$$\boxed{b_n = -\frac{A}{\pi n}}$$

Trigonometric co-efficients

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$x(t) = \frac{A}{2} + \sum_{n=1}^{\infty} -\frac{A}{n\pi}\sin nt$$

$$\boxed{x(t) = \frac{A}{2} - \frac{A}{\pi}\sum_{n=1}^{\infty}\frac{\sin nt}{n}}$$

S.No	Objective Questions
1.	Fourier series expansion of an odd function contains ------ ( ) a) **sin term** b) cos term c) cos term and constant d) sin term and constant
2	Odd function x Odd function is ----------- ( ) a) Odd function b)**Even function** c) Neither Even nor odd d) Either Even or odd
3	Sum of odd and even function is------------ ( ) a) **Odd function** b)Even function c) Neither Even nor odd d) Either Even or odd
4	Fourier series expansion of an even function contains ------ ( ) a) sin term b) **cos term** c) cos term and constant d) sin term and constant
5	A Function x(t) is said to even ,if x(t) is ( ) (a)x(-t) (b)-x(t) (c)**x(2t)** (d)x(t)
6	A Function x(t) is said to even ,if x(-t) is ( ) (a)x(-t) (b)**x(-2t)** (c)x(2t) (d)x(t)
7	Fourier Transform for the signal $e^{-at}u(t)$ does not exist if ( ) (a)a>0 (b)a=0 (c)**a=1** (d)a<0
8	The Fourier transform ( ) (a)**satisfies linearity** (b)not satisfies linearity (c) both (d)none
9	Fourier transform of the unit impulse δ(t) ( ) (a) 0 (b)Л (c)**1** (d) δ(w)
10	What is the spectrum of dc signal ( ) (a)0 (b)1 (c) **2Л δ(w)** (d)none
11	The Fourier signal of x(t) is ( ) (a)-X(w) (b)**X(-w)** (c)-X(-w) (d)X(w)

12	Time convolution property states that ( ) (a)$f_1(t)*f_2(t)$ (b) $\mathbf{f_1(t)f_2(t)}$ (c) $F_1(w)*F_2(w)$ (d) $F_1(w)/f_2(w)$	
13	Frequency convolution property states that ( ) (a)$f_1(t)*f_2(t)$ (b) $f_1(t)f_2(t)$ (c) $\mathbf{F_1(w)*F_2(w)}$ (d) $F_1(w)/f_2(w)$	
14	---------------- plot is drawn between phase angle Vs frequency a) **Phase** b) Magnitude c) log d) semi log	
15.	---------------- plot is drawn between magnitude Vs frequency a) Phase b) **Magnitude** c) log d) semi log	

UNIT-III

2 MARKS

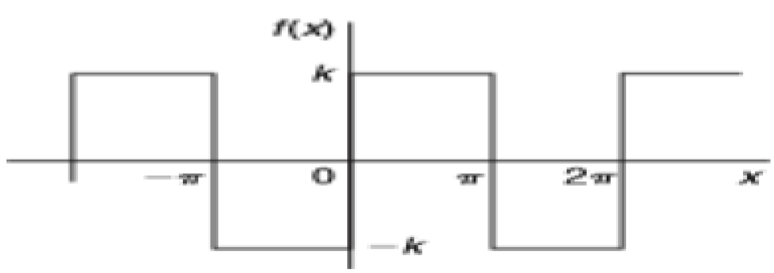
1. What do you mean by phase angle spectra?
2. Mention the properties to be satisfied by the periodic function f(t) of trigonometric form of Fourier series.
3. Mention the general properties of Fourier transforms.
4. Write the properties of Fourier transform.
5. Write the exponential form of Fourier series
6. Define Fourier series
7. What is Laplace transform?
8. Write down any two applications of Fourier transforms.

Obtain a Fourier series for the periodic function f(x) defined as:

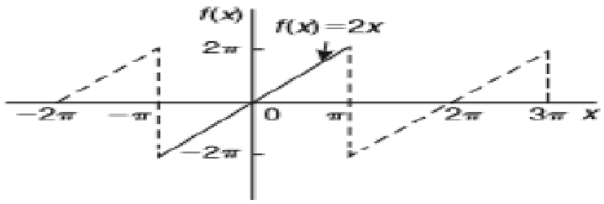
f (x)= −k, when $-\pi < x < 0$

+k, when $0 < x < \pi$

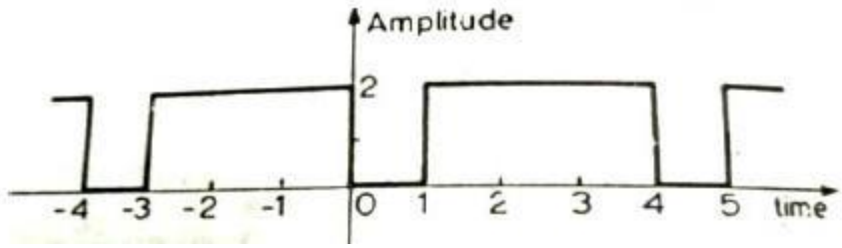
The function is periodic outside of this range with period 2π



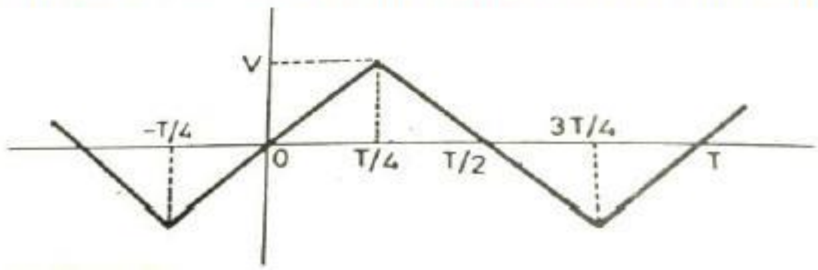
2. Determine the Fourier series to represent the function f (x)=2x in the range −π to +π



3. Obtain the Fourier Series for the waveform shown in figure below.



4. Obtain the Fourier Series for the waveform shown in figure below.



5. Derive trigonometric Fourier series representation of a periodic signal x(t) with fundamental period T.

6. List the advantages of Fourier series and express its equations with its coefficients

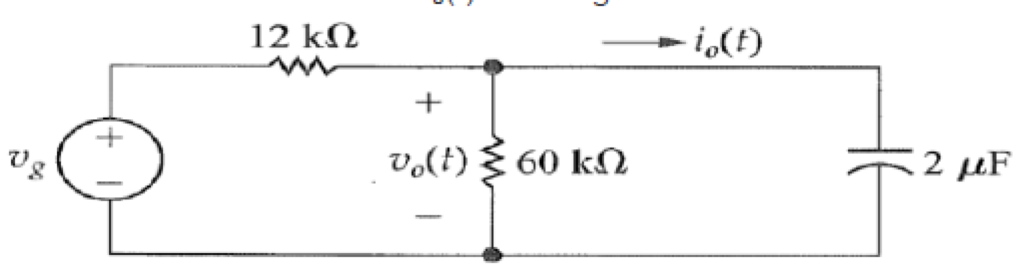
7. What method can be applied to analyze the electrical circuits with non sinusoidal periodic signal? Explain.

8. Derive the expression for exponential form of Fourier series. Mention the application of Fourier transform.

9. Explain the properties of Fourier transforms.

10.

Use the Fourier transform method to calculate $v_0(t)$ for the given circuit.



UNIT- IV NETWORK TOPOLOGY

- Definitions – Graph – Tree,
- Basic Cutset and Basic Tieset Matrices for Planar Networks
- Loop and Nodal Methods of Analysis of Networks with Dependent & Independent Voltage and Current Sources
- Duality & Dual Networks.
- Nodal Analysis for D.C Excitations.
- Mesh Analysis for D.C Excitations.
- Super Node for D.C Excitations.
- Super Mesh for D.C Excitations.

INTRODUCTION: When all the elements in a network are replaces by lines with circles or dots at both ends, configuration is called the graph of the network.

A. Terminology used in network graph:-

(i) **Path:-**A sequence of branches traversed in going from one node to another is called a path.

(ii) **Node:-**A node point is defined as an end point of a line segment and exits at the junction between two branches or at the end of an isolated branch.

(iii) **Degree of a node:-** It is the no. of branches incident to it.

2-degree

3-degree

(iv) **Tree:-** It is an interconnected open set of branches which include all the nodes of the given graph. In a tree of the graph there can‟tbe any closed loop.

(v) **Tree branch(Twig):-** It is the branch of a tree. It is also named as twig.

(vi) **Tree link (or chord):-**It is the branch of a graph that does not belong to the particular tree.

(vii) **Loop:-** This is the closed contour selected in a graph.

(viii) **Cut-Set:-** It is that set of elements or branches of a graph that separated two parts of a network. If any branch of the cut-set is not removed, the network remains connected. The term cut-set is derived from the property designated by the way by which the network can be divided in to two parts.

(ix) **Tie-Set:-** It is a unique set with respect to a given tree at a connected graph containing on chord and all of the free branches contained in the free path formed between two vertices of the chord.

(x) **Network variables:-** A network consists of passive elements as well as sources of energy . In order to find out the response of the network the through current and voltages across each branch of the network are to be obtained.

(xi) **Directed(or Oriented) graph:-** A graph is said to be directed (or oriented) when all the nodes and branches are numbered or direction assigned to the branches by arrow.

(xii) **Sub graph:-** A graph [illegible] said to be sub-graph of a graph G if every node of [illegible] is a node of G and every branch of [illegible] is also a branch of G.

(xiii) **Connected Graph:-** When at least one path along branches between every pair of a graph exits , it is called a connected graph.

(xiv) **Incidence matrix:-** Any oriented graph can be described completely in a compact matrix form. Here we specify the orientation of each branch in the graph and thenodes at which this branch is incident. This branch is called incident matrix. When one row is completely deleted from the matrix the remainingmatrix is called a reduced incidence matrix.

(xv) **Isomorphism:-** It is the property between two graphs so that both have got same incidence matrix.

B. Relation between twigs and links-

Let N=no. of nodes

L= total no. of links

B= total no. of branches

C. Properties of a Tree-

(i) It consists of all the nodes of the graph.

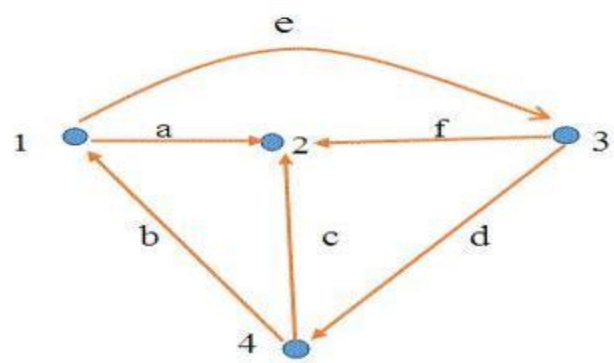
(ii) If the graph has N nodes, then the tree has (N-1) branch.

(iii) There will be no closed path in a tree

(iv) There can be many possible different trees for a given graph depending on the no. of nodes and branches.

Incidence Matrix:

Incidence matrix is that matrix which represents the graph such that with the help of that matrix we can draw a graph. This matrix can be denoted as [A_C] As in every matrix, there are also rows and columns in **incidence matrix** [A_C]. The rows of the matrix [A_C] represent the number of nodes and the column of the matrix [A_C] represent the number of branches in the given graph. If there are „n" number of rows in a given incidence matrix, that means in a graph there are „n" number of nodes. Similarly, if there are „m" number of columns in that given incidence matrix, that means in that graph there are „m" number of branches.



In the above shown graph or directed graph, there are 4 nodes and 6 branches. Thus the incidence matrix for the above graph will have 4 rows and 6 columns.

The entries of incidence matrix is always -1,0,+1. This matrix is always analogous to KCL (Krichoff Current Law). Thus from KCL we can derive that,

Type of branch Value

Outgoing branch from k_{th} node +1

Incoming branch to k_{th} node -1

Others 0

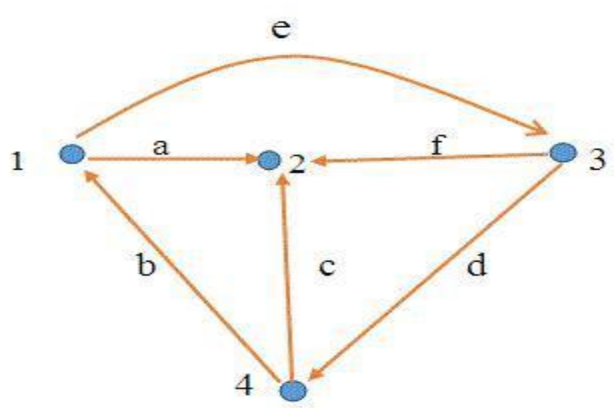
Steps to Construct Incidence Matrix

Following are the steps to draw the incidence matrix :-

1. If a given k^{th} node has outgoing branch, then we will write +1.
2. If a given k^{th} node has incoming branch, then we will write -1.

3. Rest other branches will be considered 0.

Examples of Incidence Matrix



For the graph shown above write its incidence matrix.

$[A_C] =$

nodes \ branches	a	b	c	d	e	f
1	1	-1	0	0	1	0
2	-1	0	-1	0	0	-1
3	0	0	0	1	-1	1
4	0	1	1	-1	0	0

Reduced Incidence Matrix

If from a given incidence matrix [A_C], any arbitrary row is deleted, then the new matrix formed will be reduced incidence matrix. It is represented by symbol [A]. The order of reduced incidence matrix is (n-1)*b where n is the number of nodes and b is the number of branches.

For the above shown graph, the reduced incidence matrix will be :-

[A] =

nodes \ branches	a	b	c	d	e	f
1	1	-1	0	0	1	0
2	-1	0	-1	0	0	-1
3	0	0	0	1	-1	1

[NOTE :- In the above shown matrix row 4 is deleted.]

Points to remember

- For checking correctness of incidence matrix which we have drawn, we should check sum of column.
- If sum of column comes to be zero, then the incidence matrix which we have created is correct else incorrect.
- The incidence matrix can be applied only to directed graph only.
- The number of entries in a row apart from zero tells us the number of branches linked to that node. This is also called as degree of that node.
- The rank of complete incidence matrix is (n-1), where n is the number of nodes of the graph.
- The order of incidence matrix is (n × b), where b is the number of branches of graph.
- From a given reduced incidence matrix we can draw complete incidence matrix by simply adding either +1, 0, or -1 on the condition that sum of each column should be zero.

Kirchhoff Current Law and Kirchhoff Voltage Law

There are some simple relationships between currents and voltages of different branches of an electrical circuit. These relationships are determined by some basic laws that are known as **Kirchhoff laws** or more specifically Kirchhoff Current and Voltage laws. These laws are very helpful in determining the equivalent electrical resistance or impedance (in case of AC) of a complex network and the currents flowing in the various branches of the network. These laws are first derived by Guatov Robert Kirchhoff and hence these laws are also referred as **Kirchhoff Laws**.

Kirchhoff's Current Law

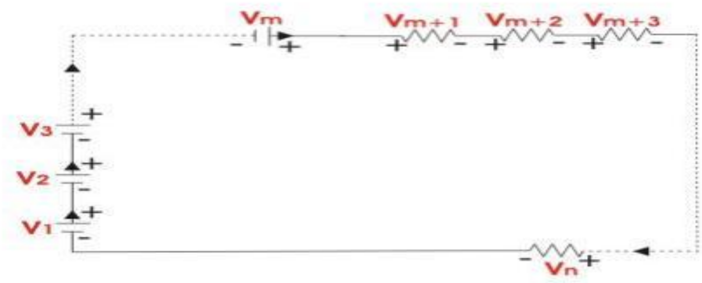
In an electrical circuit, the current flows rationally as electrical quantity.

As the flow of current is considered as flow of quantity, at any point in the circuit the total current enters, is exactly equal to the total current leaves the point. The point may be considered anywhere in the circuit.

Suppose the point is on the conductor through which the current is flowing, then the same current crosses the point which can alternatively said that the current enters at the point and same will leave the point. As we said the point may be anywhere on the circuit, so it can also be a junction point in the circuit. So total quantity of current enters at the junction point must be exactly equal to total quantity of current that leaves the junction. This is the very basic thing about flowing of current and fortunately **Kirchhoff Current law** says the same. The law is also known as **Kirchhoff First Law** and this law stated that, at any junction point in the electrical circuit, the summation of all the branch currents is zero. If we consider all the currents enter in the junction are considered as positive current, then convention of all the branch currents leaving the junction are negative. Now if we add all these positive and negative signed currents, obviously we will get result of zero

The mathematical form of **Kirchhoff First Law** is $\sum I = 0$ at any junction of electrical network.

Kirchhoff's Voltage Law



This law deals with the voltage drops at various branches in an electrical circuit. Think about one point on a closed loop in an electrical circuit. If someone goes to any other point on the same loop, he or she will find that the potential at that second point may be different from first point. If he or she continues to go to some different point in the loop, he or she may find some different potential at that new location. If he or she goes on further along that closed loop, ultimately he or she reaches the initial point from where the journey was started. That means, he or she comes back to the same potential point after crossing through different voltage levels. It can be alternatively said that net voltage gain and net voltage drops along a closed loop are equal. That is what **Kirchhoff Voltage law** states. This law is alternatively known as **Kirchhoff Second Law**.

If we consider a closed loop conventionally, if we consider all the voltage gains along the loop are positive then all the voltage drops along the loop should be considered as negative. The summation of all these voltages in a closed loop is equal to zero. Suppose n numbers of back to back connected elements form a closed loop. Among these circuit elements m number elements are voltage source and n - m number of elements drop voltage such as resistors.

According to **Kirchhoff Voltage law**, the summation of all these voltages results to zero. i.e

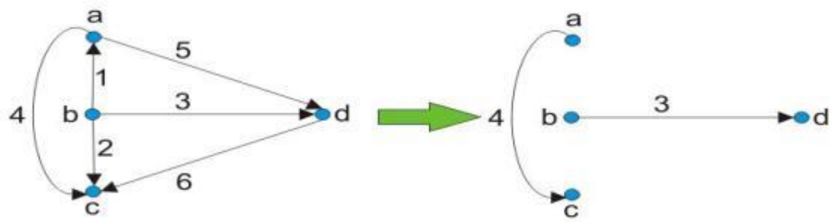
$$V_1 + V_2 + V_3 + \ldots + V_m - V_{m+1} - V_{m+2} - V_{m+3} \ldots - V_n = 0.$$

Where V_1, V_2, V_3 are voltage sources and $V_{m+1}, V_{m+2}, V_{m+3}\ldots$ are voltage drops

Cutset Matrix Concept of Electric Circuit:

When we talk of **cut-set matrix in graph theory**, we generally talk of **fundamental cut-set matrix**. A cut-set is a minimum set of branches of a connected graph such that when removed these branches from the graph, then the graph gets separated into 2 distinct parts called sub-graphs and the **cut-set matrix** is the matrix which is obtained by row-wise taking one cut-set at a time. The cut-set matrix is denoted by symbol $[Q_f]$.

Example



Two sub-graphs are obtained from a graph by selecting cut-sets consisting of branches [1, 2, 5, 6]. Thus, in other words we can say that fundamental cut set of a given graph with reference to a tree is a cut-set formed with one twig and remaining links. Twigs are the branches of tree and links are the branches of co-tree. Thus, the number of cutset is equal to the number of twigs. [Number of twigs = N-1] Where, N is the number of nodes of the given graph or drawn tree.

The orientation of cut-set is the same as that of twig and that is taken positive.

Steps to Draw Cut Set Matrix:

There are some steps one should follow while drawing the cut-set matrix. The steps are as follows-

1. Draw the graph of given network or circuit (if given).
2. Then draw its tree. The branches of the tree will be twig.
3. Then draw the remaining branches of the graph by dotted line. These branches will be links.
4. Each branch or twig of tree will form an independent cut-set.
5. Write the matrix with rows as cut-set and column as branches.

Branchase ⇒ 1 2 3 4 5 6. . b

Cutsets

C_1

C_2

C_3.

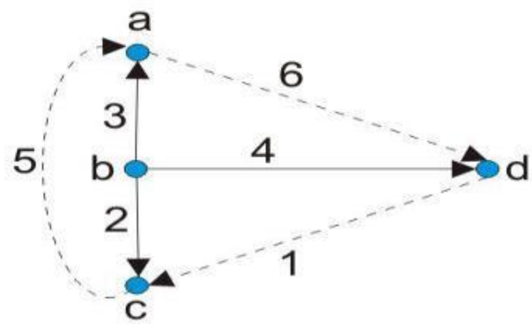
C_n

n = number of cut-set. b = number of branches.

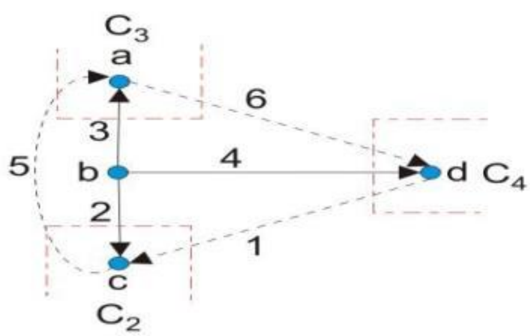
Orientation in Cut Set Matrix

Q_{ij} = 1; if branch J is in cut-set with orientation same as that of tree branch. Q_{ij} = -1; if branch J is in cut-set with orientation opposite to that of branch of tree. Q_{ij} = 0; if branch J is not in cut-set. Example 1 : Draw the cut-set matrix for the following graph.

Answer: Step 1: Draw the tree for the following graph.



Step 2: Now identify the cut-set. Cut-set will be that node which will contain only one twig and any number of links.



Here C_2, C_3 and C_4 are cut-sets. Step 3: Now draw the matrix.

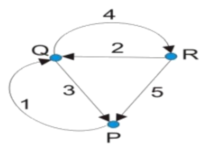
Step 3: Now draw the matrix.

Branches ⇒	1	2	3	4	5	6
Cutsets						
C_2	+1	+1	0	0	-1	0
C_3	0	0	+1	0	+1	-1

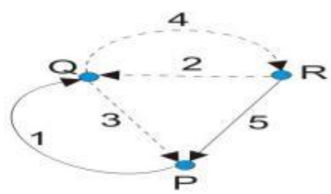
C_4	-1	0	0	+1	0	+1

This is the required matrix.

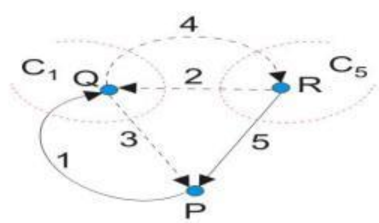
Example 2: Draw the cut-set of the given graph.



Step 1: Draw the tree for the following graph.



Step 2: Now identify the cut-set. Cut-set will be that node which will contain only one twig and any number of links.



Here C_1 and C_5 are cut-sets. Step 3: Now draw the matrix.

Branches ⇒	1	2	3	4	5
Cutsets					
C_1	+1	+1	-1	-1	0
C_5	0	-1	0	-1	+1

This is the required matrix

Points to remember There are some key points which should be remembered.

They are:-

- In cut-set matrix, the orientation of twig is taken positive
- .
- Each cut-set contains only one twig.

- Cut-set can have any number of links attached to it.

- The relation between cut-set matrix and KCL is that

$$[Q_f] \times I_b = 0$$

Mesh Analysis: The word "mesh" means a smallest loop which is closed one and formed by using circuit components. Like the other network analysis procedures, this is used to find out the voltage, current or power through a particular element or elements. This method is very simple and convenient one. It is based on one of the simplest law in the electrical circuit analysis, "The KVL". This method is very easy and may save time in most of the cases. However mesh analysis method can only be performed in case of planar circuits only, in which it is possible to draw the circuit in a plane surface in such a way that no branch passes over or under any branch.

Steps for Mesh Analysis

The steps followed in mesh analysis are very simple, they are as follows-

1. Whether the circuit is planar or non planar that we have to determine, if it is a non planar circuit we have to perform other methods of analysis such as nodal analysis method.

2. The number of meshes is counted, the number of equations to be solved is same as the number of meshes.

3. Label each of the mesh currents according to the convenience.

4. Write the KVL equation of each of the meshes, if the element lies between two meshes then the total current flowing in the element is calculated considering two meshes and if the direction of current is same then summation of current is taken as the total current flowing through the resistor and if the direction is opposite then the difference of current is taken. In that case current in the mesh under consideration is taken as the greatest among all the meshes and the procedure is followed.

5. Organize the equation according to the mesh currents.

6. Solve the mesh equations.

7. If any dependent source is there in the circuit or any unknown other than mesh currents, express that source in the suitable mesh currents.

Disadvantages of Single and Multi Mesh Analysis

1. It can be used when the circuit is planar, otherwise the method will not be useful.

2. If the network is large then the number of meshes will be large so the total number of equations will be more so it becomes inconvenient to use in that case.

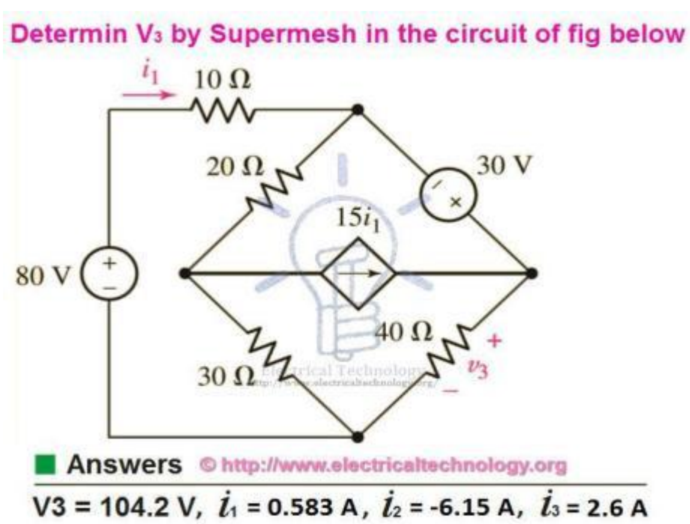
SUPERMESH Circuit Analysis | Step by Step with Solved Example

Supermesh or Supermesh Analysis is a better technique instead of using **Mesh analysis** to analysis such a complex electric circuit or network, where two meshes have a current source as a common element. This is the same where we use **Supernode circuit analysis** instead of **Node or Nodal circuit analysis** to simplify such a network where the assign supernode, fully enclosing the voltage source inside the supernode and reducing the number of none reference nodes by one (1) for each voltage source.

In supermesh circuit analysis technique, the current source is in the inner area of the supermesh. Therefore, we are able to reduce the number of meshes by one (1) for each current source which is present in the circuit.

The single mesh can be ignored, if current source (in that mesh) lies on the perimeter of the circuit. Alternatively, KVL (Kirchhoff‟s Voltage Law) is applied only to those meshes or supermeshes in the renewed circuit.

Solved Example of Supermesh Analysis:



Using KVA on Mesh 1

80 = $10i_1 + 20(i_1 – i_2) + 30$
($i_1 – i_3$) Simplifying

80 = $10i_1 + 20i_1 - 20i_2 + 30i_1 - 30i_3$

80 = $60i_1 – 20i_2 – 30i_3$ ….. → Eq 1.

Now apply **KVL** on **Supermesh** (which is integration of ***mesh 2*** and ***mesh 3***, but we have reduced it by single mesh which is known as ***supermesh***)

30 = $40i_3 + 30(i_3 – i_1) + 20(i_2 – i_1)$

30 = $40i_3 + 30i_3 – 30i_1 + 20i_2 - 20i_1$

30 = $70i_3 – 50i_1 + 20i_2$ ….. → Eq 2.

But here, we have three (3) variables i.e. i1, 12 and i3. And there are two equations. So we must need three equations as well.

The independent current source (in the **supermesh**) is related to the assumed mesh currents,
i.e. 15ix = $i_3 – i_2$

I3=15$ix + i_2$ ….. → Eq 3.

Solving equations 1, 2 and 3

i_1 = 0.583 A

i_2 = -6.15

i_3 = 2.6 A

Also, we can find the value of **v_3**

$V_3 = i_3 \times R_3$

Putting the values,

$V_3 = 2.6A \times 40\Omega$

$V_3 = 104$ V.

Nodal Analysis:

Nodal analysis is a method that provides a general procedure for analyzing circuits using node voltages as the circuit variables. **Nodal Analysis** is also called the **Node –Voltage Method**. Some Features of Nodal Analysis are as

- **Nodal Analysis** is based on the application of the Krichhoff‟s Current Law (KCL).
- Having „n‟ nodes there will be „n-1‟ simultaneous equations to solve.
- Solving „n-1‟ equations all the nodes voltages can be obtained.
- The number of non reference nodes is equal to the number of Nodal equations that can be obtained.

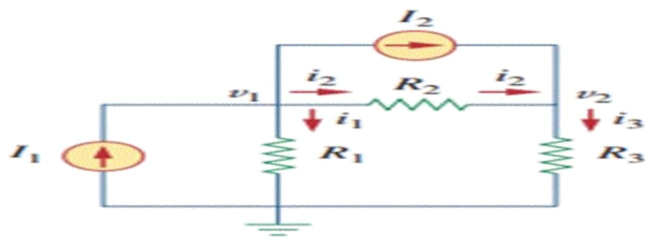
Solving of Circuit Using Nodal Analysis

Basic Steps Used in Nodal Analysis

I. Select a node as the reference node. Assign voltages V_1, V_2... V_{n-1} to the remaining nodes. The voltages are referenced with respect to the reference node.
II. Apply KCL to each of the non reference nodes.
III. Use Ohm‟s law to express the branch currents in terms of node voltages.

Node always assumes that current flows from a higher potential to a lower potential in resistor. Hence, current is expressed as follows

$$I = \frac{V_{high} - V_{low}}{R}$$



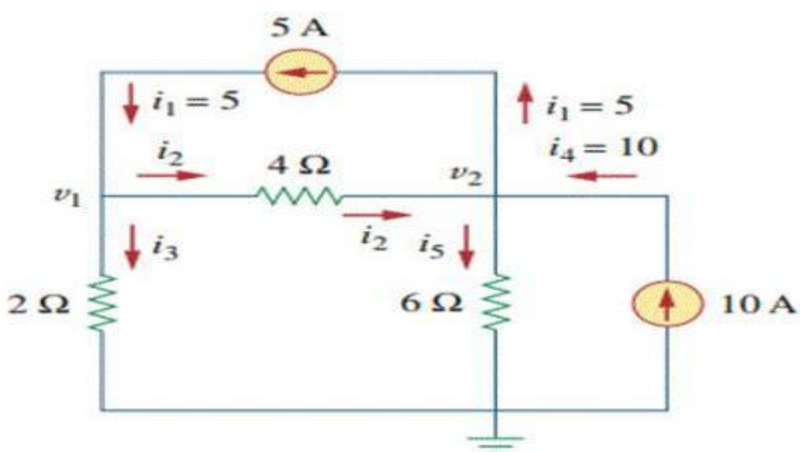
IV. After the application of Ohm‟s Law get the „n-1‟ node equations in terms of node voltages and resistances.

V. Solve „n-1‟ node equations for the values of node voltages and get the required node Voltages as result.

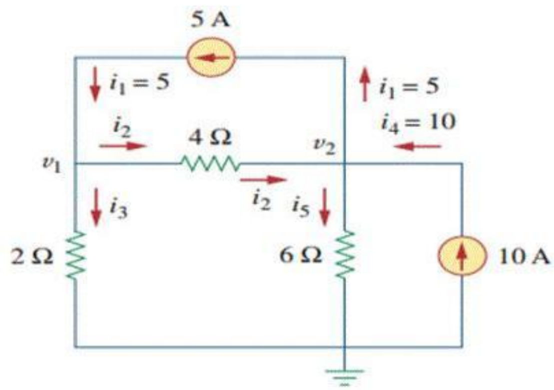
Nodal Analysis with Current Sources:

Nodal analysis with **current sources** is very easy and it is discussed with a example below. Example

Calculate NodeVoltagesinfollowingcircuit



In the following circuit we have 3 nodes from which one is reference node and other two are non reference nodes –Node1 and Node2 Step I. Assign the nodes voltages as v_1 and v_2 and also mark the directions of branch currents with respect to the r eference nodes



Step II. Apply KCL to Nodes 1 and 2 KCL at Node 1 $i_1 = i_2 + i_3 \cdots\cdots(1)$

KCL at Node 2 $i_2 + i_4 = i_1 + i_5 \cdots\cdots(2)$

Step III. Apply Ohm"s Law to KCL equations

• Ohm"s Law to KCL equation at Node 1

$$i_1 = i_2 + i_3 \Rightarrow 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

Simplifying the above equation we get, $3v_1 - v_2 = 20 \cdots\cdots(3)$
Now

$$i_2 + i_4 = i_1 + i_5 \Rightarrow \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

,Ohm"s Law to KCL equation at Node 2

Simplifying the above equation we get $-3v_1 + 5v_2 = 60 \cdots\cdots(4)$

Step IV. Now solve the equations 3 and 4 to get the values of v_1 and v_2 as, Using elimination method

$$3v_1 - v_2 = 20$$
$$-3v_1 + 5v_2 = 60$$
$$\Rightarrow 4v_2 = 80 \Rightarrow v_2 = 20\ Volts$$

And substituting value v_2=20 Volts in equation (3) we get-

$$3v_1 - 20 = 20 \Rightarrow v_1 = \frac{40}{3} = 13.333\ Volts$$

Hence node voltages are as v_1 = 13.33 Volts and v_2 = 20 Volts.

Nodal Analysis with Voltage Sources

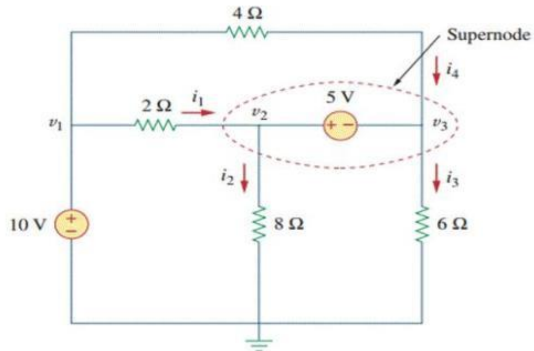
Case I - If a voltage source is connected between the reference node and a non reference node, we simply set the voltage at the non-reference node equal to the voltage of the voltage source and its analysis can be done as we done with current sources. v_1 = 10 Volts

Case II. If the voltage source is between the two non reference nodes then it forms a supernode whose analysis is done as following

Supernode Analysis

Definition of Super Node

Whenever a voltage source (Independent or Dependent) is connected between the two non reference nodes then these two nodes form a generalized node called the Super node. So, Super node can be regarded as a surface enclosing the voltage source and its two nodes.



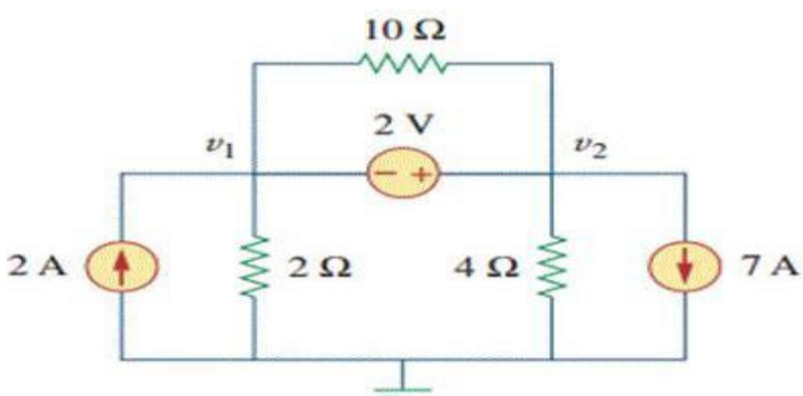
In the above Figure 5 V source is connected between two non reference nodes Node - 2 and Node - 3. So here Node - 2 and Node - 3 form the Super node.

Properties of Super node :

- Always the difference between the voltage of two non reference nodes is known at Supernode.
- A supernode has no voltage of its own
- A supernode requires application of both KCL and KVL to solve it.
- Any element can be connected in parallel with the voltage source forming the supernode .
- A Supernode satisfies the KCL as like a simple node.

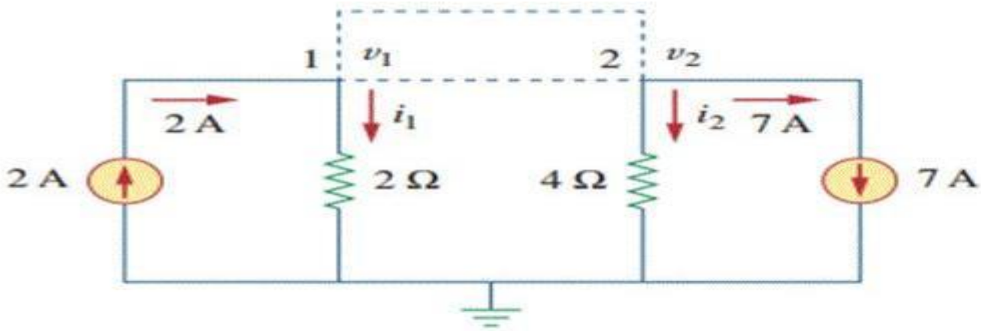
How Solve Any Circuit Containing Super node:

Let's take a example to understand how to **solve circuit containing Supernode**



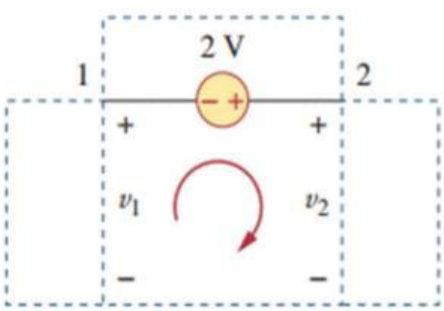
Here 2V voltage source is connected between Node - 1 and Node - 2 and it forms a Supernode with a 10Ω resistor in parallel . Note - Any element connected in parallel with the voltage source forming

Super node doesn‟t make any difference because $v_2 - v_1 = 2$ V always whatever may be the value of resistor. Thus 10 Ω can be removed and circuit is redrawn and applying KCL to the supernode as shown in figure gives,



Expressing and in terms of the node voltages

$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7 \Rightarrow 8 = 2v_1 + v_2 + 28 \Rightarrow v_2 = -2v_1 - 20 \cdots\cdots(5)$$



$$v_1 + 2 - v_2 = 0 \Rightarrow v_2 = v_1 + 2 \cdots\cdots(6)$$

From Equation 5 and 6 we can write as

$$v_2 = v_1 + 2 = -2v_1 - 20 \Rightarrow 3v_1 = -22$$

$$\Rightarrow v_1 = -7.333\ V\ \&\ v_2 = v_1 + 2 = -7.333 + 2 = -5.333\ V$$

Hence, v_1= - 7.333V and v_2 = - 5.333V.

Objective Questions and Answer

S.No	Objective Questions
1.	Given network is having N nodes and B branches, then number of twigs are [ ] a) N b) **N-1** c) B-N-1 d) B-N+1
2	Given network is having N nodes and B branches, then number of individual loops are [ ] a) N b) N-1 c) B-N-1 d) **B-N+1**
3	If N is the number of nodes and b is the number of branches in a network, then rank of cut set matrix is a) N b) **N-1** c) B-N-1 d) B-N+1 [ ]
4	If N is the number of nodes and b is the number of branches in a network, then rank of tie set matrix is a) N b) N-1 c) B-N-1 d) **B-N+1** **[ ]**
5	Basic cut set consists of [ ] a) **one branch** b) no branches c) 2 branch d) any number of branches
6	Basic Tie set consists of [ ] a) **one Link** b) no links c) 2 links d) any number of links
7	Number of branches of a graph should be --------- number of branches of a network [ ] a) greater than b) less than c) equals to d) **less than equals to**
8	The size of basic cutset incidence matrix [ ] a) **(n-1)* b** b) n*b c) n*n d) (b-1)*(b-1)
9	The size of basic Tie set incidence matrix [ ] a) **L * B** b) (L-1) *N c) L* L d) (L-1) *(L-1)
10	When we use super node technique [ ] a) current source branch is common for two meshes b**) ideal voltage source is connected between two non reference nodes** c) ideal voltage source is connected between non reference node and reference d) All of the above
11	Super mesh analysis is used in case of [ ] a)**current source branch is common for two meshes** b) ideal voltage source is connected between two non reference nodes c) Both d) either a or b
12	KVL works on the principle of which of the following [ ] a) law of conservation of charge b) **law of conservation of energy** c) both. D) None
13	KCL works on the principle of which of the following [ ] a) **law of conservation of charge** b) law of conservation of energy c) both. D) None
14	The resistivity of the conductor depends on [ ] (a).area of the conductor b) length of the conductor. C) **type of material** d)None
15.	01. If 1 A current flows in a circuit, the number of electrons flowing through this circuit is [ ] (a) 0.625×10^{19} (b) $\mathbf{0.625 \times 10^{-19}}$ (c) 6.25×10^{19} **(d)** 6.25×10^{-19}

Note: Need to prepare 15 to 20 Objective Question and Answers from each unit. Highlight the correct answer with bold.

TWO MARKS QUESTIONS AND ANSWERS

1. Define Graph?

Ans: The pictorial representation of the network is called graph that means all the passive elements are replaced by a line, voltage sources are represented by short circuited and current sources are represented by open circuit.

2. Define TREE?

ANS: It is the sub Graph of the graph it contains all nodes in the graph, but does not have any closed path or closed loop. The number of possible trees for the graph is det(AA^T) where A is reduced incidence Matrix.

3. **Define Cut-Set?**

ANS: It is that set of elements or branches of a graph that separated two parts of a network. If any branch of the cut-set is not removed, the network remains connected. The term cut-set is derived from the property designated by the way by which the network can be divided in to two parts.

4. Define Tie-Set?

ANS: It is a unique set with respect to a given tree at a connected graph containing on chord and all of the free branches contained in the free path formed between two vertices of the chord.

5. Define **Node and Degree of Node?**

ANS**:-** **Node:** A node point is defined as an end point of a line segment and exits at the junction between two branches or at the end of an isolated branch.

Degree of a node:- It is the no. of branches incident to it.

6. write the Properties of Super Node?

ANS: **Properties of Super node :**

- Always the difference between the voltage of two non reference nodes is known at Supernode.
- A supernode has no voltage of its own
  A supernode requires application of both KCL and KVL to solve it.
- Any element can be connected in parallel with the voltage source forming the supernode .
  A Supernode satisfies the KCL as like a simple node.

7. Obtain the Relation between twigs and links?

Let N=no. of nodes
L= total no. of links
B= total no. of branches
No. of twigs= N-1
Then, L= B-(N-1)

8. **Explain the Properties of a Tree?**

i) It consists of all the nodes of the graph.
ii) If the graph has N nodes, then the tree has (N-1) branch.
iii) There will be no closed path in a tree
iv) There can be many possible different trees for a given graph depending on the no. of nodes and branches.

9. What is super Node?

ANS: The region surrounding a voltage source which connects the nodes directly is called super node.

10. What is the difference between LOOP and Mesh?

ANS: A loop is any closed path of the network. A mesh is a most elementary form of a loop and not be further divided into other loops.

11. Define Duality?

ANS: Two electrical networks which are governed by the same type of equations are called duality.

12. Define dual Networks?

ANS: Two networks are called dual networks if the mesh equations of one have the same form as the nodal equations of the other. The property of duality is a mutual property.

13. State the dual elements for inductance and Mesh current.

ANS: Dual of inductance is capacitance
Dual of mesh current is node voltage.

14. **Define Super Mesh?**

ANS: The loop existing, around a current source which is common to the two loops is called super Mesh.

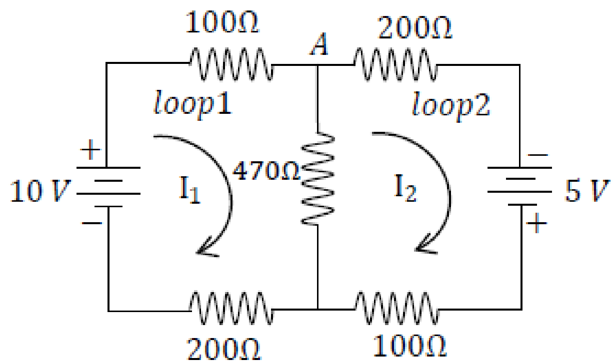
15. **Definition of Super Node?**

ANS: Whenever a voltage source (Independent or Dependent) is connected between the two non reference nodes then these two nodes form a generalized node called the Super node.

<u>10 MARKS</u>

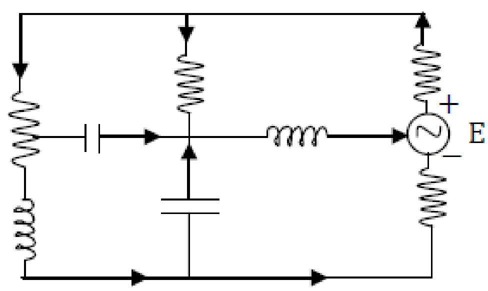
<u>1.</u>

Prove Kirchhoff's current law at node 'A' for the circuit given below.



2.

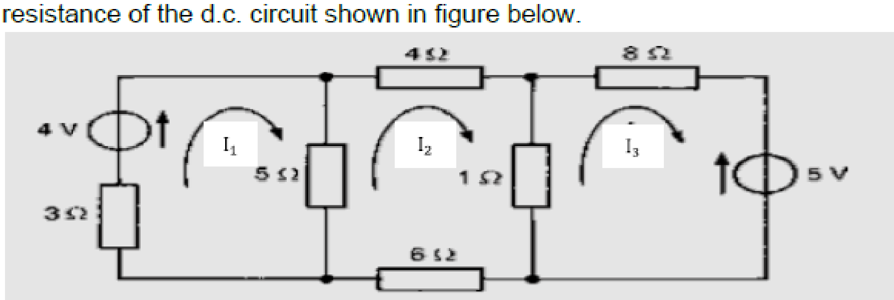
Draw the directed graph, tree and show the loops for the network shown in figure below.



3. Define and explain bandwidth, Q-factor, cutset, tieset and tree.

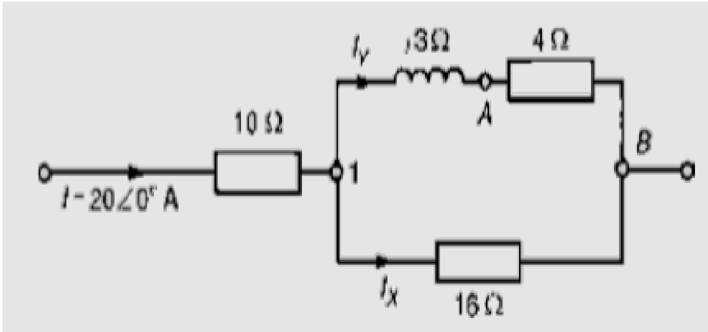
4.

Use mesh-current analysis to determine the current flowing in: (i) 5 Ohms resistance. (ii) 1 Ohm resistance of the d.c. circuit shown in figure below.



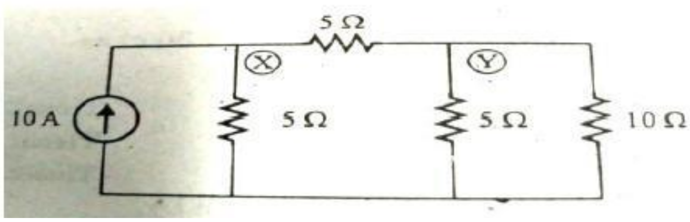
5.

For the network shown in figure below, determine the voltage V_{AB}, by using nodal analysis.



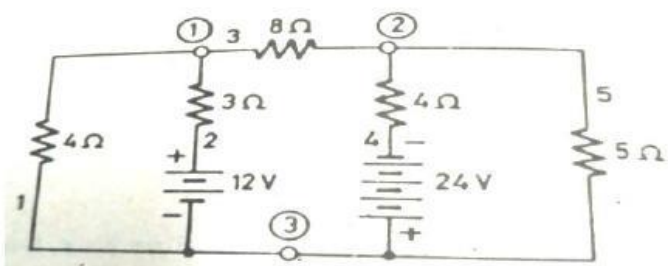
6.

Write cutest matrix, obtain the equilibrium equations using nodal equations for the network shown in figure below. Also find the Node voltages at X and Y using network topology.



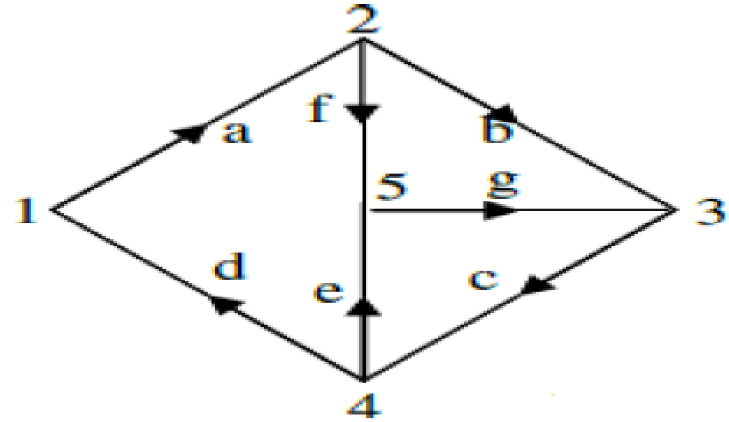
7.

For the network shown in figure below, write tie set matrix, write equilibrium equations and obtain the loop currents using network topology.



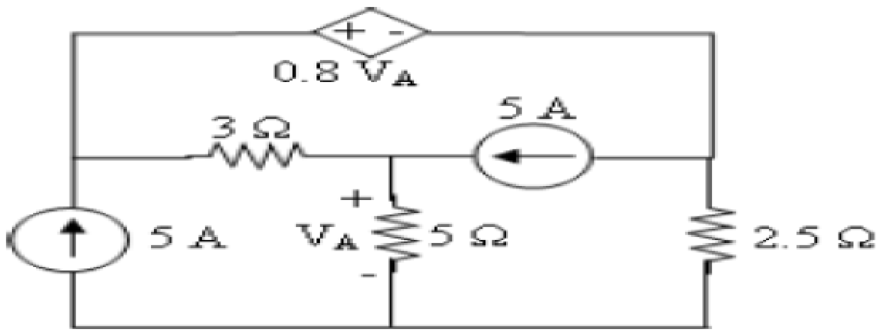
8.

Find fundamental tie-set and cut-set matrix for the graph and its tree shown below.



9.

With the help of nodal analysis on the circuit shown below: Find: (i) VA. (ii) The power dissipated 2.5 ohms resistor.



UNIT-V

FILTER DESIGN & CIRCUIT SIMULATION FILTER DESIGN & CIRCUIT SIMULATION

- Filters – Low Pass – High Pass
- Band Pass – RC, RL filters– derived filters
- composite filters design
- Circuit simulation – Description of Circuit elements, nodes, and sources, Input and Output variables –
- Modeling of the Circuit elements – DC analysis.

Introduction:

A filter is a circuit capable of passing (or amplifying) certain frequencies while attenuating other frequencies. Thus, a filter can extract important frequencies from signals that also contain undesirable or irrelevant frequencies.

In the field of electronics, there are many practical applications for filters. Examples include:

- *Radio communications*: Filters enable radio receivers to only "see" the desired signal while rejecting all other signals (assuming that the other signals have different frequency content).
- *DC power supplies*: Filters are used to eliminate undesired high frequencies (i.e., noise) that are present on AC input lines. Additionally, filters are used on a power supply's output to reduce ripple.
- *Audio electronics*: A crossover network is a network of filters used to channel low-frequency audio to woofers, mid-range frequencies to midrange speakers, and high-frequency sounds to tweeters.
- *Analog-to-digital conversion*: Filters are placed in front of an ADC input to minimize aliasing.

Low Pass Filter

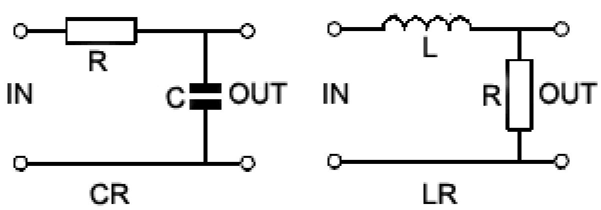
A **low-pass filter** (**LPF**) is a filter that passes signals with a frequency lower than a selected cutoff frequency and attenuates signals with frequencies higher than the cutoff frequency. The exact frequency response of the filter depends on the filter design. The filter is sometimes called a **high-cut filter**, or **treble-cut filter** in audio applications. A low-pass filter is the complement of a high-pass filter.

Low-pass filters exist in many different forms, including electronic circuits such as a **hiss filter** used in audio, anti-aliasing filters for conditioning signals prior to analog-to-digital conversion, digital filters for smoothing sets of data, acoustic barriers, blurring of images, and so on. The moving average operation used in fields such as finance is a particular kind of low-pass filter, and can be analyzed with the same signal processing techniques as are used for other low-pass filters. Low-pass filters provide a smoother form of a signal, removing the short-term fluctuations and leaving the longer-term trend.

Filter designers will often use the low-pass form as a prototype filter. That is, a filter with unity bandwidth and impedance. The desired filter is obtained from the prototype by scaling for the desired bandwidth and impedance and transforming into the desired bandform (that is low-pass, high-pass, band-pass or band-stop).

Filters are widely used to give circuits such as amplifiers, oscillators and power supply circuits the required frequency characteristic. Some examples are given below. They use combinations of R, L and C

Inductors and Capacitors react to changes in frequency in opposite ways. Looking at the circuits for low pass filters, both the LR and CR combinations shown have a similar effect, but notice how the positions of L and C change place compared with R to achieve the same result. The reasons for this, and how these circuits work will be explained in of this module.



Low pass filters.

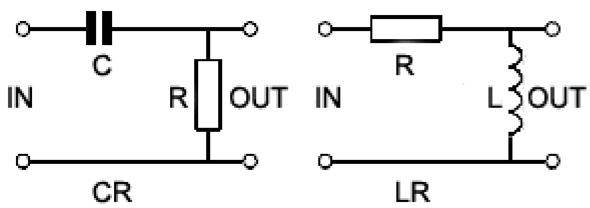
Low Pass Filters

Low pass filters are used to remove or attenuate the higher frequencies in circuits such as audio amplifiers; they give the required frequency response to the amplifier circuit. The frequency at which the low pass filter starts to reduce the amplitude of a signal can be made adjustable. This technique can be used in an audio amplifier as a "TONE" or "TREBLE CUT" control. LR low pass filters and CR high pass filters are also used in speaker systems to route appropriate bands of frequencies to different designs of speakers (i.e. ´ Woofers´ for low frequency, and ´Tweeters´ for high frequency reproduction). In this application the combination of high and low pass filters is called a "crossover filter".

Both CR and LC Low pass filters that remove practically ALL frequencies above just a few Hz are used in power supply circuits, where only DC (zero Hz) is required at the output.

High pass filters

A **high-pass filter** (HPF) is an electronic filter that passes signals with a frequency higher than a certain cutoff frequency and attenuates signals with frequencies lower than the cutoff frequency. The amount of attenuation for each frequency depends on the filter design. A high-pass filter is usually modeled as a linear time-invariant system. It is sometimes called a **low-cut filter** or **bass-cut filter**.[1] High-pass filters have many uses, such as blocking DC from circuitry sensitive to non-zero average voltages or radio frequency devices. They can also be used in conjunction with a low-pass filter to produce a band pass filter.



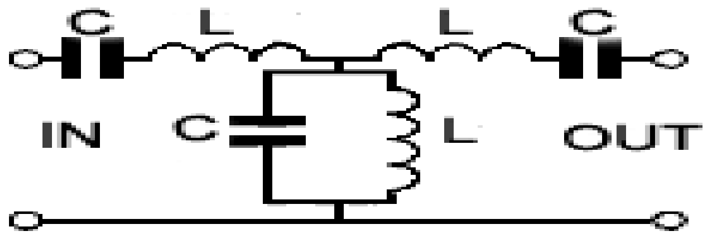
High pass filters are used to remove or attenuate the lower frequencies in amplifiers, especially audio amplifiers where it may be called a "BASS CUT" circuit. In some cases this also may be made adjustable.

BAND PASS FILTERS.

Bandpass is an adjective that describes a type of filter or filtering process; it is to be distinguished from passband, which refers to the actual portion of affected spectrum. Hence, one might say "A dual bandpass filter has two passbands." A bandpass signal is a signal containing a band of frequencies not adjacent to zero frequency, such as a signal that comes out of a bandpass filter.

An ideal bandpass filter would have a completely flat passband (e.g. with no gain/attenuation throughout) and would completely attenuate all frequencies outside the passband. Additionally, the transition out of the passband would have brickwall characteristics.

In practice, no bandpass filter is ideal. The filter does not attenuate all frequencies outside the desired frequency range completely; in particular, there is a region just outside the intended passband where frequencies are attenuated, but not rejected. This is known as the filter roll-off, and it is usually expressed in dB of attenuation per octave or decade of frequency. Generally, the design of a filter seeks to make the roll-off as narrow as possible, thus allowing the filter to perform as close as possible to its intended design. Often, this is achieved at the expense of pass-band or stop-band ripple.



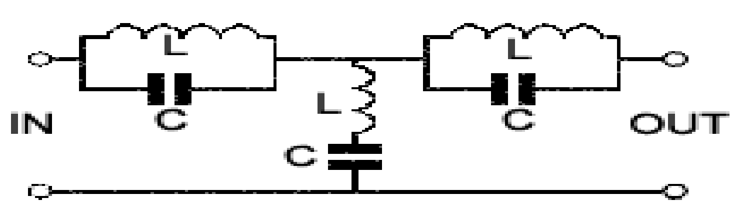
.

Band pass filters allow only a required band of frequencies to pass, while rejecting signals at all frequencies above and below this band. This particular design is called a T filter because of the way the components are drawn in a schematic diagram. The T filter consists of three elements, two series−connected LC circuits between input and output, which form a low impedance path to signals of the required frequency, but have a high impedance to all other frequencies.

Additionally, a parallel LC circuit is connected between the signal path (at the junction of the two series circuits) and ground to form a high impedance at the required frequency, and a low impedance at all others. Because this basic design forms only one stage of filtering it is also called a ´first order´ filter. Although it can have a reasonably narrow pass band, if sharper cut off is required, a second filter may be added at the output of the first filter, to form a ´second order´ filter.

Band stop filters.

Narrow notch filters (optical) are used in Raman spectroscopy, live sound reproduction (public address systems, or PA systems) and in instrument amplifiers (especially amplifiers or preamplifiers for acoustic instruments such as acoustic guitar, mandolin, bass instrument amplifier, etc.) to reduce or prevent audio feedback, while having little noticeable effect on the rest of the frequency spectrum (electronic or software filters). Other names include 'band limit filter', 'T-notch filter', 'band-elimination filter', and 'band-reject filter'.



These filters have the opposite effect to band pass filters, there are two parallel LC circuits in the signal path to form a high impedance at the unwanted signal frequency, and a series circuit forming a low impedance path to ground at the same frequency, to add to the rejection. Band stop filters may be found (often in combination with band pass filters) in the intermediate frequency (IF) amplifiers of older radio and TV receivers, where they help produce the frequency response curves of quite complex shapes needed for the correct reception of both sound and picture signals. Combinations of band stop and band pass filters, as well as tuned transformers in these circuits, require careful frequency adjustment.

COMPOSITE FILTERS DESIGN

A **composite image filter** is an electronic filter consisting of multiple image filter sections of two or more different types.

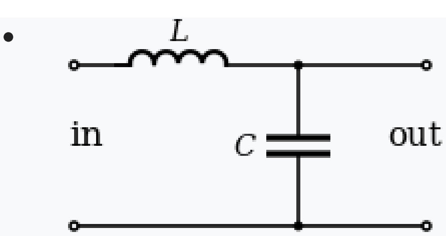
The image method of filter design determines the properties of filter sections by calculating the properties they have in an infinite chain of such sections. In this, the analysis parallels transmission line theory on which it is based. Filters designed by this method are called *image parameter filters*, or just *image filters*. An important parameter of image filters is their image impedance, the impedance of an infinite chain of identical sections.

The basic sections are arranged into a ladder network of several sections, the number of sections required is mostly determined by the amount of stopband rejection required. In its simplest form, the filter can consist entirely of identical sections. However, it is more usual to use a composite filter of two or three different types of section to improve different parameters best addressed by a particular type. The most frequent parameters considered are stopband rejection, steepness of the filter skirt (transition band) and impedance matching to the filter terminations.

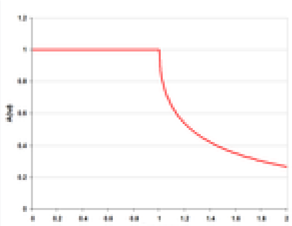
Constant k section

The **constant k** or **k-type** filter section is the basic image filter section. It is also the simplest circuit topology. The k-type has moderately fast transition from the passband to the stopband and moderately good stopband rejection.

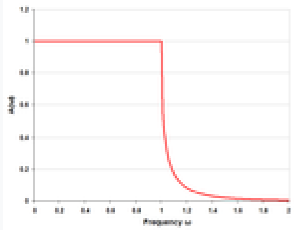
$Z_1 Z_2 = K^2$

- 

k-type low-pass filter half section

- 

k-type low-pass response, single half-section



k-type low-pass response with four (half) sections

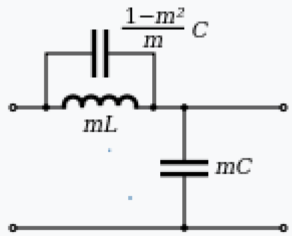
m-derived section

The **m-derived** or **m-type** filter section is a development of the k-type section. The most prominent feature of the m-type is a pole of attenuation just past the cut-off frequency inside the stopband. The parameter *m* (0<*m*<1) adjusts the position of this pole of attenuation. Smaller values of *m* put the pole closer to the cut-off frequency. Larger values of *m* put it further away. In the limit, as *m* approaches unity, the pole approaches *ω* of infinity and the section approaches a k-type section.

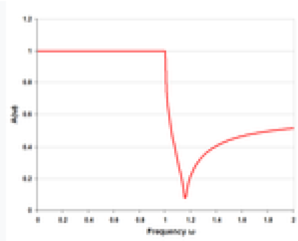
The m-type has a particularly fast cut-off, going from fully pass at the cut-off frequency to fully stop at the pole frequency. The cut-off can be made faster by moving the pole nearer to the cut-off frequency. This filter has the fastest cut-off of any filter design; note that the fast transition is achieved with just a single section, there is no need for multiple sections. The drawback with m-type sections is that they have poor stopband rejection past the pole of attenuation.

There is a particularly useful property of m-type filters with *m*=0.6. These have maximally flat image impedance in the passband. They are therefore good for matching in to the filter terminations, in the passband at least, the stopband is another story.

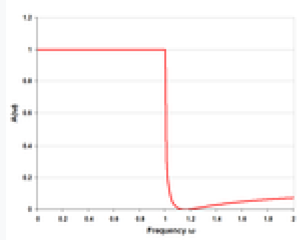
There are two variants of the m-type section, *series* and *shunt*. They have identical transfer functions but their image impedances are different. The shunt half-section has an image impedance which matches on one side but has a different impedance, on the other. The series half-section matches on one side and has on the other.

- 

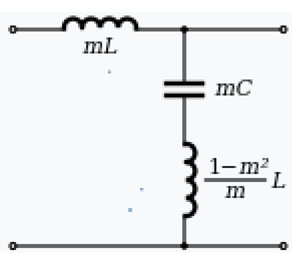
m-type low-pass filter shunt half section

- 

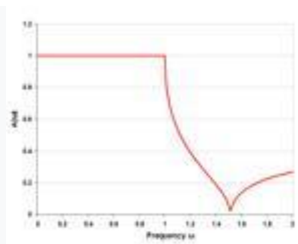
m-type low-pass response single half-section *m*=0.5

- 

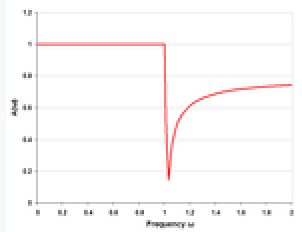
m-type low-pass response with four (half) sections *m*=0.5

- 

m-type low-pass filter series half section

- 

m-type low-pass response single half-section *m*=0.75

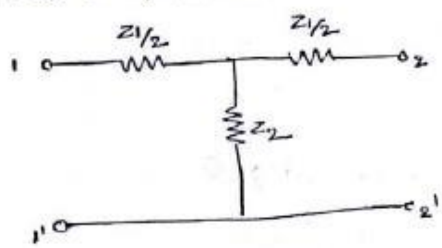


m-type low-pass response single half-section *m*=0.25

Filter Networks.

Symmetrical T-Network.

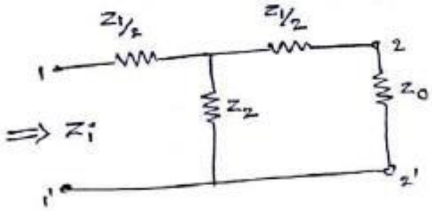
Consider a symmetrical T-Network as shown in figure.



$Z_1/2$ is the impedance in the series arm while Z_2 is the impedance in the shunt arm.

As the T-Network is symmetric, the image impedance at port 1-1' (Z_{I1}) is equal to the image impedance at port 2-2' (Z_{I2})

If the T-network is terminated in characteristic impedance (Z_0) then the input impedance (Z_1) of the networks will also show the termination of port 2-2' in Z_0.



The input impedance is given by

$$Z_i = \left(\left(\frac{Z_1}{2} + Z_0\right) \| Z_2\right) + \frac{Z_1}{2}$$

$$Z_i = \frac{\left(\frac{Z_1}{2}+Z_0\right)Z_2}{\frac{Z_1}{2}+Z_0+Z_2} + \frac{Z_1}{2} = \frac{\cancel{2}Z_2\left(\frac{Z_1+2Z_0}{\cancel{2}}\right)}{2Z_2+Z_1+2Z_0} + \frac{Z_1}{2} = \frac{Z_1Z_2+2Z_0Z_2}{2Z_2+Z_1+2Z_0} + \frac{Z_1}{2}$$

$$Z_i = Z_0$$

$$Z_0 = \frac{Z_1Z_2+2Z_0Z_2}{2Z_2+Z_1+2Z_0} \Rightarrow \cancel{4Z_0Z_2}+\cancel{2Z_1Z_0}+4Z_0^2 = 2Z_1Z_2+Z_1^2+\cancel{2Z_0Z_1}+2Z_1Z_2+\cancel{4Z_0Z_2}$$

$$4Z_0^2 = Z_1^2+4Z_1Z_2$$

$$Z_0^2 = \frac{Z_1^2+4Z_1Z_2}{4}$$

$$\boxed{Z_0 = \sqrt{\frac{Z_1^2}{4}+Z_1Z_2}} \quad -①$$

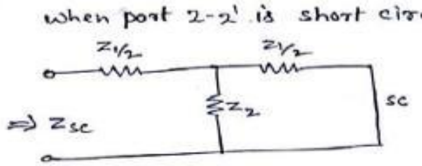
Characteristic Impedance in terms of open ckt and short ckt

consider the port 2-2' is open ckted as shown in fig ①.

The input impedance at port 1-1' is given by

$$Z_{oc} = \frac{Z_1}{2}+Z_2 \Rightarrow \boxed{Z_{oc} = \frac{Z_1+2Z_2}{2}} \quad —②$$

when port 2-2' is short circuited as shown in fig ③



$$Z_{sc} = \left(Z_2 \| \frac{Z_1}{2}\right) + \frac{Z_1}{2}$$

$$Z_{sc} = \frac{\frac{Z_1Z_2}{2}}{\frac{2Z_2+Z_1}{2}} + \frac{Z_1}{2} \Rightarrow Z_{sc} = \frac{Z_1Z_2}{2Z_2+Z_1} + \frac{Z_1}{2} = \frac{2Z_1Z_2+2Z_1Z_2+Z_1^2}{2(Z_1+2Z_2)}$$

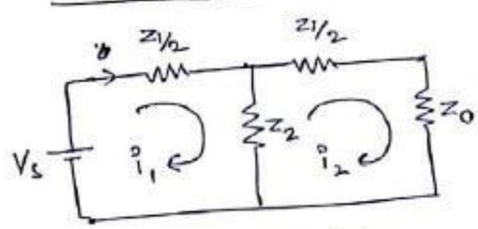
$$\boxed{Z_{sc} = \frac{Z_1^2+4Z_1Z_2}{2(Z_1+2Z_2)}} \quad —③$$

② × ③ $$Z_{oc} \times Z_{sc} = \frac{Z_1 + 2Z_2}{2} \times \frac{Z_1^2 + 4Z_1Z_2}{2(Z_1 + 2Z_2)}$$

$$= \frac{Z_1^2}{4} + Z_1Z_2$$

$$Z_0 \times Z_{sc} = Z_{OT}^2 \Rightarrow \boxed{Z_{OT} = \sqrt{Z_{oc} \times Z_{sc}}}$$

Propogation constant (γ)



let i_1, i_2 be the currents for port 1-1' and port 2-2' respectively.

Applying KVL to the loop 2

$$i_2\left(\frac{Z_1}{2}\right) + i_2 Z_0 + (i_2 - i_1)Z_2 = 0 \Rightarrow i_2\left(\frac{Z_1}{2} + Z_0 + Z_2\right) = i_1 Z_2$$

$$\frac{i_1}{i_2} = \frac{\frac{Z_1}{2} + Z_0 + Z_2}{Z_2} = \frac{Z_1}{2Z_2} + \frac{Z_0}{Z_2} + 1 \quad \text{—— ①}$$

$$\gamma = \ln\frac{i_1}{i_2} = \frac{i_1}{i_2} = e^{\gamma}$$

$$e^{\gamma} = \frac{Z_1 + 2Z_0 + 2Z_2}{2Z_2} \Rightarrow 2Z_0 e^{\gamma} = Z_1 + 2Z_0 + 2Z_2$$

$$Z_0 = Z_2(e^{\gamma} - 1) - \frac{Z_1}{2} \quad \text{—— ②}$$

$$Z_0^2 = \left(Z_2(e^{\gamma} - 1) - \frac{Z_1}{2}\right)^2 \Rightarrow Z_0^2 = Z_2^2(e^{\gamma} - 1)^2 + \frac{Z_1^2}{4} - Z_1Z_2(e^{\gamma} - 1) \quad \text{—— ③}$$

$$Z_{OT}^2 = \frac{Z_1^2}{4} + Z_1Z_2 \quad \text{—— ④}$$

③ − ④ $$0 = Z_2^2(e^{\gamma} - 1)^2 + \cancel{\frac{Z_1^2}{4}} - Z_1Z_2(e^{\gamma} - 1) - \cancel{\frac{Z_1^2}{4}} + Z_1Z_2$$

$$0 = Z_2^2(e^{2\gamma} + 1 - 2e^{\gamma}) - Z_1Z_2(e^{\gamma} - \cancel{1} + \cancel{1})$$

$$Z_2^{\cancel{2}}(e^{2\gamma}+1-2e^{\gamma}) = Z_1 \cancel{Z_2}\, e^{\gamma}$$

$$\Rightarrow \quad e^{-\gamma}(e^{2\gamma}+1-2e^{\gamma}) = \frac{Z_1}{Z_2} \quad \Rightarrow \quad e^{\gamma}+e^{-\gamma}-2 = \frac{Z_1}{Z_2}$$

$$\frac{e^{\gamma}+e^{-\gamma}}{2} - 1 = \frac{Z_1}{2Z_2} \quad \Rightarrow \quad \frac{e^{\gamma}+e^{-\gamma}}{2} = \frac{Z_1}{2Z_2}+1 \qquad \boxed{\cosh\gamma = 1+\frac{Z_1}{2Z_2}}$$

$$\sinh\gamma = \sqrt{\cosh^2\gamma - 1} \qquad \boxed{\gamma = \sinh^{-1}\left(\frac{Z_{0T}}{Z_2}\right)}$$

$$= \sqrt{\left(1+\frac{Z_1}{2Z_2}\right)^2 - 1} \qquad \boxed{\gamma = \cosh^{-1}\left(1+\frac{Z_1}{2Z_2}\right)}$$

$$= \sqrt{\cancel{1}+\left(\frac{Z_1}{2Z_2}\right)^2 + 2\times\frac{Z_1}{2Z_2} - \cancel{1}}$$

$$= \sqrt{\frac{Z_1}{Z_2}+\frac{Z_1^2}{4Z_2^2}} = \frac{1}{Z_2}\sqrt{Z_1Z_2+\frac{Z_1^2}{4}}$$

$$\sinh\gamma = \frac{1}{Z_2}\times Z_{0T}$$

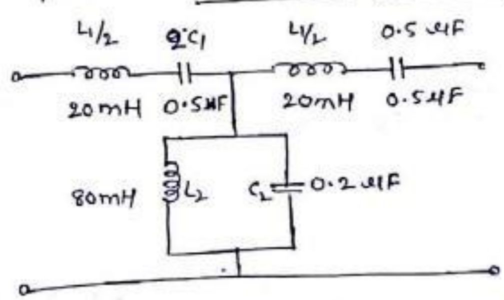
Propagation constant is very important in order to study the characteristic of filters. γ is the function of frequency and it helps to find three features of a filter

(a) pass band

(b) stop band

(c) cut-off point f_c.

Determine the bandwidth and cut off frequencies for the filter shown in figure. T- Section band pass filter.



$$\frac{L_1}{2} = 20\,mH \Rightarrow L_1 = 40\,mH$$

$$2C_1 = 0.5\,\mu F \Rightarrow C_1 = 0.25\,\mu F$$

$$L_2 = 80\,mH, \quad C_2 = 0.2\,\mu F$$

The nominal impedance R_0.

$$R_0 = \sqrt{\frac{L_1}{C_2}} = \sqrt{\frac{L_2}{C_1}}$$

$$R_0 = \sqrt{\frac{40\times10^{-3}}{0.2\times10^{-6}}} = 447.21\,\Omega$$

$$\boxed{R_0 = 447.21\,\Omega}$$

$$L_1 = \frac{R_0}{\pi(f_2-f_1)} \Rightarrow f_2-f_1 = \frac{R_0}{\pi L_1} = \frac{447.21}{\pi\times40\times10^{-3}}$$

$$\boxed{f_2-f_1 = 3558.81}$$

$$c_1 = \frac{f_2 - f_1}{4\pi R_0 f_1 f_2} \Rightarrow f_1 f_2 = \frac{f_2 - f_1}{4\pi R_0 \times C_1} = \frac{3558.81}{4\pi \times 447.21 \times 0.25 \times 10^{-6}}$$

$$\boxed{f_1 f_2 = 2.533 \times 10^6}$$

$$(f_2 + f_1)^2 = (f_2 - f_1)^2 + 4 f_2 f_1$$

$$= (3558.81)^2 + 4 \times 2.533 \times 10^6$$

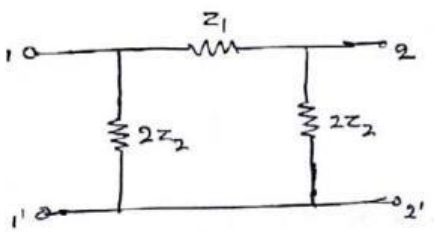
$$\boxed{f_2 + f_1 = 4774.65}$$

$$f_1 = 607.92 \text{ Hz}$$

$$f_2 = 4166.73 \text{ Hz}$$

Symmetrical π- network

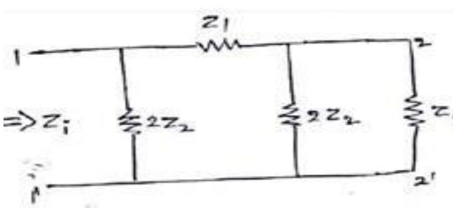
consider a symmetrical π-network as shown in figure.



z_1 is the impedance in the series arm while $2z_2$ is the impedance in each shunt arm.

$$Z_i = \left[(2Z_2 \| Z_0) + Z_1\right] \| 2Z_2$$

$\Rightarrow Z_i$



$$= \left[\frac{2Z_2 Z_0}{2Z_2 + Z_0} + Z_1\right] \| 2Z_2$$

$$Z_i = \left[\frac{2Z_2 Z_0 + 2Z_1 Z_2 + Z_0 Z_1}{2Z_2 + Z_0}\right] \| 2Z_2$$

$$Z_i = \frac{2Z_2\left[\frac{2Z_2 Z_0 + 2Z_1 Z_2 + Z_0 Z_1}{2Z_2 + Z_0}\right]}{2Z_2 + \frac{2Z_2 Z_0 + 2Z_1 Z_2 + Z_0 Z_1}{2Z_2 + Z_0}} = \frac{4Z_1 Z_2^2 + 2Z_0 Z_1 Z_2 + 4Z_0 Z_2^2}{4Z_2^2 + 2Z_0 Z_2 + 2Z_1 Z_2 + Z_1 Z_0 + 2Z_0 Z_2}$$

$$z_i = \frac{4z_1z_2^2 + 2z_0z_1z_2 + 4z_0z_2^2}{4z_2^2 + 4z_0z_2 + 2z_1z_2 + z_0z_1}$$

$$z_0\left(4z_2^2 + 4z_0z_2 + 2z_1z_2 + z_0z_1\right) = 4z_1z_2^2 + 2z_0z_1z_2 + 4z_0z_2^2$$

$$\cancel{4z_0z_2^2} + 4z_2z_0^2 + \cancel{2z_0z_1z_2} + z_0^2z_1 = 4z_1z_2^2 + \cancel{2z_0z_1z_2} + \cancel{4z_0z_2^2}$$

$$4z_0^2z_2 + z_0^2z_1 = 4z_1z_2^2$$

$$\boxed{z_{0\pi} = \sqrt{\frac{z_1z_2}{1+\frac{z_1}{4z_2}}}} \quad — (1)$$

$$z_0^2\left(z_1 + 4z_2\right) = 4z_1z_2^2$$

$$z_0 = \sqrt{\frac{\cancel{4}z_1z_2^{\cancel{2}}}{\cancel{4z_2}\left(1+\frac{z_1}{4z_2}\right)}} \qquad z_0^2 = \frac{z_1z_2}{1+\frac{z_1}{4z_2}} \times \frac{z_1z_2}{z_1z_2}$$

$$z_{0\pi} = \sqrt{\frac{z_1^2z_2^2}{z_1z_2 + \frac{z_1^2}{4}}} = \frac{z_1z_2}{\sqrt{\frac{z_1^2}{4} + z_1z_2}} = \frac{z_1z_2}{z_{0T}}$$

$$\boxed{z_{0\pi} = \frac{z_1z_2}{z_{0T}}} \quad — (2)$$

characteristic impedance in terms of open ckt and short ckt impedance.

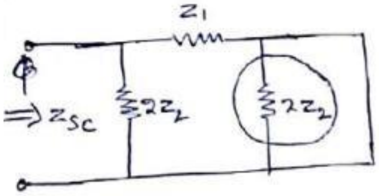
when port 2-2' is open ckted, as shown in fig (1).

The input impedance at port 1-1' is given by

$$z_{oc} = \left(z_1 + 2z_2\right) \parallel 2z_2$$

$$= \frac{2z_2\left(z_1+2z_2\right)}{2z_2 + z_1 + 2z_2} \Rightarrow \boxed{z_{oc} = \frac{2z_2\left(z_1+2z_2\right)}{z_1+4z_2}} \quad — (3)$$

When port 2-2' is short ckted as shown in figure ③.



$Z_{sc} = Z_1 \| 2Z_2$

$$\boxed{Z_{sc} = \frac{2Z_1Z_2}{Z_1+2Z_2}} \quad \text{—— ④}$$

③ × ④

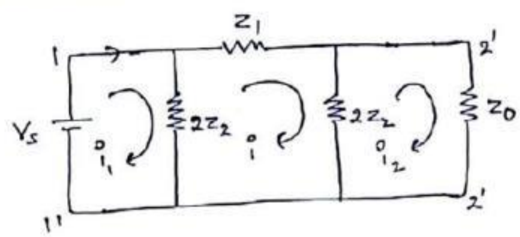
$$Z_{oc} \times Z_{sc} = \frac{2Z_2(\cancel{Z_1+2Z_2})}{Z_1+4Z_2} \times \frac{2Z_1Z_2}{\cancel{Z_1+2Z_2}} = \frac{4Z_1Z_2^2}{Z_1+4Z_2} = \frac{\cancel{4}Z_1Z_2^{\cancel{2}}}{\cancel{4}\cancel{Z_2}\left[1+\frac{Z_1}{4Z_2}\right]}$$

$$= \frac{Z_1Z_2}{1+\frac{Z_1}{4Z_2}}$$

$$= Z_{o\pi}^2$$

$$\boxed{Z_{o\pi} = \sqrt{Z_{oc} \times Z_{sc}}} \quad \text{—— ⑤}$$

Propagation constant (γ)



Let i_1, i_2 be the currents for port 1-1 and 2-2' respectively

i be the loop 2 current.

Applying KVL to loop-2

$$i_2 Z_0 + (i_2 - i)\,2Z_2 + (i - i_1)\,2Z_2 = 0$$

Applying KVL to loop-3

$$i_2(z_0+2z_2)-2iz_2=0 \quad \text{—— } ①$$

$$iz_1+2z_2(i-i_2)+2z_2(i-i_1)=0$$

$$iz_1+2z_2i-2z_2i_2+2z_2i-2z_2i_1=0$$

$$i(z_1+4z_2)=2z_2(i_1+i_2)$$

$$i=\frac{2z_2}{z_1+4z_2}(i_1+i_2) \quad -②$$

substituting ② in ①

$$i_2(z_0+2z_2)-2z_2\left[\frac{2z_2}{z_1+4z_2}(i_1+i_2)\right]=0$$

$$i_2(z_0+2z_2)-\frac{4z_2^2}{z_1+4z_2}i_1-\frac{4z_2^2}{z_1+4z_2}=0$$

$$i_2\left[z_0+2z_2-\frac{4z_2^2}{z_1+4z_2}\right]=\frac{4z_2^2}{z_1+4z_2}i_1$$

$$\frac{i_1}{i_2}=\frac{z_0z_1+4z_0z_2+2z_1z_2+4z_2^2}{4z_2^2} \quad \text{——— } ③$$

$$Y=\ln\left(\frac{i_1}{i_2}\right) \Rightarrow e^Y=\frac{i_1}{i_2}$$

$$e^Y=\frac{z_0z_1+4z_0z_2+2z_1z_2+4z_2^2}{4z_2^2}$$

$$e^Y=\frac{z_0(z_1+4z_2)+2z_2(z_1+2z_2)}{4z_2^2} \Rightarrow 4z_2^2e^Y=z_0(z_1+4z_2)+2z_2(z_1+2z_2)$$

$$z_0(z_1+4z_2)=4z_2^2e^Y-2z_2(z_1+2z_2)$$

$$z_0=\frac{4z_2^2(e^Y-1)-2z_1z_2}{z_1+4z_2}$$

squaring on both sides

$$z_{0\pi}^2=\frac{16z_2^4(e^Y-1)^2+4z_1^2z_2^2-16z_1z_2^3(e^Y-1)}{z_1^2+16z_2^2+8z_1z_2} \quad \text{——— } ④$$

$$Z_{o\pi} = \sqrt{\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}}} \quad , \quad Z_{o\pi}^2 = \frac{4Z_1 Z_2^2}{Z_1+4Z_2} \quad \text{--- } (5)$$

(4)−(5)

$$e^{2\gamma}\left[16 Z_1 Z_2^4 + 64 Z_2^5\right] + 16 Z_1 Z_2^4 + 64 Z_2^5 = e^{\gamma}\left[32 Z_1 Z_2^4 + 128 Z_2^5 + 16 Z_1^2 Z_2^3 + 64 Z_1 Z_2^4\right]$$

$$\cancel{16}\left[e^{2\gamma}\left[Z_1 Z_2^4 + 4 Z_2^5\right] + Z_1 Z_2^4 + 4 Z_2^5\right] = \cancel{16}\left(e^{\gamma}\left(2 Z_1 Z_2^4 + 8 Z_2^5 + Z_1^2 Z_2^3 + 4 Z_1 Z_2^4\right)\right)$$

$$Z_2^4\left(\left(Z_1 + 4Z_2\right)e^{2\gamma} + Z_1 + 4Z_2\right) = e^{\gamma}\left(\left(Z_1 + 4Z_2\right)\left(2Z_2^4 + Z_1 Z_2^3\right)\right)$$

$$Z_2^4\left(\cancel{Z_1 + 4Z_2}\right)\left(e^{2\gamma}+1\right) = e^{\gamma}\left[\left(\cancel{Z_1 + 4Z_2}\right)\left(2Z_2^4 + Z_1 Z_2^3\right)\right]$$

$$Z_2^4\left(e^{2\gamma}+1\right) = e^{\gamma}\left(2Z_2 + Z_1\right)Z_2^3$$

$$Z_2\left(e^{2\gamma}+1\right) = e^{\gamma}\left(2Z_2 + Z_1\right)$$

$$\frac{e^{2\gamma}+1}{e^{\gamma}} = \frac{Z_1 + 2Z_2}{Z_2}$$

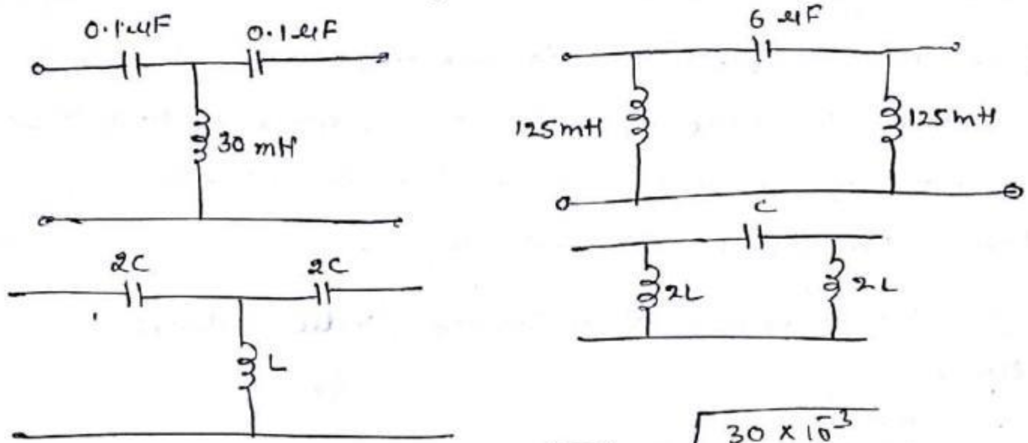
$$e^{\gamma} + e^{-\gamma} = 2 + \frac{Z_1}{Z_2}$$

$$\frac{e^{\gamma} + e^{-\gamma}}{2} = 1 + \frac{Z_1}{2Z_2}$$

$$\cosh\gamma = 1 + \frac{Z_1}{2Z_2}$$

$$\boxed{\gamma = \cosh^{-1}\left(1 + \frac{Z_1}{2Z_2}\right)}$$

Ex.
Determine the cut off frequency for the high pass filter



$2C = 0.1\,\mu F$ T-network $K = \sqrt{\frac{L}{C}} = \sqrt{\frac{30\times10^{-3}}{0.05\times10^{-6}}}$

$C = \frac{0.1}{2} = 0.05\,\mu F.$

$\boxed{K = 774.6\,\Omega}$

$L = 30\,mH.$

$L = \frac{K}{4\pi f_c} \Rightarrow f_c = \frac{K}{4\pi\times L} = \frac{774.6}{4\times\pi\times30\times10^{-3}}$

$\boxed{f_c = 2.054\,KHz}$

π- network

$C = 6\,\mu F$

$2L = 125\,mH \Rightarrow L = \frac{125}{2} = 62.5\,mH.$

$K = \sqrt{\frac{L}{C}} = \sqrt{\frac{62.5\times10^{-3}}{6\times10^{-6}}} = 102.06\,\Omega$

$L = \frac{K}{4\pi f_c} \Rightarrow f_c = \frac{K}{4\pi L} = \frac{102.06}{4\times\pi\times62.5\times10^{-3}} = 129.94\,Hz.$

$\boxed{f_c = 0.13\,KHz}$

M-derived T-Section Filters

It is possible to designe a filter to have rapid attenuation in the stop band and the same characteristic Impedance as the prototype at all frequencies. such a filter is called m-derived filter.

m-derived Low Pass T-section network.

The general representation of m-derived T-section network is shown in figure.

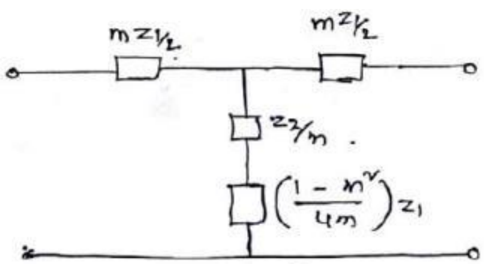


Figure ①.

For an m-derived low-pass T-Section network, the Impedances are given as.

$Z_1 = j\omega L$

$\frac{mZ_1}{2} = \frac{j\omega L \times m}{2}$

$= j\omega \frac{mL}{2}$

= Impedance of Inductance, $\frac{mL}{2}$.

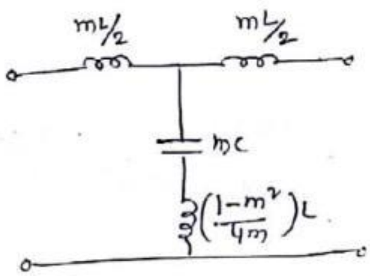
$Z_2 = \frac{1}{j\omega C}$

$\frac{Z_2}{m} = \frac{1}{j\omega C \times m} = \frac{1}{j\omega mC}$

= Impedance of capacitance, mC

$$\left(\frac{1-m^2}{4m}\right)Z_1 = \frac{1-m^2}{4m} \times j\omega L = j\omega \times \left(\frac{1-m^2}{4m}\right)L$$

= Impedance of inductance, $\left(\frac{1-m^2}{4m}\right)L$.

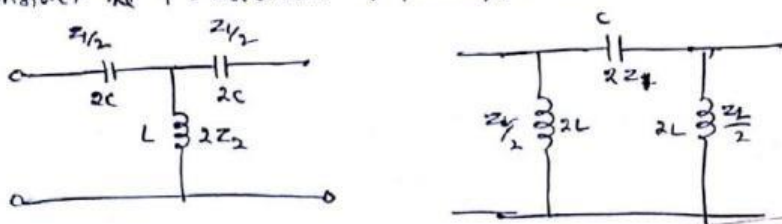


The values of m, L and C are selected in such a way, so as to obtain resonance in the shunt arm at a frequency (f_r) above the cut-off frequency (f_c). $(f_r > f_c)$

for a Series LC ckt, at resonance, the inductive and capacitive reactances will be equal and so the net ~~reactance~~ impedance of shunt arm becomes zero. Thus at resonance frequency, the shunt arm acts as dead short ckt and so the output will be zero.

Constant-K High Pass filter

Consider the T-network of prototype HPF is shown in figure.



$$Z_1 = \frac{1}{j\omega C}, \quad Z_2 = \frac{1}{j\omega C} \qquad Z_1 Z_2 = \sqrt{\frac{L}{C}} \Rightarrow \boxed{K = \sqrt{\frac{L}{C}}} \quad (1)$$

cut off frequency

$$\frac{1}{j\omega C} = 0 \Rightarrow \omega \to \infty, \; f_c \to \infty.$$

The other cut of frequency

$$\frac{Z_1}{2} = -2Z_2 \Rightarrow Z_1 + 4Z_2 = 0 \Rightarrow \frac{1}{j\omega C} + 4j\omega L = 0$$

$$1 - 4\omega^2 LC = 0 \Rightarrow 4\omega^2 LC = 1 \Rightarrow \omega^2 = \frac{1}{4LC}$$

$$\omega = \frac{1}{2\sqrt{LC}} \Rightarrow f_c = \frac{1}{4\pi\sqrt{LC}}$$

Inductance L

$$\sqrt{C} = \frac{\sqrt{L}}{K}. \qquad f_c = \frac{1}{4\pi\sqrt{L}\cdot\frac{\sqrt{L}}{K}}$$

$$= \frac{K}{4\pi L}$$

$$\boxed{L = \frac{K}{4\pi f_c}}$$

$$f_c = \frac{1}{4\pi K\sqrt{C}\cdot\sqrt{C}} = \frac{1}{4\pi K C}$$

$$\sqrt{L} = K\cdot\sqrt{C} \Rightarrow$$

$$\boxed{C = \frac{1}{4\pi K f_c}}$$

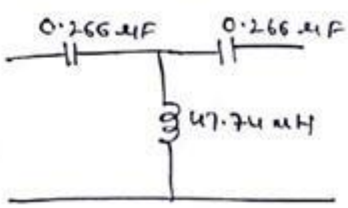
Design a high pass filter having a cut off frequency with a load resistance of 600 Ω

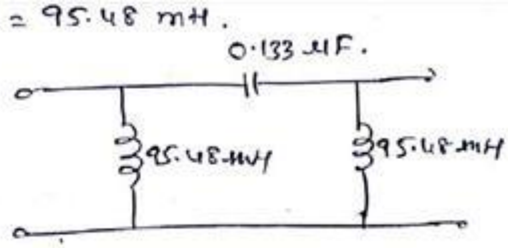
$R_L = K = 600\,\Omega$ and $f_c = 1\,KHz$

$$L = \frac{K}{4\pi f_c} = \frac{600}{4\times\pi\times1000} = 47.74\ mH.$$

$$C = \frac{1}{4\pi\cdot K f_c} = \frac{1}{4\times\pi\times600\times1000} = 0.133\ \mu F.$$

$2C = 0.266\ \mu F$ $\qquad$ $2L = 95.48\ mH.$





Limitations of constant-K filter

→ The characteristic impedances varies varies widely over the pass band frequencies so that impedance matching over the entire pass band becomes impossible

→ Constant-K filters do not have sharp cut off frequencies.

→ In order to obtain sharp cut off frequencies, two or more constant-K filters having identical characteristics must be connected in cascade. Due to this, the ckt size will increase.

→ In the pass band region, the characteristic impedance is not absolutely constant and hence is a function of frequency.

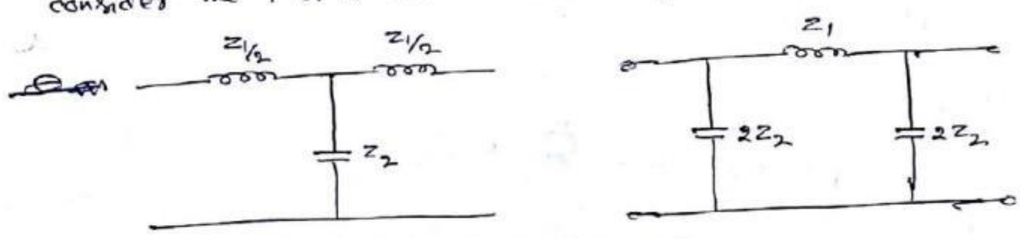
Constant - K Low Pass filter

Constant - K Filter.

Any symmetrical network either T-type or π-type having impedances Z_1 and Z_2 of opposite sign as network elements and also if its product is constant i.e., the filters that satisfy the condition, $Z_1 Z_2 = K^2$ then it is known as constant-K filter and are also known as prototype filters.

Constant-K low pass filter:

consider the T or π-network of prototype LPF are shown in figures.



T-type

$Z_1 = j\omega L$ $\quad Z_1 Z_2 = j\omega L \times \frac{1}{j\omega C}$

$Z_2 = \frac{1}{j\omega C}$ $\quad Z_1 Z_2 = \frac{L}{C} \Rightarrow K^2 = \frac{L}{C}$

condition for constant-k filter $K = \sqrt{\frac{L}{C}}$ — ①

$Z_1 Z_2 = K^2$

The characteristic impedance Z_0.

$$Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$= \left[\frac{Z_1^2}{4} + Z_1 Z_2\right]^{1/2}$$

$$= \left(\frac{(j\omega L)^2}{4} + j\omega L \cdot \frac{1}{j\omega C}\right)^{1/2}$$

$$= \left[\frac{-\omega^2 L^2}{4} + \frac{L}{C}\right]^{1/2}$$

$$Z_0 = \left[\frac{L}{C}\left(1 - \frac{\omega^2 LC}{4}\right)\right]^{1/2}$$

$$Z_0 = \sqrt{\frac{L}{C}}\sqrt{1 - \frac{\omega^2 LC}{4}}$$

At cut off frequency pass band is given by

$$-1 < -\frac{\omega^2 LC}{4} < 0$$

$$-\frac{\omega^2 LC}{4} = 0, \quad -\frac{\omega^2 LC}{4} = -1$$

$f = 0$ $\quad \omega^2 = \frac{4}{LC} \Rightarrow \omega = \sqrt{\frac{4}{LC}} = \frac{2}{\sqrt{LC}}$

$$2\pi f_c = \frac{2}{\sqrt{LC}} \Rightarrow \boxed{f_c = \frac{1}{\pi\sqrt{LC}}}$$

cut-off frequency ranges from 0-to- $\frac{1}{\pi\sqrt{LC}}$

Inductance L

$$K = \sqrt{\frac{L}{C}} \Rightarrow \sqrt{C} = \frac{\sqrt{L}}{K}$$

$$f_c = \frac{1}{\pi\sqrt{LC}} = \frac{1}{\pi\sqrt{L}\cdot\sqrt{C}} = \frac{1}{\pi\sqrt{L}\cdot\frac{\sqrt{L}}{K}} = \frac{K}{\pi L}$$

Capacitance

$$K = \sqrt{\frac{L}{C}} \Rightarrow \sqrt{L} = K\cdot\sqrt{C}$$

$$f_c = \frac{1}{\pi\sqrt{LC}} = \frac{1}{\pi\sqrt{L}\cdot\sqrt{C}} = \frac{1}{\pi K\sqrt{C}\cdot\sqrt{C}} = \frac{1}{\pi K C} \quad \boxed{C = \frac{1}{\pi K f_c}}$$

$f_c = 2\,KHz$, $K = \sqrt{\frac{L}{C}} = 400\Omega$ to operate with the load resistance of $400\,\Omega$.

$$L = \frac{K}{\pi f_c} = \frac{400}{\pi \times 2\times 10^3} = 0.063\ H.$$

$$C = \frac{1}{\pi K f_c} = \frac{1}{\pi \times 400 \times 2000} = 0.397\ \mu F$$

$$L/2 = 0.015\ H,$$

$$C/2 = 0.198\ \mu F.$$

It produces infinite attenuation.

$$f_r = \frac{1}{2\pi\sqrt{L'C'}} \qquad L' = \left(\frac{1-m^2}{4m}\right)L$$

$$C' = mC$$

$$= \frac{1}{2\pi\sqrt{\left(\frac{1-m^2}{4m}\right)L \times mC}}$$

The cut off frequency of a low-pass filter

$$f_c = \frac{1}{\pi\sqrt{LC}} \quad \text{—— ②}$$

$$\pi\sqrt{LC} = \frac{1}{f_c}.$$

$$f_r = \frac{1}{\pi\sqrt{(1-m^2)LC}} \quad \text{— ①}$$

$$= \frac{1}{\pi\sqrt{LC} \times \sqrt{(1-m^2)}}$$

$$f_r = \frac{f_c}{\sqrt{(1-m^2)}} \Rightarrow \sqrt{(1-m^2)} = \frac{f_c}{f_r} \Rightarrow (1-m^2) = \left(\frac{f_c}{f_r}\right)^2$$

$$m = \sqrt{1-\left(\frac{f_c}{f_r}\right)^2} \quad \text{— ③}.$$

nominal or characteristic impedance value is given by

$$R_0 = \sqrt{Z_1 Z_2}$$

$$= \sqrt{j\omega L \times \frac{1}{j\omega C}} = \sqrt{\frac{L}{C}}$$

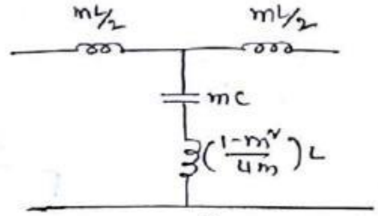
$$\boxed{R_0 = \sqrt{\frac{L}{C}}}$$

** Design a m-derived low-pass filter having a cut off frequency of 1 KHz, design impedance of 400 Ω and the resonant frequency is 1100 Hz.

$f_c = 1$ KHz

$f_r = 1100$ Hz

$R_L = K = 400\ \Omega$



$$C = \frac{1}{\pi K f_c} = \frac{1}{\pi \times 400 \times 1000}$$

$C = 0.795\ \mu F$

$$m = \sqrt{1 - \left(\frac{f_c}{f_r}\right)^2}$$

$$m = \sqrt{1 - \left(\frac{1000}{1100}\right)^2} = 0.4165$$

$$L = \frac{K}{\pi f_c} = \frac{400}{\pi \times 1000} = 127.3\ mH$$

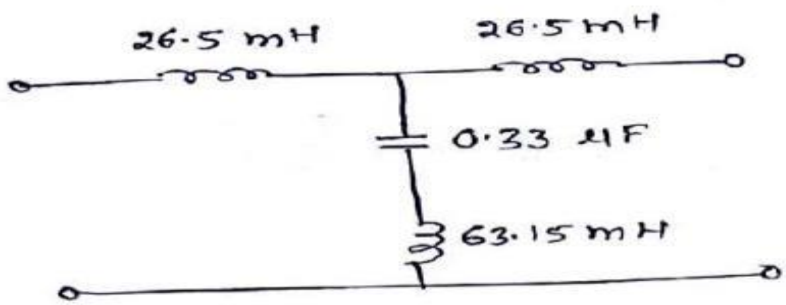
calculation of T-section.

$$\frac{mL}{2} = \frac{0.4165 \times 127.3 \times 10^{-3}}{2} = 26.5\ mH$$

$$mC = 0.4165 \times 0.795 = 0.33\ \mu F$$

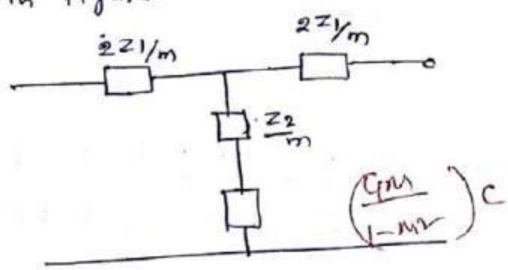
$$\left(\frac{1-m^2}{4m}\right)L = \frac{1-(0.4165)^2}{4 \times 0.4165} \times 127.3$$

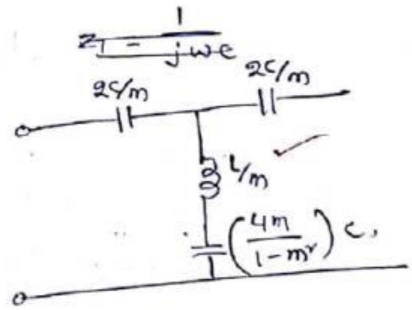
$$= 63.15\ mH$$



m-derived High Pass T- Section filter

General representation of m-derived High pass T- Section filter as shown in figure.





$$f_r = \frac{1}{2\pi\sqrt{L'C'}} \Rightarrow \quad L' = \frac{L}{m}, \quad C' = \left(\frac{4m}{1-m^2}\right)C$$

$$= \frac{1}{2\pi\sqrt{\frac{L}{m}\left(\frac{4m}{1-m^2}\right)C}}$$

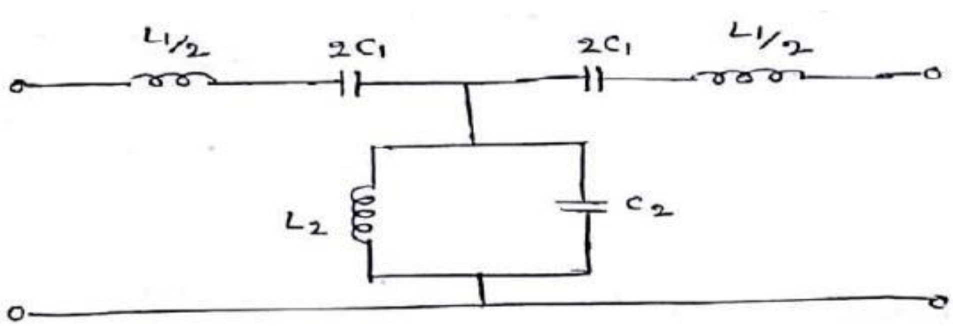
$$= \frac{1}{4\sqrt{LC}\cdot\sqrt{\frac{1}{1-m^2}}} = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$

$$\boxed{f_r = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}} \quad \text{--- (1)}$$

Band Pass Filters

Basically Band Pass filter can be obtained by connecting a low pass filter in series with a high pass filter

The cut-off frequency of a low pass filter (f_1) must be greater than the cut off frequency of a high pass filter (f_2). ∴ $f_1 > f_2$



The values of inductance and capacitance in series as well as shunt arms are designed in order to obtain same resonance frequency in series and shunt arms.

Designing of band pass filter

Designing of filter means to determine the following characteristics.

1. Design Impedance $\varnothing$.
2. Cut-off frequencies
3. Filter components.

1. Design Impedance (or) Nominal characteristic Impedance.

The resonance frequency of series arm.

$$f_{r1} = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

The resonance frequency of shunt arm

$$f_{r2} = \frac{1}{2\pi\sqrt{L_2 C_2}}$$

$$f_{r1} = f_{r2} = f_r = \frac{1}{2\pi\sqrt{L_1 C_1}} = \frac{1}{2\pi\sqrt{L_2 C_2}} \qquad \omega_r^2 = \frac{1}{L_1 C_1} = \frac{1}{L_2 C_2} \quad \text{—} (1).$$

Series arm Impedance

$$z_1 = j\omega\frac{L_1}{2} + \frac{1}{j\omega \times 2C_1} + j\omega\frac{L_1}{2} + \frac{1}{j\omega \times 2C_1}$$

$$= j\omega L_1 + \frac{1}{j\omega C_1} = \frac{j^2\omega^2 L_1 C_1 + 1}{j\omega C_1}$$

$$\boxed{z_1 = \frac{j(\omega^2 L_1 C_1 - 1)}{\omega C_1}} \quad \text{—} (2).$$

Shunt arm impedance

$$z_2 = (j\omega L_2) \parallel \left(\frac{1}{j\omega C_2}\right) = \frac{j\omega L_2 \times \frac{1}{j\omega C_2}}{j\omega L_2 + \frac{1}{j\omega C_2}} = \frac{j\omega L_2}{j^2\omega^2 L_2 C_2 + 1}$$

$$\boxed{Z_2 = \frac{-j\omega L_2}{\omega^2 L_2 C_2 - 1}} \quad \text{— ③}$$

② × ③

$$Z_1 \times Z_2 = \frac{j(\omega^2 L_1 C_1 - 1)}{\omega C_1} \times \frac{-j\omega L_2}{\omega^2 L_2 C_2 - 1}$$

$$= \frac{-j^2(\omega^2 L_1 C_1 - 1) L_2}{C_1(\omega^2 L_2 C_2 - 1)} = \frac{-j^2(\cancel{\omega^2 L_1 C_1 - 1}) L_2}{C_1(\cancel{\omega^2 L_1 C_1 - 1})}$$

$$\boxed{Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2}} \quad \text{— ④}$$

$R_0^2 \quad K^2 = Z_1 Z_2 \quad , \quad R_0 = \sqrt{Z_1 Z_2}$

$$\boxed{R_0 = \sqrt{\frac{L_2}{C_1}} = \sqrt{\frac{L_1}{C_2}}} \quad \text{— ⑤}$$

3. Cut-off Frequencies.

The pass band is given by,

$$-1 < \frac{Z_1}{4Z_2} < 0.$$

$$\frac{Z_1}{4Z_2} = -1$$

$$Z_1 = -4Z_2$$

$$Z_1^2 = -4 Z_1 Z_2$$

$$= -4 R_0^2$$

$$Z_1 = \sqrt{-4R_0^2}$$

$$Z_1 = \pm j 2 R_0$$

At lower cut off frequency (f_1)

$$Z_1 = j 2 R_0 \quad \text{— ⑥}$$

At higher cut off frequency (f_2)

$$Z_1 = -j 2 R_0 . \quad \text{— ⑦}$$

From ⑥ & ⑦

$$Z_1\Big|_{f=f_1} = -Z_1\Big|_{f=f_2}$$

$$\frac{j(\omega^2 L_1 C_1 - 1)}{\omega C_1}\Big|_{f=f_1} = -\frac{j(\omega^2 L_1 C_1 - 1)}{\omega C_1}\Big|_{f=f_2}$$

$$\Rightarrow \frac{(2\pi f_1)^2 L_1 C_1 - 1}{2\pi f_1 C_1} = \frac{-((2\pi f_2)^2 L_1 C_1 - 1)}{\omega C_1} \quad , \quad \frac{4\pi^2 f_1^2 L_1 C_1 - 1}{\cancel{2\pi} f_1 \cancel{C_1}} = \frac{-4\pi^2 f_2^2 L_1 C_1 + 1}{\cancel{2\pi} f_2 \cancel{C_1}}$$

$$(4\pi^2 f_1^2 L_1 C_1 - 1) f_2 = (-4\pi^2 f_2^2 + 1) f_1 \quad , \quad 4\pi^2 f_1^2 f_2 L_1 C_1 - f_2 = -4\pi^2 f_2^2 f_1 L_1 C_1 + f_1$$

$$4\pi^2 f_1 f_2 L_1 C_1 (\cancel{f_1 + f_2}) = (\cancel{f_1 + f_2}) \quad , \quad L_1 C_1 = \frac{1}{4\pi^2 f_1 f_2}$$

$$\frac{1}{\omega_r^2} = \frac{1}{4\pi^2 f_1 f_2} \qquad \left(\because L_1C_1 = \frac{1}{\omega_r^2}\right)$$

$$\frac{1}{(2\pi f_r)^2} = \frac{1}{4\pi^2 f_1 f_2}, \qquad \frac{1}{\cancel{4\pi^2} f_r^2} = \frac{1}{\cancel{4\pi^2} f_1 f_2}$$

$$f_r^2 = f_1 f_2 \Rightarrow \boxed{f_r = \sqrt{f_1 f_2}} \quad — (8)$$

Hence, the common resonance of shunt and series arms is the geometric mean of the lower and upper cut off frequencies

3) Filter components.

$$Z_1 = -j2R_0$$

$$\frac{j(\omega^2 L_1 C_1 - 1)}{\omega C_1} = -j2R_0$$

$$\omega_1^2 L_1 C_1 - 1 = -2\omega_1 R_0 C_1$$

$$\left(\omega_1^2 \times \frac{1}{\omega_r^2}\right) - 1 = -2\omega_1 R_0 C_1$$

$$\frac{(2\pi f_1)^2}{(2\pi f_r)^2} - 1 = -2 \times 2\pi f_1 \times R_0 C_1$$

$$\frac{4\pi^2 f_1^2}{4\pi^2 f_r^2} - 1 = -4\pi f_1 R_0 C_1$$

$$\frac{f_1^{\cancel{2}}}{\cancel{f_1} f_2} - 1 = -4\pi f_1 R_0 C_1$$

$$\frac{f_1}{f_2} - 1 = -4\pi f_1 R_0 C_1$$

$$\frac{f_1 - f_2}{f_2} = -4\pi f_1 R_0 C_1$$

$$\boxed{C_1 = \frac{f_2 - f_1}{4\pi f_1 f_2 R_0}} \quad — (9)$$

From.

$$\frac{1}{\omega_r^2} = L_1 C_1$$

$$L_1 = \frac{1}{\omega_r^2 C_1} = \frac{1}{(2\pi f_r)^2 \times \frac{f_2 - f_1}{4\pi f_1 f_2 R_0}} = \frac{1}{\cancel{4}\pi^2 f_r^2 \times \frac{f_2 - f_1}{\cancel{4\pi} f_1 f_2 R_0}}$$

$$L_1 = \frac{\cancel{f_1 f_2} R_0}{\pi \cancel{f_1 f_2} (f_2 - f_1)} \Rightarrow \boxed{L_1 = \frac{R_0}{\pi (f_2 - f_1)}} \quad — (10)$$

From (5). $R_0^2 = \frac{L_1}{C_2} \Rightarrow C_2 = \frac{L_1}{R_0^2} = \frac{\cancel{R_0}}{\pi (f_2 - f_1) \times R_0^{\cancel{2}}}$

$$\boxed{C_2 = \frac{1}{\pi (f_2 - f_1) R_0}} \quad \text{——— (11)}$$

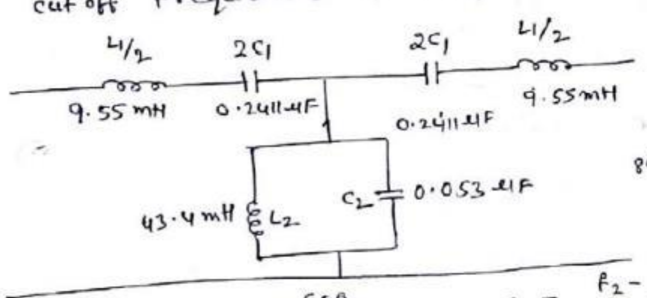
$$\boxed{L_2 = \frac{(f_2 - f_1) R_0}{4\pi f_1 f_2}} \quad \text{——— (12)}.$$

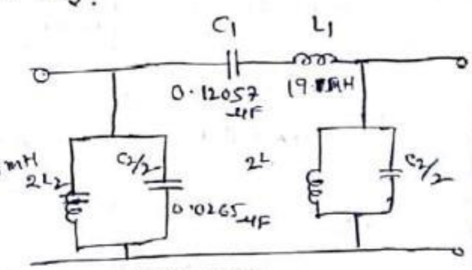
$$R_0^2 = \frac{L_2}{C_1}$$

$$L_2 = R_0^2 \times C_1$$

$$= R_0^{\cancel{2}} \times \frac{f_2 - f_1}{4\pi f_1 f_2 \cancel{R_0}}$$

Design a band pass filter having a design impedance of 600Ω and cut off frequencies $f_1 = 1\ KHz$ and $f_2 = 11\ KHz$.





$$L_1 = \frac{R_0}{\pi (f_2 - f_1)} = \frac{600}{\pi(11000 - 1000)}, \quad C_1 = \frac{f_2 - f_1}{4\pi R_0 f_1 f_2} = \frac{11000 - 1000}{4\pi \times 600 \times 11000 \times 1000}$$

$L_1 = 19.1\ mH$, $\frac{L_1}{2} = \frac{19.1}{2} = 9.55\ mH$ $\quad C_1 = 0.12057\ \mu F$, $2C_1 = 0.2411\ \mu F$.

$$L_2 = C_1 R_0^2$$

$$= (0.12057 \times 10^{-6}) \times 600^2$$

$$L_2 = 43.4\ mH$$

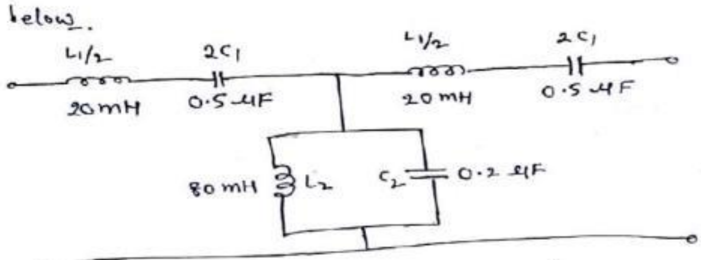
$$2L_2 = 86.8\ mH$$

$$\frac{L_1}{C_2} = R_0^2 \Rightarrow C_2 = \frac{L_1}{R_0^2} = \frac{19.1 \times 10^{-3}}{600^2}$$

$$C_2 = 0.053\ \mu F.$$

$$\frac{C_2}{2} = 0.0265\ \mu F.$$

*** Determine the bandwidth and cut-off frequencies for the filter shown below.



$\frac{L_1}{2} = 20\ mH$, $\quad 2C_1 = 0.5\ \mu F$ $\quad L_2 = 80\ mH$, $\quad C_2 = 0.2\ \mu F$

$L_1 = 40\ mH$ $\quad C_1 = \frac{0.5}{2} = 0.25\ \mu F$

Nominal impedance Ro is given by

$$R_0 = \sqrt{\frac{L_1}{C_2}} = \sqrt{\frac{L_2}{C_1}}$$

$$= \sqrt{\frac{40\times10^{-3}}{0.2\times10^{-6}}} = 447.21\ \Omega$$

$$\boxed{R_0 = 447.21\ \Omega}$$

Band width $f_2 - f_1$

$$L_1 = \frac{R_0}{\pi(f_2 - f_1)}$$

$$f_2 - f_1 = \frac{R_0}{\pi L_1}$$

$$= \frac{447.21}{\pi \times 40\times10^{-3}}$$

$$\boxed{\text{band width} = 3558.81}$$

$$C_1 = \frac{f_2 - f_1}{4\pi f_1 f_2 R_0} \Rightarrow \frac{3558.81}{4\times\pi\times\ 0.25\times10^{-6}\times 447.21} = f_1 f_2$$

$$f_1 f_2 = 2.533\times10^{6}$$

$$(f_1+f_2)^2 = (f_2-f_1)^2 + 4 f_2 f_1$$

$$= (3558.81)^2 + 4\times 2.533\times10^6$$

$$(f_1+f_2)^2 = 2.279\times10^7 \Rightarrow$$

$$f_2 + f_1 = 4774.65$$

$$f_2 - f_1 = 3558.81$$

$$2f_2 = 8333.46$$

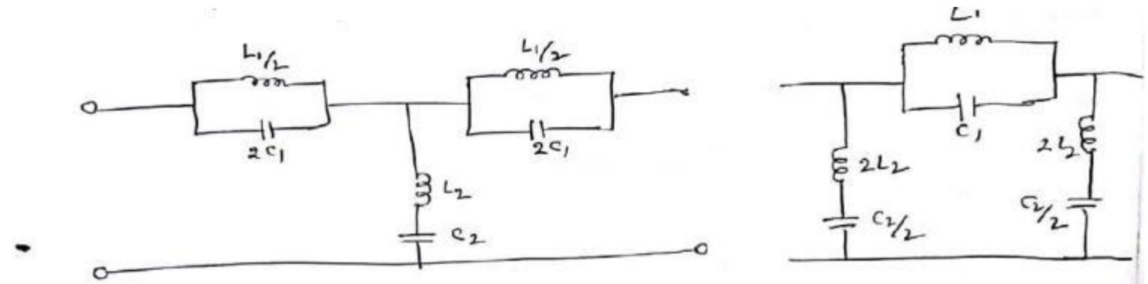
$$f_2 = \frac{8333.46}{2} = 4166.73\ Hz$$

$$f_1 = 607.92\ Hz.$$

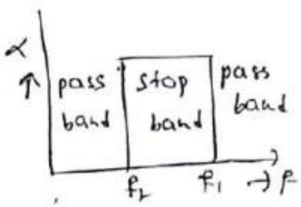
Band stop Filter

Basically band stop filter can be obtained by connecting a low pass filter in parallel with a high pass filter. The cut off frequency of high pass filter must be greater than cut off frequency of low pass filter.

i.e., if f_2 is the frequency of high pass filter and f_1 is the frequency of low pass filter. then $\boxed{f_2 > f_1}$



A band ~~pass~~ stop filter is a filter which attenuates a band of frequencies and allows the transmission of frequencies below and above the band



Design of T-section band ~~pass~~ stop filter

Designing of filter means to determine the following characteristics.

1. Design Impedance
2. Cut off frequency
3. Filter components.

1. Design Impedance

The values of inductance and capacitance in series as well as shunt arms are designed in order to obtain same resonance frequency in series and shunt arms.

F. The resonance frequency of series arm

$$\frac{\omega_r L_1}{2} = \frac{1}{2\omega_r C_1} \Rightarrow \omega_r^2 = \frac{1}{L_1 C_1} \quad \text{--- (1)}$$

The resonance frequency of shunt arm.

$$\omega_r L_2 = \frac{1}{\omega_r C_2} \Rightarrow \omega_r^2 = \frac{1}{L_2 C_2} \quad \text{--- (2)}$$

From (1) & (2) $\quad L_1 C_1 = L_2 C_2$ ——— (3).

The series arm impedance is given by

$$Z_1 = \frac{j\omega L_1 \times \frac{1}{j\omega C_1}}{j\omega L_1 + \frac{1}{j\omega C_1}} = \frac{j\omega L_1}{1 + j^2\omega^2 L_1 C_1}$$

$$Z_1 = \frac{j\omega L_1}{1 - \omega^2 L_1 C_1} \quad \text{——— (4)}.$$

The shunt arm impedance is given by.

$$Z_2 = j\omega L_2 + \frac{1}{j\omega C_2} = \frac{j^2\omega^2 L_2 C_2 + 1}{j\omega C_2}, \quad Z_2 = \frac{j(\omega^2 L_2 C_2 - 1)}{\omega C_2} \quad \text{— (5)}$$

(4) × (5)

$$Z_1 Z_2 = \frac{j\omega L_1}{1 - \omega^2 L_1 C_1} \times \frac{j(\omega^2 L_2 C_2 - 1)}{\omega C_2}$$

$$= \frac{j^2 \cancel{\omega} L_1 (\omega^2 L_2 C_2 - 1)}{\cancel{\omega} C_2 (1 - \omega^2 L_1 C_1)}.$$

$$= \frac{L_1 (\omega^2 L_2 C_2 - 1)}{C_2 (\omega^2 L_1 C_1 - 1)}$$

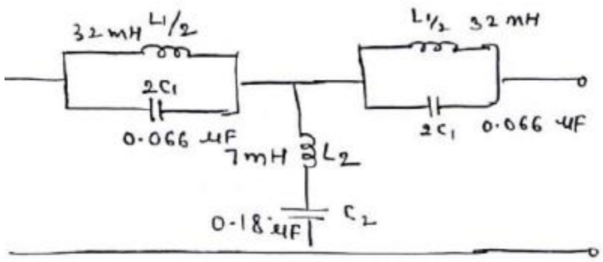
$$= \frac{L_1 (\cancel{\omega^2 L_2 C_2 - 1})}{C_2 (\cancel{\omega^2 L_2 C_2 - 1})} \qquad \boxed{L_1 C_1 = L_2 C_2}$$

$$Z_1 Z_2 = \frac{L_1}{C_2} = \frac{L_2}{C_1}$$

$$R_0 = \sqrt{Z_1 Z_2} \Rightarrow R_0^2 = Z_1 Z_2$$

$$\boxed{R_0 = \sqrt{\frac{L_1}{C_2}} = \sqrt{\frac{L_2}{C_1}}} \quad \text{—— (6)}.$$

Determine the bandwidth and cut off frequency for the filter shown in figure.



$\frac{L_1}{2} = 32\,mH$, $L_1 = 64\,mH$.

$2C_1 = 0.066\,\mu F$, $C_1 = 0.033\,\mu F$

$L_2 = 7\,mH$

$C_2 = 0.18\,\mu F$

The nominal impedance

$$R_0 = \sqrt{\frac{L_1}{C_2}} = 596.28\,\Omega$$

$$\cancel{L_1} = \cancel{\frac{(f_2-f_1)R_0}{\pi f_1 f_2}} \Rightarrow \cancel{f_2 - f_1} = \cancel{\pi f_1 f_2}$$

$$C_1 = \frac{1}{4\pi R_0 (f_2 - f_1)} \Rightarrow f_2 - f_1 = \frac{1}{4\pi C_1 R_0}.$$

$$f_2 - f_1 = 4019.06 \quad (1).$$

$$L_1 = \frac{(f_2-f_1)R_0}{\pi f_1 f_2} \Rightarrow f_1 f_2 = \frac{(f_2-f_1)R_0}{\pi L_1} = 1199.35\times10^4$$

$$(f_2+f_1)^2 = (f_2-f_1)^2 + 4 f_1 f_2$$

$$= (4019.06)^2 \times 4 \times 1199.35\times10^4$$

$$= 6412.69\times10^4$$

$$f_2+f_1 = 8007.92 \quad (2)$$

$$f_2 - f_1 = 4019.06 \quad (1)$$

$$f_2 = \frac{12026.98}{2}$$

$$f_1 = 8007.92 - 6013.49 = 1994.43$$

$f_2 = 6013.49 \Rightarrow 6.013\,KHz$

$f_1 = 1994.43 \Rightarrow 1.994\,KHz.$

S.No	Objective Questions
1.	Filter Networks having only___ Elements [ ] a)**Reactive** b) resistor c) Active elements d) None
2	Low pass Filter allows the frequencies [ ] a) $>f_c$ b) **$<f_c$** c) $=f_c$ d) all
3	High pass Filter allows the frequencies [ ] a) **$>f_c$** b) $<f_c$ c) $=f_c$ d) all
4	A ________ filter rejects all frequencies within a specified band and passes all those outside this band. A) Low Pass b) High Pass c) Band Pass **d) Band Elimination**
5	A ________ filter significantly attenuates all frequencies below f_c and passes all frequencies above f_c. A) Low Pass b) **High Pass** c) Band Pass **d)** Band Elimination
6	The bandwidth in a ________ filter equals the critical frequency. A) **Low Pass** b) High Pass c) Band Pass **d)** Band Elimination
7	The critical frequency is defined as the point at which the response drops ________ from the passband. A) -20dB **B) -3dB** C) -6 dB D) -40 dB
8	A low-pass filter has a cutoff frequency of 1.23 kHz. Determine the bandwidth of the filter. A) 2.46 KHz B) **1.23KHz** C) 644 Hz D) not enough information given
9	One important application of a state-variable ________ filter with a summing amplifier is to minimize the 60 Hz "hum" in audio systems. A) Low Pass b) High Pass c) Band Pass **d) Band Stop**
10	A ________ filter passes all frequencies within a band between a lower and an upper critical frequency and rejects all others outside this band. A) Low Pass b) High Pass **c) Band Pass** d) Band Stop
11	Super mesh analysis is used in case of [ ] a)**current source branch is common for two meshes** b) ideal voltage source is connected between two non reference nodes c) Both d) either a or b
12	What is an ideal value of attenuation for the frequencies in pass band especially for a cascade configuration a. **Zero** b. Unity c. Infinity d. Unpredictable
13	It is possible to overcome the drawback of m-derived filter by connecting number of sections in addition to prototype & m-derived sections with terminating __________

	a. One-fourth sections b. **Half sections** c. Square of three-fourth sections d. Full sections
14	In band elimination filter, the frequency of resonance of individual arms is geometric __________ a. **Mean of two cut-off frequencies** b. Difference of two cut-off frequencies c. Product of two cut-off frequencies d. Division of two cut-off frequencies
15.	What do the high pass filters generally comprise of? A. Capacitive series arm B. Capacitive shunt arm C. Inductive series arm D. Inductive shunt arm a**. A & D** b. A & C c. B & C d. B & D

2 MARKS

1. **Give the applications of filter.**

High-pass and low-pass filters are also used in digital image processing to perform transformations in the spatial frequency domain.

Most high-pass filters have zero gain (-*inf* dB) at DC. Such a high-pass filter with very low cutoff frequency can be used to block DC from a signal that is undesired in that signal (and pass nearly everything else). These are sometimes called DC blocking filters.

2. What is a high pass filter?

A high-pass filter is a filter that passes high frequencies well, but attenuates

(Reduces the amplitude of) frequencies lower than the cutoff frequency.

3. What is a filter network?

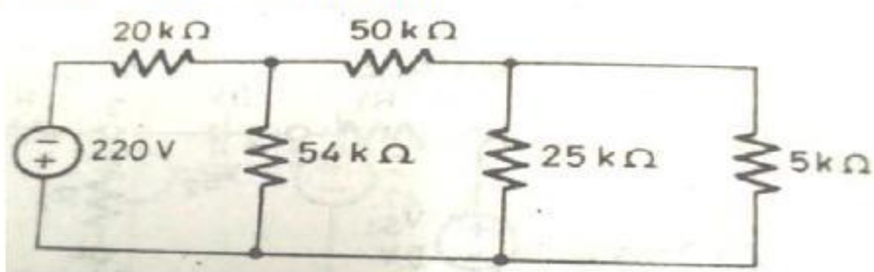
Filter is a reactive network which allows all the desired band of frequencies and attenuates the remaining all other band of frequencies.

5. Draw a circuit which acts as a low pass filter.
6. What is duality in electrical engineering?
7. Sketch the low pass filter with its quadrant of operation
8. What is simulation?
9. Compare derived and composite filters.
10. State any two applications of notch filter
11. A n-section filter comprises a series arm inductance of 20 mH & two shunt capacitors each of 0.16 micro farad. Calculate the attenuation at 15 KHz.
12. A second order band pass filter has a value of 10 for the ratio of center frequency to bandwidth. The filter can be realized with ___________.

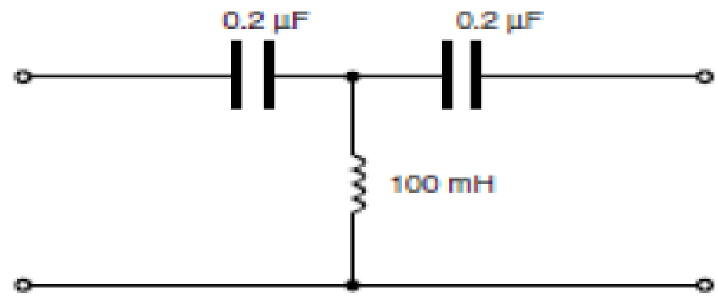
10 MARKS

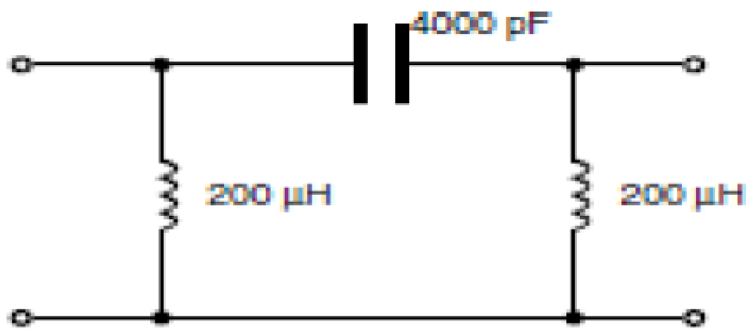
1. Discuss about low pass and band pass filters in detail.
2. Draw the circuit of high pass active filter and discuss its operation and characteristics.
3. Design a constant K-low pass filter having cut-off frequency 2.5 kHz and design resistance R_0 = 700 Ω. Also find the frequency at which this filter produces attenuation of 19.1dB. Find its characteristic impedances and phase constant at pass band and stop or attenuation band.
4.

Solve the circuit using PSpice program.



5.
Determine for each of the high-pass filter sections shown in figure:
(i) The cut-off frequency. (ii) The nominal impedance.





6
Elucidate the following terminologies with an example:
(i) Node. (ii) Linear graph. (iii) Tree. (iv) Twig. (v) Path.

.
7. Design a high pass filter with a cut-off frequency of 1 KHz with a terminated design impedance of 800 Ω.

8.
(a) What is the different between constant –k and m-derived filters?
(b) Design an m-derived π section high pass filter with a cut-off frequency of 10 KHz, R_k = 600 Ω and infinite attenuation frequency of 8 KHz.

9.
(a) Explain what is meant by constant k-filters. Classify them.
(b) Design an m-derived T section low pass filter having a design impedance of 600 Ω, cut-off frequency of 2,400 Hz and infinite attenuation at 2,500 Hz.