APLLIED PHYSICS

(20A56201T)

LECTURER NOTES

I – BTECH & I-SEM

Prepared by

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	R20 Regulations **JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY ANANTAPUR** **(Established by Govt. of A.P., ACT No.30 of 2008)** **ANANTAPUR – 515 002 (A.P) INDIA**

Course Code	**APPLIED PHYSICS**	**L**	**T**	**P**	**C**
20A56201T		**3**	**0**	**0**	**3**

Course Objectives

- To make a bridge between the physics in school and engineering courses
- To identify the importance of the optical phenomenon i.e. interference, diffraction and polarization related to its Engineering applications.
- To understand the mechanisms of emission of light, the use of lasers as light sources for low and high energy applications, study of propagation of light wave through optical fibres along with engineering applications
- To explain the significant concepts of dielectric and magnetic materials that leads to potential applications in the emerging micro devices.
- To enlighten the concepts of Quantum Mechanics and to provide fundamentals of de'Broglie waves, quantum mechanical wave equation and its applications, the importance of free electron theory and band theory of solids.
- Evolution of band theory to distinguish materials, basic concepts and transport phenomenon of charge carriers in semiconductors. To give an impetus on the subtle mechanism of superconductors using the concept of BCS theory and their fascinating applications.

Course outcomes (CO) : After completion of the course, the student can able to

CO-1: Study the different realms of physics and their applications in both scientific and technological systems through physical optics.

CO-2: Identify the wave properties of light and the interaction of energy with the matter & Asses the electromagnetic wave propagation and its power in different media.

CO-3: Understands the response of dielectric and magnetic materials to the applied electric and magnetic fields.

CO-4: Study the quantum mechanical picture of subatomic world along with the discrepancies between the classical estimates and laboratory observations of electron transportation phenomena by free electron theory and band theory.

CO-5: Elaborate the physical properties exhibited by materials through the understanding of properties of semiconductors and superconductors.

Unit-I Wave Optics

Interference- Principle of superposition , Interference of light , Conditions for sustained interference, Interference in thin films (Reflection Geometry), Colors in thin films , Newton's Rings, Determination of wavelength and refractive index.
Diffraction- Introduction, Fresnel and Fraunhofer diffraction, Fraunhofer diffraction due to single slit, double slit and N-slits (qualitative), Grating spectrum.
Polarization- Introduction, Types of polarization, Polarization by reflection, refraction and double Refraction, Nicol's Prism, Half wave and Quarter wave plates with applications.

Unit-II Lasers and Fiber optics

Lasers- Introduction, Characteristics of laser, Spontaneous and Stimulated emission of radiation Einstein's coefficients, Population inversion, Lasing action, Pumping mechanisms, Nd-YAG Laser, He-Ne laser, Applications of lasers.
Fiber optics- Introduction, Principle of optical fiber, Acceptance Angle, Numerical Aperture, Classification of optical fibers based on refractive index profile and modes, Propagation of electromagnetic wave through optical fibers, Propagation Losses (Qualitative), Applications.

Unit-III Dielectric and Magnetic Materials

Dielectric Materials- Introduction, Dielectric polarization, Dielectric polarizability, Susceptibility and Dielectric constant, Types of polarizations: Electronic, Ionic and Orientation polarization (Qualitative), Lorentz internal field, Clausius-Mossotti equation.
Magnetic Materials- Introduction, Magnetic dipole moment, Magnetization, Magnetic susceptibility and permeability, Origin of permanent magnetic moment, Classification of magnetic materials: Dia, para & Ferro, Domain concept of Ferromagnetism (Qualitative), Hysteresis, Soft and Hard magnetic materials.

Unit-IV Quantum Mechanics, Free Electron Theory and Band theory of Solids

Quantum Mechanics- Dual nature of matter, Schrodinger's time independent and dependent wave equation, Significance of wave function, Particle in a one-dimensional infinite potential well.
Free Electron Theory- Classical free electron theory (Merits and demerits only), Quantum free electron theory, Equation for electrical conductivity based on quantum free electron theory, Fermi-Dirac distribution, Density of states – Fermi

energy. **Band theory of Solids**- Bloch's Theorem (Qualitative), Kronig-Penney model (Qualitative), E vs K Diagram, Classification of crystalline solids, Effective mass of electron, m* vs K diagram – Concept of hole.
Unit-V Semiconductors and Superconductors
Semiconductors- Introduction, Intrinsic semiconductors, Density of charge carriers, Electrical Conductivity, Fermi level, Extrinsic semiconductors, Density of charge carriers, Dependence of Fermi energy on carrier concentration and temperature, Drift and diffusion currents, Einstein's Equation, Direct and indirect band gap semiconductors, Hall effect, Hall coefficient, Applications of Hall effect. **Superconductors**- Introduction, Properties of superconductors, Meissner effect Type I and Type II superconductors, BCS theory, Josephson effects (AC and DC), High Tc superconductors, Applications of superconductors.
Text Books
1. Engineering Physics – Dr. M.N. Avadhanulu & Dr. P.G. Kshirsagar, S. Chand and Company 2. Engineering Physics – B.K. Pandey and S. Chaturvedi, Cengage Learning.
Reference Books
1. Engineering Physics – Shatendra Sharma, Jyotsna Sharma, Pearson Education, 2018 2. Engineering Physics – K. Thyagarajan, McGraw Hill Publishers 3. Engineering Physics - Sanjay D. Jain, D. Sahasrambudhe and Girish, University Press 4. Semiconductor physics and devices- Basic principle – Donald A, Neamen, Mc Graw Hill

UNIT -I: WAVE OPTICS

Introduction

Have you ever observed that the beautiful colors in a soap bubbles, mica and film of oil floating on the surface of water when a sun light falls on them? Did you ever try to find out the reason? This is due to the phenomenon of interference in thin films by reflection of light**.** This was first explained by Thomas young on basis of Hugens' wave concept and principle of superposition.

Principle of Superposition of Waves

When two or more light waves traveling through a medium superimpose one another then the resultant displacement at any point is equal to the algebraic sum of the individual displacements at that point. This is called principle of superposition.

i.e., $$\vec{y} = \vec{y_1} \pm \vec{y_2} \pm \vec{y_3} \pm \ldots\ldots \qquad \longrightarrow(1.1)$$

where $\vec{y}$ is the resultant displacement and $\vec{y_1}, \vec{y_2}, \vec{y_3}$ ... are the displacements of individual waves.

$+$ ve sign is taken when displacements are in same direction.

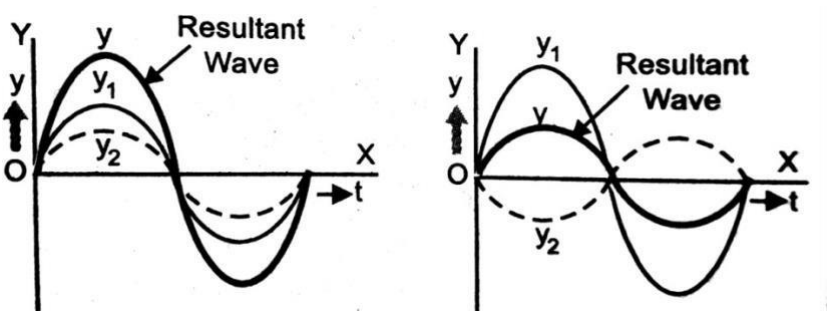
$-$ ve sign is taken when displacements are in opposite direction.

(i) If two light waves are in same phase, the resultant displacement is the sum of displacements of two waves. This is called constructive interference.

i.e., $$\vec{y} = \vec{y_1} + \vec{y_2} \qquad \longrightarrow(1.2)$$

(ii) If two light waves are out of phase, the resultant displacement is the difference of displacements of two waves. This is called destructive interference

i.e., $$\vec{y} = \vec{y_1} - \vec{y_2} \qquad \longrightarrow(1.3)$$



(a) (b)

Fig.1.1 Superposition of waves

Interference of light

When two or more light waves traveling through a medium superimpose one another then the resultant intensity at different points of the medium undergoes change from point to point. The change in the intensity of light in a medium from point to point is called interference of light.

Thus, the phenomenon of modification or change in uniform redistribution of light energy or intensity or amplitude due to the super position of two or more light waves is called interference.

Theory

According to Young's Double slit experiment,

The amplitude of resultant waves is

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\emptyset} \quad \longrightarrow(1.4)$$

The resultant intensity at point 'P' is given by the square of the amplitude

i.e., $I = A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\emptyset$ $\longrightarrow(1.5)$

Case1: Condition for constructive interference

- If the two waves are in phase with each other, then they undergo constructive interference producing maximum intensity of light called bright band or fringe.

The resultant intensity will be maximum when $\cos\emptyset = +1$

i.e., $\emptyset = 0, 2\pi, 4\pi, 6\pi \ldots\ldots$

From equation (1.1) $\emptyset = n(2\pi)$ where n= 0,1, 2, 3…

$$I_{max} = a_1^2 + a_2^2 + 2a_1a_2 \quad \longrightarrow(1.6)$$

$$\boxed{I_{max} = (a_1 + a_2)^2} \quad \longrightarrow(1.7)$$

Let $a_1 = a_2 = a$ (if two light waves are coherent

$$I_{max} = I_o = (4a)^2 \quad \longrightarrow(1.8)$$

Therefore, the intensity maximum when the phase difference is an integral multiple of 2□ or path difference is an integral multiple of □ because phase difference of 2□ corresponds to a path difference of □.

Path difference $\Delta = n\lambda$ or
Phase difference $\emptyset = 2n\pi$

$\longrightarrow(1.9)$

Case: 2 Condition for destructive interference

If the two waves are out of phase, then they undergo destructive interference producing zero intensity of light called dark band or fringe.

The resultant intensity will be maximum when $\cos\emptyset = -1$

i.e., $\emptyset = \pi, 3\pi, 5\pi \ldots\ldots$

$\emptyset = (2n-1)\pi$

Where n= 1, 2, 3…

From equation (1.1)

$$I_{min} = a_1^2 + a_2^2 - 2a_1a_2 \quad \longrightarrow (1.10)$$

$$\boxed{I_{min} = (a_1 - a_2)^2} \quad \longrightarrow (1.11)$$

Let $a_1 = a_2 = a$ (if two light waves are coherent)

$$I_{min} = 0 \quad \longrightarrow (1.12)$$

Therefore, the intensity minimum when the phase difference is an odd multiple of π or path difference is an odd multiple of $\lambda/2$ because phase difference of π corresponds to a path difference of $\lambda/2$.

$$\boxed{\begin{array}{l} \text{Path difference } \Delta = (2n-1)\frac{\lambda}{2} \text{ or} \\ \text{Phase difference } \emptyset = (2n-1)\pi \end{array}} \quad \longrightarrow (1.13)$$

Conditions for sustained interference

To observe a well defined interference pattern, the following conditions must be fulfilled.

a) *Conditions for Sustained Interference*
   i. To produce interference, we require two coherent light sources. i.e., the two sources must emit light of same frequency or wavelength, same amplitude and in the same phase or with a constant phase.
   ii. The two sources must be perfectly monochromatic, emitting light of a single wavelength or frequency.

b) *Conditions for good visibility or observation*
   i. The distance between the two sources (2d) must be small.
   ii. The distance between the two coherent sources and the screen (D) must be large.

c) *Conditions for good contrast*

i. The amplitudes of the two waves must be nearly the same or equal.

ii. The two sources must be narrow and parallel.

iii. The two sources must emit light in same direction

iv. To view interference fringes, the background must be dark Cas shown in Fig1.2.

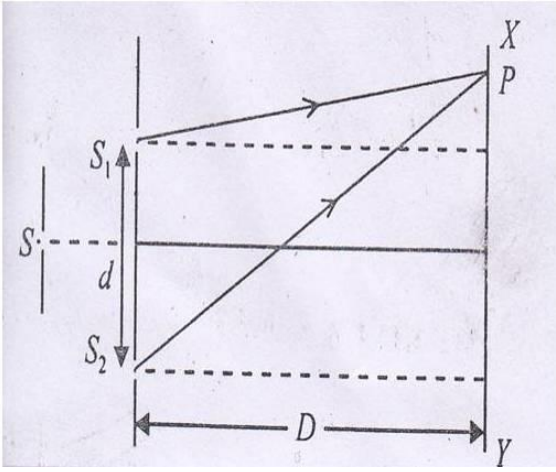


Fig.1.2 Schematic representation of Interference due to two slits

Where 'S' is a source of monochromatic light, S_1 &S_2 are two pin holes and XY is background.

Interference in thin films

If a light incident on the thin film, a small part of light is reflected from the top surface and the reaming portion is transmitted into the film. Again, a small part of the transmitted component light is reflected back into the film by the bottom surface and emerges through the top surface. A small portion of the light thus is reflected partially several times in succession within the film. These reflected light waves superimpose with each other, producing interference and forming interference patterns.

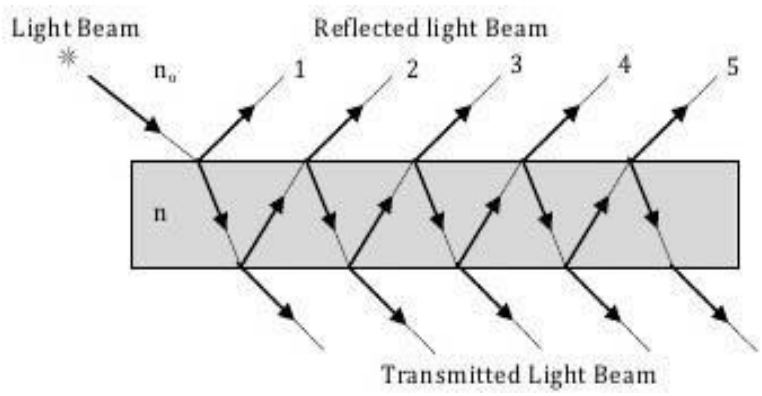


Fig.1.3 Interference in thin films by reflection

Interference in thin films by reflection of light

When light is incident on plane parallel thin film, some portion of light gets reflected from the top surface and the reaming portion is transmitted into the film. Again, some portion of the transmitted light is reflected back into the film by the bottom surface and emerges through the top surface. These reflected light waves superimpose with each other, producing interference and forming interference patterns. This is the principle of interference in thin films by reflection of light.

- Let us consider a thin transparent film of uniform thickness t with refractive index μ and is surrounded by air on both sides. Let the refractive index of the air be μ_{air} as shown in Fig 1.4.

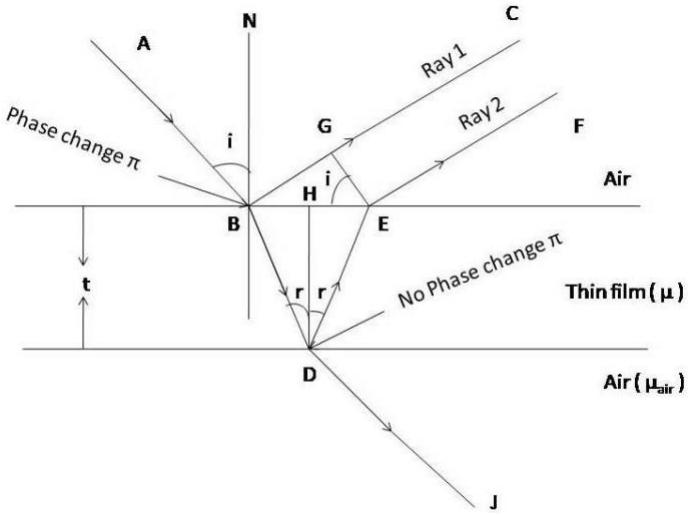


Fig1.4 Interference in thin films by reflection of light

- When a monochromatic light ray AB is incident on the top surface at an angle i, it is partly reflected along BC and partly refracted into the film along BD makes an angle ***'r'*** with the normal DH and meets the lower surface.
- At D, it is again partly reflected back into the film along DE and emerges along EF which is parallel to the ray BC. The major portion of light refracts into the outer medium along DJ.
- Since the two reflected light rays BC and EF are derived from the incident ray AB, so they act as coherent light rays. These two coherent light rays superimpose and produce interference patterns. Condition of interference depends on the optical path difference between the rays 1 and 2.
- To calculate let us draw normals from EG to BC and DH to BE and the rays GC and EF equal distances.

- The Optical path difference (OPD**)** between the rays (1) and (2) is

Δ = μ x geometrical path BDE in film- μ_{air} x geometrical path BG in air

$= \mu$ (BD+DE) - μ_{air} (BG) [Since $\Delta = \mu$ L and $\mu_{air} = 1$]

$= \mu$ (BD+DE) – BG $\longrightarrow$(1.14)

Step 1: Calculation of geometrical path BDE in film

Let us calculate path BD+ DE in film,

From fig, ΔBDH, $\cos r = \frac{DH}{BD}$

$$BD = \frac{t}{\cos r} \quad \longrightarrow(1.15)$$

Similarly,

From ΔEDH, $\cos r = \frac{DH}{DE}$

$$DE = \frac{t}{\cos r} \quad \longrightarrow(1.16)$$

$$\therefore BD+DE = \frac{t}{\cos r} + \frac{t}{\cos r}$$

$$= \frac{2t}{\cos r} \quad \longrightarrow(1.17)$$

Step 2: Calculation of geometrical path BG in film

To calculate BG air, first BE which is equal to (BH+HE) has to be obtained.

From ΔBDH, $\tan r = \frac{BH}{DH} = \frac{BH}{t}$

$$BH = t.\tan r \quad \longrightarrow(1.18)$$

Similarly,

FromΔEDH, $\tan r = \frac{HE}{DH} = \frac{HE}{t}$

$$HE = t.\tan r \quad \longrightarrow(1.19)$$

$\therefore$ BE=BH+HE = t.tan r +t. tan r

$$=2t \tan r \quad \longrightarrow(1.20)$$

From ΔBGE, $\sin i = \frac{BG}{BE}$

BG= BE Sin i

$$= 2t \tan r \sin i \quad \longrightarrow(1.21)$$

From Snell's law, $\mu = \frac{\sin i}{\sin r}$

$$\sin i = \mu \sin r \quad \longrightarrow(1.22)$$

From the equations (1.21) and (1.22)

BG= 2t tan r μ Sin r

$= 2\mu t \tan r. \sin r$

$=2\mu t \frac{\sin r}{\cos r}. \sin r$

$$=2\mu t \frac{\sin^2 r}{\cos r} \qquad \longrightarrow(1.23)$$

Substituting the equations (1.17) and (1.23) in equation (1.14); we have

$$\Delta = \frac{2\mu t}{\cos r} - 2\mu t \frac{\sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r}(1 - \sin^2 r)$$

$$= \frac{2\mu t}{\cos r} \cos^2 r$$

$$\Delta = 2\mu t \cos r \qquad \longrightarrow(1.24)$$

This is called Cosine law.

According to stoke principle, if a light wave traveling from rarer medium to denser medium under goes a phase of **π** or path change $\frac{\lambda}{2}$ (i.e., the wave loses half of wavelength) when it gets reflected at the boundary of a rarer to denser medium.

$$\boxed{\Delta = 2\mu t \cos r - \frac{\lambda}{2}} \qquad \longrightarrow(1.25)$$

Case 1. Condition for constructive interference or bright band

Bright band occurs when the path difference $\Delta = n\lambda$ $\longrightarrow(1.26)$

From the above equations (1.25) and (1.26); we have

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$2\mu t \cos r = n\lambda + \frac{\lambda}{2}$$

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2} \qquad \longrightarrow(1.27)$$

where n= 0,1, 2, 3…

This is the condition for constructive interference. The film appears bright under this condition.

Case2. Condition for destructive interference or dark band

Dark band occurs when the path difference $\Delta = (2n-1)\frac{\lambda}{2}$ $\longrightarrow(1.28)$

From the above equations (12) and (15); we have

$$2\mu t \cos r - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$2\mu t \cos r = (2n-1)\frac{\lambda}{2} + \frac{\lambda}{2}$$

$$2\mu t \cos r = n\lambda \quad \text{where } n=0, 1, 2, 3 \qquad \longrightarrow(1.29)$$

This is the condition for destructive interference. The film appears dark under this condition.

Interference in Non-uniform thin films - Newton's Rings

Interference in non-uniform thin films by reflection of light was first experimentally observed by Newton.

Definition

When a Plano-convex lens of large radius of curvature is placed convex surface on a plane glass plate, an air film is formed between the lower surface of the Plano-convex lens and the upper surface of the glass plate. The thickness of the air film is zero at point of contact and gradually increases from the point of contact outwards. If a monochromatic light is allowed to fall normally on this air film, then interference pattern is formed in the form of alternate concentric bright and dark circular rings are formed in the air film due to superposition of reflected light, known as Newton's rings because these rings were discovered by Newton.

Principle

Newton's rings are formed due to interference between the light rays reflected from the top and bottom surfaces of air film between the glass plate and the Plano-convex lens.

Formation of Newton's rings

- The formation of Newton's rings can be explained with help of Fig 1.5.
- When a light ray (AB) is incident on the systems, a part of the light is reflected by the curved surface of the lens (L) and a part is transmitted which is reflected back from the plane surface of the glass plate (P). Ray 1 undergoes no phase change but ray 2 under goes a phase of π or path change $\frac{\lambda}{2}$ (i.e., the wave loses half of wavelength) when it gets reflected at the boundary of a rare to denser medium.
- These reflected light rays superimpose with each other producing interference and forming interference and forming interference patterns in the form of bright dark circular rings.

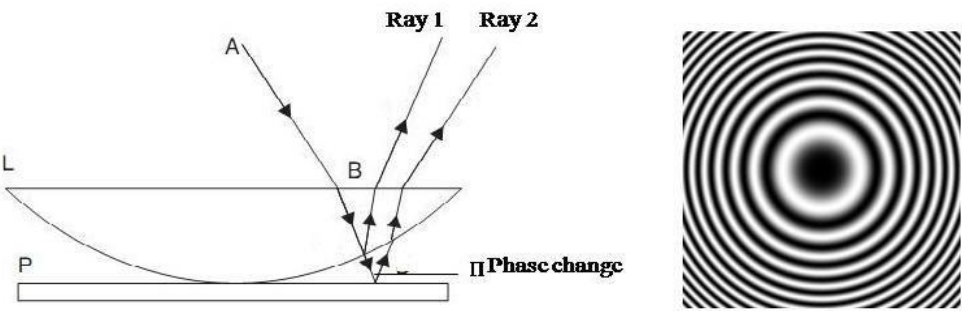


Fig.1.5 (a) Formation of Newton's rings (b) Newton's rings

- We know that, the path difference between two reflected rays in uniform thin film(plane parallel thin film) is

$$\Delta = 2\mu t \cos r - \frac{\lambda}{2} \qquad \longrightarrow (1.30)$$

- *Similarly for non-uniform thin film,*

  Let refractive index of air film $\mu = 1$ and r =0 for normal incidence.

Thus the path difference $\boxed{\Delta = 2t - \frac{\lambda}{2}}$ $\longrightarrow (1.31)$

This is the path difference between two reflected rays in non-uniform thin film.

Case 1. Dark central spot

At the point of contact 'O' of the lens and glass plate, the thickness of air film is approximately is zero.

i.e., $t \approx 0$

$\therefore$ The path difference $\Delta = 2t - \frac{\lambda}{2}$

$$= 2(0) - \frac{\lambda}{2}$$

$$\Delta = -\frac{\lambda}{2} \qquad \longrightarrow (1.32)$$

Thus, at point of contact two waves are out of phase and interference destructively. Hence dark spot is formed at centre.

Case 2.Condition for bright band

Bright band occurs when the path difference $\Delta = n\lambda$ $\longrightarrow (1.33)$

From equations (1.32) and (1.33), we have

$$2t - \frac{\lambda}{2} = n\lambda$$

$$2t = n\lambda + \frac{\lambda}{2}$$

$$2t = (2n+1)\frac{\lambda}{2} \qquad \longrightarrow (1.34)$$

where n= 0,1, 2, 3…

This is the condition for maxima. The film appears bright under this condition.

Case3. Condition for dark band

Dark band occurs when the path difference $\Delta = (2n-1)\frac{\lambda}{2}$ $\longrightarrow (6)$

From the above equations (1.32) and (1.35), we have

$$2t - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$2t = (2n-1)\frac{\lambda}{2} + \frac{\lambda}{2}$$

$$2t = n\lambda \qquad \rightarrow (1.35)$$

where n=0, 1, 2, 3…

This is the condition for minima. The film appears dark under this condition.

Experimental Arrangement

- The experimental arrangement for producing Newton rings is as shown in Fig 1.6
- Keep the convex surface of the Plano-convex lens over the plane glass plate and arrange glass plate G at an angle of 45^0 over the base set.
- Switch on the monochromatic light source 'S' (Sodium vapor lamp) and it is focus on the Double convex lens (L). This sends parallel beam of light. This beam of light falls on the glass plate B at 45^0.
- The glass plate 'G' reflects a part of light towards the air film enclosed by the Plano-convex lens and the plane glass plate.
- A part of the light is reflected by the curved surface of the Plano-convex lens (P) and a part is transmitted which is reflected back from the plane surface of the plane glass plate (E).
- These reflected light rays superimpose with each other producing interference and forming interference and forming interference patterns in the form of bright dark circular rings.
- These rings are seen with a microscope (M) focused on the air film

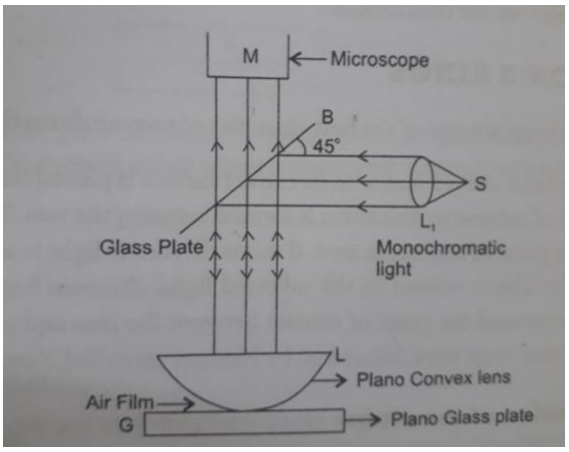


Fig.1.6 The experimental arrangement of Newton rings

Theory of Newton's rings

To find the diameters (radii) of dark and bight rings, let 'L' be a Plano convex lens placed on a glass plate P. The convex surface of the lens is the part of spherical surface with centre at C as shown in Fig.1.7.

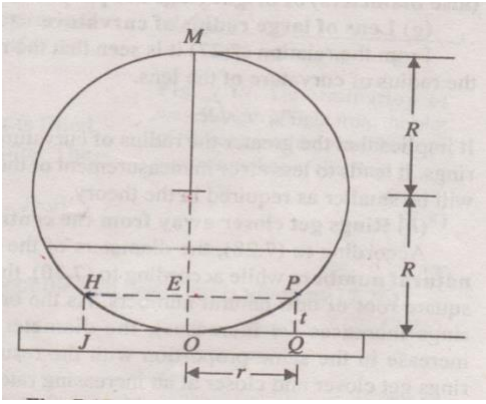


Fig.1.7 Theory of Newton's rings

Case1. Diameters (Radii) of the dark rings

Let 'R' be the radius of curvature of the lens.

Let a dark be located at the point Q is PQ= t.

The radius of the ring at Q is OQ= r.

By the theorem of intersecting chords,

$$EP \times HE = OE \times EM \qquad \rightarrow (1.36)$$

But EP=OQ=HE= r; OE=PQ= t

and EM= OM-OE =2R-t

From equation (1.36)

$$r \times r = t \times (2R-t)$$

$$\text{(Or)} \quad r^2 = 2Rt - t^2$$

As $2Rt \gg t^2$; t^2 can be neglected.

$$r^2 = 2Rt$$

Thus, the radius of the nth dark ring will be given by $r_n^2 = 2Rt$ $\rightarrow$ (1.37)

We know that, the condition of the dark ring is $2t = n\lambda$ $\rightarrow$ (1.38)

From the above equations (2) and (3)

$$r_n^2 = n\lambda R$$

$$r_n = \sqrt{n\lambda R} \qquad \rightarrow (1.39)$$

This is the condition for radii of the dark rings.

The diameter of the dark ring is therefore given by $D_n = 2r_n$

$$D_n = 2\sqrt{n\lambda R} \qquad \rightarrow (1.20)$$

∴ The radii (diameters) of the dark rings is directly proportional to (i) $\sqrt{n(\text{ natural nuber})}$

(ii) $\sqrt{\lambda}$

(iii) $\sqrt{R}$

Case2. Diameters (Radii) of the bright rings

Let us now suppose that a bright ring be located at the point Q.

Therefore, the radius of the nth bright ring will be given by $r_n^2 = 2Rt$ $\longrightarrow$ (1.21)

We know that, the condition of the bright band is $2t = (2n+1)\frac{\lambda}{2}$ $\longrightarrow$ (1.22)

From the above equations (1.21) and (1.22)

$$r_n^2 = (2n+1)\frac{\lambda}{2}R = (n+\frac{1}{2})\lambda R$$

$$r_n = \sqrt{(n+\frac{1}{2})\lambda R} \qquad \longrightarrow (1.23)$$

This is the condition for radii of the bright rings.

The diameter of the bright ring is therefore given by $D_n = 2r_n$

$$D_n = 2\sqrt{(n+\frac{1}{2})\lambda R} \qquad \longrightarrow (1.24)$$

∴ The radii (diameters) of the bright rings is directly proportional to (i) $\sqrt{n+\frac{1}{2}}$

(ii) $\sqrt{\lambda}$

(iii) $\sqrt{R}$

Applications

The theory of Newton's rings can be used

(i) To determine the wavelength of monochromatic light

(ii) To determine the radius of curvature of Plano-convex lens and

(iii) To determine the refractive index of a given liquid.

Determination of wavelength of monochromatic light source.

Let 'R' be the radius of curvature of a Plano-convex lens, λ be the wavelength of monochromatic light source.

Let D_m and D_n are the diameters of mth and n^{th} dark rings respectively.

Then,

$$D_m^2 = 4m\lambda R,$$

$$D_n^2 = 4n\lambda R,$$

and $D_n^2 - D_m^2 = 4n\lambda R - 4m\lambda R$

$$= 4(n-m)\lambda R$$

$$\lambda = \frac{D_n^2 - D_m^2}{4(n-m)R}$$

A graph is drawn with the number of rings on the - axis and the square of the diameter of the ring (D^2) on the y-axis. The graph is straight line passing through the origin as shown in Fig.1.8. Form the graph the values of D_m^2 and D_n^2 corresponding to n^{th} and m^{th} rings are found. From the graph, the slope is calculated.

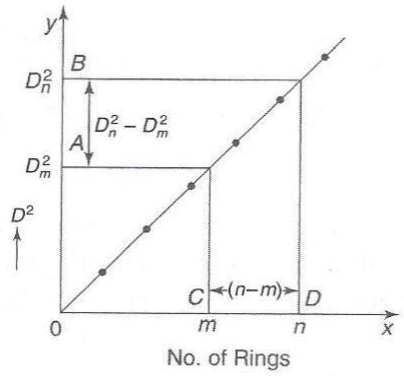


Fig.1.8 A graph between (D^2) and number of rings

From graph, the slope

$$\frac{D_n^2 - D_m^2}{(n-m)} = \frac{AB}{CD} \qquad \longrightarrow (1.26)$$

The radius of curvature 'R' of the Plano-convex lens is found by Boy's method or spherometer. Substituting the values of R and $D_n^2 - D_m^2$ in the above formula (1.26) λ can be calculated.

Determination of refractive index of a given liquid

To find the refractive index of a given liquid, the Plano convex lens and glass plate set up is placed in a small container 'C'. The transparent liquid of refractive index 'μ'is introduced between the lens L and the glass Plate 'G' as shown as Fig 1.9. Then a film of liquid formed between the lens and glass plate. The diameters of mth and n^{th} dark rings are determined with help of travelling microscope.

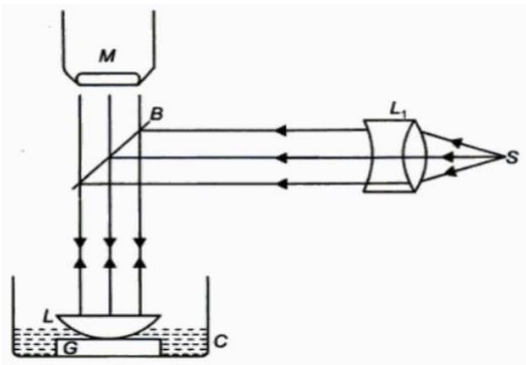


Fig.1.9 Determination of refractive index of a given liquid

We know that,

The diameters of mth and n^{th} dark rings in the air film are given by

$$D^2_m = 4m\lambda R,$$

$$D^2_n = 4n\lambda R,$$

$$\text{and } D^2_n - D^2_m = 4n\lambda R - 4m\lambda R$$

$$= 4(n-m)\lambda R \qquad \rightarrow (1.27)$$

With liquid film, the diameters of mth and n^{th} dark rings are determined;

$$D^2_{m\,I} = \frac{4m\lambda R}{\mu}$$

$$D^2_{n\,I} = \frac{4n\lambda R}{\mu}$$

$$\text{and } D^2_{n\,I} - D^2_{m\,I} = \frac{4n\lambda R}{\mu} - \frac{4m\lambda R}{\mu}$$

$$= \frac{4\lambda R(n-m)}{\mu} \qquad \rightarrow (1.28)$$

From the above equations (1.27) and (1.28), $D^2_{n\,I} - D^2_{m\,I} = \frac{D^2_n - D^2_m}{\mu}$

$\mu = \frac{D^2_n - D^2_m}{D^2_{n\,I} - D^2_{m\,I}}$. Using the above formula μ can be calculated.

Engineering applications of Interference

The phenomenon of interference can be used in various engineering applications; some of the applications are given below.

i. It is used in anti-reflecting coatings.
ii. It is used to determination of thickness of a thin film coating.
iii. It is used in testing of flatness of surfaces.
iv. It is used in interference filters to transmit a very narrow range of wavelengths.

DIFFRACTION

Introduction

Diffraction is one of the natural phenomena. The effect of diffraction is usually seen in everyday life. For example, the iridescent colors of peacock feathers, butterfly wings and some other insects, rainbowlike diffraction is produced by CD OR DVD.

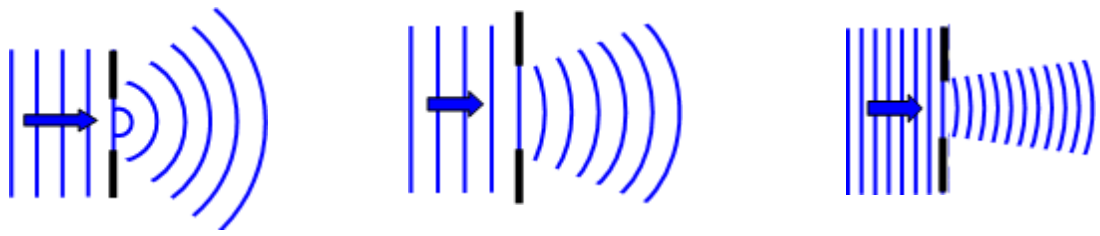
The English word diffraction was coined by the *diffractus* which means to spread out or to break out. This phenomenon was first discovered by Francesco Grimaldi.

When light waves encounter an obstacle they bend at the corners or edges of the obstacle and spreading into the geometrical shadow of an obstacle. This phenomenon is known as diffraction.

(or)

The phenomenon of redistribution of light intensity due to the superposition of secondary wavelets from the same wavefront is called diffraction.

The effect of diffraction depends on the size of the obstacle (d) and the wavelength (λ) of light wave as illustrated in Figure 1.10. Light waves are very small in wavelength, i.e., from 4×10^{-7} m to 7×10^{-7} m. If the size of opening or obstacle is near to this limit, only then we can observe the phenomenon of diffraction.



(a) $d \approx \lambda$ (b) $d > \lambda$ (c) $\lambda \ll d$

Figure 1.10 Illustration of diffraction phenomenon

From Fig.1.12 it is observed that

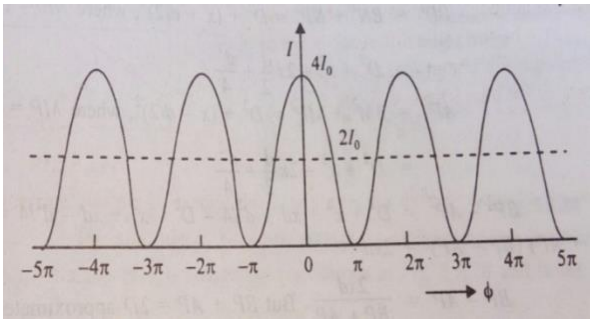
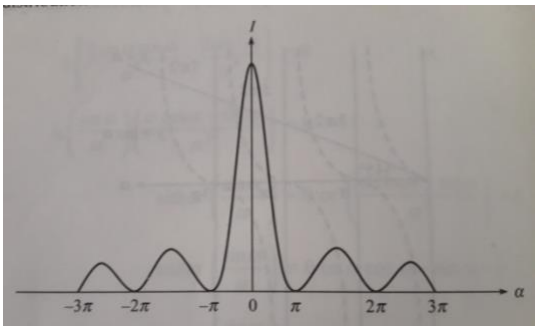
i. The effect of diffraction is more noticeable if the size of the obstacle is equal to the wavelength of light ($i.e. d \approx \lambda$).
ii. The effect of diffraction is not noticeable if the size of the obstacle is greater than the wavelength of light ($i.e. d > \lambda$).
iii. The effect of diffraction is less noticeable if the size of the obstacle is smaller than the wavelength of light ($i.e. \lambda \ll d$).

Conditions for diffraction

To observe diffraction

i. Light must be monochromatic and
ii. The wavelength of light must be comparable to the obstacle. ($\lambda \approx d$)

Differences between interference and diffraction

S.No	Interference	Diffraction
1	It is arises due to the superposition of waves from the two coherent sources.	It is arises due to the superposition of secondary wavelets from the different parts of the same wavefront.
2	It is formed due to the uniform distribution of light intensity.	It is formed due to the non uniform distribution of light intensity.
3	In interference pattern, all bright fringes are of the same intensity	In diffraction pattern, all bright fringes are not of the same intensity
4	The intensity of all dark fringes is zero.	The intensity of all dark fringes is no zero.
5	In interference pattern, the width of fringes is equal.	In diffraction pattern, the width of fringes is not equal.
6	The intensity distribution graph of interference as shown in Fig.1.13. 	The intensity distribution graph of diffraction as shown in Fig1.14. 

Types of Diffraction

There are two types of diffraction.

a) Fresnel diffraction and
b) Fraunhofer diffraction.

(a) Fresnel diffraction

- In this class of diffraction, the source of light and the screen are finite from the diffraction slit (or) aperture.
- No double convex lenses (or) mirrors are used to observe diffraction effect.
- The diffraction pattern is the image of the slit.
- The centre of diffraction pattern is either bright (or) dark.

- The incident wave front is spherical (or) cylindrical.
- It is very easy to observe experimentally and complex mathematically.

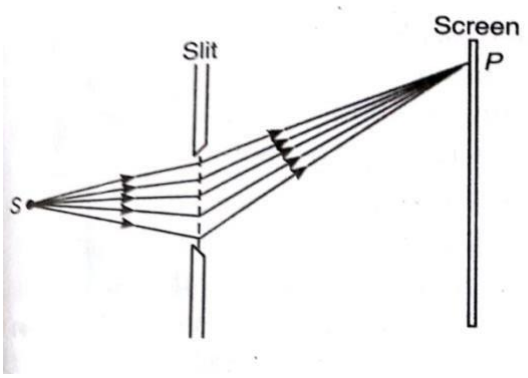


Fig.1.11 Fresnel diffraction

(b) Fraunhofer diffraction

- In this class of diffraction, the source of light and the screen are infinite distance from the diffraction slit (or) aperture.
- Two double convex lenses (or) mirrors are used to observe diffraction effect.
- The incident wave front is plane.
- The diffraction pattern is the image of source of itself.
- The centre of the diffraction pattern is bright.
- It is very simple and easy to observe both experimentally and mathematically.

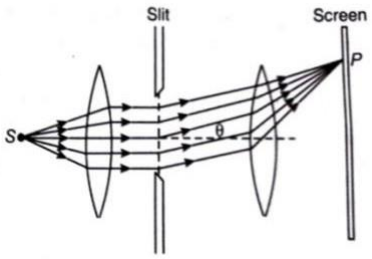


Fig.1.12 Fraunhofer diffraction

Fraunhofer diffraction due to single slit

The arrangement of fraunhofer diffraction due to single slit as shown in Fig 1.13.

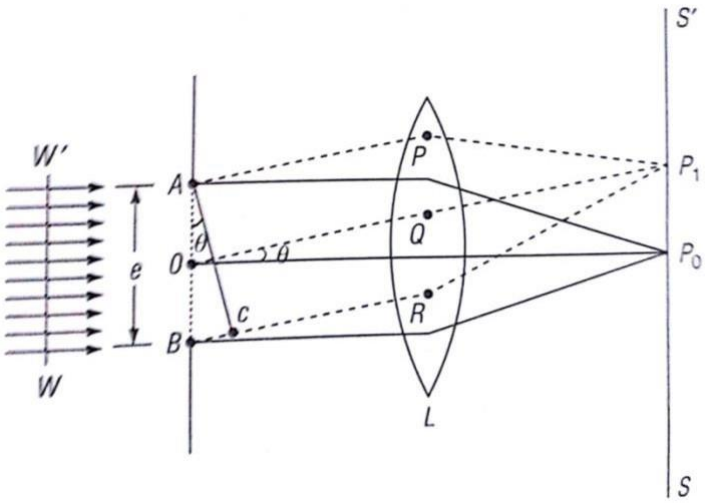


Fig 1.13 Fraunhofer diffraction due to single slit

- In Fig S is a point source of monochromatic light with wave length λ and AB is the single slit of width "e"
- Let WW' be the plan wave front and is incident on the single slit AB.
- According to Hygen's principle, each point on wave front is a source of fresh disturbance called secondary wavelet that spread out to right angles in all directions.

Without diffraction ($\theta = 0$)

- Let us consider the wave front WW' travelling along OP_0 through the slit AB without diffraction and focus at P_0 by using lens L_2. Since all these wave fronts have same phase and no path difference between them. Therefore, they undergo constructive interference and producing maximum intensity called central maximum or principal maximum.

With diffraction ($\theta \neq 0$)

- Let us consider the plain wave front WW'(secondary wave lets) travelling along OP_1 and diffract at an angle θ and focused at P_1 . The intensity at P_1 either maximum or minimum depending up on the path difference between the two wavelets (A and B). To calculate path difference let us draw a normal AC to BR as shown in fig.

From fig, ΔABC,

$$\text{Sin } \theta = \frac{BC}{AB}$$

BC=AB Sinθ

∴The Path difference BC=e Sinθ ⟶(1.29)

And corresponding phase difference = $\frac{2\pi}{\lambda} \times$ path difference

$$= \frac{2\pi}{\lambda} \times esin\theta \quad \longrightarrow(1.30)$$

Let us consider the width of slit AB is divided into n parts and amplitude of each part is "a." Therefore, the phase difference between any two successive parts is

$$\frac{1}{n} \times \text{phase difference} = \frac{1}{n} \times \text{path difference}$$

$$\frac{1}{n} \times \text{phase difference} = \frac{1}{n}\left[\frac{2\pi}{\lambda} \times e\sin\theta\right] = d \qquad \longrightarrow (1.31)$$

By using the method of vector addition of amplitudes,

The resultant amplitude is given by $R = \dfrac{a \operatorname{Sin}\frac{nd}{2}}{\operatorname{Sin}\frac{d}{2}}$ $\longrightarrow$ (1.32)

Substituting equation (1.31) in equation (1.32)

$$R = \frac{a \operatorname{Sin} n \times \frac{1}{n}\left(\frac{\frac{2\pi}{\lambda} \times e\operatorname{Sin}\theta}{2}\right)}{\operatorname{Sin} \times \frac{1}{n}\left(\frac{\frac{2\pi}{\lambda} \times e\operatorname{Sin}\theta}{2}\right)}$$

$$= \frac{a \operatorname{Sin}\left(\frac{\pi e \operatorname{Sin}\theta}{\lambda}\right)}{\operatorname{Sin}\left(\frac{\pi e \operatorname{Sin}\theta}{n\lambda}\right)}$$

$$R = \frac{a \operatorname{Sin}\alpha}{\operatorname{Sin}\frac{\alpha}{n}}$$

where $\alpha = \frac{\pi e \sin\theta}{\lambda}$ $\longrightarrow (1.33)$

Since $\operatorname{Sin}\frac{\alpha}{n}$ is very small , $\operatorname{Sin}\frac{\alpha}{n} \simeq \frac{\alpha}{n}$

$$= \frac{a \operatorname{Sin}\alpha}{\frac{\alpha}{n}}$$

$$= \frac{na \operatorname{Sin}\alpha}{\alpha} \qquad [\because na = A]$$

$$\boxed{R = A\left(\frac{\sin\alpha}{\alpha}\right)} \qquad \longrightarrow (1.34)$$

This equation represents the resultant amplitude of the wave fronts at P_1.

And intensity $I = R^2$

$$I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$\boxed{I = A^2 \left(\frac{\sin\alpha}{\alpha}\right)^2} \quad \longrightarrow(1.35)$$

Case 1.Principle maximum or Central maximum at P_0

The resultant amplitude (R) can be expressed in ascending power of α,

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \dots\dots\dots.\right]$$

$$= \frac{A}{\alpha} \times \alpha\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \dots\dots\dots.\right] \quad \longrightarrow(1.36)$$

Intensity at P_0 will be maximum when the values of R is maximum.

For maximum value of R the negative terms must be vanish in equation (1.36)

i.e., $\alpha = 0$ [From the equation (1.33)]

$$\frac{\pi e \, \mathrm{Sin}\theta}{\lambda} = 0$$

$$\pi e \, \mathrm{Sin}\theta = 0$$

$$\mathrm{Sin}\theta = 0$$

$$\boxed{\theta = 0}$$

Thus we can say that, the condition *θ=0* means that the wave fronts passing through the slit AB without diffraction and focus at P_0.Since all these wave fronts have same phase and no path difference between them. Therefore, they undergo constructive interference and producing maximum intensity called central maximum or principal maximum (i.e., $I_{max} = R^2 = A^2 = I_0$).

Case.2 Minimum intensity positions

Intensity I will be minimum when Sinα = 0.

i.e., $\alpha = \pm m\pi$

Where m = 1, 2, 3 …

$$\frac{\pi e \sin\theta}{\lambda} = \pm m\pi$$

$$\boxed{e\sin\theta = \pm m\lambda} \quad \longrightarrow(1.37)$$

This is the condition for minimum intensity. Thus, we obtain minimum intensity positions either side of the principle maximum.

Case.3 Maximum intensity positions (or) Secondary maximum

In addition to principle maximum, there exist week secondary maximum between equally spaced minima. These secondary maximum intensity positions can be obtained by differentiating the expression of intensity with respect to "α" and equating to zero.

$$I = A^2 \left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2 \left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

$$= A^2.\frac{d}{d\alpha}\left(\frac{\sin\alpha}{\alpha}\right)^2 = 0 \qquad \left[\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\right]$$

$$= A^2.\frac{2\sin\alpha}{\alpha}.\frac{d}{d\alpha}\left(\frac{\sin\alpha}{\alpha}\right) = 0$$

$$= A^2.\frac{2\sin\alpha}{\alpha} \times \frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0 \qquad \longrightarrow (1.38)$$

In the above equation (1.38) either Sinα=0 (or) α Cosα−sinα=0

We know that, Sinα = 0 gives the minimum intensity positions. Thus αcosα−sinα=0 gives the maximum intensity positions.

i. e, α cosα−sinα=0

α cosα= sinα

$$\boxed{\alpha = \tan\alpha} \qquad \longrightarrow (1.39)$$

The value of α satisfying the above equations are obtained graphically by plotting the curves y=α and y= tanα on the same graph as shown in Fig 1.14.

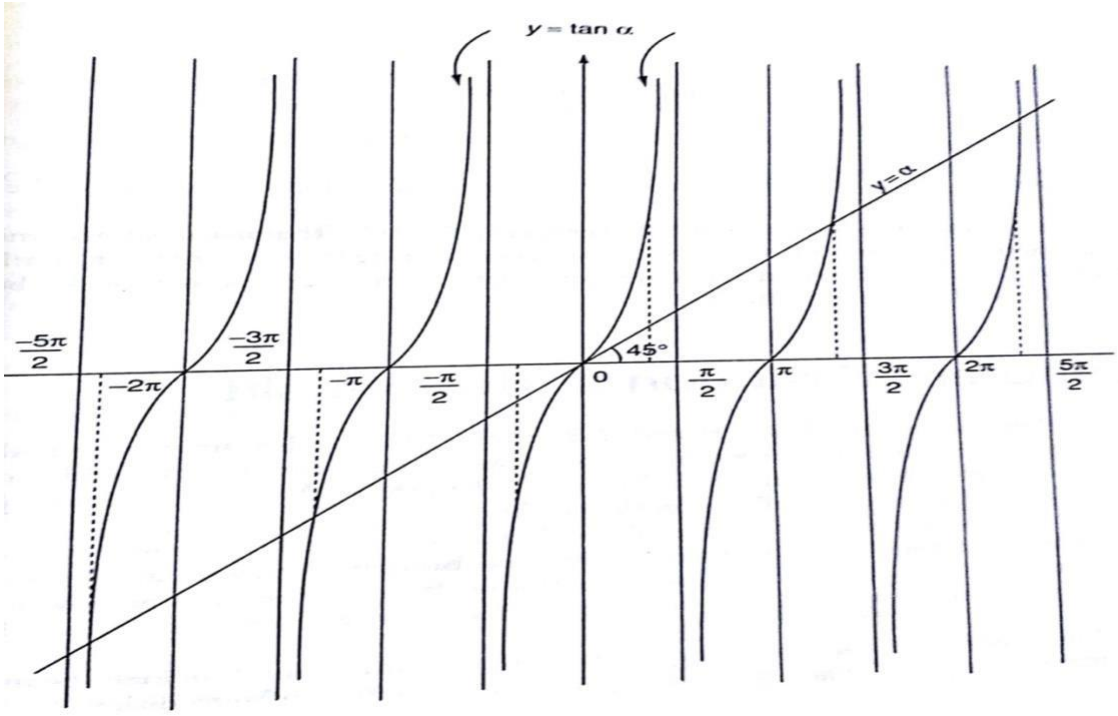


Fig 1.14 The plots of y=α and y= tanα

The points of intersections of the two curves gives the value of α which satisfying the above equation(1.39). The points of intersections are α= 0, $\pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$…….

The first value of α gives the principle maximum and remaining values of α give the secondary maxima.

The values are α= $\pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$…….

$$\alpha = \pm(2m+1)\frac{\pi}{2} \qquad \rightarrow(1.40)$$

where m = 1,2,3,4……..

∴ From the equations (5) & (12)

$$\frac{\pi e sin\theta}{\lambda} = \pm(2m+1)\frac{\pi}{2}$$

$$\boxed{e\ \sin\theta = = \pm(2m+1)\frac{\lambda}{2}} \qquad \rightarrow(1.41)$$

This is the condition for maxima.

(i) For the principle maximum α = 0

$$\therefore \text{ The intensity I} = R^2 = A^2 = \left(\frac{sin\alpha}{\alpha}\right)^2$$

$$I = R^2 = A^2 = I_0$$

(ii) For the secondary maxima $\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$.....

The intensity of first secondary maxima, $\alpha = \pm\frac{3\pi}{2}$

$$\therefore \text{ Intensity } (I_1) = A^2 \left(\frac{\sin\left(\frac{3\pi}{2}\right)}{\frac{3\pi}{2}}\right)^2$$

$$= \frac{4}{9}\pi^2 . A^2$$

$$= \frac{A^2}{22}$$

$$I_1 = \frac{I_0}{22}$$

$$\frac{I_1}{I_0} = \frac{1}{22} = 4.5\%$$

∴ The intensity of first secondary maxima is about $\frac{1}{22}$ of intensity of the principle maximum.

$$\boxed{i.e., I_1 \text{ is } 4.5\% \text{ of } I_0}$$

The intensity of second secondary maxima $\alpha = \pm\frac{5\pi}{2}$

$$I_2 = A^2 \left(\frac{sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right)^2$$

$$= A^2 \left(\frac{4}{25\pi^2}\right)$$

$$= \frac{A^2}{62} = \frac{I_0}{62}$$

$$\frac{I_2}{I_0} = \frac{1}{62} = 1.61\%$$

∴ The intensity of second secondary maxima is about $\frac{1}{62}$ of intensity of princiole maxima.

$$\boxed{i.e., I_2 \text{ is } 1.61\% \text{ of } I_0}$$

∴ In the same way the intensity of secondary maxima goes on decreasing very rapidly.

Intensity distribution graph

A graph is plotted between intensity of light (I) and α is shown in Fig1.15.

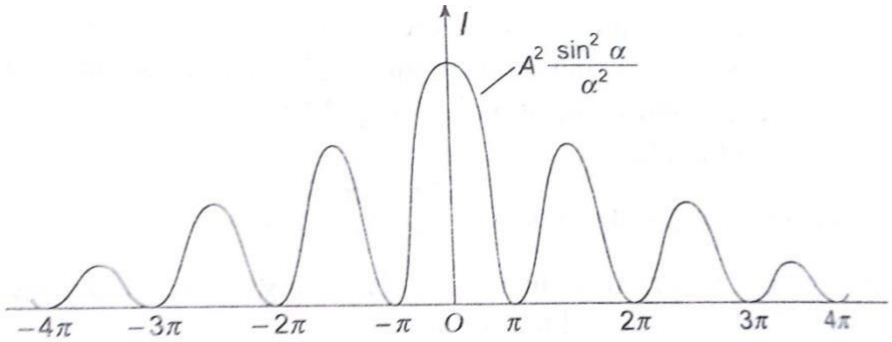


Fig1.15 The intensity of diffraction pattern due to single slit

From the graph the following conclusions can be observed,

- At α=0, the intensity is very high called principle maximum.
- At $\alpha = \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \pm \frac{7\pi}{2}$ ....., the intensity is maximum called secondary maxima.
- At $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi$..... ,the intensity is minimum.
- The most of the light is concentrated in the principle maximum & the intensity of secondary maximum decreasing on either sides of principle maximum as shown in Fig 1.15.

Fraunhofer diffraction due to double slit

The arrangement of fraunhofer diffraction due to double slit as shown in Fig 1.16.

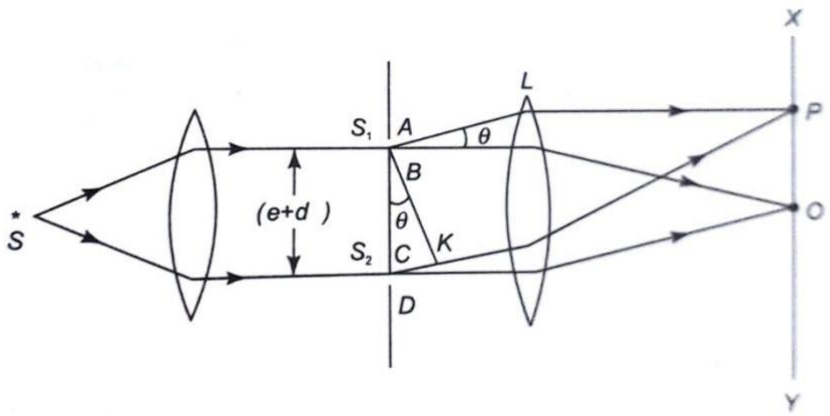


Fig 1.16 Fraunhofer diffraction due to double slit

- In Figure1.16, S is a point source of monochromatic light with wavelength λ and s_1 and s_2 are the two slits of same widthe "e" and separated by a distance 'd'.
- Let WW' be the plan wave front and is incident on the two slits s_1 and s_2.
- According to Hygen's principle, each point on wave front is a source of fresh disturbance called secondary wavelet that spread out to right angles in all directions.

Without diffraction ($\theta = 0$)

- Let us consider the plane wave front ww' travelling along OP_0, through the slits s_1 and s_2 without diffraction ($\theta = 0^0$) and focused at P_0 by using lens L_2.Since all these wave fronts have same phase and no path difference between them .So they under go constructive interference and producing maximum intensity at P_0 called central maximum or principle maximum.

With diffraction ($\theta \neq 0$)

- Let us consider the plain wave front ww' (secondary wavelets) travelling along OP_1 through the slits s_1 and s_2 and diffract at an angle θ with respect to the normal and focused at P_1.The intensity at P_1 is the combination of both interference and diffraction.
- *Interference:* It is due to the super position of secondary diffracted wavelets from the two slits s_1 and s_2.
- *Diffraction:* It is due to the single slit, the intensity at P_1 either maximum or minimum depending up on the path difference between secondary wavelets s_1 and s_2.

From $\Delta\ s_1 s_2 k$,

$$\sin\theta = \frac{s_2 k}{s_1 s_2}$$

$$s_1 s_2 sin\theta = s_2 k \qquad \longrightarrow(1.42)$$

∴ The path difference $s_2 k = (e + d) in\theta$ and corresponding phase difference

$$\delta = \frac{2\pi}{\lambda}(e + d) in\theta \qquad \longrightarrow(1.43)$$

To find out resultant amplitude at P_1 we use vector addition method (as shown in figure) in which two sides of a triangle are represented by the amplitudes of s_1 and s_2 and third side gives the resultant amplitude.

According to vector addition method,

From the figure,

$$(OH)^2 = (OG)^2 + (GH)^2 + 2(OG)(GH)Cos\,\delta$$

$$R^2 = A^2\left(\frac{in\,\alpha}{\alpha}\right)^2 + A^2\left(\frac{Sin\,\alpha}{\alpha}\right)^2 + 2.A\left(\frac{sin\alpha}{\alpha}\right).A\left(\frac{sin\alpha}{\alpha}\right)Cos\,\delta$$

$$= 2A^2\left(\frac{Sin\,\alpha}{\alpha}\right)^2 + 2A^2\left(\frac{Sin\,\alpha}{\alpha}\right)^2 Cos\,\delta$$

$$= 2A^2\left(\frac{Sin\,\alpha}{\alpha}\right)^2[1 + Cos\,\delta\,]$$

$$= 2A^2\left(\frac{Sin\,\alpha}{\alpha}\right)^2 2Cos^2\frac{\delta}{2}$$

$$= 4A^2\left(\frac{Sin\,\alpha}{\alpha}\right)^2 Cos^2\frac{\delta}{2}$$

$$= 4A^2\left(\frac{Sin\,\alpha}{\alpha}\right)^2 Cos^2\left(\frac{2(e+d)sin\theta}{2}\right)$$

$$= 4A^2\left(\frac{SIN\alpha}{\alpha}\right)^2 Cos^2\frac{\pi}{\lambda}(e+d)in\theta$$

$$\boxed{R^2 = 4A^2\left(\frac{SIN\alpha}{\alpha}\right)^2 os^2\beta} \qquad \longrightarrow(1.44)$$

Where $\beta = \frac{\pi}{\lambda}(e+d)in\theta$ $\longrightarrow(1.45)$

The above equation (1.44) represents the resultant amplitude of the wave fronts S_1 and S_2 at P_1.

And intensity $I = R^2$

$$\boxed{I = R^2 = 4A^2\left(\frac{SIN\alpha}{\alpha}\right)^2 os^2\beta} \qquad \longrightarrow(1.46)$$

From the above equation (1.46), we can say that the resultant intensity (I) at P_1 is the product of two factors.

(i) $A^2\left(\frac{SIN\alpha}{\alpha}\right)^2$ which represents the diffraction pattern due to a single slit.

(ii) $Cos^2\beta$ which represents the interference pattern due to the super position of secondary diffracted wavelets from two slits s_1 and s_2 .

Therefore, the resultant intensity is the combination of both diffraction and interference effects.

Diffraction effect

In the above equation (1.46), $A^2 \left(\frac{SIN\alpha}{\alpha}\right)^2$ gives the principle maximum at the centre of diffraction pattern with alternative minima and secondary maxima positions on the either side of it (Fig.1.17).

Case (i) The principle maximum occurs at $\alpha = 0$

$$\boxed{\text{i.e., } I = R^2 = A^2 = I_0}$$

Case (ii) The minimum intensity positions occurs at

$$\alpha = \pm m \text{ , where m=1, 2, 3,4…….}$$

$$\frac{\pi e sin\theta}{\lambda} = \pm m\pi$$

$$\boxed{\text{e sinθ==} \pm m\lambda} \qquad \longrightarrow(1.47)$$

Case (iii) The secondary maxima positions occurs at

$$\alpha = \pm(2m+1)\frac{\pi}{2} \text{ , where m=1,2,3,4…….}$$

$$\frac{\pi e sin\theta}{\lambda} = \pm(2m+1)\frac{\pi}{2}$$

$$\boxed{\text{e sinθ} = \pm(2m+1)\frac{\lambda}{2}} \qquad \longrightarrow(1.48)$$

Interference effect

In the above equation (1.46), $Cos^2\beta$ gives the alternative maximum and minimum intensity positions with equal magnitude of intensity.

Case (i) The Minimum intensity positions will occur at

$$Cos^2\beta = 0 \text{ When } \beta = \pm\frac{\pi}{2}, \pm\frac{3}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2} \ldots\ldots\ldots$$

$$\therefore \beta = \pm(2m+1)\frac{\pi}{2} \qquad \longrightarrow(1.49)$$

where m= 0,1,2,3,4……..

But $\beta = \frac{\pi}{\lambda}(e+d)in\theta$ $\longrightarrow(1.50)$

From the equations (1.49) & (1.50)

$$\frac{\pi}{\lambda}(e+d)in\theta = \pm(2m+1)\frac{\pi}{2}$$

$$(e+d)\sin\theta = \pm (2m+1)\,\lambda/2 \qquad \longrightarrow(1.51)$$

Case (ii) The maximum intensity positions will occur at

$$Cos^2\beta = 1 \text{ When } \beta = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \ldots\ldots\ldots\ldots\ldots\ldots \pm m\pi$$

$$\therefore \beta = \pm m\pi \qquad \longrightarrow(1.52)$$

$$\text{But } \beta = \frac{\pi}{\lambda}(e+d)in\theta \qquad \longrightarrow(1.53)$$

From the equations (1.52)&(1.53)

$$\frac{\pi(e+d)sin\theta}{\lambda} = \pm m\pi$$

$$\boxed{(e+d)sin\theta = \pm m\lambda} \qquad \longrightarrow(1.54)$$

Intensity distribution graph

Fig 1.21 shows the resultant intensity distribution due to the combination of both diffraction and interference effects. From the graph, it is observed that the resultant minima are not equal to zero; still they have some minimum intensity due to interference effect.

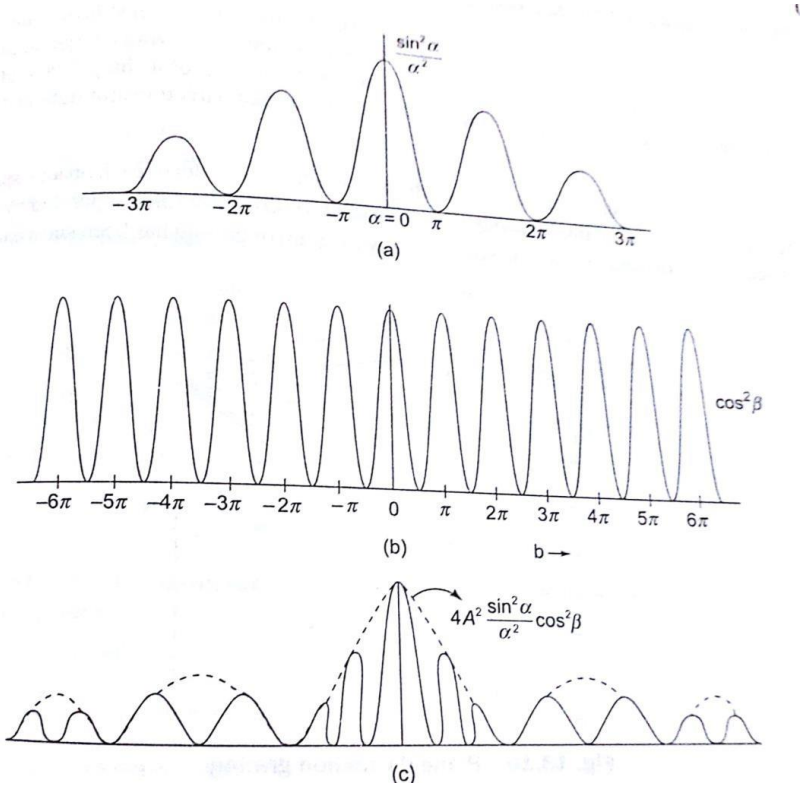


Fig.1.17 Intensity distribution due to diffraction double slit

Diffraction Grating

A diffraction grating is an optical device consisting of a large number of equidistant narrow parallel, rectangular slits of equal width and separated by opaque portions. This was first constructed by the German physicist Joseph von Fraunhofer in 1821.

Construction

- A diffraction grating is prepared by ruling a large number of equidistant narrow, parallel lines on an optically plane glass plate with a fine diamond point.
- The ruled lines are opaque to light called opaque portions while the space between any two successive ruled lines is transparent to light called transparent portions and act as slits.
- The distance between any two successive slits (or) ruled lines is called grating element as shown in Fig.1.18.

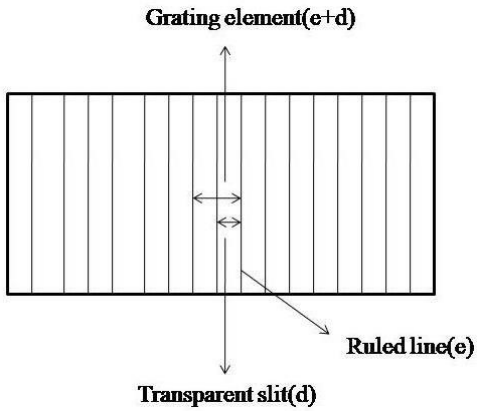


Fig.1.18. Construction of diffraction grating

- The Commercial gratings are made up a thin layer of colloidal solution (ex: Solution of cellulose Acetate) is poured on the ruled surface and allowed to dry to form a thin film.
- The colloidal thin film is peeled carefully from the ruled surface. The film retains impression of the ruling of ruled surface.
- The ruled lines acts as opaque portions where as the space between them act as transparent portions which transmit incident light. The film is mounted between two glass plates called grating.
- When the light is incident on the grating surface then light is transmitted through transmission portions or slits and obstructed by the opaque portions or ruled lines. Such

a grating is known as transmission grating.

Theory

Let' e' be the width of each ruled line (or) opaque portion and 'd' be the width of transparent portion (or) slit then (e + d) represents the grating element. The relation between grating element and the angles of diffracted beams of light is known as grating equation. It is given by

$$(e+d)sin\theta = n\lambda, \quad \text{Where n = 0, 1, 2, 3....} \qquad \longrightarrow(1.55)$$

This expression is known as grating equation.

$$sin\theta = (\frac{1}{e+d})\, n\lambda$$

$$sin\theta = \mathrm{N}\, n\lambda$$

Where $\frac{1}{e+d}$ = N is the number of grating elements or lines per unit width of the grating.

$$\mathrm{N}\,(e+d) = 1\text{inch} = 2.54 \text{ cm}$$

$$e+d = \frac{2.54}{\mathrm{N}} \text{ cm} \qquad \longrightarrow(1.56)$$

Grating Spectrum

The diffraction pattern formed with a grating is known as grating spectrum.

The positions of principle maxima in grating are given by

$$(e+d)sin\theta = n \text{ , Where n = 0, 1, 2, 3….}$$

This expression is known as grating equation.

For a particular wavelength , the diffraction angle is different for different for principal maxima of different orders.

The first order maxima is obtained for n=1, then $(e+d)in\theta_1 = \lambda$

The second order maxima is obtained for n=2, then $(e+d)in\theta_2 = 2\lambda$

The different orders of principal maxima are obtained on both sides of zero order maxima as shown Fig.1.19.

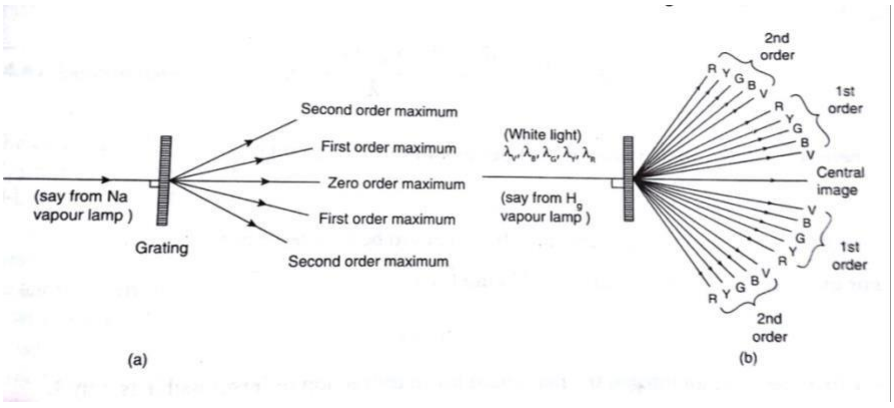


Fig.1.19. Grating spectrum (a) for monochromatic source (b) for white light source

Determination of Wavelength (λ) of given source of light

The positions of principle maxima in grating are given by

$$(e+d)sin\theta = n \text{ , Where n = 0, 1, 2, 3….}$$

$$sin\theta = (\frac{1}{e+d})\, n\lambda$$

$$sin\theta = \mathrm{N}\, n\lambda$$

$$\lambda = \frac{sin\theta}{\mathrm{N}\, n} \qquad \longrightarrow(1.57)$$

Where $\frac{1}{e+d}$ = N is the number of grating elements or lines per unit width of the grating.

By knowing the values of angle of diffraction, order of the principal maxima and grating element, the wavelength can be determined.

Engineering applications of diffraction

The phenomenon of diffraction can be used in various engineering applications; some of the applications are given below.

1. It is used in x-ray diffraction studies of crystals.
2. It is also used in holography for reconstructing 3D images of objects using laser light.
3. Diffraction gratings are used in spectroscopes to separate a light source into its component wavelengths.
4. Diffraction grating can be chosen to specifically analyze a wavelength of light emitted by molecules in diseased cells in a biopsy sample.
5. Diffraction gratings are used in optical fiber technologies where fibers are designed to provide optimum performance at specific wavelengths.

POLARIZATION

Introduction

The phenomena of interference and diffraction tells us that light is a form of wave. But they do not tell us whether light is transverse or longitudinal waves in nature. The phenomenon polarization confirms that light waves are transverse in nature.

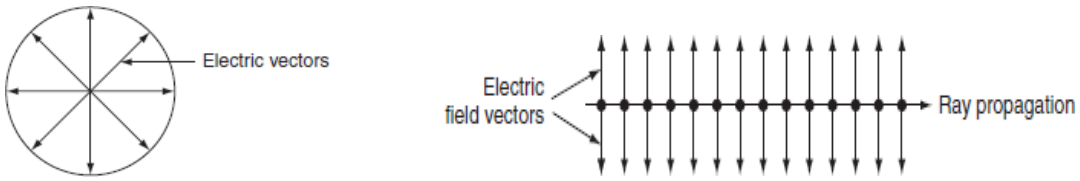
In general, light waves are electromagnetic waves and contain vibrations of electric and magnetic field vectors (*E*&*B*) which are perpendicular to each other and also perpendicular to the direction of propagation of light waves .i.e., transverse waves. Therefore, light is a transverse wave motion. It has been found experimentally that the electric field vector (E) component of light is mainly responsible for polarization and other optical effects but not due to the magnetic field vector. Therefore, the electric field vector E is also called the light vector and vibration of light means vibrations of electric field vector of light.

The word polarization comes from the Greek word *"polos"* which means orientation. *Therefore the phenomenon of restricting or orienting the vibrations of the electric field vector in a particular direction is called polarization of light.*

Unpolarized light

If the light vector (electric field vector E) vibrates in all possible directions which are perpendicular to the direction of propagation then the light is said to be unpolarized light.

Let us consider an ordinary light ray passing perpendicular to the plane of the paper and into the paper. The electric field vectors are perpendicular to the ray propagating with equal amplitude in all possible directions as shown in Fig.1.20. This is the nature of unpolarized light. A double headed arrows show the to and fro vibrations of the electric vector.



(a) (b)

Fig 1.20. Representation of unpolarized light

Polarized light

The phenomenon of restricting the vibrations of the light vector or electric field vector (E) in a particular direction is called polarization of light. The polarized light can be classified into three types. They are

i. Linearly (or) Plane polarized light
ii. Circularly polarized light and
iii. Elliptically polarized light

Linearly (or) Plane polarized light

If the vibrations of light vector (electric field vector E) restricting along straight line only in a plane perpendicular to the direction of propagation of light, it is said to be plane polarized light.

The linearly polarized light is shown in 1.25. The arrowed lines represent in the plane of paper and the dots, the vibrations at right angles (perpendicular) to the paper. In Fig. 1.21(a), the vibrations of electric field vectors lie in the plane of the paper and in Fig. 1.21(b) the vibrations of electric field vectors are perpendicular to the plane of the paper.

(a) (b)

Fig 1.21. Representation of plane polarized light

Circularly polarized light and elliptically polarized light

When two plane polarized light waves are superimposed each other then under certain conditions the resultant light vector may rotates with a constant magnitude in a plane perpendicular to the direction of propagation of light.

If the magnitude of the resultant light vector remains constant while its orientation varies regularly, the tip of the vector traces a circle. Thus the light is said to be circularly polarized.

If however, both magnitude and orientation of light vector vary, the tip of the vector traces an ellipse. Thus the light is said to be elliptically polarized.

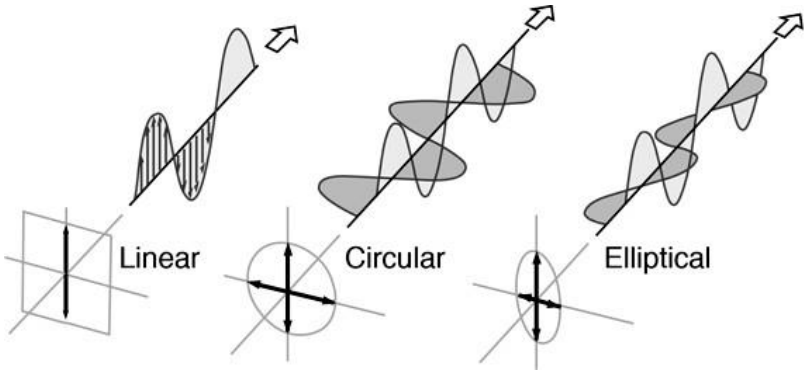


Fig 1.22. Representation of plane, circular and elliptically polarized light

Methods of producing polarized light

Polarized light can be produced by any one of the following.

i. Polarization by reflection
ii. Polarization by refraction
iii. Polarization by double refraction
iv. Polarization by scattering
v. Polarization by selective absorption

Present section we only discuss only polarization by double refraction.

Polarization by double refraction

When a light ray is passed through a glass plate, we get only one refracted ray. But when a light ray is passed through a certain crystals like calcite or quartz, we get two refracted rays, one refracted light ray which obeys the laws of refraction is called ordinary or *o*-ray. The other ray does not obey the laws of refraction and is extraordinary ray or *e*-ray. This phenomenon is called double refraction or birefringence. This was first discovered by Erasmus Bartholinus in 1869.

Explanation

When an ink dot is marked on a white paper and is seen through a calcite crystal, then two images (dots) are observed. If the crystal is rotated slowly with incident ray as vertical axis, then it is observed that one image remains fixed and the other image rotates with the rotation of the calcite crystal. The fixed image is called ordinary image and its refracted ray is called *o*-ray which has vibrations perpendicular to plane of the white paper. The other image is called extraordinary image and its refracted ray is called *e*-ray which has vibrations in the plane of the paper as shown in Fig .1.23.

Inside the crystal the *o*-ray travels with the same velocity in all direction. But the *e*-ray has different velocities in different directions. If the incident ray strikes the crystal along a certain direction called the **optical axis** of the crystal, there will be no double refraction. Hence optical axis of the crystal is a direction along which both *o*-ray and *e*-ray travel in the same direction with the same velocity.

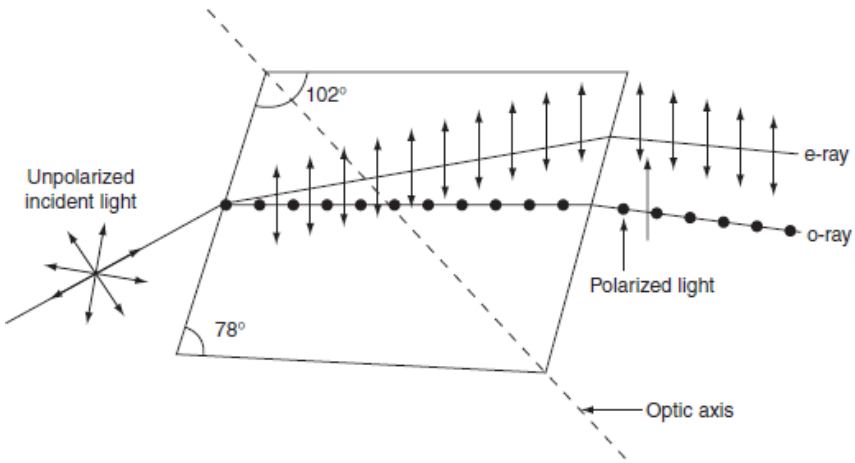


Fig.1.23. Double refraction in calcite crystal

Double refracting crystals

The crystals which exhibit double refraction are called double refracting crystals.

There are two types of double refracting crystals

i. uni-axial crystals
ii. biaxial crystals

Uni-axial crystals

If only one optic axis is present in the crystals, then they are called uni-axial crystals. *Examples:* Calcite, quartz and tourmaline etc.

These crystals further dived into two crystals

i. Uni-axial negative crystals
ii. Uni-axial positive crystals

Uni-axial negative crystals

If the velocity of *e*-ray is more than that of *o*-ray in the crystals, then they are called negative crystals.

i.e., $v_e > v_o$ and $\mu_e < \mu_o$

Examples: Calcite, tourmaline, ruby and emerald etc

Uni-axial positive crystals

If the velocity of *e*-ray is less than that of *o*-ray in the crystals, then they are called negative crystals.

i.e., $v_e < v_o$ and $\mu_e > \mu_o$

Examples: Quartz and iron oxide.

Biaxial crystals

If two optic axes are present in the crystals, then they are called biaxial crystals.

Examples: Topaz and aragonite.

Note:
Optical axis

The direction in which the ray of transmitted light does not suffer double refraction inside the crystal is known as the optic axis. Along this axis both the velocities of *o*-ray and *e*-ray are the same and also refractive indices are same. i.e., $v_e = v_o$ and $\mu_e = \mu_0$. So, there no double refraction.Optical axis is a direction but not line.

Wave Plates or Retardation Plates

Wave plates are the doubly refracting uniaxial (calcite) crystals whose refracting faces are cut parallel to the direction of the optic axis and are used to produce a phase difference between ordinary and extraordinary rays when they emerging from the doubly refracting crystals.

These are of mainly two types

i. Quater-wave plate
ii. Half-wave plate

Quarter-wave plate

A quater-wave plate is a thin double refracting crystal (calcite) having a thickness 't', cut and polished with its refracting faces parallel to the direction of optic axis such that it produces a path difference of $\lambda/4$ or phase difference of $\pi/2$ between the o-ray and e-ray when plane polarized light incident normally on the surface and passes through the plate.

Consider a calcite crystal of thickness of 't'. When a plane polarized light is incident normally on the surface of calcite crystal, then the light will split up into o-ray and e-ray. These rays travel with different velocities in the crystal. As a result, when *o*-ray and *e*-ray emerging from the crystal, they have a phase or path difference between them due to varaition in their velocities.

Let $\mu_o$and μ_e are the refractive indices of *o*-ray and *e*-ray respectively. Let 't' be the thickness of the crystal. Hence the path difference between the two rays is

Δ = Optical path for *o*-ray − Optical path for *e*-ray

$= \mu_o t - \mu_e t$

$= (\mu_o - \mu\ )$ ⟶(1.57)

As the crystal is a quater-wave plate, it introduces a path difference of $\lambda/4$ between o-ray and e-ray.

$\Delta = \lambda/4$ ⟶(1.58)

From Eqs.(1.57) and (1.58),we get

$$(\mu_o - \mu_e)t = \frac{\lambda}{4}$$

Therefore $t = \dfrac{\lambda}{4(\mu_o - \mu_e)}$

For positive crystal $\mu_e > \mu_o$, then $t = \dfrac{\lambda}{4(\mu_e - \mu_o)}$

For negative crystal $\mu_o > \mu_e$, then $t = \dfrac{\lambda}{4(\mu_o - \mu_e)}$

Applications

1. A quarter wave plate is used to produce circularly and elliptically polarized light.
2. Quarter wave plate converts plane-polarized light into elliptically or circularly polarized light depending upon the angle that the incident light vector makes with the optic axis of the quarter wave plate.

Half-wave plate

A half-wave plate is a thin double refracting crystal (calcite) having a thickness 't', cut and polished with its refracting faces parallel to the direction of optic axis such that it produces a path difference of $\lambda/2$ or phase difference of π between the o-ray and e-ray when plane polarized light incident normally on the surface and passes through the plate.

As the crystal is a haf-wave plate, it introduces a path difference of $\lambda/2$ between *o*-ray and *e*-ray.

$$(\mu_e - \mu_o)t = \frac{\lambda}{2}$$

Therefore $t = \dfrac{\lambda}{2(\mu_e - \mu_o)}$

For positive crystal $\mu_e > \mu_o$, then $t = \dfrac{\lambda}{2(\mu_e - \mu_o)}$

For negative crystal $\mu_o > \mu_e$, then $t = \dfrac{\lambda}{2(\mu_o - \mu_e)}$

Applications

1. A half wave plate is used to produce plane polarized light.
2. It produces a phase difference of π between the ordinary and extraordinary ray.

Nicol's Prism

Nicol prism is an optical device used to produce and analyze plane polarized light. This was invented by William Nicol in the year 1828 and is known as Nicol prism. It is made from a double refracting calcite crystal.

Principle

It is based on the phenomenon of double refraction. When a light ray is passed through a calcite crystal, it splits up into two refracted rays such as ordinary (*O*-ray) and extraordinary ray (*E*-ray). Nicol prism transmits the extraordinary rays and eliminates ordinary rays with the help of the phenomenon of total internal reflection.

Construction

- It is constructed from the calcite crystal ABCD having length three times of its width.
- The end faces AB and CD are cut down such that the angles of principal section are 68^0 and 112^0 instead of 71^0 and 109^0.
- The crystal is then cut diagonally into two parts. The surfaces of these parts are ground to make optically flat and then these are polished.

- These polished surfaces are connecting together with special cement known as Canada balsam which is a transparent liquid material.
- Canada balsam is optically (refractive index) more dense than *e*-ray and less dense for *o*-ray ($\mu_o = 1.6584, \mu_{ca} = 1.55\ and\ \mu_e = 1.4864$).

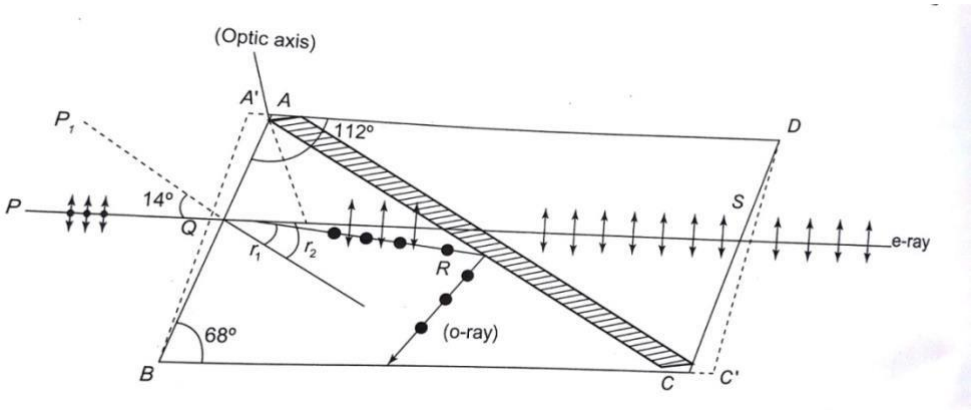


Fig.1.24.Nicol prism

Action or Working

- When a beam of unpolarized light PQ is incident on the face of AB, it splits into two refracted rays *o*-ray (QR) and e-ray (QS) due to double refraction of calcite crystal.
- These two rays are plane polarized.
- From the refractive index values, we know that the Canada balsam acts as a rarer medium for the ordinary ray and it acts as a denser medium for extraordinary ray.
- When *o*-ray of light travels in the calcite crystal and enters the Canada balsam cement, it passes from denser to rarer medium. When the angle of incidence for ordinary ray on the Canada balsam is greater than the critical angle then the incident ordinary ray is totally internally reflected from the crystal and only e-ray is transmitted through the prism and emerges out of Nicol prism.
- In this way, plane polarized light is produced.

Nicol prism as polarizer and analyzer

- Nicol prism can be used as polarizer and analyzer.
- In order to produce and analyse the plane polarized light, two Nicol prisms are arranged adjacently as shown in Fig.1.25.
- The first Nicol prism is used to produce plane polarized light and is called polarizer. The second Nicol prism is used to test the emerging light and is called analyzer.

- In the parallel positions, the extraordinary ray passes through both the prisms as shown in Fig.1.29 (a). In this case, the intensity of emergent extraordinary light is maximum.
- If the second prism is slowly rotated, then the intensity of the extraordinary ray decreases. When they are perpendicular to each other, no light come out of the second prism because the e-ray that comes out from first prism will enter into the second prism and act as an ordinary ray. So, this light is reflected in the second prism as shown in Fig1.29(b).

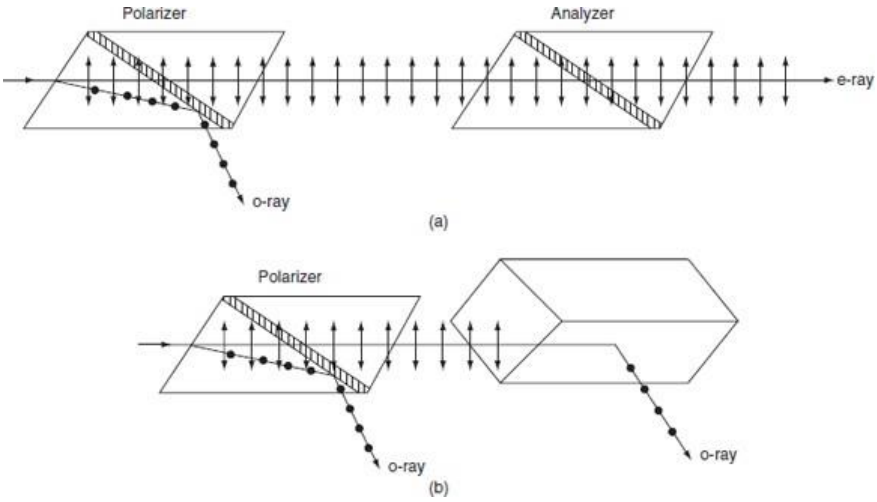


Fig.1.26. Nicol prism as polarizer and analyzer

Engineering applications of Polarization

The phenomenon of Polarization can be used in various engineering applications; some of the applications are given below.

1. Polaroid glasses are used to reduce the amount of light that is approachable to eye.
2. Polarization is useful in receiving and transmitting wave signals.
3. Laser is an outcome of polarization of waves.
4. 3D movies (or stereoscopic movies) are possible because of polarization of light.
5. Photographic filters.
6. Photo elasticity – To study the objects with irregular boundaries and stress analysis.
7. LCD’s.

Unit II: Lasers & Fiber optics

Introduction:

Laser:

- Laser is one of the outstanding inventions of the 20th century.
- A laser is a ***photonic device*** that emits light(electromagnetic radiation) through a process of optical amplification based on the **stimulated emission of electromagnetic radiation.**
- The term "laser" originated as an acronym for **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation.

Characteristics of Lasers:

Laser has certain unique properties when compared to ordinary sources of light. They are

- Monochromatic
- Directional
- Intensity
- Coherent

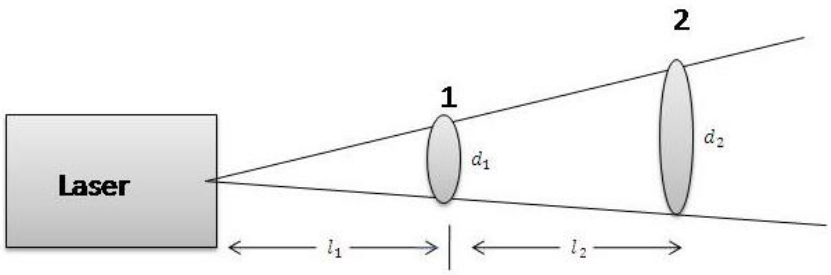
1. Monochromaticity:

- Mono means single and chromo means color. i.e., Monochromatic means that it consists of single color or wavelength or frequency.
- The light emitted from a laser is highly monochromatic, i.e., it is consists of one color/wavelength/frequency. But ordinary white light is a combination of many colors (or wavelengths) of light.

2. Directionality:

- Directional means that the beam is well collimated (very parallel) and travels over long distances with very little spread.
- The light emitted from a laser is highly directional,i.e, laser light is emitted as a relatively narrow beam in a specific direction with very little spread
- An ordinary source of light (bulb) emits light waves in all the directions and is highly divergent.

Fig:



Measurement of beam divergence.

The angular spread or divergence (θ) = $\frac{d_2-d_1}{l_2-l_1}$degrees

Where: d_1 = Beam diameter at point 1.
d_2 = Beam diameter at point 2.
l_1 = Distance from laser to point 1.
l_2 = Distance from laser to point 2.

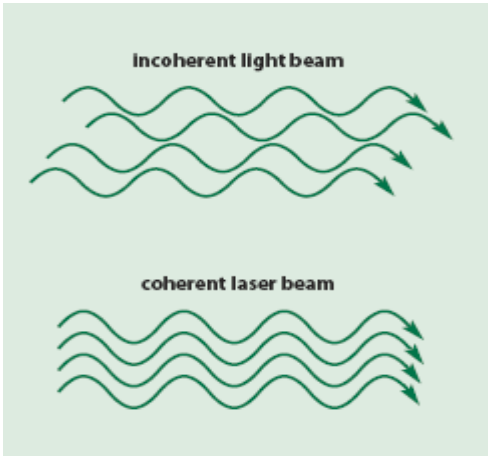
The angular spread of ordinary light is 1meter/meter or 1 radian.
The angular spread of laser light is 1mm/meter or 10^{-3} radians.

3. Intensity:

- An ordinary light spreads in all directions; the intensity reaching the target is very less.
- But in the case of laser, due to high directionality many beams of light incident in smallarea, therefore the intensity of light high. Hence its brightness is more.

4. Coherence:

- The light from a laser is said to be coherent, which means that the wavelengths of the laser light are in phase in space and time.
- Ordinary light can be a mixture of many wavelengths.



Interaction of radiation with matter:

Introduction:

When the incident radiation (Photon) interacts with atoms in the energy levels then three three distinct processes can takes place.

- Absorption of radiation

- Spontaneous emission of radiation
- Stimulated emission of radiation

1. **Absorption of radiation:**

Suppose If an atom in the lower energy level (or) ground state energy level E_1 and absorbs the incident photon radiation of energy then it goes to the higher energy level (or) excited state E_2 as shown in fig(1). This process is called absorption of radiation.

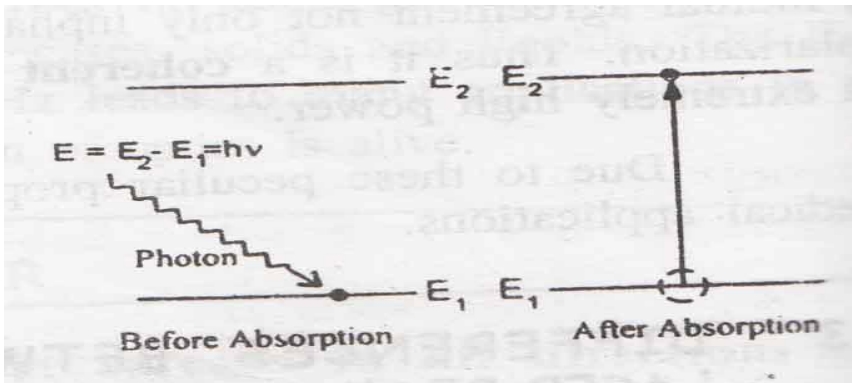


Fig: Absorption of radiation

The process may be expressed as

$$A + h\upsilon \longrightarrow A^*$$

Where A=Atom in the ground state

A^*= Excited atom

$h\upsilon$ = Incident photon

- The number of absorptions depend upon the number of atoms per unit volume (N_1) in lower energy level (E_1) and the number of photons per unit volume of radiation i.e. incident radiation density ρ_υ. The rate of absorption (R_{12}) is proportional to the following factors

i.e., $R_{12} \propto$ incident radiation density (ρ_υ)

$\propto$ No. of atoms in the ground state (N_1)

$$\therefore \boxed{R_{12} = B_{12}\, \rho_\upsilon\, N_1} \longrightarrow (1)$$

Where B_{12} is a constant and is known as Einstein's coefficient of absorption of radiation.

2.**Spontaneous emission of radiation:**

Normally the atom in the excited state will not stay there, for a long time i.e., it can stay up to 10^{-9} second. This called life time of atom. After the life time of the excited atom it returns to the ground state by emitting photon energy $E= E_2-E_1 = h\upsilon$, spontaneously without any external energy as shown in fig (2).

This process is known as Spontaneous emission of radiation.

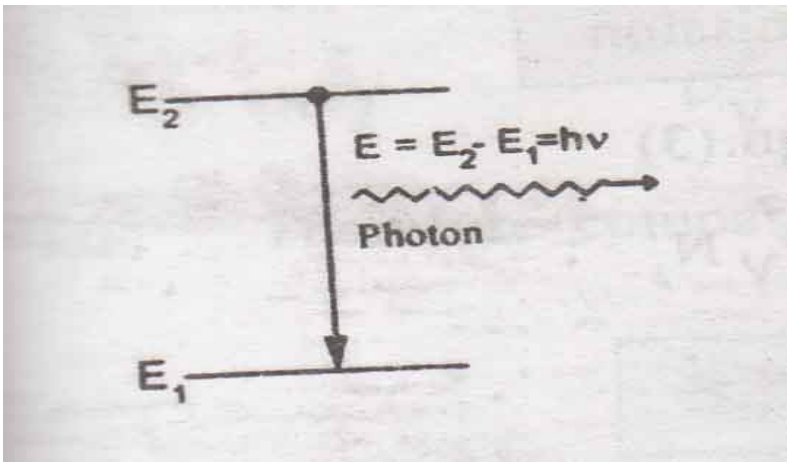


Fig: Spontaneous emission of radiation.

The process may expressed as

$$A^* \longrightarrow A + h\upsilon$$

Where A=Atom in the ground state

A^*= Excited atom

$h\upsilon$ = Incident photon.

The number of spontaneous emission of radiation depends on the number of atoms per unit volume in higher energy level i.e. N_2

∴The rate of spontaneous emission is $R_{21(SP)} \propto N_2$

$$\therefore \quad R_{21(SP)} = A_{21}N_2 \longrightarrow (2)$$

Where A_{21} is a constant called Einstein's coefficient of spontaneous emission of radiation.

3. Stimulated emission of radiation:

Suppose if we incident some suitable form of energy on the atom in the excited state, then it can also return to the ground by emitting a photon, known as stimulated emission.

In this process two photons are released. They have same frequency, wavelength and in phase difference and of same directionality as shown in fig.

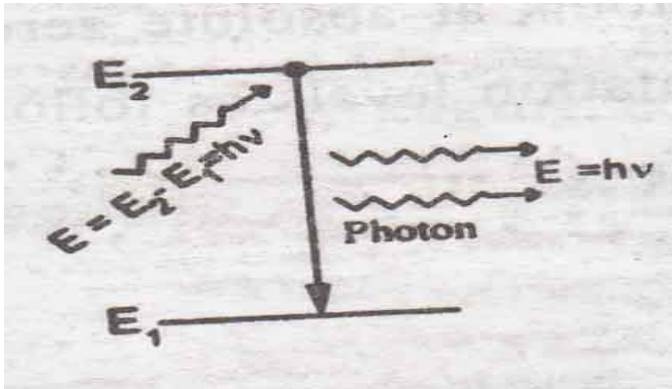


Fig: Stimulated emission of radiation.

The process may be expressed as

$$A^* + h\upsilon \longrightarrow A + 2\,h\upsilon$$

The number of stimulated emission depends on the number of atoms in the energy level (E_2) ,N_2 and the incident radiation density ρ_υ.

∴ The rate of stimulated emission R_{21} is given by

$$R_{21(St)} \propto N_2$$

$$\propto \rho_\upsilon$$

$$R_{21(St)} \propto N_2\, \rho_\upsilon$$

$$\therefore \boxed{R_{21(St)} = B_{21}\ \rho_\upsilon\ N_2} \longrightarrow (3)$$

Where B_{21} is a constant called Einstein coefficient of stimulated emission of radiation

<u>Difference between spontaneous and stimulated emission of radiation</u>

Spontaneous Emission of radiation	**Stimulated Emission of radiation**
1. This emission is postulated by Bhor	1. This emission is postulated by Einstein.
2. Emission of radiation takes place without any inducement or stimulus energy.	2. Emission of radiation takes place with help of inducement or stimulus energy.
3. The emitted photons move in all directions and are random.	3. The emitted photons move in same direction and is highly direction
4.Incoherent radiation	4.Coherent radiation

5. Low intense and less directional	5. High intense and more directional
6.Polychromatic radiation	6.Monochromatic radiation
7.It is an uncontrollable process	7.It is controllable process
8.The rate of spontaneous emission is $R_{12(SP)} = A_{21}N_2$	8. The rate of stimulated emission is $R_{21(St)} = B_{21}\ \rho_{\upsilon}\ N_2$
9. Example: Light from sodium vapor lamp and mercury vapor lamp	9. Example: Light from Ruby laser, He-Ne laser and GaAs laser etc.

Population:

The number of atoms per unit volume in an energy level is known as population of that energy level**.**

According to Boltzmann`s distribution law; if N is the number of atoms per unit volume in an energy state E, at temperature T , then the population of that energy level E is given by

$$N=N_O \exp^{\frac{-E}{K_B T}}$$

Where, N_O is the population of lower energy level and K_B is Boltzmann`s constant (1.3807 × 10−23 J K−1)

Einstein's coefficient's and their relations:-

In 1917 Einstein proposed a mathematical relation between absorption and emission of radiation based on Boltzmann's distribution law and Planck's theory of radiation.

Consider two energy levels of energies E_1 and E_2 ($E_2 > E_1$) .Let N_1 and N_2 be the number of atoms per unit volume of E_1 and E_2.

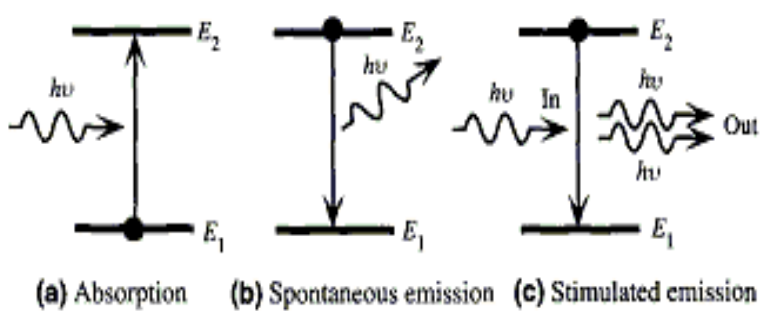


Fig: Three different processes during the interaction of light with matter.

We know that when the incident radiation (photon) interacts with atoms in the energy levels then three distinct processes takes place.

1) **Absorption :-**

The rate of absorption (R_{12}) $= B_{12}\ \rho_\upsilon\ N_1 \longrightarrow (1)$

2) **Spontaneous emission :-**

The rate of spontaneous emission is given by

$$R_{21\,(SP)} = A_{21}N_2 \longrightarrow (2)$$

3) **Stimulated Emission:-**

The rate of stimulate emission is given by

$$R_{21(St)} = B_{21}\ \rho_\upsilon\ N_2 \longrightarrow (3)$$

Under thermal equilibrium,

The rate of absorption = The rate of emission

i.e., Eq(1) = Eq(2) +Eq(3)

$$B_{12}\ \rho_\upsilon\ N_1 = A_{21}N_2 + B_{21}\ \rho_\upsilon\ N_2$$

$$B_{12}\ \rho_\upsilon\ N_1 - B_{21}\ \rho_\upsilon\ N_2 = A_{21}N_2$$

$$\rho_\upsilon(B_{12}\ N_1 - B_{21}\ N_2) = A_{21}N_2$$

$$\rho_\upsilon = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}\ N_2}$$

$$= \frac{A_{21}N_2}{N_2(B_{12}\frac{N_1}{N_2} - B_{21})}$$

$$\rho_\upsilon = \frac{A_{21}}{B_{12}\ \left(\frac{N_1}{N_2}\right) - B_{21}} \rightarrow (4)$$

$$= \frac{A_{21}}{B_{12}\left[\frac{N_1}{N_2} - \frac{B_{21}}{B_{12}}\right]} \rightarrow (5)$$

We know that; Boltzmann distribution law

$$N_1 = N_0 \exp\frac{-E_1}{K_{BT}} \rightarrow (6)$$

Similarly $N_2 = N_0 \exp\frac{-E_2}{K_{BT}} \rightarrow (7)$

And $\frac{N_1}{N_2} = \exp\frac{-E_1}{K_{BT}} \cdot \exp\frac{E_2}{K_{BT}}$

i.e., $\frac{N_1}{N_2} = \exp\frac{(E_2 - E_1)}{K_{BT}}$

Since $E_2 - E_1 = h\upsilon$, we have

$$\frac{N_1}{N_2} = \exp\frac{h\upsilon}{K_{BT}} \rightarrow (8)$$

Substituting Eq (8) in Eq (5) we have

$$\rho_\upsilon = \frac{A_{21}}{B_{21}\left[\exp\frac{h\upsilon}{K_{BT}} - \frac{B_{21}}{B_{12}}\right]} \quad \rightarrow (9)$$

According to Planck's quantum theory of radiation, the incident radiation density is given by,

$$\rho_\upsilon = \frac{8\pi h\nu^3}{C^3}\left[\frac{1}{\exp^{\frac{h\nu}{K_{BT}}} - 1}\right] \quad \rightarrow (10)$$

Therefore comparing equations (9) and (10), we can write

$$\boxed{\frac{B_{21}}{B_{12}} = 1 \text{ or } B_{21}=B_{12}=1 \text{ and}} \quad \rightarrow (11)$$

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{C^3}} \quad \rightarrow (12)$$

These results were obtained by Einstein in 1917, and that is why the coefficients A_{21} B_{21} and B_{12} are called Einstein's coefficients.
From eq. (11), we conclude that the coefficient of absorption B_{12} is equal to the coefficient of stimulated emission B_{21}.
From eq. (11), we conclude that the coefficient of spontaneous versus stimulated emission is proportional to the third power of frequency of the radiation.

i.e., $\frac{A_{21}}{B_{21}} \alpha\, \nu^3$

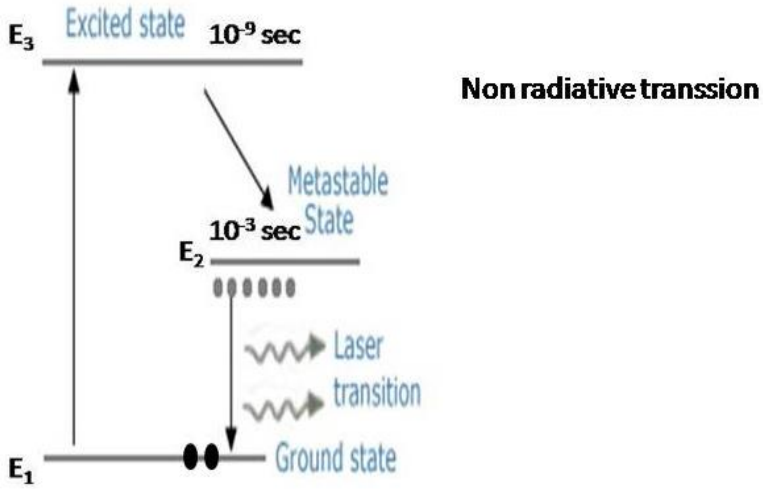
Thus, the spontaneous emission of radiation dominates the stimulated emission of radiation at normal conditions. This is why it is difficult to achieve laser action. The spontaneous emission produces incoherent light, while stimulated emission produces coherent light. In an ordinary conventional light source, the spontaneous emission is dominated. For, laser action stimulated emission should be predominant over spontaneous emission and absorption. To achieve this, an artificial condition, known as population inversion, is required.

Population Inversion:

In general, the population of lower energy level will be greater than that of the higher energy level. To get stimulated emission of radiation, the population of higher energy level (E_2) should be greater than the population of the lower energy level (E_1).i.e., $N_2>N_1$.

The process of making a state in which the population of higher energy level (E_2) is greater than the population of the lower energy level (E_1) is known as population inversion.

Fig: $N_1 > N_2$ **Fig: $N_2 > N_1$**



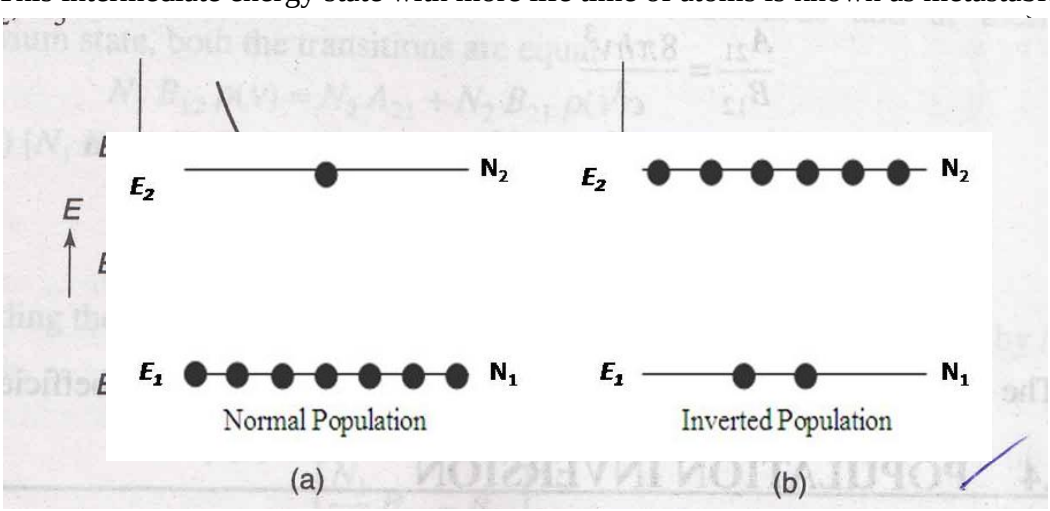
- To Consider three present levels are
- In $N_1 > N_2 >$
- E_1 is the of an with less is the life time of E_3.

Explanation:

explain Population Inversion, let us a there energy level system in which energy levels E_1, E_2 and E_3 are and populations in those energy N_1, N_2 and N_3 respectively. normal conditions $E_1 < E_2 < E_3$ and N_3 obeying Boltzmann's distribution law.

lower energy state with more time atom, E_3 is the higher energy state lifetime of an atom (10^{-9}sec) and E_2 intermediate energy state with more of an atom (10^{-3}sec) compare to that

- This intermediate energy state with more life time of atoms is known as metastable state.



This state provides necessary population inversion for the laser action.

When suitable form of energy is supplied to the system, then the atoms excite from ground state E_1 to higher energy state E_3 and E_2.

Graphically this has been as shown in fig.

Fig: (a) Boltzmann's distribution Fig: (b) Population inversion between E_1 and E_2

- Let the atoms in the system be excited from E_1 state to E_3 state by supplying energy equal to $E_3 - E_1 = h\upsilon$ from an external source.
- The atoms in E_3 state are unstable; they can stay up to 10^{-9} s. This called life time of atoms. After the life time of the excited atoms, they can returns to the meta stable state E_2 without emission of any radiation .This process is called *non-radiative transition.*

- In E_2 state, the atoms can stay for a very long time(10^{-3}s).
- As atoms in E_1 state are continuously exciting to E_3 , so the population in E_1 energy state goes decreasing.
- A state will reach at which the population in E_2 State is greater than E_1 state (i.e.$N_2>N_1$). This situation is known as population inversion.

<u>Excitation mechanisms</u>:

<u>Pumping</u>:

The population inversion cannot be achieved thermally. To achieve population inversion suitable form of energy must be supplied. The process of supplying suitable form of energy to a system to achieve population inversion is called pumping. There're several methods for achieving the condition of population inversion necessary for laser action. Some of the most commonly used pumping methods are,

(i) Optical pumping method
(ii) Electrical discharge(Direct electron excitation) pumping method
(iii)Inelastic atom-atom collision pumping method
(iv)Direct conversion pumping method
(v) Chemical reactions pumping method.

<u>(i). Optical pumping method</u>:

- The process of supplying suitable form of optical energy to a system to achieve population inversion is called optical pumping.
- In this method, light source is used to supply suitable form of optical energy to excite the atoms to higher energy level to achieve population inversion.
- This type of pumping is used in solid state lasers (Ex: Ruby laser and Nd-YAG Laser).

<u>(ii). Electrical discharge (Direct electron excitation) pumping method:</u>

- In this method, a high voltage or electric field is applying to electrodes at both sides of the discharge tube containing the gas causes Electrons are ejected from the cathode, accelerated toward the anode, and collide with the gas molecules along the way.
- During the collision, the mechanical kinetic energy of the electrons is transferred to the gas molecules, and excites them. (This same method of energy transfer is used in common fluorescent lights).
- This type of pumping is used in gaseous ion lasers (Ex: He-Ne laser and co_2 Laser).

<u>(iii). Inelastic atom-atom collision pumping method</u>

- In this method a combination of two types of gases are used say A and B, both having same or nearly coinciding excited states A* and B* .
- In the first step ,during electric discharge , A gets excited to A* (meta stable state) due to collision with electrons .The excited atom now collide with the B atoms so that B goes to excited state B* .

$$e^- + A \longrightarrow A^*$$
$$A^* + B \longrightarrow B^* + A$$

- For example, in the helium-neon laser the electrons from the discharge collide with the <u>helium</u> atoms, exciting them. The excited helium atoms then collide with <u>neon</u> atoms, transferring energy so that Ne atoms go the excited state.

<u>(iv).Direct conversion pumping method</u>

- In this method, when a p-n junction diode is forward biased and then the recombination of electrons and holes across the junction emits the radiation.

Electron + hole $\longrightarrow$ photon

- This method is used in semiconductor lasers.

(v).Chemical reactions pumping method

- In this method, due to some chemical reactions, the atoms may be raised to excited state.
- For example, hydrogen fluoride chemical laser, in which hydrogen can react with fluorine to produce hydrogen fluoride liberating heat energy. This heat energy will try to excite the atoms to higher energy level.

$$H_2 + F_2 \longrightarrow 2HF + \text{Heat energy}$$

Block diagram of a laser system:

The block diagram of laser system contains three parts, they are

(i) Source of energy
(ii) Active medium and
(iii) Optical resonator as shown in fig.

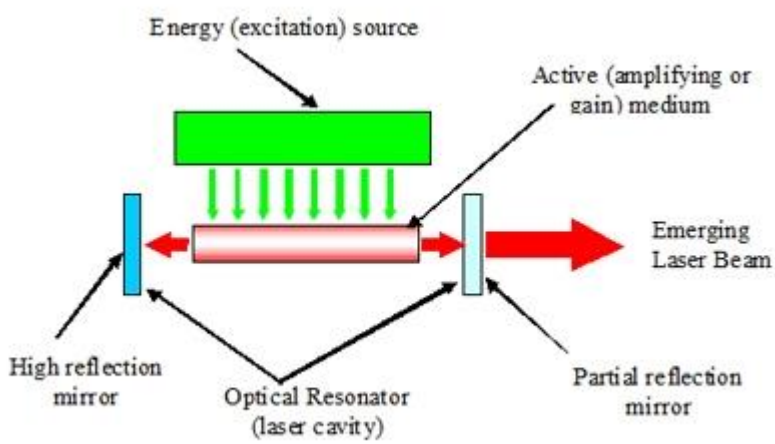


Fig: Components of LASER system.

(i) Source of energy:

- To achieve population inversion suitable form of energy must be supplied. It supplies suitable form of energy by using any one of the pumping methods.
- For example in ruby laser, helical xenon flash tube used as pumping source.
- In helium-neon laser, electrical discharge tube used as pumping source.

(ii) Active medium:

- To achieve population inversion medium is necessary.
- The material medium in which population inversion takes place is called as active medium. In which metastable state is present.
- In metastable state only the population inversion takes place. It can be a solid, liquid, gas or semiconductor diode junction.
- The material medium in which the atoms are raised to exited state to achieve population inversion is called as active centers.

- For example in ruby laser, the active medium is aluminum oxide (Al_2O_3) dopedwith chromium oxide (Cr_2O_3). In which chromium ions (Cr^{3+}) act as active centers.
- In helium -neon laser it is the combination of helium and neon in the ratio of 10:1 in which Ne atoms act as active centers.

(iii)<u>Optical resonator:</u>

- An optical resonator which consists of two mirrors. One mirror is fully reflective and other is partially reflective.
- An active medium is kept between in them. The light emitted due to the stimulated emission of radiation bounces back and forth between the two mirrors and hence the intensity of the light is increased enormously.
- Finally the intense, amplified beam called laser is allowed to come out through the partial mirror as shown in fig.

<u>Types of lasers :</u>

<u>Nd-YAG [Neodymium-Yttrium Aluminum Garnet] laser:-</u>

<u>Characteristics of laser:-</u>

Type : Solid state laser (4-level solid state laser)

Active medium : Yttrium Aluminum Garnet [$Y_3Al_50_{12}$]

Active centre : Nd_3^+ ions

Pumping method : Optical pumping

Pumping source : Xenon flash lamp

Optical resonator : two ends of the rod polished with silver

Nature output : pulsed

Power output : $2x10^4$ watts

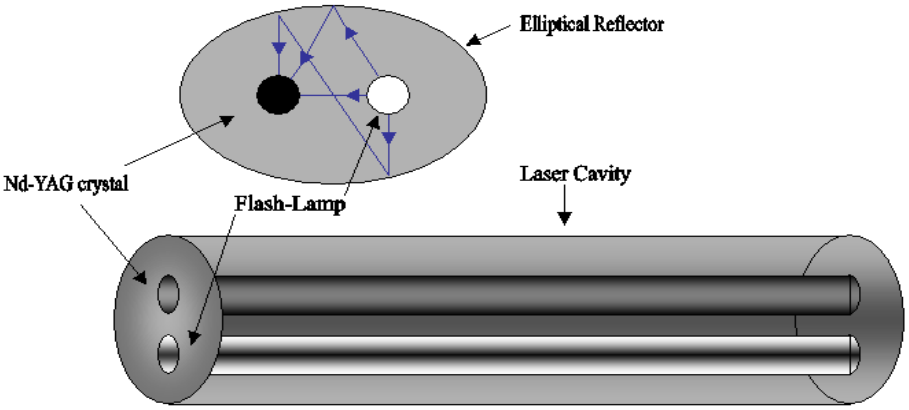
Wave length : 1.064 µm.

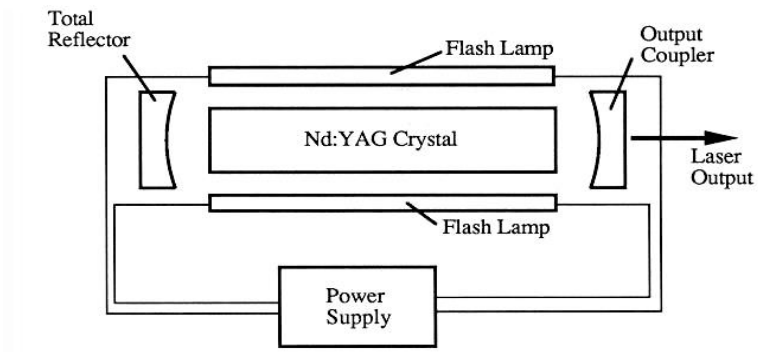
<u>Principle</u>:-

The neodymium ions are raised to exited states optical pumping using xenon flash lamp. Then the ions are accumulated at Meta stable state by non radiative transition. Due to stimulated emission the transition of ions takes place from Meta stable state to ground state, the laser beam of wavelength1.064 µm emitted.

<u>Construction</u>

- A Nd-YAG laser consists of a cylindrical Nd-YAG rod [$Y_3Al_50_{12}$].
- In the Nd-YAG rod, Nd^{3+} ions are the active ions taking part in the laser action.
- The Nd-YAG rod will act as an active medium.
- One end of the Nd-YAG rod is fully silvered and the other end is partially silvered so that the two ends will act as optical resonator (or) cavity.
- The Nd-YAG rod surrounded by elliptical glass cavity which in turn is enclosed by xenon flash lamp filled with xenon gas s shown in fig.





Fig; Construction of Nd-YAG laser.

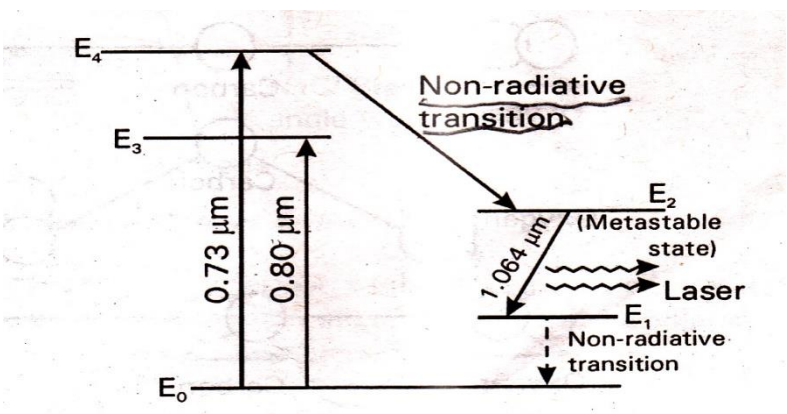


Fig: Energy level diagram of Nd-YAG laser.

Working:-

- The xenon flash lamp is switched on.
- A few thousand joules of light energy are discharged in a few milliseconds.

- A part of this light energy will be flashes on the Nd-YAG rod.
- Then the Nd^{3++} ions in the rod absorbs the particular wavelength of the incident light energy and are excited to higher energy states as shown in fig(2).
- The Nd^{3+} ions absorbs the light of photon of wavelength 0.73 µm and go to E_4 excited state and by absorbing wavelength 0.80 µm they go to E_3 excited state as shown in the energy level diagram.
- The excited Nd^{3+} ions then make a transition from these energy levels.
- The Nd^{3+} ions remain for about 10^{-9} second in these energy levels and makes non-radiative transition to the Meta stable state (E_2).
- In Meta stable state, the Nd^{3+} ions remain for longer duration of the order 10^{-3} second, so population inversion takes place between Meta stable and ground state.
- As a result, stimulated emission takes place and Nd^{3+} ions translated from Meta stable state to ground state.
- Hence, pulsed form of laser beam of wavelength 1.064 µm is emitted during transition from $E_2$to E_1.

Applications of Nd-YAG Laser

- These lasers are widely used for cutting, drilling, welding in the industrial products.
- It is used in long haul communication systems.
- It is also used in the endoscopic applications.

Applications of laser: - Lasers find applications in various fields of science technology. They are described below.

Medical applications :-

- Lasers are used in eye surgery.
- Lasers are used for treatments such as plastic surgery, skin injuries and to remove moles and tumours developed in skin tissue.
- Lasers are used in cancer diagnosis and therapy.

Scientific field:-

- Lasers are used in counting of isotopes separation and to separate isotopes of uranium.
- Lasers are used to estimate size and shape of biological cells such as erythrocytes.
- Lasers are used to create plasma.
- Lasers are used to produce chemical reaction
- Lasers are used in recording and reconstruction of a hologram.

3) Industry applications:-

- Lasers are used to cut glass and quartz.
- Lasers are used to drill holes in ceramics.
- Lasers are used to drill aerosol nozzles.
- Lasers are used for heat treatment in the tooling and automotive industry.

Fiber optics

Introduction

Fibre optics deals with the light propagation through thin glass fibres. Fibre optics plays an important role in the field of communication to transmit voice, television and digital data signals from one place to another. The transmission of light along the thin cylindrical glass fibre by total internal reflection was first demonstrated by John Tyndall in 1870 and the application of this phenomenon in the field of communication is tried only from 1927. Today the applications of fibre optics are also extended to medical field in the form of endoscopes and to instrumentation engineering in the form of optical sensors.

Generally, communication is transferred through carrier waves in any communication system. When the frequencies of the carrier waves are high then the information carrying capacity also enhances. As the propagation of light takes place in the form of high frequency waves, these light waves can be used to carry information, i.e., as carrier waves. For the proper guiding of information carrying light waves, we need a proper guiding medium or material. That material is the optical fibre.

1. Optical fibre

Optical fibre is a guiding medium or material which guides the information carrying light waves. To guide the light waves, optical fibre should be transparent. To minimize the transmission losses through the optical fibre it is made thin.

Thus ***Optical fibre is a thin transparent guiding medium or material which guides the information carrying light waves***

2. The Basic principle of optical fibre (Or) Working principle of optical fibre (or) Total internal reflection

The transmission of light in an optical fibre is based on the phenomenon of total internal reflection. Optical fibre consists of inner most layer known as core, a denser medium and next layer is known as cladding a rarer medium.

Total internal reflection:-

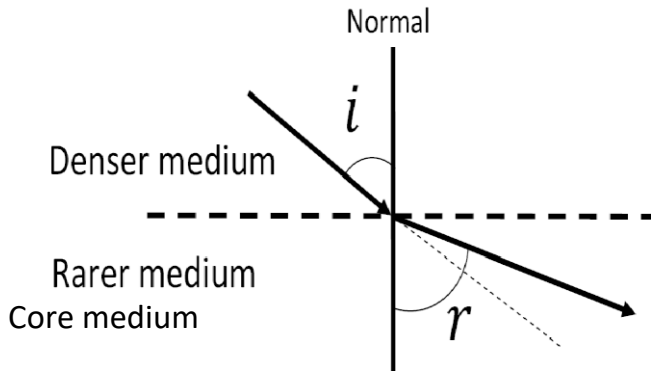
Definition: *When the light ray travels from denser medium to rarer medium the refracted ray bends away* from the normal. When the angle of incidence is greater than the critical angle, the refracted ray again reflects into the same medium. This phenomenon is called total internal reflection.

Let us consider n_1 and n_2 are refractive indices of core and cladding mediums. Let, a light ray traveling from core medium to cladding medium, then the refracted ray bends away from the normal with i is the angle of incidence and r is the angle of refraction.

In this we get three cases

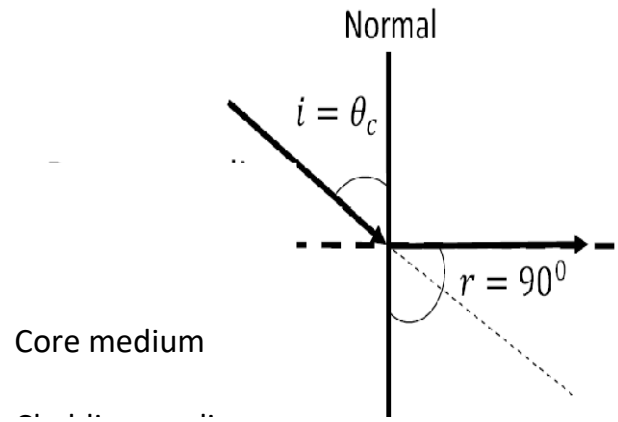
Case I:

When i € 8_c, the light ray refracts into cladding medium. [Figure 1]



Case II:-

When i € 8_c, the light ray travels along the interface of core and cladding, θ_C is known as critical angle. [Figure 2]



When the angle of incidence is increased angle of reflection also increases and for a particular angle of incidence $(i = \theta_C)$ the refracted ray travels along the interface of two mediums. This angle of incidence is known as ***critical angle (θ_C).***

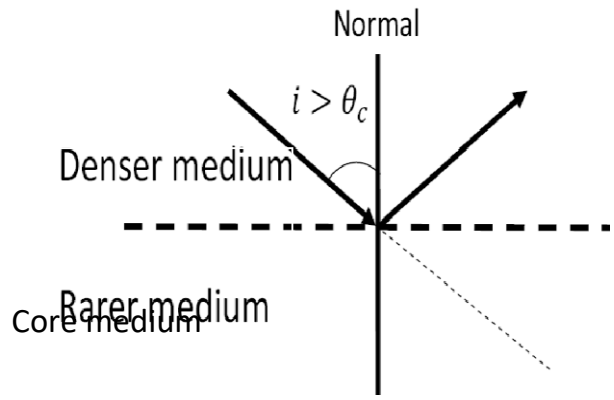
$$n_1 \sin \theta_C = n_2 \sin 90$$

$$n_1 \sin \theta_C = n_2 \Rightarrow \sin \theta_C = n_2 / n_1$$

$$\theta_c = Sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Case III:-

When i Σ 8_c, then the light ray will be reflected back into the core medium and undergoes total internal reflection. [Figure 3]

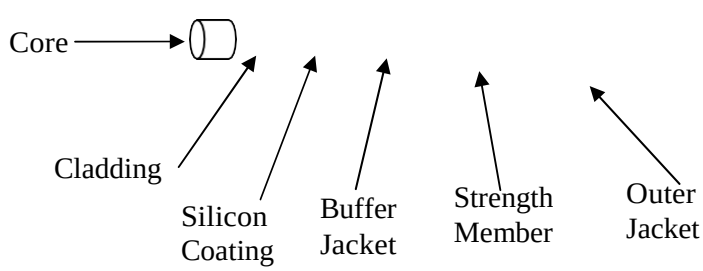


When the angle of incidence is greater than the critical angle $(i > \theta_C)$, the refracted ray again reflects into the same medium. This phenomenon is called total internal reflection

3. Construction of optical fibre

The optical fibre mainly consists the following six parts as shown in figure

1. Core
2. Cladding
3. Silicon coating
4. Buffer jacket
5. Strength member
6. Outer jacket



Core:

A typical glass fibre consists of a central core material. Generally core diameter is ranges from 5 to 600 μN. The core is surrounded by cladding. The core medium refractive is always greater than the cladding refractive index.

Cladding

Cladding refractive index is lesser than the cores refractive index. The overall diameter of cladding is 125 μN to 750μN.

Silicon Coating

Silicon coating is provided between buffer jacket and cladding. It improves the quality of transmission of light.

Buffer Jacket

Silicon coating is surrounded by buffer jacket. Buffer jacket is made of plastic and protects the fibre cable from moisture.

Strength Member

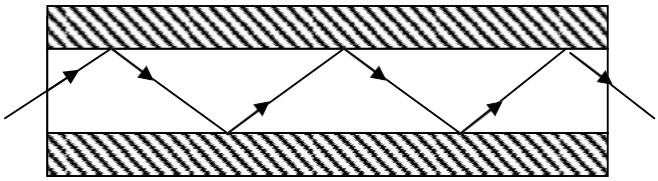
Silicon coating is surrounded by strength member. It provides strength to the fibre cable.

Outer Jacket

Finally the fibre cable is covered by polyurethane outer jacket. Because of this arrangement fibre cable will not be damaged during pulling, bending, stretching and rolling through the fibre cable is made up of glasses.

4. Dimensions of optical fibre

Optical fibres are made in lengths of 1km. optical fibres can be connected with suitable connectors. Generally its outer diameter ranges from 0.1 to 0.15 mm. Naturally the diameter of core and cladding ranges from 5 to 600 μN and 125 to 750 μN respectively. Due the outer jacket 100 μN may add to diameter of the total optical fibre. The optical signal passes through the core medium of the optical fibre. The propagation of optical signal in the optical fibre in the form of multiple total internal reflection is shown in figure.



5. Classification of fibres

Based on the refractive index of core medium, optical fibres are classified into two categories.

i. Step index fibre
ii. Graded index fibre

Based on the number of modes of transmission, optical fibres are classified into two categories

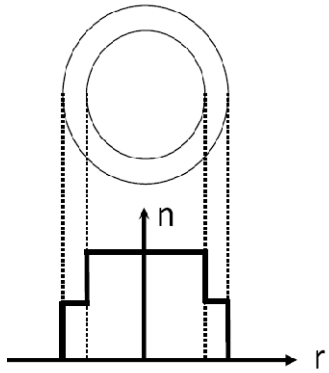
i. Single modefibre
ii. Multi modefibre

Based on the material used, optical fibres are may broadly classified into four categories

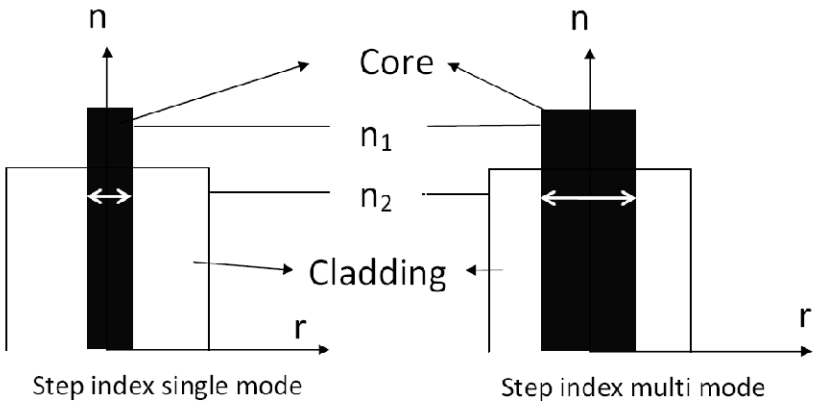
i. All glass fibre
ii. All plastic fibre
iii. Glass core with plastic cladding fibre
iv. Polymer clad silica fibre.

6. Step index fibre- refractive index profile

In step index fibre the refractive index of the core medium is uniform and undergoes an abrupt change at the interface of core and cladding as shown in figure.



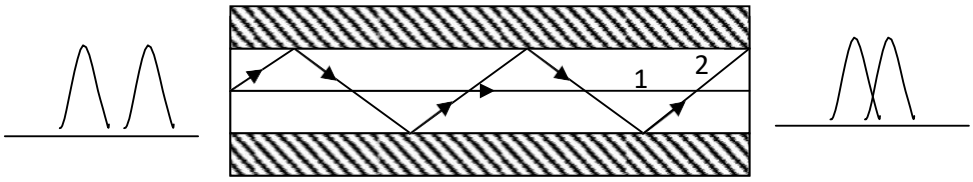
The diameter of core is about 50 to 200 micrometers in case of multi mode and 10 micrometers in single mode fibre.



Attenuation is more for step index multi mode fibres but less in step index single mode fibres
Numerical aperture is more for step index multi mode fibres but it is less in step index single mode fibres.
This fibre is called reflective type fibre.

Transmission of signal in step index fibre

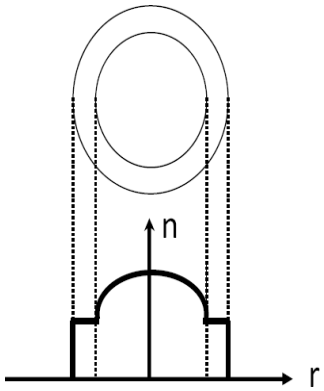
Generally, the optical signal is transmitted through the fibre in the digital form i.e., in the form of 1's and 0's. the propagation of signals through the multi mode fibre is shown in fibre. The transmitted optical signal will cross the fibre axis during every reflection at the core cladding boundary. The shape of propagation of the optical signal is in zigzag manner. Generally the signal through the fibre is in digital form i.e. in the form of pulses representing 0s and 1s.



From figure the ray 1 follows shortest path (i.e. travels along the axis of fibre) and the ray 2 follows longer path than ray 1. Hence the two rays reach the received end at different times. Therefore, the pulsed signal received at other end gets broadened. This is called intermodal dispersion. This difficulty is over come in graded index fibres.

7. Graded index fibre - refractive index profile:-

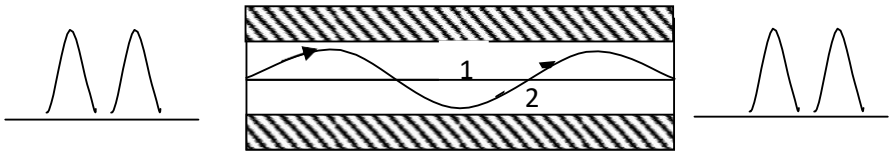
In graded index fibres, the refractive index of the core medium is varying in the parabolic manner such that the maximum refractive index is present at the center of the core.



The diameter of the core is about 50 micro meters.
Attenuation is very less in graded index fibres
Numerical aperture is less in graded index fibres
This fibre is called reflective type fibre.

Transmission of signal in graded index fibre:-

The shape of propagation of the optical is in helical or spiral manner.
The transmitted optical signal will never cross the fibre axis during every reflection at the core cladding boundary.



To discuss intermodal dispersion, we consider two rays as shown in figure, the ray 1 is traveling along the axis of the core and the other ray 2 traveling away from the axis undergoes refraction and bent. Since, ray 2 is traveling in the lesser refractive index medium, so ray 2 moves slightly faster than ray 1. Hence the two rays reach the other end simultaneously. Thus the problem of intermodal dispersion can be overcome by using graded index fibre.

8. Single mode optical fibre:-

- In single mode optical fibres only one mode of propagation is possible.
- These fibres have small core diameter and the difference between the refractive indices of core and cladding is very small.
- In single mode fibres there is no dispersion, so these are more suitable for communication.
- The single mode optical fibres are costly, because the fabrication is difficult.
- The process of launching of light into single mode fibres is very difficult.
- Fabrication is very difficult and the fibre is costly.
- The condition for single mode operation is

$$V = \frac{2\pi}{\lambda} a\, NA$$

Where a is the radius of the core of the fibre, n is the refractive of the core, NA is the numerical aperture and λ is the wave length of light traveling through the fibre

9. Multi mode optical fibre:-

- In multi mode optical fibres many mummer of modes of propagation are possible.
- These fibres have large core diameter and the difference between the refractive indices of core and cladding is also large compared to the single mode fibres.
- Due to multi mode transmission, the dispersion is large, so these fibres are not used for communication purposes.
- The multi mode optical fibres are cheap than single mode fibres, because the fabrication is difficult.
- The process of launching of light into single mode fibres is very easy.
- Fabrication is very easy and the fibre is cheaper.
- The condition for multi mode propagation is

$$N = 4.9\left(\frac{d \bullet NA}{\pi}\right)^2$$

Where d the radius of the core of the fibre and NA is is the numerical aperture.

10. Glass fibre

If the optical fibres are made by fusing mixtures of metal oxides and silica glasses, thin it is known as glass fibre. The most common material used in glass fibre is silica (oxide glasses). It has a refractive index of

1.458 at 850 nm. To vary the refractive indices of core and cladding either fluorine or various of oxides such as B_2O_3, GeO_2 or P_2O_5 are added to silica.

Example of glass fibre compositions are
GeO_2 – SiO_2 core SiO_2 cladding.
P_2O_5 – SiO_2 core SiO_2 cladding.
SiO_2 core, P_2O_5 – SiO_2 cladding

11. Plastic fibre

If the optical fibres are made by plastic, thin it is known as Plastic fibre. The plastic fibres ar low cost and can be handled without special care due to their toughness and boro durability.

Example of glass fibre compositions are
a polystyrene core (n=1.60) and methylmethacryalate cladding (1.49).
a poly methylmethacryalate core (1.49) and cladding made of its co-polymer (n=1.40)

12. Acceptance angle

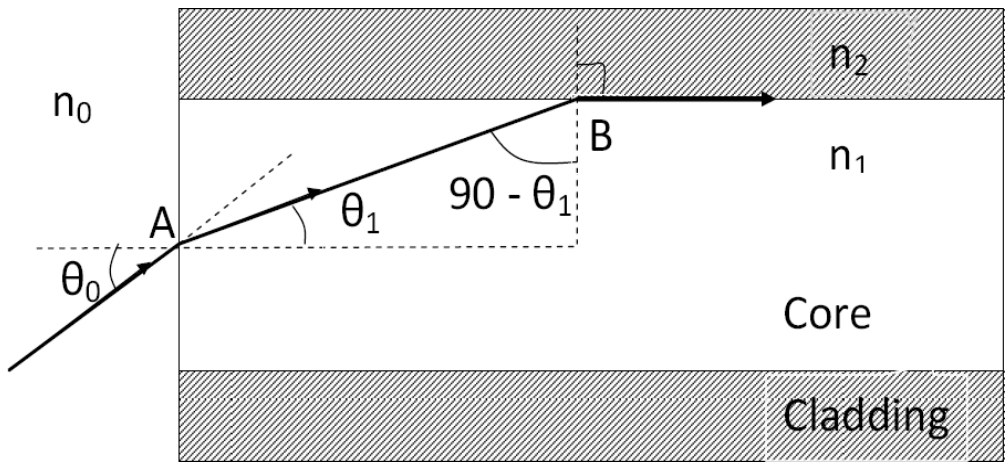
Definition:-

Acceptance angle is defined as the maximum angle of incidence at the interface of air medium and core medium for which the light ray enters into the core and travels along the interface of core and cladding.

Let n_0, n_1 and n_2 be the refractive indices of air, core and cladding media. Let a light ray OA is incident

on the interface of air medium and core medium with an angle of incidence θ_0 then the light ray refracts into the core medium with an angle of refraction θ_1, and the refracted ray AB is again incidenting on the interface of core and cladding with an angle of incident $(90^0 - \theta_1)$.

If $(90^0 - \theta_1)$ is equal to the critical angle of core and cladding media then the ray travels along the interface of core and cladding along the path BC. If the angle of incident at the interface of air and core $\theta_1 < \theta_0$, then $(90^0 - \theta_1)$ will be greater than the critical angle. Therefore, the total internal reflection takes place.



According to Snell's law at point A

$n_0 \sin 8_0 = n_1 \sin 8_1$

$\sin 8_0 = \frac{n_1}{n_0} \sin 8_1$

According to Snell's law at point B

$n_1 \sin(90 - 8_1) = n_2 \sin 90$

$n_1 \cos 8_1 = n_2$

$\cos 8_1 = \frac{n_2}{n_1}$

$\sin 8_1 = \sqrt{(1 - \cos^2 8_1)}$

$\sin 8_1 = \sqrt{(1 - \frac{n_2^2}{n_1^2})} = \frac{\sqrt{(n_1^2 - n_2^2)}}{n_1}$

$\sin 8_0 = \frac{n_1}{n_0} \sin 8_1 = \frac{n_1}{n_0} \frac{\sqrt{(n_1^2 - n_2^2)}}{n_1} = \frac{\sqrt{(n_1^2 - n_2^2)}}{n_0}$

$\sin 8_0 = \frac{\sqrt{(n_1^2 - n_2^2)}}{n_0}$

$8_0 = \sin^{-1}\left(\frac{\sqrt{(n_1^2 - n_2^2)}}{n_0}\right)$

Acceptance angle $8_0 = \sin^{-1}\left(\frac{\sqrt{(n_1^2 - n_2^2)}}{n_0}\right)$

n_0

13. Acceptance cone

Definition:-

A cone obtained by rotating a ray at the end face of an optical fibre, around the fibre axis with acceptance angle is known as acceptance cone.

14. Numerical aperture

Definition: -

Numerical aperture is defined as the light gathering capacity of an optical fibre and it is directly proportional to the acceptance angle.

Numerically it is equal to the sin of the acceptance angle.

$$\text{NA} = \sin(\text{acceptance angle})$$

$$NA = Sin\left(Sin^{-1}\left(\frac{\sqrt{(n_1^2 - n_2^2)}}{n_0}\right)\right)$$

$$NA = \frac{\sqrt{(n_1^2 - n_2^2)}}{n_0}$$

If the refractive index of the air medium is equal to unity then

$$NA = \sqrt{(n_1^2 - n_2^2)}$$

Fractional change in refractive index

$$\Delta = \frac{(n_1 - n_2)}{n_1}$$

$$n_1\Delta = (n_1 - n_2)$$

$$NA = \sqrt{(n_1 - n_2)(n_1 + n_2)}$$

$$n_1\Delta = (n_1 - n_2)$$

$$NA = \sqrt{n_1\Delta(n_1 + n_2)}$$

$$n_1 \approx n_2 ; \quad n_1 + n_2 = 2n_1$$

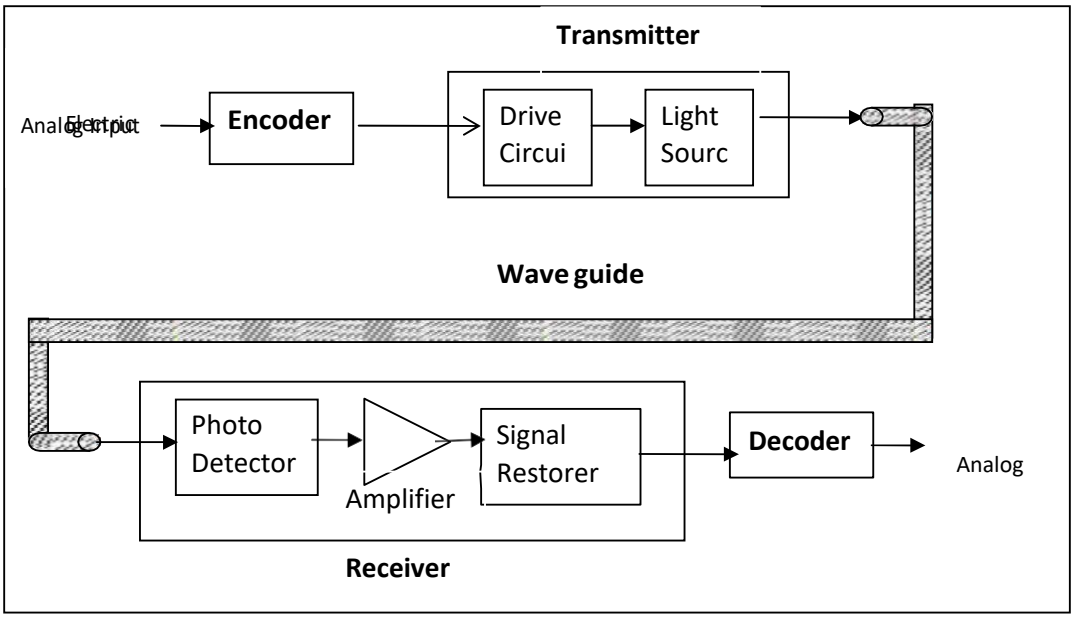
$$NA = \sqrt{n_1\Delta\, 2n_1}$$

$$NA = n_1\sqrt{2\Delta}$$

The above equation gives a relationship between numerical aperture and fractional change in relative refractive index.

15. Optical fibre communication system

An efficient optical fibre communication system requires high information carrying capacity, fast operating speed over long distances with a minimum number of repeaters. An optical fibre communication system mainly consists of the following parts as shown in figure.



1. Encoder

Encoder is an electronic system that converts the analog information like voice, figures, objects etc., into binary data.

2. Transmitter

It contain two parts, they are drive circuit and light source. Drive circuit supplies the electric signals to the light source from the encoder in the required form. The light source converts the electrical signals into optical form.

With the help of specially made connector optical signals will be injected into wave guide from the transmitter.

3. Wave guide.

It is an optical fibre which carriers information in the form of optical signals over distances with the help of repeaters. With the help of specially made connector optical signals will be received by the receiver from the wave guide.

4. Receiver.

It consists of three parts; they are photo detector, amplifier and signal restorer. The photo detector converts the optical signal into the equivalent electric signals and supply to hem to amplifier. The amplifier amplifies the electric signals as they become weak during the long journey through the wave guide over longer distance. The signal restorer deeps the electric signals in a sequential form and supplies to the decoder in the suitable way.

5. *Decoder*

It converts electric signals into the analog information.

16. Advantages of fibre optic communication

The optical fibre communication has more advantages than convectional communication.

1. Enormous bandwidth
2. low transmission loss
3. electric isolation
4. signal security
5. small size and less weight
6. low cost
7. immunity cross talk

1. *Enormous bandwidth*

The information carrying capacity of a transmission system is directly proportional to the frequency of the transmitted signals. In the coaxial cable (or convectional communication system) transmission the bandwidth range is up to around500MHZ. only. Where as in optical fibre communication, the bandwidth range is large as 10^5 GHZ.

2. *Low transmission loss:-*

The transmission loss is very low in optical fibres (i.e. $0.2Km^{-1}$) than compare with the convectional communication system. Hence for long distance communication fibres are preferred.

3. *Electric isolation*

Since fibre optic materials are insulators, they do not exhibit earth and interface problems. Hence communicate through fibre even in electrically danger environment.

4. *Signal security*

The transmitted signal through the fibre does not radiate, unlike the copper cables, a transmitted signal cannot be drawn from fibre with out tampering it. Thus the optical fibre communication provides 100% signal security.

5. *Small size and less weight*

The size of the fibre ranges from $10\mu m$ to $50\mu m$, which is very small. The space occupied by the fibre cable is negligibly small compared to convectional electrical cables. Optical fibres are light in weight.

6. *Low cost*

Since optical fibres made up of silica which is available in abundance, optical fibres are less expensive.

7. *Immunity cross talk*

Since the optical fibres are dielectric wave guides, they are free from any electromagnetic interference and radio frequency interference. Since optical interference among different fibres is not possible, cross talk is negligible even many fibres are cabled

together.

17. Differences between step index fibres and graded index fibres:-

Step index fibre	Graded index fibre
1. In step index fibres the refractive index of the core medium is uniform through and undergoes an abrupt change at the interface of core and cladding.	1. In graded index fibres, the refractive index of the core medium is varying in the parabolic manner such that the maximum refractive index is present at the center of the core.
2. The diameter of core is about 50 to 200 micrometers in case of multi mode and 10 micrometers in single mode fibre	2. The diameter of the core is about 50 micro meters.
3. The transmitted optical signal will cross the fibre axis during every reflection at the core cladding boundary.	3. The transmitted optical signal will never cross the fibre axis at any time.
4. The shape of propagation of the optical signal is in zigzag manner.	4. The shape of propagation of the optical signal appears in the helical or spiral manner
5. Attenuation is more for multi mode step index fibres but Attenuation is less in single mode step index fibres	5. Attenuation is very less in graded index fibres
6. Numerical aperture is more for multi mode step index fibres but it is less in single mode step index fibres	6. Numerical aperture is less in graded index fibres

18. Differences between single mode and multi mode fibres:-

Single mode fibre	Multi mode fibre
1. In single mode optical fibres only one mode of propagation is possible	1. In multi mode optical fibres many mummer of modes of propagation are possible.
2. These fibres have small core diameter and the difference between the refractive indices of core and cladding is very small.	2. These fibres have large core diameter and the difference between the refractive indices of core and cladding is also large compared to the single mode fibres.
3. In single mode fibres there is no dispersion, so these are more suitable for communication.	3. Due to multi mode transmission, the dispersion is large, so these fibres are not used for communication purposes.
4. The process of launching of light into single mode fibres is very difficult	4. The process of launching of light into single mode fibres is very easy.
5. The condition for single mode operation is $V = \frac{2\pi}{\lambda} a\, NA$ $V = \frac{2\pi}{\lambda} a\, n_1 \quad 2\Delta$ Multi mode optical fibre	5. The condition for multi mode propagation is $N = 4.9\left(\frac{d \bullet NA}{\pi}\right)^2$ Where d the radius of the core of the fibre and NA is the numerical aperture.

6. Fabrication is very difficult and the fibre is costly.	6. Fabrication is very easy and the fibre is cheaper.

UNIT-III-Dielectric and Magnetic properties of materials

Dielectric materials

Introduction

Dielectrics are insulating or non-conducting ceramic materials and are used in many applications such as capacitors, memories, sensors and actuators. Dielectrics are insulating materials that exhibit the property of electrical polarization, thereby they modify the dielectric function of the vacuum. A dielectric material is any material that supports charge without conducting it to a significant degree. In principle all insulators are dielectric, although the capacity to support charge varies greatly between different insulators. Although these materials do not conduct electrical current when an electric field is applied, they are not inert to the electric field. The field may cause a slight shift in the balance of charge within the material to form an electric dipole.

Thus the materials is called dielectric material.

Dielectric materials are used in many applications, from simple electrical insulation to sensors and circuit components.

Faraday was carried out the first numerical measurements on the properties of insulating materials when placed between the two parallel plates (capacitor), those materials, he called as dielectrics. He has found that the capacity of a condenser was dependent on the nature of the material separating the conducting surface. This discovery encouraged further empirical studies of insulating materials aiming at maximizing the amount of charge that can be stored by a capacitor. In search of suitable dielectric materials for specific applications, these materials have become increasingly concerned with the detailed physical mechanism governing the behavior of these materials.

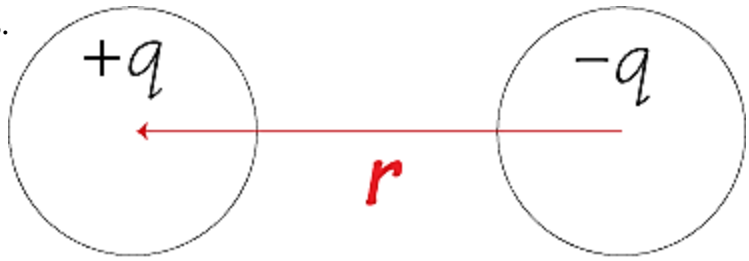
The difference between dielectric material and insulator depends on its application. Insulating materials are used to resist flow of current through it, on the other hand dielectric materials are used to store electrical energy. In contrast to the insulation aspect, the dielectric phenomena have become more general and fundamental, as it has the origin with the dielectric polarization.

Electric dipoles:

Upon application of a dc or static electric field, there is a long range migration of charges. However, there is a limited movement of charges leading to the formation of charge dipoles and the material, in this state, is considered as polarized. These dipoles are aligned in the direction of the applied field. The net effect is called Polarization of the material.

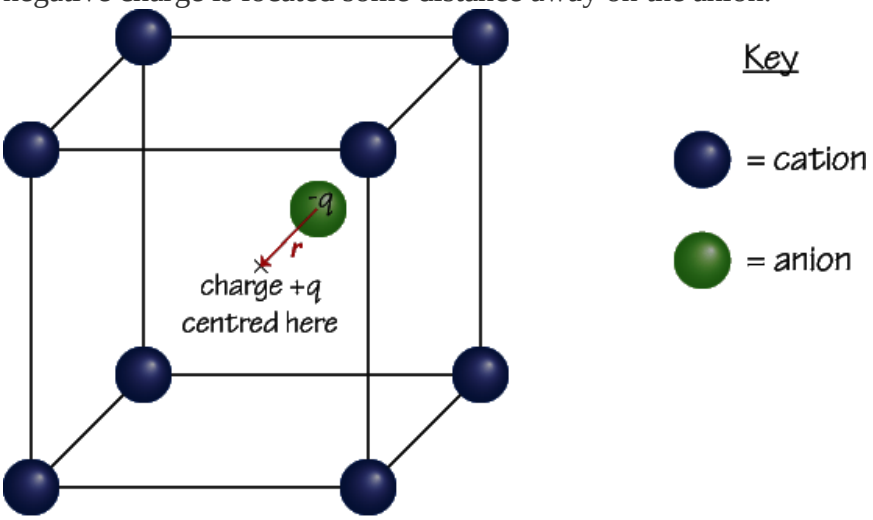
A dielectric supports charge by acquiring a polarisation in an electric field, whereby one surface develops a net positive charge while the opposite surface develops a net negative charge. This is made possible by the presence of electric dipoles – two opposite charges separated by a certain distance – on a microscopic scale.

1. If two discrete charged particles of opposite charges are separated by a certain distance, a dipole moment μ arises.



$$\mu = qr$$

2. If the centre of positive charge within a given region and the centre of negative charge within the same region are not in the same position, a dipole moment μ arises. For example, in the diagram below the centre of positive charge from the 8 cations shown is at X, while the centre of negative charge is located some distance away on the anion.



$\mu = qr$

The second view of dipole moment is more useful, since it can be applied over a large area containing many charges in order to find the net dipole moment of the material.

The dipoles can be aligned as well as be induced by the applied field.

Note that in the equation for dipole moment, ***r*** is a vector (the sign convention is that ***r*** points from negative to positive charge) therefore the dipole moment $\boldsymbol{\mu}$ is also a vector

Electric field intensity or electric field strength (E)

The force experienced by a unit test charge is known as electric field strength E

$$E = \frac{Q}{4\pi\varepsilon r^2} \qquad \text{.....(1)}$$

where ε is the permittivity or dielectric constant of the medium in which electric charge is placed. For vacuum $\varepsilon = \varepsilon_o = 8.854 \text{ X}10^{-12}\text{Fm}^{-1}$

Electric flux density or electric displacement vector (D)

The electric flux density or electric displacement vector is the number of flux lines crossing normal to a unit surface area. The electric flux density at a distance from the point charge Q is

$$D = \frac{Q}{4\pi r^2} \qquad \text{.....(2)}$$

then from (1) and (2) $\quad D = \varepsilon E \qquad$ ..... (3)

Dielectric constant (ε_r)

The dielectric constant of a material is defined as the ratio of the permittivity of the medium (ε) to the permittivity of free space (ε_o). It can also defined as the ratio of the capacitance with dielectric (C_d) and with air (C_A) between the plates.

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_o} = \frac{C_d}{C_A} \qquad \text{.....(4)}$$

Capacitance: The property of a conductor or system of conductor that describes its ability to store electric charge.

C = q / V = A ε / d where

C is capacitance of capacitor
q is charge on the capacitor plate
V is potential difference between plates
A is area of capacitor plate
ε is permittivity of medium
d is distance between capacitor plates

Units: Farad .

Polarization

When an electric field is applied to a material with dielectrics, the positive charges are displaced opposite to the direction of the field and negative charges displaced in the direction of the field. The displacement of these two charges create a local dipole, creation of dipole by applying electric field is called as polarization.

Polarization is defined as induced dipole moment per unit volume.

$$P = \frac{\mu}{Volume} \qquad \ldots(5)$$

Polarisability

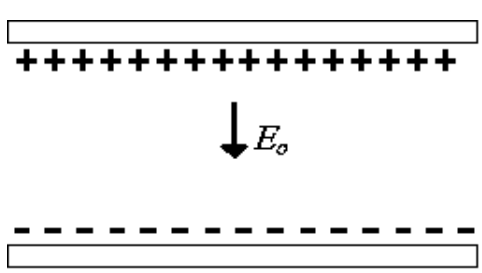
The polarization *P* is directly proportional to the electric field strength E

$$P \propto E$$
$$P = \alpha E \qquad \ldots(6)$$

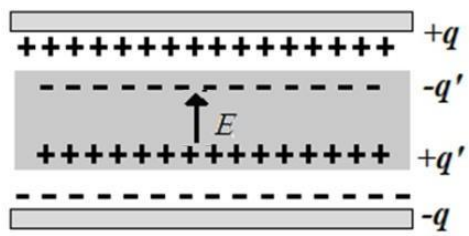
Where α proportionality is constant called as polarisability. The polarisability is defined as polarization per unit applied electric field. If the material contains N number of dipoles per unit volume then

$$P = N\alpha E \qquad \ldots.(7)$$

Relation between polarization and dielectric constant



Parallel plate condensor without dielectrics



Parallel plate condensor with dielectrics

Let us apply Gauss theorem for parallel plate condenser.

$$\int E_0 dA = \frac{q}{\varepsilon_0}$$

$$E_0 A = \frac{q}{\varepsilon_0}$$

$$E_0 = \frac{q}{A\varepsilon_0} = \frac{\sigma}{\varepsilon_0}. \qquad \ldots(1)$$

Where σ is the charge per unit area.

Let a dielectric slab placed between two plates. Due to polarization, charges appear on the two faces of the slab, and establish yet another field within the dielectric media. Let this field be E′. The direction of E′ will be opposite to that of E_0.

The resultant field E in the material can be written as,

$$E = E_0 - E' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

If σ_p is the charge/unit area on the inserted dielectric slab surfaces, then by following equation (1), we write,

$$E' = \frac{q'}{A\varepsilon_0} = \frac{\sigma_p}{\varepsilon_0}. \quad \textbf{.....(3)}$$

From (1), (2) and (3),

`

$$E = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_p}{\varepsilon_0}.$$

or

$$\varepsilon_0 E = \sigma - \sigma_p \quad .....(4)$$

Since the magnitude of polarization P = dipole moment/ Unit Volume

But dipole moment = induced charge X distance

Therefore

P = induced charge/ Area= σ_p

We know that electric displacement fieldor electric flux density D is given by charge /unit area

D = q/A= σ

Therefore Equation 4 becomes

$$\varepsilon_0 E = D - P$$

$$P = D - E\varepsilon_0 \quad .....(5)$$

in free space where there is no dielectric P=0

$$D = E\varepsilon_0$$

But in dielectric media the D changes. From electrostatics

$$D = \varepsilon_0 \varepsilon_r E$$

From (3)

$$P = E\varepsilon_o \varepsilon_r - E\varepsilon_0$$

$$\boxed{P = E\varepsilon_0(\varepsilon_r - 1)} \quad .....(6)$$

$$(\varepsilon_r - 1) = \frac{P}{E\varepsilon_0} = \chi$$

Where χ is electric susceptibility of the dielectric medium. It doesn't have any units.

Since P and E are vectors eqn (6) can be written as

$$\vec{P} = \vec{E}\varepsilon_0(\varepsilon_r - 1) \ldots(7)$$

This equation represents polarization vector.

Types of polarization

Dielectric polarization is the displacement of charge particles with the applied electric field. The displacement of electric charges results in formation of electric dipole moment in atoms, ions or molecules of the material. There are four different types of polarization, they are listed below.

1. Electric polarization,
2. Ionic polarization,
3. Orientation polarization
4. Space charge polarization

Electric polarization

The displacement of the positively charged nucleus and the negatively charged electrons of an atom in opposite directions, on application of an electric field, result in electronic polarization.

On applying a field, the electron cloud around the nucleus shifts towards the positive end of the field. As the nucleus and electron cloud are separated by a distance, dipole moment is created within each atom. The extent of this shift is proportional to the field strength.

Induced dipole moment

$$\mu_e \propto E$$

$$\mu_e = \alpha_e E$$

Where α_e is called electronic polarizability. The dipole moment per unit volume is called electronic polarization.

- It increases with increase of volume of the atom.
- This kind of polarization is mostly exhibited in monoatomic gases.(e.g. He, Ne, Ar, Kr, Xe etc..)

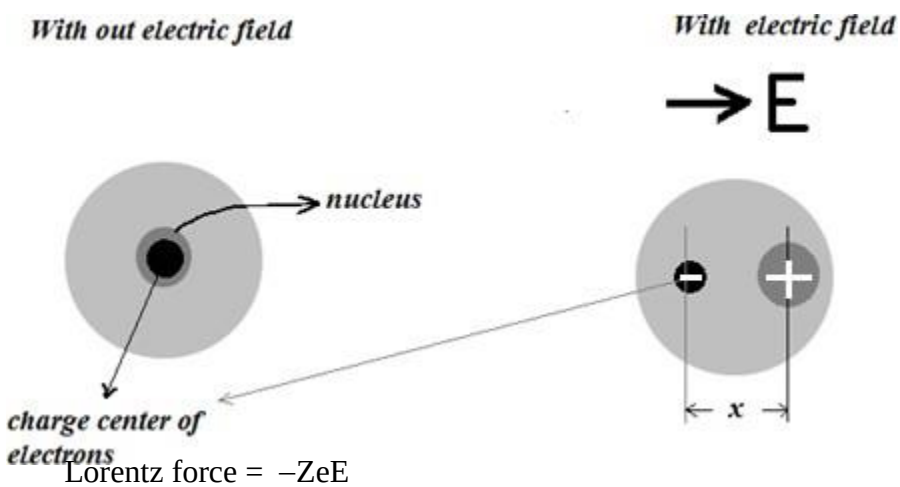
- It is independent of temperature.
- It occurs only at optical frequencies (10^{15}Hz)
- Vast fast process: 10^{-15}~10^{-16}s.

Calculation of electronic polarizability:

Electronic polarization can be explained by classical model of an atom in gasses. In gases the atoms are assumed that the interaction among the atoms is negligible. Here the nucleus of charge Ze is surrounded by an electron cloud of charge –Ze distributed in the sphere of radius R.

Charge density, $$\rho = -\frac{Ze}{(4/3)\pi R^3} \qquad \ldots(1)$$

When an electric field E is applied, the nucleus and electrons experience Lorentz force of magnitude ZeE in opposite direction. Therefore the nucleus and electrons are pulled apart. As they are pulled apart a Coulomb force develops between them. At equilibrium these two forces are equal and nucleus and electron cloud are separated by a small distance x.



Lorentz force = –ZeE $\qquad \ldots(2)$

Coulomb Force = Ze X $\dfrac{Ch\text{ arg }eenclosed\text{ int }hesphereofradiusx}{4\pi\varepsilon_0 x^2}$

The charge enclosed = $\frac{4}{3}\pi x^3 \rho$

From equation (1) $$= \frac{4}{3}\pi x^3\left[-\frac{3}{4}\left(\frac{Ze}{\pi R^3}\right)\right]$$

$$= -\frac{Zex^3}{R^3}$$

Hence Coulomb force is = $\frac{Ze}{4\pi\varepsilon_0 x^2} \times -\frac{Zex^3}{R^3} = -\frac{Z^2 e^2 x^2}{4\pi\varepsilon_0 R^3}$ …(3)

At equilibrium Lorentz force = Coulomb force (equation (2) equal to (3))

$$-ZeE = -\frac{Z^2 e^2 x}{4\pi\varepsilon_0 R^3}$$

$$x = -\frac{4\pi\varepsilon_0 R^3 E}{Ze} \qquad \ldots(4)$$

The displacement of the electron cloud is proportional to applied electric field.

The electric dipole moment $\mu_e = Zex = \frac{Ze4\pi\varepsilon_0 R^3 E}{Ze}$

$$\mu_e = 4\pi\varepsilon_0 R^3 E$$

$$\mu_e \propto E$$

$$\mu_e = \alpha_e E \qquad \ldots(5)$$

Where $\alpha_e = 4\pi\varepsilon_0 R^3$ is called electronic polarizability.

$P_e = N\mu_e = N\alpha_e E$

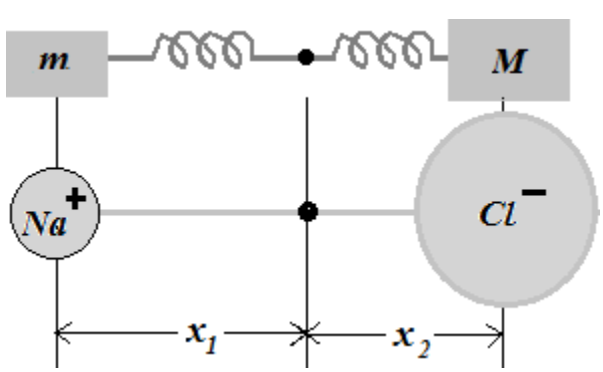
Where N is the number of atoms/m^3

But polarization $P = E\varepsilon_0(\varepsilon_r - 1) = N\alpha_e E$

$$(\varepsilon_r - 1) = \frac{N\alpha_e}{\varepsilon_0} \quad \text{or } \alpha_e = \frac{(\varepsilon_r - 1)\varepsilon_0}{N}$$

Ionic Polarization

Ionic polarization occurs in ionic solids such as NaCl, KBr, and LiBr. When an electric field is applied to an ionic solid the positive and negative ions displace to their respective polarities creating an electric dipole this is called as ionic polarization.



In the absence of an electric field there is no displacement of ions. When an electric field is applied an induced dipole moment μ_i is produced.

Let x_1 and x_2 be the displacement of positive and negative ion respectively. Then the induced dipole moment.

$$\mu_i = e(x_1 + x_2) \qquad \ldots(6)$$

Let F be restoring force F

$$\propto x_1$$
$$\propto x_2$$
$$= \beta_1 x_1$$
$$= \beta_2 x_2$$

From mechanics the spring constant of mass attached to a spring is given by $\beta = m\omega^2$

At equilibrium the Lorentz force = restoring force

$$eE = m\omega_0^2 x$$

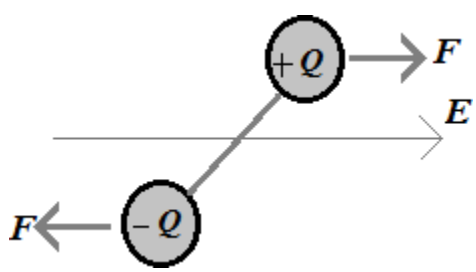
Therefore $x = \dfrac{eE}{m\omega_0^2}$ Then $x_1 = \dfrac{eE}{m\omega_0^2}$ $x_2 = \dfrac{eE}{M\omega_0^2}$

$$\mu = \frac{e^2}{\omega_0^2}\left[\frac{1}{m} + \frac{1}{M}\right]E = \alpha_i E \qquad \ldots(7)$$

Where $\alpha_i = \dfrac{e^2}{\omega_0^2}\left[\dfrac{1}{m} + \dfrac{1}{M}\right]$ is called as ionic polarisability …(8)

Orientation Polarization

Orientation polarization occurs only in polar molecules (the molecules which have permanent dipole moment eg H_2O, Phenol, etc.). When an electric field is applied to a polar molecule, the dipoles experience a torque and try to align parallel to the applied field.



Consider a polar molecule subjected to an electric field *E*. The alignment of electric dipole with the electric field is similar to the alignment of magnetic dipole with the applied magnetic field in paramagnetic material.

The expression for polarization can be obtained from the theory of paramagnetism.

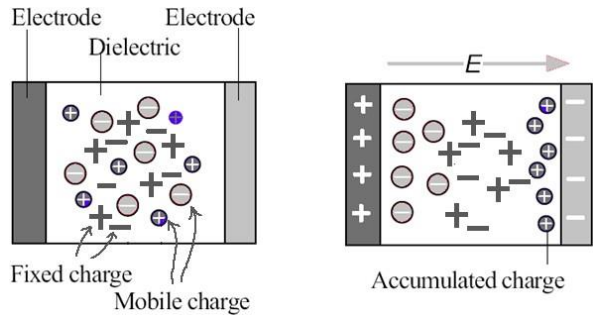
The orientation polarization is given as

$$P_0 = \frac{N\mu_o^2 E}{3KT} = \alpha_o E \qquad \ldots(9)$$

Where $\alpha_0 = \dfrac{\mu^2}{3KT}$ is called as orientation polarisability …(10)

Space charge polarization

Space charge polarization occurs due to the accumulation of charges at the electrodes or at interfaces in a multiphase materials.



In the presence of an applied field, the mobile positive ions and negative ions migrate toward the negative electrode and positive electrode respectively to an appreciable distance giving rise to redistribution of charges , but they remain remains in the dielectric material (electrode is blocking). The space charge polarization can be defined as the redistribution of charges due to the applied electric field and the charges accumulate on the surface of the electrodes. It occurs when the rate of charge accumulation is different from rate of charge removal. Space charge polarization is not significant in most of the dielectric materials.

Internal field in liquids and solids (one dimensional)

In gases state the atoms are separated by large distances and the interaction between the atoms can be neglected. When an external electric field E is applied, the intensity of the electric field experienced by an atom in gases state will be equal to the applied electric field E.

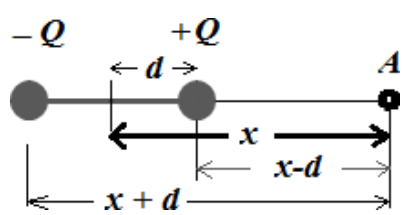
In solids and liquids, the atoms are close to each other leading to strong interaction between them. In solids and liquids the intensity of the electric field at a given point of the material is not equal to the applied electric field but equal to internal field which is the sum of applied electric field and field due to other dipoles present in the material.

Internal field $E_i = E + E`$ …(1)

The internal field can be calculated by Epstein model in the case of one dimensional atomic array.

Electric field along the axis of an electric dipole

Consider an electric dipole of length ***2d*** and charge *Q*, the field along the axis of the dipole at point A is the sum of the electric field due to +Q and –Q.



The electric field due to +Q at point A is $E^{+} = \dfrac{Q}{4\pi\varepsilon_o (x-d)^2}$

The electric field due to +Q at point A is $E^{-} = \dfrac{Q}{4\pi\varepsilon_o (x+d)^2}$

Electric field of dipole at A is $E_A = E^{+} - E^{-} = \dfrac{Q}{4\pi\varepsilon_o}\left[\dfrac{1}{(x-d)^2} - \dfrac{1}{(x+d)^2}\right]$

$$= \frac{2Q}{4\pi\varepsilon_o}\left[\frac{2dx}{(x-d)^2(x+d)^2}\right]$$

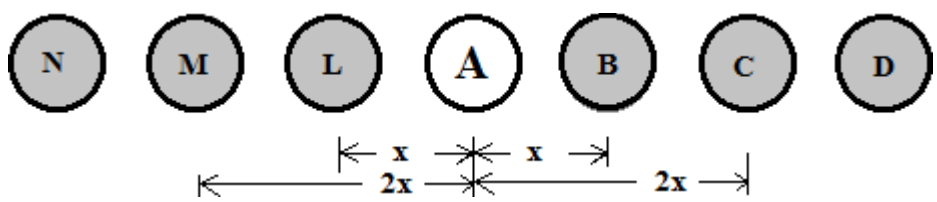
since $x >> d$ $(x-d)^2 \approx (x+d)^2 \approx x^2$ *then*

$$= \frac{2Q}{4\pi\varepsilon_o}\left[\frac{2dx}{x^4}\right] = \frac{4dQ}{4\pi\varepsilon_o x^3}$$

since 2dQ $=\mu_i$

$$E_A = \frac{2\mu_i}{4\pi\varepsilon_o x^3} \qquad \ldots(2)$$

Consider an array of one dimensional atoms along x- axis. The all the atoms are similar, equally spaced and have induced electric dipole moment μ_i in an applied electric field *E*. The electric field experienced at the A is the sum of electric fields of other dipoles and applied electric field E.



The electric field at A due to the induced dipole B and L which are at a distance x is

$$E_B = E_L = \frac{2\mu_i}{4\pi\varepsilon_o x^3}$$

The electric field at A due to the induced dipole C and M which are at a distance 2x is

$$E_C = E_M = \frac{2\mu_i}{4\pi\varepsilon_o (2x)^3}$$

Therefore the field due to other dipoles is

$$E' = E_B + E_L + E_C + E_M + E_D + E_N + \ldots \qquad \ldots(3)$$

$$E' = \frac{2\mu_i}{4\pi\varepsilon_o x^3} + \frac{2\mu_i}{4\pi\varepsilon_o x^3} + \frac{2\mu_i}{4\pi\varepsilon_o (2x)^3} + \frac{2\mu_i}{4\pi\varepsilon_o (2x)^3} + \frac{2\mu_i}{4\pi\varepsilon_o (3x)^3} + \frac{2\mu_i}{4\pi\varepsilon_o (3x)^3} + \ldots$$

$$E' = \frac{4\mu_i}{4\pi\varepsilon_o x^3} + \frac{4\mu_i}{4\pi\varepsilon_o (2x)^3} + \frac{4\mu_i}{4\pi\varepsilon_o (3x)^3} + \ldots$$

$$E' = \frac{\mu_i}{\pi\varepsilon_o x^3}\left[1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \frac{1}{5^3} + \ldots\right] \quad \text{where } 1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \frac{1}{5^3} + \ldots = 1.2$$

$$E' = \frac{1.2\mu_i}{\pi\varepsilon_o x^3}$$

Therefore the internal field $E_i = E + \frac{1.2\mu_i}{\pi\varepsilon_o x^3}$ $\qquad \ldots(4)$

The local field in a three dimensional solid is similar the above equation the number density N of atoms replaces $1/a^3$. Since $N\mu_i=P$ and $1.2/\pi$ is replaced by γ. Then the internal field is

$$E_i = E + \frac{1.2N\mu_i}{\pi\varepsilon_o} = E + \frac{1.2P}{\pi\varepsilon_o} = E + \frac{\gamma P}{\varepsilon_o}$$

γ depends on the internal structure For a cubic symmetry crystal γ value is1/3

$$\boxed{E_i = E + \frac{P}{3\varepsilon_o}} \qquad \ldots(5)$$

The field given by the above equation is called Lorentz field.

Clausius - Mosotti equation

Let us consider elemental solid dielectric which exhibits only electronic polarization. If α_e is the electronic polarisability per atom, it is related to the bulk polarization P through the relation

$$P = N\alpha_e E_i \qquad \ldots(6)$$

$$\alpha_e = \frac{P}{NE_i} \qquad \ldots(7)$$

Where N is the number of atoms per unit volume and E_i is the local field using the relation (5)

$$\alpha_e = \frac{P}{N\left[E + \frac{P}{3\varepsilon_o}\right]} \qquad \ldots(8)$$

By using the relation between the polarization and permittivity we have

$$P = E\varepsilon_0(\varepsilon_r - 1) \qquad \ldots(9)$$

$$E = \frac{P}{\varepsilon_0(\varepsilon_r - 1)} \qquad \ldots(10)$$

Substitute the value of E from (10) in (8)

$$\alpha_e = \frac{P}{N\left[\frac{P}{\varepsilon_0(\varepsilon_r - 1)} + \frac{P}{3\varepsilon_o}\right]}$$

$$\frac{N\alpha_e}{\varepsilon_0} = \frac{1}{\left[\frac{1}{(\varepsilon_r - 1)} + \frac{1}{3}\right]} = \frac{1}{\left[\frac{\varepsilon_r + 2}{3(\varepsilon_r - 1)}\right]}$$

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha_e}{3\varepsilon_0} \qquad \ldots(11)$$

The above equation is known as Clausius Mosotti equation which is valid for nonpolar solids

Dielectric loss:

Dielectric loss is the dissipation of energy through the movement of charges in an alternating electromagnetic field as polarisation switches direction.

An efficient dielectric supports a varying charge with minimal dissipation of energy in the form of heat is called dielectric loss. There are two main forms of loss that may dissipate energy within a dielectric. In conduction loss, a flow of charge through the material causes energy dissipation.

Dielectric loss is especially high around the relaxation or resonance frequencies of the polarisation mechanisms as the polarisation lags behind the applied field, causing an interaction between the field and the dielectric's polarisation that results in heating. This is illustrated by the diagram below (recall that the dielectric constant drops as each polarisation mechanism becomes unable to keep up with the switching electric field.)

Dielectric loss quantifies a dielectric material's inherent dissipation of electromagnetic energy into, e.g., heat.

It can be represented in terms loss tangent tan δ and is defined:

$$\tan \delta_e = \frac{\epsilon''}{\epsilon'}$$

Dielectric Breakdown : The dielectric breakdown is the sudden change in state of a dielectric material subjected to a very high electric field , under the influence of which , the electrons are lifted into the conduction band causing a surge of current , and the ability of the material to resist the current flow suffers a breakdown .

Or

When a dielectric material loses its resistivity and permits very large current to flow through it, then the phenomenon is called dielectric breakdown

Or

At high electric fields, a material that is normally an electrical insulator may begin to conduct electricity – i.e. it ceases to act as a dielectric. This phenomenon is known as dielectric breakdown.

Frequency dependence of polarizability:

On application of an electric field, polarization process occurs as a function of time. The polarization P(t) as a function of time. The polarization P(t) as a function of time t is given by

$P(t) = P[\ 1- \exp(\ - t / t_r\ )]$

Where P – max. Polarization attained on prolonged application of static

field. t_r - relaxation time for particular polarization process

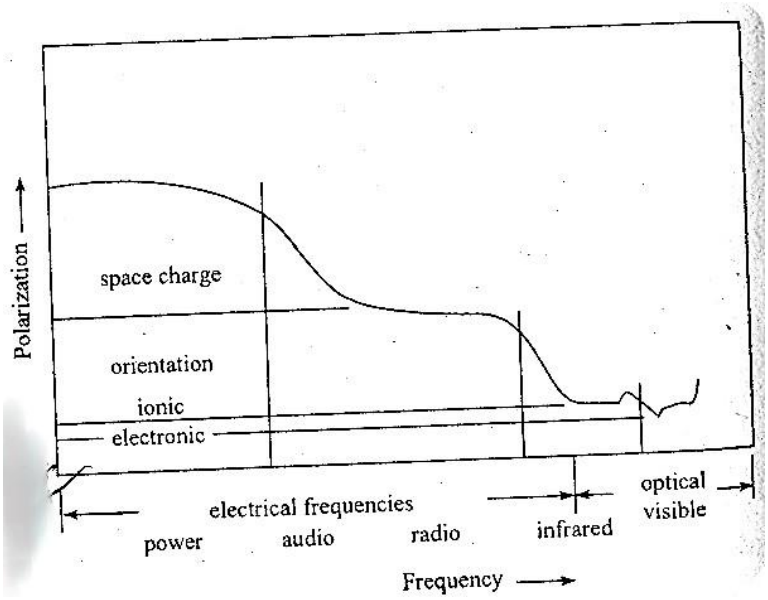
The relaxation time t_r is a measure of the time scale of polarization process. It is the time taken for a polarization process to reach 0.63 of the max. value.

Electronic polarization is extremely rapid. Even when the frequency of the applied voltage is very high in the optical range ($\approx 10^{15}$ Hz), electronic polarization occurs during every cycle of the applied voltage.

Ionic polarization is due to displacement of ions over a small distance due to the applied field. Since ions are heavier than electron cloud, the time taken for displacement is larger. The frequency with which ions are displaced is of the same order as the lattice vibration frequency (≈1013Hz). Hence, at optical frequencies, there is no ionic polarization. If the frequency of the applied voltage is less than 1013 Hz, the ions respond.

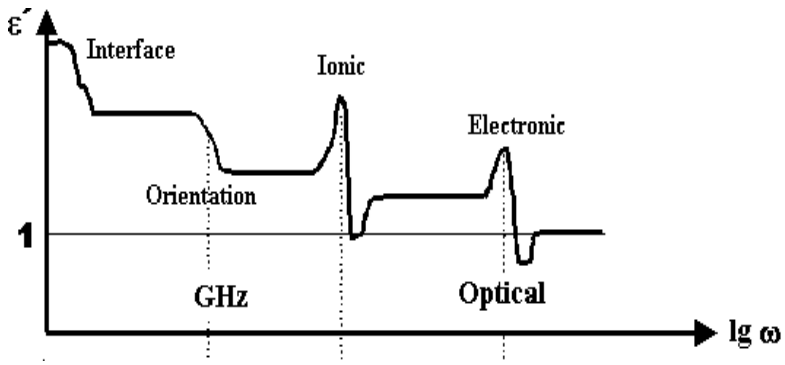
Orientation polarization is even slower than ionic polarization. The relaxation time for orientation polarization in a liquid is less than that in a solid. Orientation polarization occurs, when the frequency of applied voltage is in audio range (1010 Hz).

Space charge polarization is the slowest process, as it involves the diffusion of ions over several interatomic distances. The relaxation time for this process is related to frequency of ions under the influence of applied field. Space charge polarization occurs at power frequencies (50-60 Hz).



Frequency Dependence of dielectric constant

When a dielectric material is subjected to an alternating field, the polarization component required to follow the field in order to contribute to the total polarization of the dielectrics. The relative permittivity which is a measure of the polarization also depends on the frequency. The dependence of ε_r on frequency of the electric field is shown in the figure.



At very low frequency, the dipoles will get sufficient time to orient themselves completely with the field and all types of polarization exist. Since the dielectric is characterized by polarisability α ($\alpha = \alpha_e + \alpha_i + \alpha_o$) at low frequency i.e at radiofrequency region the dielectric constant will be due to all polarisability.

The orientation polarization, which is effective at low frequencies, is damped out for higher frequencies. In the microwave region the dipoles fail to follow the field and the polarisability reduces to ($\alpha = \alpha_e + \alpha_i$), as a result ε_r decreases to some amount.

In the IR region the ionic polarization fails to follow the field so the contribution of ionic polarization dies away. In this region only electronic polarization contributes to the total polarization. Therefore ($\alpha = \alpha_e$) the ε_r still decreases and only electronic polarization exist.

We know that $$P = N\alpha_e E = \varepsilon_o(\varepsilon_r - 1)E$$

Then the relative permittivity is $$\varepsilon_r = 1 + \frac{N\alpha_e}{\varepsilon_o} \qquad \ldots(1)$$

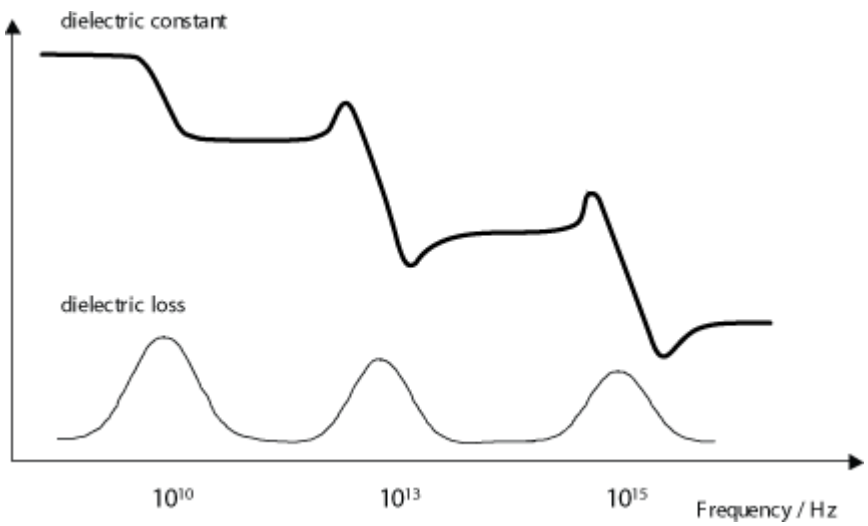
In the ultraviolet region even the electron cloud could not follow the field and electronic polarizability becomes almost zero and the permittivity becomes one.

$$[\varepsilon_r]_{X-ray} = 1$$

For example at low frequency the dielecric constant of water at room temperature is about 80, but it fall to about 1.8 in the optical region.

Frequency Dependence of dielectric loss:

Dielectric loss tends to be higher in materials with higher dielectric constants. This is the downside of using these materials in practical applications. Dielectric loss is utilised to heat food in a microwave oven: the frequency of the microwaves used is close to the relaxation frequency of the orientational polarisation mechanism in water, meaning that any water present absorbs a lot of energy that is then dissipated as heat. The exact frequency used is slightly away from the frequency at which maximum dielectric loss occurs in water to ensure that the microwaves are not all absorbed by the first layer of water they encounter, therefore allowing more even heating of the food.



Ferroelectrics

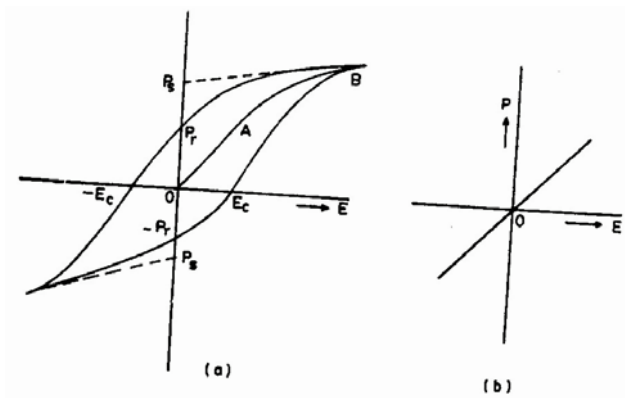
Below certain temperature it is found that some materials spontaneously acquire an electric dipolemoment. These materials are called as ferroelectric materials or ferroelectrics.The temperature at which ferroelectric property of the material disappears is called as ferroelectric Curie temperature.

Ferroelectric materials are anisotropic crystals which exhibit a hysteresis curve P versus E which can be explained by domain hypothesis.

Ferro electricity: Ferro electric materials are an important group not only because of intrinsic Ferro electric property, but because many possess useful piezo electric, birefringent and electro optical properties.

The intrinsic Ferro electric property is the possibility of

reversal or change of orientation of the polarization direction by an electric field. This leads to hysteresis in the polarization P, electric field E relation , similar to magnetic hysteresis. Above a critical
temperature, the Curie point T_c, the spontaneous polarization is destroyed by thermal disorder. The permittivity shows a characteristic peak at T_c.



Piezo – Electric Materials and Their Applications: Single crystal of quartz is used for filter, resonator and delay line applications. Natural quartz is now being replaced by synthetic material.

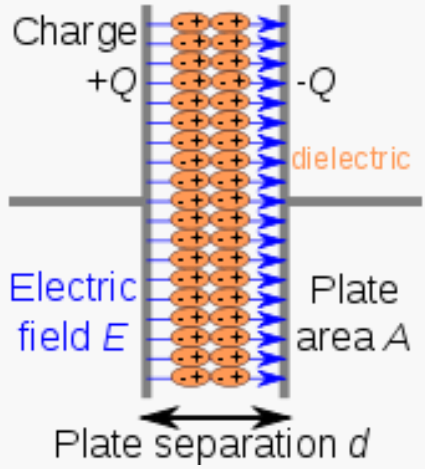
Rochelle salt is used as transducer in gramophone pickups, ear phones,
hearing aids, microphones etc. the commercial ceramic materials are based on barium titanate, lead zirconate and lead titanate. They are used for high voltage generation (gas lighters), accelerometers, transducers etc. Piezo electric semiconductors such as GaS, ZnO & CdS are used as amplifiers of ultrasonic waves.

Applications of Dielectric Materials:

Almost any type of electrical equipment employs dielectric materials in some form or another. Wires and cables that carry electrical current, for example, are always coated or wrapped with some type of insulating (dielectric) material. Sophisticated electronic equipment such as rectifiers, semiconductors, transducers, and amplifiers contain or are fabricated from dielectric materials. The insulating material sandwiched between two conducting plates in a capacitor is also made of some dielectric substance.

Liquid dielectrics are also employed as electrical insulators. For example, transformer oil is a natural or synthetic substance (mineral oil, silicone oil, or organic esters, for example) that has the ability to insulate the coils of a transformer both electrically and thermally.

1. **Capacitors**



Charge separation in a parallel-plate capacitor causes an internal electric field. A dielectric (orange) reduces the field and increases the capacitance.

Commercially manufactured capacitors typically use a solid dielectric material with high permittivity as the intervening medium between the stored positive and negative charges. This material is often referred to in technical contexts as the *capacitor dielectric*.

The most obvious advantage to using such a dielectric material is that it prevents the conducting plates, on which the charges are stored, from coming into direct electrical contact. More significantly, however, a high permittivity allows a greater stored charge at a given voltage. This can be seen by treating the case of a linear dielectric with permittivity ε and thickness d between two conducting plates with uniform charge density σ_ε. In this case the charge density is given by

$$\sigma_\varepsilon = \varepsilon \frac{V}{d}$$

and the capacitance per unit area by

$$c = \frac{\sigma_\varepsilon}{V} = \frac{\varepsilon}{d}$$

From this, it can easily be seen that a larger ε leads to greater charge stored and thus greater capacitance.

Dielectric materials used for capacitors are also chosen such that they are resistant to ionization. This allows the capacitor to operate at higher voltages before the insulating dielectric ionizes and begins to allow undesirable current.

2. **Dielectric resonator**

A *dielectric resonator oscillator* (DRO) is an electronic component that exhibits resonance of the polarization response for a narrow range of frequencies, generally in the microwave band. It consists of a "puck" of ceramic that has a large dielectric constant and a low dissipation factor. Such resonators are often used to provide a frequency reference in an oscillator circuit. An unshielded dielectric resonator can be used as a Dielectric Resonator Antenna (DRA).

3. **Insulators-**

Required Qualities of Good Insulating Materials: The required qualities can be classified as under electrical, mechanical, thermal and chemical applications.

i) Electrical: 1. electrically the insulating material should have high electrical resistivity and high dielectric strength to withstand high voltage.

2 .The dielectric losses must be minimum.

3. Liquid and gaseous insulators are used as coolants. For example transformer oil, hydrogen and helium are

ii) Mechanical: 1. insulating materials should have certain mechanical properties depending on the use to which they are put.

2. When used for electric machine insulation, the insulator should have sufficient mechanical strength to withstand vibration.

iii) Thermal: Good heat conducting property is also desirable in such cases. The insulators should have small thermal expansion and it should be non-ignitable.

iv) Chemical: 1. chemically, the insulators should be resistant to oils, liquids, gas fumes, acids and alkali's.

2. The insulators should be water proof since water lowers the insulation resistance and the dielectric strength.

Other Applications:

Solid dielectrics are perhaps the most commonly used dielectrics in electrical engineering, as very good insulators. Some examples include porcelain, glass, and most plastics.
Air, nitrogen and sulfur hexafluoride are the three most commonly used gaseous dielectrics.

Industrial coatings such as parylene provide a dielectric barrier between the substrate and its environment.

Mineral oil is used extensively inside electrical transformers as a fluid dielectric and to assist in cooling. Dielectric fluids with higher dielectric constants, such as electrical grade castor oil, are often used in high voltage capacitors to help prevent corona discharge and increase capacitance.

Because dielectrics resist the flow of electricity, the surface of a dielectric may retain stranded excess electrical charges. This may occur accidentally when the dielectric is rubbed (the triboelectric effect). This can be useful, as in a Van de Graaff generator or electrophorus, or it can be potentially destructive as in the case of electrostatic discharge.

Piezoelectric materials are another class of very useful dielectrics which are used for transducers and sensors.

Ferroelectric materials often have very high dielectric constants, making them quite useful for capacitors.

Magnetic Materials

Classification of dia, para and ferromagnetic materials. Curie Temperature Hysterisis in ferromagnetic materials. Soft and Hard magnetic materials. Applications.

Introduction

The materials that can be magnetised are called as magnetic materials

Magnetic dipoles and magnetic dipole moment

Any two opposite poles separated by distance constitute an magnetic dipole. A magnet is a dipole which has north pole and south pole and the length of the magnet is the distance of separation.

Magnetic dipole moment is the product of magnetic pole strength (m) and length of the magnet(l)

$$\mu_m = m.l$$

Magnetic field intensity (H)

The force experienced by a unit north pole (of strength 1 Wb) placed at a point in a magnetic field is a measure of the` field intensity' or `field strength'

Magnetisation or Intensity of magnetisation (M)

Magnetization may be defined as the process of converting a non magnetic bar into a magnetic bar

Magnetic Induction Or Flux Density*(B)*

Magnetic induction or magnetic flux density in an any material is the number of lines of magnetic force passing through unit area perpendicular . Wb/m^2

$$B = \frac{\phi}{A} = \mu_0 (M + H)$$

Magnetic Susceptibility *(χ)*

The ratio of the magnetization to the field strength

$$\chi = \frac{M}{H}$$

Permeability ***(μ)***

The ration of the amount of magnetic density B to the applied magnetic field . It is used to measure magnetic lines of forces passing through the material

$$\mu = \frac{B}{H}$$

<u>Origin of Magnetism:</u>

Magnetism originates from magnetic dipole moment. This magnetic dipole moment arises due to the rotational motion of charged particles.

<u>According to modern view;</u>

- All substances are made of atoms or molecules. An atom which consists of '+' vely charged nucleus at the centre and negatively charged electrons revolving around the nucleus in different orbits. This motion of electrons is called orbital motion as shown in fig. The orbiting elections constitute tiny current loops. These loops behave as the magnetic dipoles.
- The orbital motion of electrons around the nucleus gives rise to the orbital magnetic dipole moment (μ_{orbit}).
- The electrons also rotate around their own axes. This motion of electrons is called spin motion as shown fig.
  The spinning motion of electrons around their axes gives rise to the spin magnetic dipole m oment (μ_{spin}).
- The motion of the protons and neutrons within the nucleus also contributes to the total magnetic moment (($\mu_{nucleus}$). But the magnitude of the nuclear magnetic moment is (about 10^{-3} times) very small compared with the magnetic moment of electron and is usually neglected.

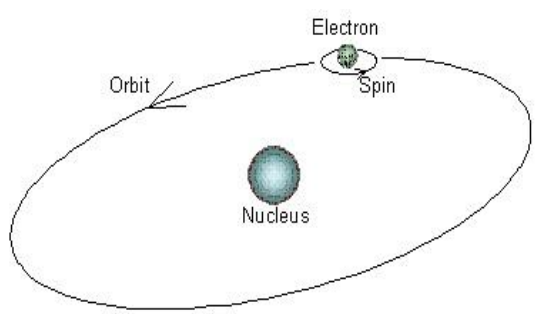


Fig: Motion of electron

<u>Orbital magnetic dipole moment of electron (μ_{orbit}):</u>

The magnetic dipole moment arises due to the orbital motion of electrons around the nucleus is called orbital magnetic dipole moment (μ_{orbit}) .

$$\boxed{\mu_{Orbit,z} = -\mu_B . m_l}$$

<u>Proof:</u>

Let us consider an electron of mass **'m'** and charge e revolving around the nucleus in a circular orbit o radius **'r'** with linear velocity **'v'** as shown in fig.

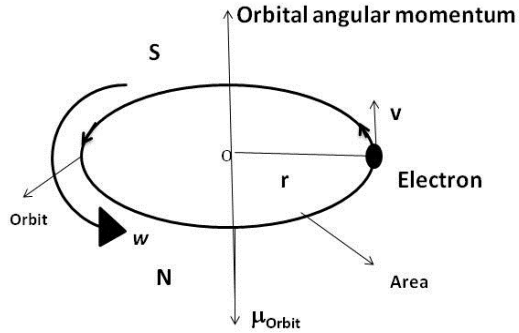


Fig : Orbital motion of electron

The revolving electron in circular orbit establishes a current is given by

$$I = \frac{\text{Charge of electron}}{\text{time period}} = \frac{-e}{T} \longrightarrow (1)$$

Where 'T' is the time taken by the electron to make one revolution around the nucleus

$$\text{i.e., } T = \frac{2\pi}{w} \longrightarrow (2)$$

Where *'w'* is the angular frequency of the electron

But relation between linear velocity 'v' and angular velocity can be written as

$$v = r\,w$$

$$\text{and } w = \frac{v}{r} \longrightarrow (3)$$

Substituting the equation (3) in (2),

$$T = \frac{2\pi r}{v} \longrightarrow (4)$$

Further, substituting the equation (4) in (1),

$$I = \frac{-ve}{2\pi r} \longrightarrow (5)$$

The current 'I' establishes a magnetic field around the circular orbit, so that the upper surface acts as South Pole and the lower surface acts as North Pole.

The Area of the orbit is **A** = $\pi r^2 \longrightarrow$ (6)

Then the corresponding magnetic dipole moment is given by

$$\mu_{Orbit} = IA$$

$$= \frac{-ve}{2\pi r} x \pi r^2$$

$$= \frac{-evr}{2} \longrightarrow (7)$$

Dividing and multiplying the equation (7) by the mass "m' of electron.

$$\mu_{Orbit} = \frac{-evr}{2} x \frac{m}{m}$$

$$= \frac{-e(mvr)}{2m}$$

$$= \frac{-e(\vec{L})}{2m} \qquad \text{(But } \vec{L} = m\vec{v}r\text{)}$$

$$\boxed{\mu_{Orbit} = \frac{-e}{2m}\vec{L} \text{ (Orbital angular momentum)}}$$

The – ve sign indicates that the orbital angular momentum and orbital magnetic dipole moment are in opposite directions.

*An electron has an intrinsic orbital angular momentum (**L**) itself cannot be measured. However, its component along any axis can be measured.*

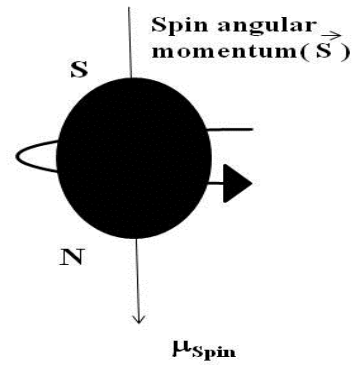


Fig: Spin motion of electron

Let us assume that the component of orbital angular momentum (**L**) is measured along the z- axis of a coordinate system. Then the measured component $\mathbf{L_Z}$ can have only the values is given by

$$\mathbf{L_Z = m_l \frac{h}{2\pi}}$$

Where m_s is called orbital magnetic quantum number = $0, \pm1, \pm2, \pm3$ ....

The orbital magnetic dipole moment (μ_{Orbit}) of electron itself also cannot be measured. Only; its component along any axis can be measured.

Let us assume that the component of orbital magnetic dipole moment (μ_{Orbit}) of electron is measured along the z- axis of a coordinate system. Then the measured component μ_{orbit}, **z** can have only the two values is given by

$$\mu_{Orbit,z} = \frac{-e}{2m} \mathbf{L_z}$$

$$\mu_{Orbit,\,z} = -\left(\frac{e}{2m}\right) \mathbf{m_l} \frac{h}{2\pi}$$

$$\mu_{Orbit,\,z} = -\left(\frac{eh}{4\pi m}\right) \mathbf{m_l}$$

$$\boxed{\mu_{Orbit,z} = -\mu_B . m_l}$$

Where $\mu_B = \frac{eh}{4m\pi}$ is known as Bohr magneton and its value is 9.27x 10-^{24} A-m^2.

<u>Spin magnetic dipole moment of electron (μ_{spin}):</u>

The magnetic dipole moment arises due to its spin motion is called spin magnetic moment (μ_{spin}) and is given by

$$\mu_{Spin} = -g\left(\frac{e}{2m}\right)\vec{S} \text{ (Spin angular momentum)}$$

The – ve sign indicates that the spin angular momentum and spin magnetic dipole moment are in opposite directions.

Where g is called land's g factor or Spectroscopic splitting factor

$$g = \frac{1+J(J+1)+S(S+1)-L(L+1)}{J(J+1)}$$

g_l=1 for orbital motion

g_s =2 for spin motion

An electron has an intrinsic spin angular momentum($\vec{S}$) itself cannot be measured. However, its component along any axis can be measured..

Let us assume that the component of spin angular momentum ($\vec{S}$) is measured along the z- axis of a coordinate system. Then the measured component $\mathbf{S_Z}$ can have only the values is given by

$$\mathbf{S_Z = m_s \frac{h}{2\pi}}$$

Where m_s is called magnetic spin quantum number = $\pm\frac{1}{2}$

$m_s = +\frac{1}{2}$ for spin up and

$m_s = -\frac{1}{2}$ for spin down

The spin magnetic dipole moment (μ_{Spin}) of electron itself also cannot itself be measured. Only; its component along any axis can be measured. Let us assume that the component of spin magnetic dipole moment ((μ_{Spin}) of electron is measured along the z- axis of a coordinate system. Then the measured component μ_{spin}, **z** can have only the two values is given by

$$\mu_{Spin,\,z} = -g\left(\frac{e}{2m}\right) S_z$$

$$\mu_{Spin,\,z} = -g\left(\frac{e}{2m}\right) \mathbf{m_s} \frac{h}{2\pi}$$

$$\mu_{Spin,\,z} = -2\left(\frac{eh}{4\pi m}\right) \mathbf{m_s}$$

$$\boxed{\mu_{Spin,\,z} = -2\,(\mu_B)\,\mathbf{m_s}}$$

Where $\mu_B = \frac{eh}{4m\pi}$ is known as Bohr magneton and its value is 9.27x 10-^{24} A-m^2.

<u>Nuclear spin magnetic dipole moment ($\mu_{Nuclear}$):</u>

The atomic nucleus contains protons and neutrons. They have intrinsic spin.

The spin motion of the protons and neutrons within the nucleus also contributes to the total spin magnetic dipole moment and is given by

$$\mu_{nuclear,\,spin} = \frac{eh}{4\pi M_N} = 5.525X1O^{-27}\ A\text{-}m^2$$

Where M_N is the Mass of the proton

But the magnitude of the nuclear magnetic dipole moment is(about 10 $^{-3}$ times) very small compared with the magnetic dipole moment of electron and is usually neglected.

Therefore, the magnetism mainly arises due to the orbital and spin magnetic dipole moments of electron.

Bohr Magneton

The orbital angular momentum of an electron in an atom can be expressed in terms of atomic unit of magnetic moment called Bohr Magneton.

$$\mu_B = \frac{eh}{4\pi m}$$

Classification of the magnetic materials

The magnetic materials are broadly classified in to two types. They are 1. Those atoms or molecules do not have permanent dipole moments and 2. Those atoms or molecules have permanent dipole moments even in the absence of external magnetic field.

Based on the magnetic moments the materials are classified as

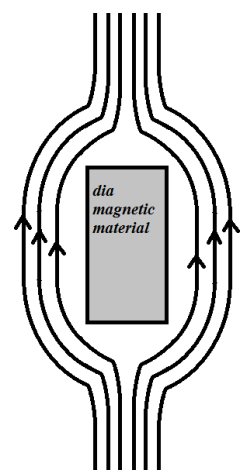
Diamagnetic materials
Paramagnetic materials
Ferromagnetic materials
Anti ferromagnetic materials
Ferri magnetic materials

Diamagnetic materials

Dia magnetic materials has completely filled sub shell electronic structure resultant magnetic moment is zero. There are no permanent dipoles and hence the magnetic effect are small. When a diamagnetic material is placed in a magnetic field, there will be a small induced magnetic moment which always oppose the applied field(accordance with Lenz`s Law). Due to this effect the magnetic lines of forces expelled from the materials.

Mostly the covalent and ionic crystals exhibits the diamagnetic properties. The magnetic susceptibility is small and negative and is independent of temperature**.**

The examples of diamagnetic materials are 1. Covalent materials such as Si,Ge, diamond, ii) some metals such as copper, silver, gold.

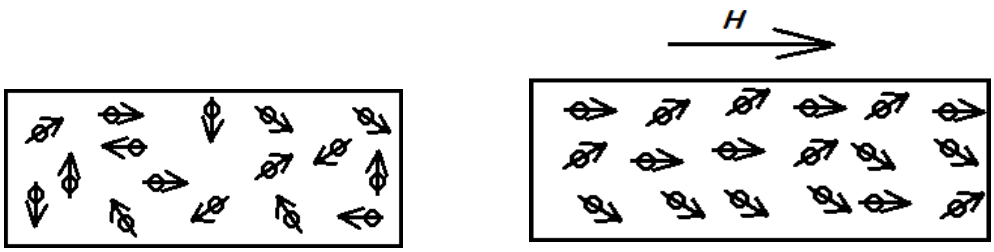


Para magnetic materials.

Atoms or molecules of paramagnetic materials have permanent magnetic moment oriented in random direction. The magnetic interaction between the dipoles try to align themselves but the thermal agitation disturb the alignment**.** In paramagnetic materials vector sum of magnetic moments is zero in the absence of field.

When an external magnetic field is applied the partial alignment of permanent atomic magnetic moments occur

When a magnetic field is applied , the individual magnetic moment takes the alignment along the applied field as shown in figure . The magnetization of a paramagnetic material increases with the increase in the applied field. Increase in temperature it reduces the magnetization and destroys the alignment of dipoles with applied field.



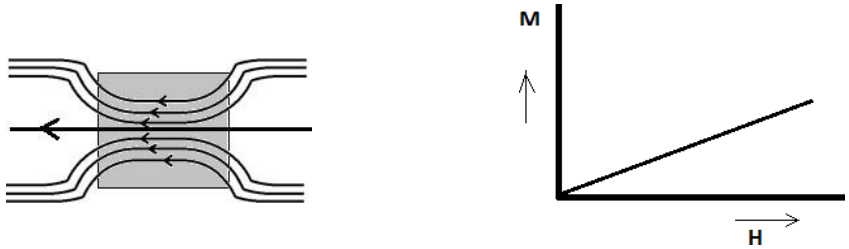
Consider a paramagnetic material placed in non-uniform magnetic field. The paramagnetic materials experience a net magnetic force towards the greater field. The magnetic susceptibility is small and positive and is dependent on temperature**.** The susceptibility of the magnetic field is given by

$$\chi = \frac{C}{T}$$

Where C is the curie temperature and T is the temperature in Kelvin scale.

The magnetization in ferromagnetic material is linear and gets saturated when a large magnetic field is applied at low temperature.

The examples of paramagnetic materials are Mg, gaseous and liquid oxygen , ferromagnetic material (Fe), and anti-ferromagnetic materials at high temperature and



ferromagnetic material (Fe_3O_4) at high temperature.

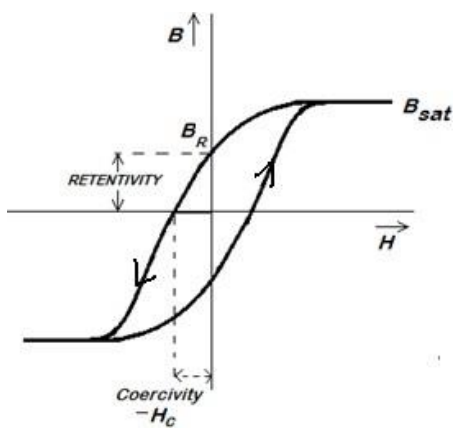
Ferromagnetic materials

Atoms or molecules of ferromagnetic materials have permanent magnetic moment. In ferromagnetic materials all the dipoles are aligned parallel as shown in the figure if a small value of magnetic field is applied, a large value of magnetization is produced. As the ferromagnetic material have permanent magnetic dipole moment and the susceptibility is positive. The magnetization in ferromagnetic material is non linear and gets saturated when a large magnetic field is applied.

A ferromagnetic materials exhibits two different properties. It behaves as a ferromagnetic material below a certain temperature known as ferromagnetic ***curie temperature***. Above the temperature it behaves as a paramagnetic material. In the ferromagnetic region, it exhibits well known curve known as hysteresis curve as shown in the figure.

The susceptibility of a ferromagnetic material above the ferromagnetic curie temperature is given by $\chi = \dfrac{C}{T - \theta_f}$

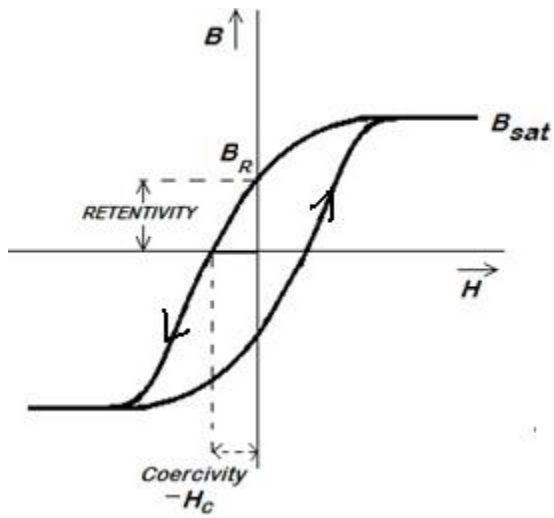
Where C is the Curie constant and θ_f is the ferromagnetic Curie temperature.The transition and rare earth metals such as Fe ,Co, Ni ,Gd are the examples of ferromagnetic materials.



Hysteresis in ferromagnetic materials (B-H curve)

Below the ferromagnetic Curie temperature (T < θ_f) Ferromagnetic material exhibits a well known curve called hysteresis curve. The variation of B(magnetic induction) with H(applied field) can be represented by a closed curve called hysteresis loop or curve. This refers lagging of magnetization behind the magnetising field.

If a magnetic field is increased gradually, the flux density increases and it becomes maximum. The maximum value of flux density is called saturated magnetization.If the field is reversed, the ferromagnetic materials is found to have magnetization in the absence opf external field. This is called as retentivity or remanent magnetization (B_R) and this property is called as spontaneous magnetization. If the field is further reduced the flux density reduces to zero. The field requird in the opposite direction to bring magnetization to zero is called as coercive field or coercivity ($-H_c$). If the field is increased in oposite direction it attains saturation magnetization. If an alternating field is applied a closed loop as shown in the figure is obtained.



According to Weiss, a virgin specimen of ferromagnetic material consist of number of regions or domains($\approx 10^{-6}$ m or above) which are spontaneously magnetized. When magnetic field is not applied the direction of spontaneous magnetization varies from domain to domain. The resultant magnetization may hence be zero or nearly zero.

These domains are separated from other by a wall known as domain wall or Bloch wall The domain concept is used to explain the hysteresis property. When an external field is applied two possible ways of alignment domain growth are possible one by domain wall motion and other by rotation of domain wall and domain growth is also reversible. Hysteresis curve is explained by domain concept.

Antiferromagnetic matériels : These are the ferromagnetic materials in which equal no of opposite spins with same magnitude such that the orientation of neighbouring spins is in antiparallel manner are present.

Susceptibility is small and positive and it is inversely proportional to the temperature.

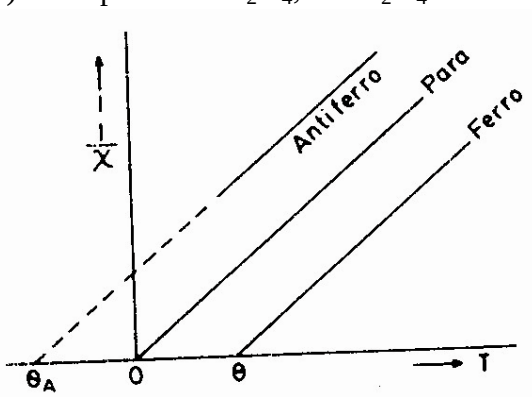
$$\chi = C/(T+\theta)$$

the temperature at which anti ferromagnetic material converts into paramagnetic material is known as Neel's temperature.

Examples: FeO, Cr_2O_3.

Ferrimagnetic materials: These are the ferromagnetic materials in which equal no of opposite spins with different magnitudes such that the orientation of neighbouring spins is in antiparallel manner are present.
Susceptibility positive and large, it is inversely proportional to temperature

$$\chi = C/(T \pm \theta) \qquad T > T_N$$ (Neel's temperature) Examples : $ZnFe_2O_4$, $CuFe_2O_4$



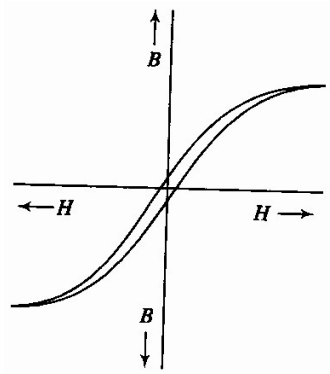
Soft and Hard magnetic materials

Soft magnetic materials

The magnetic materials that are easy to magnetize and demagnetize are called as soft magnetic materials.

Properties:

1. Low remanent magnetization
2. Low coercivity
3. Low hysteresis energy loss
4. Low eddy current loss
5. High permeability
6. High susceptibility



HYSTERESIS CURVE FOR SOFT MAGNETIC MATERIAL

Examples of soft magnetic materials are

i) Permalloys (alloys of Fe and Ni)
ii) Si – Fe alloy
iii) Amorphous ferrous alloys (alloys of Fe, Si, and B)
iv) Pure Iron (BCC structure)

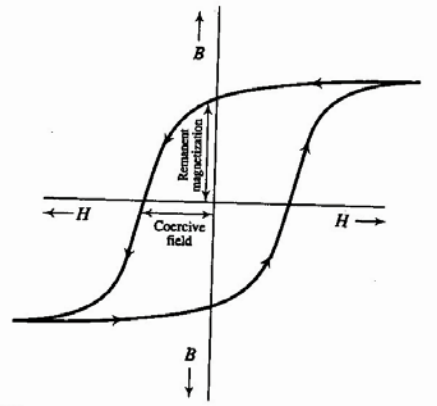
Applications of soft magnetic materials: Mainly used in electro- magnetic machinery and transformer cores. They are also used in switching circuits, microwave isolators and matrix storage of computers.

Hard magnetic materials

The magnetic materials that are difficult to magnetize and demagnetize are called as hard magnetic materials.

Properties:

1. High remanent magnetization
2. High coercivity
3. High saturation flux density
4. Low initial permeability
5. High hysteresis energy loss
6. High permeability



HYSTERESIS CURVE FOR HARD MAGNETIC MATERIAL

7. The eddy current loss is low for ceramic type and large for metallic type.

Examples of hard magnetic materials are, i) Iron- nickel- aluminum alloys with certain amount of cobalt called Alnico alloy. ii) Copper nickel iron alloys. iii) Platinum cobalt alloy.

Applications of hard magnetic materials: For production of permanent magnets, used in magnetic detectors, microphones, flux meters, voltage regulators, damping devices and magnetic separators.

Hard magnetic materials	**Soft magnetic materials**
Difficult to magnetize and demagnetize	Easy to magnetize and demagnetize
large hysteresis loop area	small hysteresis loop area
Have large hysteresis loss	Have very low hysteresis loss
The domain wall movement is difficult and it is irreversible in nature	Domain wall movement is relatively easier. Even for small change in the magnetizing field ,magnetization changes by large amount
The coercivity and retentivity are large	The coercivity and retentivity are small
Magnetostatic energy is large	Magnetostatic energy is small.
Small values of permeability and susceptibility	Large values of permeability and susceptibility
Used to make permanent magnets Examples- *Iron-nickel-aluminium alloys (alnicol)* *Copper nickel iron (cunife)*	Used to make electromagnet Examples- *Fe-Si , Ferrous nickel alloys ,Ferrites,Garnets*

Sample Questions:

Dielectric Materials:

1. Explain the phenomenon of electric polarization in dielectric materials?.
2. Describe in brief
   (a)electronic polarization
   (b) ionic polarization
   (c) Orientation polarization
   (d) Space charge polarisation with diagrams. Also find the total polarizability.
3. Differentiate between polar and nonpolar dielectrics.
4. Drive an expression for internal field in solids and liquids. Or Derive an experession for for internal field by Lorentz method?
5. Derive Clausius-Mossoti equation.
6. What are ferroelectric materials? Describe in detail the Ferroelectric hysteresis.
7. Applications of Dielectric materials.
8. Derive the relation between dielectric polarization and dielectric constant?
9. What is its dielectric constant? Explain its importance in dielectric materials?
10. Which of the following properties are typical for dielectrics?
11. Write a note on
    (a) Ferroelectricity (b) Dielectric constant (c) internal field in solids
12. Discuss the frequency dependence of various polarization processes in dielectric materials.
13. What is dielectric loss? And also explain *Dielectric Breakdown*.
14. Describe the frequency dependence of dielectric constant.
15. Explain the applications of dielectric materials as insulators.

Magnetic Materials:

1. Define magnetization and show that B= $\mu_0(H+M)$
2. Explain the classification of magnetic materials in detail?
3. Give important features of ferromagnetic materials. Explain the hysteresis curve on the basis of domains.
4. What is hysteresis loss? Explain.
5. Distinguish between soft and hard magnets.
6. What is ferromagnetic Curie temperature? Discuss the behaviour of a ferromagnetic material below the Curie temperature.
7. What are ferrites? Explain the magnetic properties of ferrites and mention their industrial applications.
8. Write the importance of hard magnetic materials in engineering applications?
9. What are the applications of soft and hard magnetic materials?

Important formulae:

1. Dipole moment, $\mu = ql$.

2. Flux density, $D = \varepsilon_0 \varepsilon_r E$.

3. $$D = \varepsilon_0 E + P.$$

4. Dielectric susceptibility, $\chi = (\varepsilon_r - 1)$.

5. Polarization, $P = \varepsilon_0(\varepsilon_r - 1)E$, $P = N\alpha E$.

6. Induced dipole moment, $\mu = \alpha E$; $\mu_e = \alpha_e E$.

7. Electronic polarizability, $\alpha_e = \dfrac{\varepsilon_0(\varepsilon_r - 1)}{N}$.

8. Orientational polarizability, $\alpha_o = \dfrac{\mu^2}{3kT}$.

9. Internal field, $E = E + \left(\dfrac{\gamma}{\varepsilon_0}\right)P$

10. Lorentz field, $E_{\text{Lorentz}} = E + \dfrac{P}{3\varepsilon_0}$

11. Clausius - Mossotti relation

$$\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \frac{N\alpha_e}{3\varepsilon_0}$$

12. The energy loss in a dielectric material is due to the phase lag of voltage behind the current under a.c. conditions. It is also called tangent loss given by, $\tan\delta = \dfrac{\varepsilon_r''}{\varepsilon_r'}$.

13. Curie-Weiss law, $\varepsilon_r = \dfrac{C}{T - \theta}$ for $T > T_c$.

2. $B = \mu_o \mu_r H$, $\mu_o = 4\pi \times 10^{-7}$ H/m.

3. $\chi = \dfrac{M}{H}$.

4. $B = \mu_o (H + M)$.

5. Curie Weiss law, $\chi = \dfrac{C}{T - \theta}$ for $T > \theta$.

Problems

An air-filled parallel plate capacitor has a capacitance of 1.5 pF. If the separation between the plates is doubled and wax is inserted between them, the capacitance increases to 3 pF. Compute the dielectric constant of wax.

Data :

Capacitance when air is dielectric, $C_1 = 1.5 \text{ pF} = 1.5 \times 10^{-12}$ F.

Capacitance when wax is dielectric, $C_2 = 3 \text{ pF} = 3 \times 10^{-12}$ F.

To find :

For wax, $\varepsilon_r = ?$

Solution :

Let the capacitance be C_1 while air is the dielectric,

$$\therefore \quad C_1 = \frac{\varepsilon_0 \varepsilon_r s}{d}.$$

But for air, $\varepsilon_r = 1$.

$$\therefore \quad 1.5 = \frac{\varepsilon_0 \times 1 \times s}{d} \qquad \text{.....(1)}$$

When d is doubled and wax is inserted,

let the capacitance be C_2.

$$\therefore \quad C_2 = \frac{\varepsilon_0 \varepsilon_r s}{2d}.$$

$$\text{or,} \quad 3 = \frac{\varepsilon_0 \varepsilon_r s}{2d}. \qquad \text{.....(2)}$$

Dividing Eq(2) by Eq(1), we have,

$$\frac{3}{1.5} = \frac{\varepsilon_0 \varepsilon_r s}{2d} \times \frac{d}{\varepsilon_0 s}.$$

$$\therefore \quad \varepsilon_r = 4.$$

$\therefore$ **Dielectric constant of wax is 4.**

2. **A parallel plate capacitor has an area of 7.45×10^{-4} m^2 and its plates are separated by a distance of 2.45×10^{-3} m across which a potential of 10 V is applied. If a material with dielectric constant 6 is introduced between the plates, determine the capacitance, the charge stored in each plate, the dielectric displacement D and the polarization.**

Data :

Area of the plates, $s = 7.45 \times 10^{-4}$ m^2.

Distance of separation of the plates, $d = 2.45 \times 10^{-3}$ m.

Applied potential, $V = 10$ V.

Dielectric constant of the material, $\varepsilon_r = 6$.

To find :

Capacitance of the capacitor, $C = ?$,

Displacement, $D = ?$

Charge stored, $Q = ?$

Polarization, $P = ?$

Solution :

We have for capacitance of a capacitor,

$$C = \frac{\varepsilon_0 \varepsilon_r s}{d} = \frac{8.85 \times 10^{-12} \times 6 \times 7.45 \times 10^{-4}}{2.45 \times 10^{-3}},$$

$$= 16.147 \times 10^{-12} \text{ F} = 16.147 \text{ pF}.$$

Charge stored on the plates,

$$Q = CV = 16.147 \times 10^{-12} \times 10,$$

$$= 16.147 \times 10^{-11} \text{ C}.$$

Polarization is given by,

$$P = \varepsilon_0 (\varepsilon_r - 1) E.$$

But,

$$E = \frac{V}{d}.$$

$\therefore$

$$P = \varepsilon_0 (\varepsilon_r - 1) \frac{V}{d},$$

$$= \frac{8.85 \times 10^{-12} (6-1) 10}{2.45 \times 10^{-3}}$$

$$= 1.806 \times 10^{-7} \text{ Cm}^{-2}.$$

Displacement D is given by,

$$D = \varepsilon_0 \varepsilon_r E = \varepsilon_0 \varepsilon_r \frac{V}{d} = \frac{8.85 \times 10^{-12} \times 6 \times 10}{2.45 \times 10^{-3}},$$

$$= 2.167 \times 10^{-7} \text{ Cm}^{-2}.$$

$\therefore$ **The capacitance, the charge stored, the polarization and the displacement are 16.147 pF, 16.147×10^{-11} C, 1.806×10^{-7} Cm^{-2}, and 2.167×10^{-7} Cm^{-2} respectively.**

3. The dielectric constant of Sulphur is 3.4. Assuming a cubic lattice for its structure, calculate the electronic polarizability for Sulphur.
Given: Sulphur density= 2.07 gm/cc, and atomic weight =32.07

Given data:

Dielectric constant, $\varepsilon_r = 3.4$.

Density, $D = 2.07$ gm/cc $= 2.07 \times 10^3$ kg/m^3.

Atomic weight = 32.07.

Crystal structure of sulphur is cubic.

find :

Electronic polarizability for sulphur, $\alpha_e = ?$

Solution :

Since the crystal structure of sulphur is cubic we can apply Clausius - Mossotti equation,

$$\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \frac{N\alpha_e}{3\varepsilon_0}.$$

$$\therefore \quad \alpha_e = \frac{3\varepsilon_0}{N}\left[\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right]. \quad \text{.....(1)}$$

Now, N the number off atoms/unit volume can be written as,

$$N = \frac{N_A \times 10^3 \times D}{\text{atomic weight}}.$$

$$\therefore \quad N = \frac{6.025 \times 10^{26} \times 2.07 \times 10^3}{32.07}.$$

$$\therefore \quad N = 3.89 \times 10^{28} / \text{m}^3.$$

By substituting the values for various parameters, Eq(1) becomes,

$$\alpha_e = \frac{3 \times 8.854 \times 10^{-12}}{3.89 \times 10^{28}}\left[\frac{3.4 - 1}{3.4 + 2}\right]$$

$$\therefore \quad \alpha_e = 3.035 \times 10^{-40} \text{ Fm}^2.$$

$\therefore$ **Electronic polarizability of sulphur is 3.035×10^{-40} Fm^2.**

4. **Find the polarization produced in a dielectric medium of relative permittivity 15 in presence of an electric field of 500 V/m.**

Given data:

Relative permittivity, $\varepsilon_r = 15$.

Strength of electric field, $E = 500$ V/m.

find :

Polarization, $P = ?$

We have,

$$P = \varepsilon_0(\varepsilon_r - 1)E,$$

$$= 8.854 \times 10^{-12}(15-1)500,$$

$$= 6.198 \times 10^{-8}\ \text{cm}^{-2}.$$

Polarization produced in the medium is 6.198×10^{-8} cm^{-2}.

1. In a magnetic material, the field strength is 10^6 Am^{-1}. The magnetic susceptibility of the material is $0.5X10^{-5}$. Calculate the intensity of magnetization and the flux density of the material.

$$\chi = \frac{M}{H}$$

$$M = \chi H = 0.5 \times 10^{-5} \times 10^6 = 5$$

$$B = \frac{\phi}{A} = \mu_0 (M + H)$$

$$= 4\pi \times 10^{-7}(5 + 10^6)$$

$$= 1.257 Wbm^{-2}$$

2. If a magnetic field of 1800Am^{-1} produces a magnetic field 3 x 10^{-5} Wb in an iron barof cross sectional area 0.2cm^2, Calculate permeability.

$$B = \frac{\phi}{A} = 1.5 Wbm^{-2}$$

$$B = \mu_0 \mu_r H$$

$$\mu_r = \frac{B}{H\mu_0} = \frac{1.5}{4\pi \times 10^{-7} \times 1800} = 663.14$$

3. Calculate the saturation magnetization for Ni ferrite. The lattice parameter of a cubic unit cell of Ni ferrite is 0.835nm and the magnetic moment per unit cell is 18.4μ_B .

($\mu_B = 9.27 \times 10^{-24}$)

$$M = \frac{Magneticmoment}{volume} = \frac{18.4 \times 9.27 \times 10^{-24}}{(0.8353 \times 10^{-9})^3} = 2.929 \times 10^5$$

HYSTERISIS:-

When a Ferro magnetic substance (e.g. iron) is subjected to a cycle of magnetization, it is found that flux density B in the material lags behind the applied magnetizing force H. This phenomenon is known as hysteresis.

The term hysteresis is derived from the Greek word hysterein meaning to lag behind.

Hysterisis loop:

- If a piece of ferromagnetic material is subjected to one cycle of magnetization, the resultant B-H curve is a closed loop "a b c d e f a" is Called hysteresis loop.
- Consider an iron-cored toroid carrying current I. If N is the total number of turns and ℓ the length of toroid, then magnetizing force is H= NI/ℓ .The value of H can be changed by varying current in the coil.

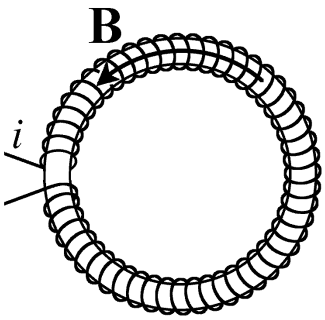


Fig: Toroid

i. To start with, the toroid is unmagnetised and its situation is represented by point

ii. in fig. As H is increased (by increasing current I), B increases along oa and reaches its saturation value B_{max} at a .At this stage, all the domains are aligned.

iii. If now H is gradually reduced by decreasing the current in the toroid, it is found that curve follows the path ab instead of ao .At point b, H=0 but flux density in the material has a finite value $+B_r$ (=ob) called *residual flux density*. It is also called **remanence** or **retentivity.** Note that *B* lags behind H. This effect is called *hysteresis.*

iv. In order to reduce flux density in the material to zero, it is necessary to apply *H* in the reverse direction. This can be done by reversing the current I the toroid .When *H* is gradually increased in the reverse direction, he curve follows the path bc. At point c, B=0 and H= $-H_C$ .The value of *H* needed to wipe out residual magnetism is called coercive force (H_C).

v. Now *H* is further increased in the reverse direction until point d is reached where the sample is saturated in the reverse direction ($-B_{max}$).If *H* is now reduced to zero point e is reached and the sample again retains magnetic flux density ($-B_r$).The remaining part of the loop is obtained by increasing current to produce *H* in the original direction. The curve "*a b c d e f a"* is called hysteresis loop. Thus hysteresis loop results because the domains do not become completely unaligned when *H* is made zero. The area enclosed by the hysteresis loop represents loss in energy. This energy appears in the material as heat.

vi. Based on the area of the hysteresis loop, the magnetic materials are classified into soft and hard magnetic materials.

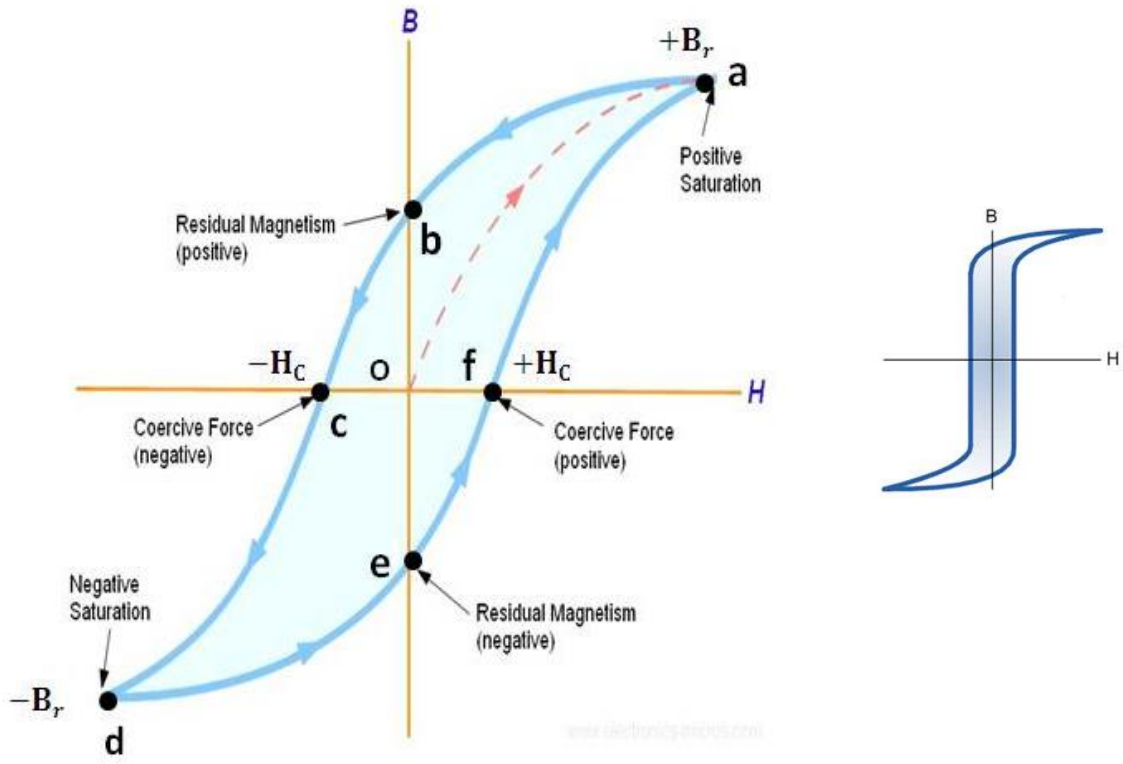


Fig: B-H Curve

Soft magnetic materials:-

The materials which can be easily magnetized and demagnetized are called Soft magnetic materials.

Properties:

- They can be easily magnetized and demagnetized and hence they show high values of susceptibility and permeability.
- Movement of domain wall is easy and hence even for small applied field large magnetization occurs.
- The nature of hysteresis loop is very narrow
- The hysteresis loop area is very small hence the hysteresis loss is also small as shown in fig.
- The coercivity and retentivity values are small
- These materials are free from irregularities or impurities or imperfections
- **Examples:**
  Fe- Si alloys, Ni-Fe alloys, Fe-Co alloys, Ferrities and Garnets etc
- **Applications:**
  - They are used in switching devices, electromagnets,
  - They are used in matrix storage of computers.
  - They are used in motors, relays and sensors
  - They are used to make the temporary magnets.

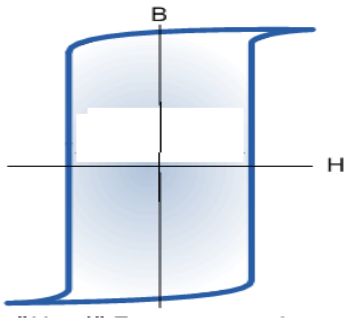
Hard magnetic materials:-

The materials which can't be easily magnetized and demagnetized are called hard

magnetic materials.

Properties:

- They can't be easily magnetized and demagnetized and hence they show low values of susceptibility and permeability.
- Movement of domain wall is not easy due to presence of impurities and hence large magnetic field is required for magnetization
- The nature of hysteresis loop is very broad.
- The hysteresis loop area is large hence the hysteresis loss is also large as shown in fig.
- The coercivity and retentivity values are high
- These materials are have irregularities or impurities or imperfections
- **Examples:**
  Carbon steel, tungsten steel, chromium steel,
  Cu-Ni-Fe alloys
  Cu-Ni-Co alloys
  Al-Ni-Co alloys
- **Applications:**
  - They are used in magnetic detectors ,
  - They are used in microphones.
  - They are used in magnetic separators.
  - They are used to make the permanent magnets.

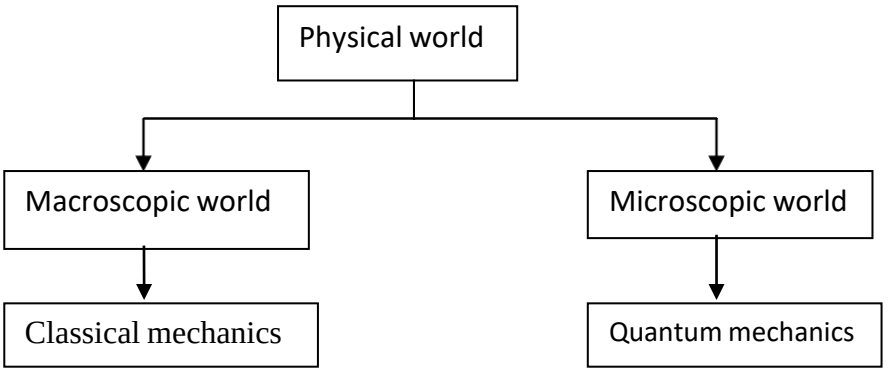


UNIT - IV

QUANTUM MECHANICS, FREE ELECTRON THEORY OF METALS AND BAND THEORY OF SOLIDS

QUANTUM MECHANICS

Introduction:



In the late seventeen century, Isaac Newton discovered classical mechanics, the law of motion of macroscopic objects.

In the early twentieth century, physicists found that classical mechanics does not correctly describe the behavior of very small particles such as the electrons nuclei (protons & neutrons etc.) and molecules. The behavior of such particles is described by a set of laws called **Quantum mechanics.**

I.e. Classical mechanics applies only to macroscopic particles. For microscopic "particles" .We require a new form of mechanics, called **Quantum mechanics.**

Classical mechanics:

- Classical mechanics deals with the motion and behavior of **macroscopic particles**.
  Ex: Motion of planets.

- Classical mechanics is expressed in the language of un-probabilities, certainties.

Quantum mechanics:

- Quantum mechanics deals with the motion and behavior of **microscopic particles**
  Ex: Motion of Electrons
- Quantum mechanics is expressed in the language of probabilities, not certain.

DUAL NATURE OF MATTER

de-Broglie hypothesis:

According to de-Broglie hypothesis, every moving particle has always associated with a wave. The particle is controlled by this wave in every aspect.

The wave associated with moving particle of matter (like electrons, protons etc) are known as matter wave or pilot waves or de-Broglie waves.

Figure .1 Motion of particle (Electron)

If a particle of mass *m* moving with velocity *v*, then the wavelength of matter wave is given by

$$\lambda = \frac{h}{mv}$$

Where

h→Planck's constant,

m →mass of particle,

v→velocity of the particle

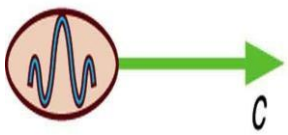
Derivation of de – Broglie's wave length:

According to Planck's quantum theory of radiation (light) the energy of photon is given by

$$E = h\nu \longrightarrow (1)$$

Where h -> Planck's constant,

ν -> frequency of radiation



According to Einstein's energy-mass relation, the energy of photon is given by

$$E = mc^2 \longrightarrow (2)$$

Where m-> mass of photon
C-> velocity of light or photon

Figure .2. Motion of photon

From the above equations (1) and (2); we can write

$h\nu = mc^2$ $[\because C = \nu\lambda \text{ and } \nu = \frac{c}{\lambda}]$

$$\frac{hc}{\lambda} = mc^2 \quad h = mc$$

$$\boxed{\lambda = \frac{h}{mc} = \frac{h}{p}} \longrightarrow (3)$$

Where p = mc is the momentum of photon having mass *m* and travelling with velocity *c*.

de - Broglie suggested that equation (3)can be applied both for photos and material particles. If m is mass of the particle and v is the velocity of the particle, then

Momentum (p) = mv

$$\boxed{\therefore \text{de – Broglie wave length } \lambda = \frac{h}{mv} \quad (4)} \longrightarrow$$

This equation represents that the de-Broglie wave length associated with a particle and also known as de –Broglie's wave equation.

<u>OTHER FORMS OF de –Broglie's WAVE LENGTH:</u>

<u>1. de –Broglie's wave length in terms of energy</u>

If m is mass of the particle and v is the velocity of the particle, then the kinetic energy of the particle;

$$E = \frac{1}{2} mv^2$$

Multiplying by 'm' on both sides; we get

$$m\,E = \frac{1}{2} m^2v^2$$

$$2\,m\,E = m^2v^2$$

(Or) $m\,v = \sqrt{2\,m\,E}$

$\therefore$ de – Broglie wave length $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2\,m\,E}}$ (5) $\longrightarrow$

2. *de –Broglie's wave length in terms of voltage*

If a charged particle of charge 'q' is accelerated through a potential difference 'v', then

$$E = q\,v \longrightarrow (6)$$

Also we know that the K.E of the particle $E = \frac{1}{2}mv^2$ (7) $\longrightarrow$

Equating equations (7)and (6) we get

$$\frac{1}{2}mv^2 = q\,v$$

Multiplying by 'm' on both sides we get

$$\frac{1}{2}m^2v^2 = m\,q\,v$$

$$\therefore m^2v^2 = 2\,mqv$$

$$m\,v = \sqrt{2\,m\,q\,v} \longrightarrow (8)$$

Substituting equation 8 in equation 4 , we get

$\therefore$ de – Broglie wave length $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2\,mqv}}$ (9) $\longrightarrow$

3.*de –Broglie's wave length for electron*

Substituting the values of m, q and h in equation 9 we get

$$\lambda = \frac{6.626\ X10^{-34}JS}{\sqrt{2\ X9.1\ X10^{-31}\ KgX1.6\ X10^{-19}CXV}}$$

$$= \frac{12.27\ X10^{-10}m}{\sqrt{V}} \qquad [\because 1\ A^0 = 10^{-10}m\ ]$$

$$\lambda = \frac{12.27}{\sqrt{V}}A^0$$

This equation represents the de – Broglie wave length associated with an electron.

4.*Wave velocity:*

We know that

$$E = h\nu \text{ and } E = mc^2$$

$$h\nu = mc^2$$

$$\nu = \frac{mc^2}{h}$$

The wave velocity is given by

$$\omega = \nu\lambda$$

$$\omega = \frac{mc^2}{h} \cdot \frac{h}{mv}$$

$$\therefore \omega = \frac{c^2}{V}$$

Thus matter waves travel faster than velocity of light.

Properties of de-Broglie waves:

The following are the properties of matter waves.

1. The de-Broglie wavelength of particle of mass (m) moving with a velocity (V) is given by $\lambda = \frac{h}{mv}$

If v = 0, λ□□ and if v□□, λ□0.

This implies that matter waves are associated with material particles only if they are in motion

2. Lesser the velocity of the particle, larger the wave length associated with it.

3. Smaller the mass of the particle, the greater the length associated with it.

4. The matter wave of particle is independent of charge (or) nature of the particle.

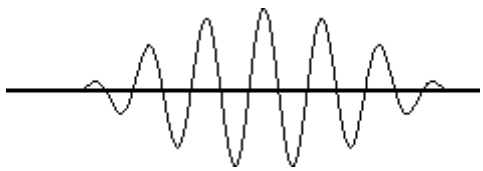
5. de-Broglie waves are pilot waves. The word, pilot means that these waves pilot (or) guide the particle.

6. Matter waves travel faster than velocity of light, i.e., $\omega = \frac{c^2}{V}$.

7. Mater waves are not electromagnetic waves.

8. The wave nature of matter introduces an uncertainty in the location of the position of the particle because a wave cannot be set exactly at this position (or) at that position.

9. de-Broglie waves are called probability waves. The amplitude of the wave tells us about the probability of finding particle in space at a particular instant. Large wave amplitude means large probability of finding the particle at that position.



NOTE:

1. Mention an application of matter waves?

A. The electron microscope is an application of matter waves, associated with electrons

2. Why cannot we experience the existence of matter waves in our day to day life?

A. $\lambda = \frac{h}{mv}$, if m is large, λ will be small, thus the wave length will be very small, compared to the size of the particle, hence it is not observable.

<u>Heisenberg uncertainty principle</u>:

In 1927, Heisenberg proposed a very interesting principle, which is direct consequence of the dual nature of matter, known as uncertainty principle.

According to Heisenberg uncertainty principle, it is impossible to determine accurately and simultaneously the values of position and momentum of a particle at any time. If Δx is the error (uncertainty) in measuring position, Δp is the error (uncertainty) in measuring momentum; then according to this principle

$$\boxed{\Delta x . \Delta p \geq \frac{h}{4\pi}}$$

NOTE:Similarly such relations are $\Delta E.\ \Delta t \geq \frac{h}{4\pi}$ and $\Delta J.\ \Delta\theta \geq \frac{h}{4\pi}$, Where ΔE and Δt are the uncertainties in determining the energy and time, while ΔJ and $\Delta\theta$ areuncertainties in determining the angular momentum and angle.

Application of uncertainty principle:

1. It is verified Nonexistence of Electrons inside the nucleus.
2. It is also verified Existence of protons and neutrons inside the nucleus
3. It is calculated binding energy of electron in an

atom4.It is also calculated Mass of Π -meson

5. Emitted radiation of excited atom is confirmed by this principle.

Physical significance of wave function:

The wave nature of matter introduced an uncertainty in the location of the position of the particle because a wave cannot be set exactly at this position (or) at that position.

According to Heisenberg uncertainty principle tells us that we cannot detect the particle nature and wave nature at the same time. To resolve this uncertainty, Eddington once upon a time humorously suggested that the primordial entity is neither particle nor wave but a 'wavicle'. Thus, de-Broglie waves are called probability waves. The amplitude of the wave tells us about the probability of finding particle in space at a particular instant. A large wave amplitude means a large probability of finding the particle at that position. Mathematically, to explain it, scientists introduced a new physical quantity called wave function.

<u>Wave function:</u> - it is a mathematical tool used in quantum mechanics to describe any physical system, and it is denoted by 'ψ'.

- It is variable quantity i.e., associated with a moving particle at any position (x,y,z) and at any time t and it relates probability of a finding the particle at that point and at that time.

- It must be well behaved, that is single valued and continuous everywhere.
- It usually complex quantity and individually it has no meaning.
- To explain it, Max born suggested a new idea about the physical significance of ψ which is generally accepted now a days.
- According Max born $\psi\ \psi^{*}=|\psi|^{2}$ is a real and positive, it has physical meaning. It represents the probability of finding the particle in the state ψ.
- Since the wave function is a complex quantity, it may be expressed in the form

of $\Psi_{(x,y,z)} = (a+ib)$

Where a,b are real function of the variables (x,y,z,t) and $i=\sqrt{-1}$.

- The complex conjugate of ψ is given by

$\Psi^{*}_{(x,y,z)} = (a-ib)$

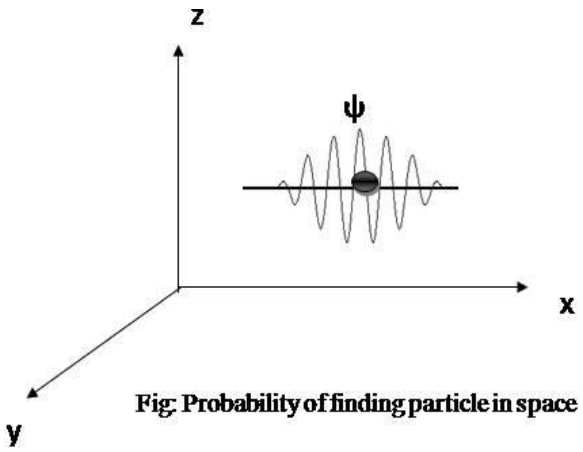


Fig: Probability of finding particle in space

Multiplying the above two equations, we have

$\Psi \Psi^* = (a+ib)(a-ib) = a^2+b^2$

According to Max born

$P = \Psi \Psi^* = |\psi_{(x,y,z)}|^2 = a^2+b^2$

Thus the product of ψ and ψ^* is real and positive if $\psi \neq 0$ and is known as probability density of the particle associated with the de-Broglie wave

- The probability of finding the particle in a volume dv=dx.dy.dz is given by

  Probability (p) $= |\psi|^2 dx\, dy\, dz$
- For the total probability of finding the particle somewhere is(space).

  $P = \iiint |\psi|^2\, dx\, dy\, dz = 1$
- A wave function ψ satisfying the above relation is called a normalized wave function.
- For motion of a particle in one dimensional, the quantity
  $Pdx = |\psi|^2 dx$ is the probability of finding the particle over a small distance dx at position x.

<u>Schrodinger wave equation:-</u>

<u>Definition:</u> **-** The equation that describes the wave nature of a particle in mathematical form is known as Schrodinger's wave equation.

There are two types of Schrodinger wave equations they are

1) Time independent wave equation and

2) Time dependent wave equation

<u>Schrodinger one dimensional Time independent wave equation :-</u>

Let us consider a particle of mass m moving with a velocity v. Let φ be the wave function of a particle along X-direction at any time (as shown in figure).

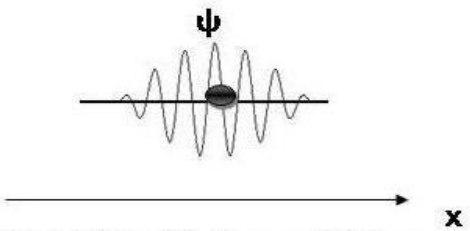


Fig: Probability of finding particle in space

The classical differential equation of wave motion is given by

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \qquad (1) \longrightarrow$$

The solution for equation (1) is assumed to be

$$\psi = \psi_0\, e^{-i\omega t} \qquad (2) \longrightarrow$$

Where ψ_0 is the amplitude of the wave at any point..

Differentiating equation (2) partially with respect to‘t’; we get

$$\frac{\partial\psi}{\partial t} = \psi_0\, e^{-i\omega t} \times (-\, i\omega\,) \qquad (3) \longrightarrow$$

Differentiating once again equation (3) partially with respect to‘t’; we get

$$\frac{\partial^2\psi}{\partial t^2} = \psi_0\, e^{-i\omega t} \times (-\, i\omega\,)\, x(-\, i\omega\,)$$

$$= \psi_0\, e^{-i\omega t}(\ -\omega^2\ ) \qquad [\text{ from eq (2)}]$$

$$= -\ \omega^2\psi \ (4) \longrightarrow$$

Substituting equation (4) in (1) we get

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}(-\omega^2\psi)$$

$$= -\ \frac{\omega^2}{v^2}\psi \ (5) \longrightarrow$$

But $\omega = 2\pi\nu = 2\pi\, v/\lambda$ $\quad$ ($v = \nu\lambda$ and $\nu = v/\lambda$)

(or) $\frac{\omega}{v} = \frac{2\pi}{\lambda} \rightarrow$ (6)

Substituting equation (6) in (5), we get

$$\frac{\partial^2\psi}{\partial x^2} + \frac{4\pi^2\psi}{\lambda^2} = 0 \quad (7) \longrightarrow$$

This is the classical wave equation in terms of wavelength.

Schrodinger connected the expression for the de Broglie wave the length in to the classical wave equation in terms of wavelength for a moving a particle and obtained a new wave equation.

We know that debroglie wavelength is given by

$$\lambda = \frac{h}{mv} \rightarrow (8)$$

Substituting equation (8) in (7), we get

$$\frac{\partial^2\psi}{\partial x^2} + \frac{4\pi^2 m^2 v^2}{h^2}\psi = 0 \rightarrow (9)$$

If E is the total energy of a particle, 'V' is the potential energy and $\frac{1}{2}mv^2$ is the kinetic energy; then

Total energy (E) =P.E+K.E

$$E = V + \frac{1}{2}mv^2$$

(Or) $E - V = \frac{1}{2}mv^2$

$$2(E-V) = mv^2$$

Multiplying by 'm' on both sides in the above equation, we get

$$2m(E-V) = m^2 v^2 \rightarrow (10)$$

Substituting equation (10) in (9) we get

$$\boxed{\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E-V)\,\psi = 0}$$

$$\text{Or } \frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\,\psi = 0 \qquad (12)$$

Where $\hbar = \frac{h}{2\pi}$ is called plank's reduced constant.

This equation is known as Schrodinger time independent one-dimensional wave equation.

For 3dimensional:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{8\pi^2 m}{h^2}(E-V)\,\psi = 0$$

$$\text{Or } \nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-V)\,\psi = 0$$

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is called Laplace operator.

<u>Time dependent wave equation:</u>

Let us consider a particle of mass m moving with a velocity v. Let ψ be the wave function of a particle along x-direction at any time (as shown in figure).

The classical differential equation of wave motion is given by

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \qquad (1)$$

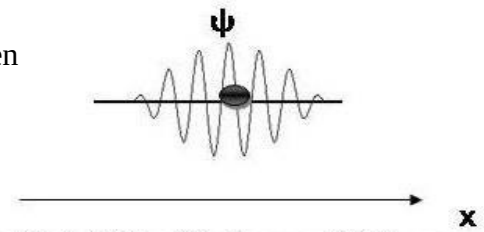


Fig: Probability of finding particle in space

The solution for equation (1) is assumed to be

$$\psi = \psi_0\, e^{-i\omega t} \qquad (2)$$

Where ψ_0 is the amplitude of the wave at any point.

Differentiating equation (2) partially with respect to 't'; we get

$$\frac{\partial\psi}{\partial t} = \psi_0\, e^{-i\omega t} \times (-\, i\omega\,)$$

$$= -i\omega\psi \qquad (3)$$

But $E = h\nu = h\,\frac{\omega}{2\pi}$

$$= \frac{h}{2\pi}\omega \quad \left(\omega = 2\pi\nu \text{ and } \nu = \frac{\omega}{2\pi}\right)$$

$\therefore E = \hbar\omega$

$$\text{And } \omega = \frac{E}{\hbar} \qquad (4)$$

Substituting equation (4) in eq (3), we get

$$\frac{\partial\psi}{\partial t} = -\,i\,\frac{E}{\hbar}\psi$$

$$= \frac{E}{i\hbar}\psi$$

$$E\psi = i\hbar\frac{\partial\psi}{\partial t} \longrightarrow (5)$$

Substituting equation (5) inSchrodinger time independent one-dimensional wave equation, we get

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\,\psi = 0 \qquad (6)$$

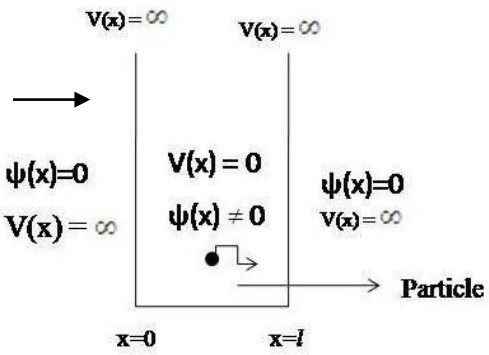


Fig : Particle in one dimensional box

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E\psi - V\psi) = 0$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}\left(i\hbar\frac{\partial\psi}{\partial t} - V\psi\right) = 0$$

Multiplying $\frac{\hbar^2}{2m}$ on both sides, we get

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}\cdot\frac{\hbar^2}{2m}\left(i\hbar\frac{\partial\psi}{\partial t} - V\psi\right) = 0$$

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \left(i\hbar\frac{\partial\psi}{\partial t} - V\psi\right) = 0$$

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} - V\psi = -i\hbar\frac{\partial\psi}{\partial t}$$

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}}$$

This equation is known as Schrodinger time dependent one-dimensional wave equation.

Or $\qquad H\psi = E\psi$

Where H $\quad -\hbar^2\,\partial^2 + V$

$$= \frac{}{2m}\frac{}{\partial x^2}$$ is called Hamilton operator

$$E = i\hbar\frac{\partial}{\partial t}$$ is called energy operator.

Particle in one dimensional box or well:

- Let us consider a particle of mass '*m*' moving with velocity '*v*' along x-direction and is confined between to infinite potential rigid walls, so that the particle has nochance of escaping from them. Therefore, the particle bounces back and forth between two walls as shown in fig.

- Let the potential energy of electron inside the box is constant and can be taken as zero for simplicity.
- From fig , we consider two regions :
  (i) Outside the box
  (ii) Inside the box

Outside the box:

The particle cannot exist outside and on the walls of the box; therefore the probability of finding the electron outside the wall is zero and the potential energy is∞.

i.e., $\boxed{\psi(x)=0}$ When $x \leq 0$ and $x \geq l$

Inside the box:

The particle exists inside the box; therefore the probability of finding the electron inside the box is no equal zero and the potential energy is zero.

i.e., $\boxed{\psi(x)\neq 0}$ When $x>0$ and $x< l$

- To calculate the probability of finding particle within the box , let us consider one dimensional time independent Schrödinger wave equation

$$\text{i.e., } \frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-V(x))\,\psi=0 \longrightarrow (1)$$

Inside the box, the potential energy V(x) = 0

Therefore, equation (1) becomes

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 mE}{h^2}\psi=0$$

$$\text{(Or) } \frac{d^2\psi}{dx^2} + k^2\psi=0 \longrightarrow (2)$$

$$\text{Where } k^2 = \frac{8\pi^2 mE}{h^2} \qquad (3)$$

Equation (2) is a second order differential equation; therefore it should have solution with two arbitrary constants.

∴ The solution for equation (2) is given by

$\Psi(x) = A\sin Kx + B\cos Kx$ ~~(4)~~ →

Where A and B are called as arbitrary constants, which can be found by applying the boundary conditions.

Boundary Condition at x=0:

At x = 0, the probability of finding the electron is zero, i.e., $\psi(x) = 0$

Equation (4) becomes

$O = A \sin K(0) + B \cos K(0)$

$O = 0 + B$

$\boxed{\therefore B = 0}$ → (5)

Boundary Condition at x=*l*:

At x = *l*, the probability of finding the electron is zero, i.e., $\psi(x) = 0$

Equation (4) becomes

$O = A \sin K(l) + B \cos K(l)$

$O = A \sin K(l) + (0) \cos K(l)$ [Since B = 0]

$A \sin K(l) = 0$

Since $A \neq 0$; $\sin K(l) = 0$ → (6)

We know that, $\sin n\pi = 0$ → (7)

Comparing these two equations, we can write

$K(l) = n\pi$

$$K = \frac{n\pi}{l} \quad (8) \rightarrow$$

$$k^2 = \frac{n^2\pi^2}{l^2} \quad (9) \rightarrow$$

Substituting the values of '*B*' and '*K*' in equation (4),

Equation (4) becomes

$$\boxed{\Psi_n(x) = A\sin\frac{n\pi x}{}} \rightarrow \quad (10)$$

This equation represents the wave function associated with moving free electron inside the box.

<u>Normalization of the wave function:</u>it is the process by which the probability (P) of finding the particle or electron inside the box can be done.

The total probability that the particle is somewhere in the box must be unity

i.e ., $P = \int_0^l |\psi_n(x)|^2 \longrightarrow (11)$

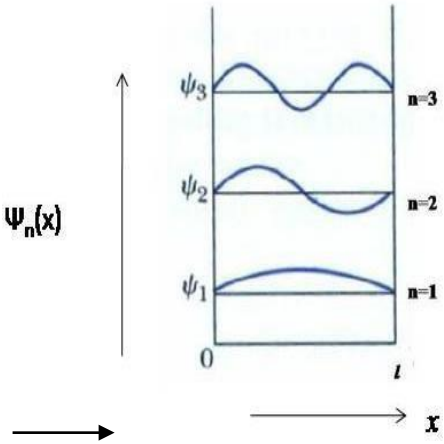
[Since the particle present inside the box between the length 0 to *l*, thenthe limits are chosen between 0 to *l*]

Substitutingequation (10)in equation (11), we get

$$P = \int_0^l A^2 \sin^2 \frac{n\pi x}{l} dx = 1$$

$$A^2 \int_0^l \sin^2 \frac{n\pi x}{l} dx = 1 \qquad \left[\text{Since } \sin^2 x = \frac{1-\cos 2x}{2}\right]$$

$$A^2 \int_0^l \frac{1-Cos2n\pi x}{2} = 1$$ [Since $\int \frac{Cosn\pi x}{l} = \frac{Sinn\pi x}{\frac{n\pi}{l}}$]



$$\frac{A^2}{2} \int_0^l \frac{1-Cos2n\pi x}{l} dx = 1$$

$$\frac{A^2}{2} \int_0^l \frac{1-Cos2n\pi x}{l} dx = 1$$

$$\frac{A^2}{2} [\int_0^l dx - \int_0^l \frac{l\, Cos2n\pi x}{l}] = 1$$

$$\frac{A^2}{2}[l] - \frac{A^2}{2}[\frac{\frac{Sin2n\pi l}{l}}{\frac{2n\pi}{l}}] = 1 \quad (12)$$

Since Sin nπ = 0, Sin 2nπ is also= 0

Equation (13) can be written as

$$\frac{A^2}{2}[l] = 1$$

$$A^2 = \frac{2}{l}$$

(13) $$\boxed{A = \sqrt{\frac{2}{l}}}$$

Substituting the values of '*A*'in equation (10),

$$\Psi_n(x) = \sqrt{\frac{2}{l}} Sin \frac{n\pi x}{l} \quad (14)$$

Each value of wave function is known as Eigen function .The various Eigen functions of an electron enclosed in a1D box as shown in fig.

Case 1: If n=1, then $\Psi_1(x)$hastwo nodes at x=0 and x= *l.*

Case 2: If n=2, then $\Psi_2(x)$hastwo nodes at x=0 and $x = \frac{l}{2}$ $and\ x = l.$

Case 3: If n=3, then $\Psi_3(x)$hasthree nodes at x=0 and $x = \frac{l}{3}$ $and\ x = \frac{l2}{3}$ $and\ x = l.$

Therefore $\Psi_n(x)$ has (n+1) nodes.

Probability of the location of the particle:

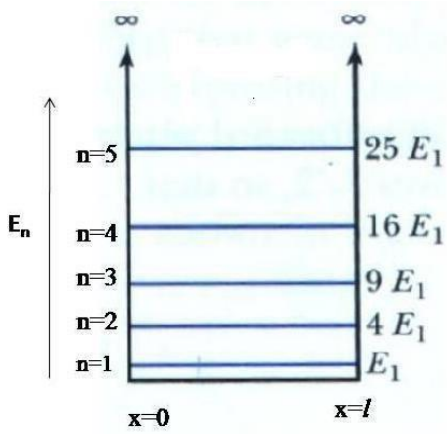
The probability of finding a particle over a small distance *dx* at x is given by

$P(x)\,dx = |\psi_n(x)|^2 dx$

$$P(x)\,dx = \frac{2}{l}\sin^2\frac{n\pi x}{l}$$

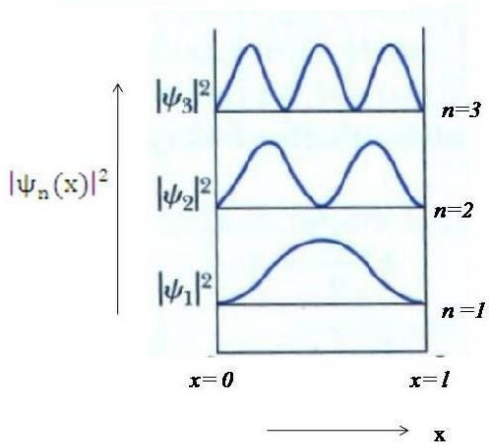
Thus, the probability density for one dimensional motion is

$$P(x) = \frac{2}{l}\sin^2\frac{n\pi x}{l}$$



The probability density is maximum when $\frac{n\pi x}{l} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

$$x = \frac{l}{2}, \frac{3l}{2n}, \frac{5l}{2n} \ldots\ldots$$



Case 1: If n=1, then the probability of position of the particle is at $x = \frac{l}{2}$

Case 2: If n=2then the probability of position of the particle is at $x = \frac{l}{4}$ and $\frac{3l}{4}$.

Case 3: If n=3, then the probability of position of the particle is at $x = \frac{l}{6}$ and $\frac{3l}{6}$ and $\frac{5l}{6}$.these positions are shown in fig.

Energy of the Particle:

From the equations (3) & (9)

$$k^2 = \frac{8\pi^2 mE}{h^2} \longrightarrow (3)$$

$$k^2 = \frac{n^2\pi^2}{l^2} (9) \longrightarrow \frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{l^2}$$

(1 $\boxed{E_n = \frac{n^2 h^2}{8ml^2}}$ $\longrightarrow$

This is an expression for the energy of the particle.

Each value of E_n is known as Eigen value.

The various Eigen values of an electron enclosed in a 1D box as shown in fig

Case 1: If n=1, then $E_1 = \frac{h^2}{8ml^2}$.

Case 2: If n=2, then then $E_2 = \frac{4h^2}{8ml^2} = 4E_1$

Case 3: If n=3, then $E_3 = \frac{9h^2}{8ml^2} = 9E_1$ and so on. Therefore, energy levels of electron are

discrete.

Free Electron Theory and Band Theory of Solids

Introduction:

The study of behavior of free electrons and various properties such as electrical, thermal and magnetic properties etc., in a solid is known as free electron theory of solids.

The electron theory of solids has been developed in three main stages.

(i). The classical free electron theory:

- This theory was developed by Drude and Lorentz in 1900.
- According to this theory, metals consist of positive ion cores and free electrons.
- The free electrons bound to move within the metal in a completely uniform potential field among these ion cores and obey the laws of classical mechanics.

(ii). The Quantum free electron theory:

- This theory was developed by Sommerfeld in 1928.
- The free electrons bound to move within the metal in a completely uniform potential field among the ion cores and obey quantum laws.

(iii). The Zone theory:

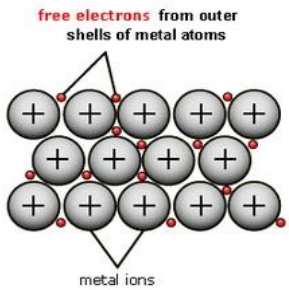
Bloch stated this theory in 1928.
According to this theory, the free electrons move in a periodic field provided by the lattice and obey quantum laws. This theory is also called “Band theory of solids”.

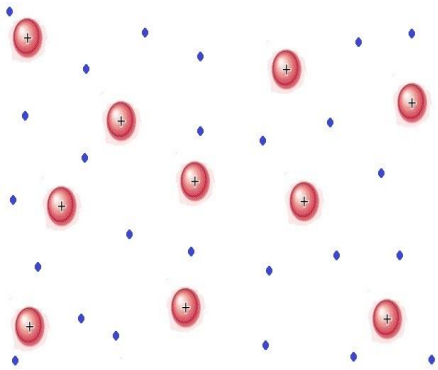
Classical free electron theory (Free electron gas model):

After discovery of electron by J.J.Thomson in 1897, Paul Drude developed the free electron theory of metals in 1900 and later it was extended by Hendrik Antoon Lorentz.

- According to this theory, metals consist of positive ion cores and free electrons.
- The free electrons bound to move within the metal in a completely uniform potential field among these ion cores.

- The behavior of free electrons moving within the metal is considered to be similar to that of atoms or molecules in perfect gas. These electrons are, therefore, also referred to as free electron gas and the theory is accordingly named as free electron gas model.
- The movement of free electrons obeys the laws of classical kinetic theory of gases.
- These free electrons move in random directions and collide with either positive ions fixed to the lattice or free electrons .All collisions are elastic, i.e., there is no lose of energy.
- The electrostatic force of attraction between the free electrons and the metallic ions are neglected.
- The mutual repulsion (electrostatic force of repulsion) among the free electrons is ignored, so that they move in all the directions with all possible velocities and obey the Maxwell-Boltzmann distribution of velocities.





Electrical conduction:

Consider a metallic conductor of length *l* and area of cross-section *A*.

Let *n* be the number of free electrons per unit volume and e be the charge of an electron.

<u>Case 1:</u>

When there is no electric field, all free electrons move in randomly in all directions. So the net current flow is zero.

<u>Case 2:</u>

When an electric field is applied to the metallic conductor, it exerts a force on the free electrons (Lorentz force) which start accelerating towards the positive terminal.

As the free electrons move, they collide again and again with positive ions of the metal. Each collision destroys the extra velocity gained by the free electrons.

The average distance travelled by a free electron between two successive collisions is called mean free path (λ). The average time interval between two successive collisions is called the relaxation time (τ). Its value is of the order of 10^{-14} second.

Although the free electrons are continuously accelerated by the electric field, collisions prevent their velocity becoming large. Finally free electrons acquire a constant average velocity is called drift velocity (V_d).

The constant average velocity acquired by the free electrons in a conductor subjected to an electric field is called drift velocity (V_d).

The drift velocity of free electron is of the order of 10^{-5}m/s.

<u>Under an electric field:</u>

The electron gains acceleration $a = \frac{Drift\ velocity\ (V_d)}{Relaxation\ time(\tau)}$

∴ The drift velocity $V_d = a\tau \longrightarrow (1)$

Each electron experiences a Lorentz force $F_L = e\,E \longrightarrow (2)$

From Newton's Second law of motion,

The force on the electron $F = ma \longrightarrow (3)$

Equating equation (2) and equation (3), we have

$$ma = e\,E$$

$$a = \frac{e\,E}{m} \longrightarrow (4)$$

Substituting equation (4) in (1), we get

$$V_d = \left(\frac{e\,E}{m}\right)\tau \longrightarrow (5)$$

The current flowing through a conductor $I = neAV_d \longrightarrow (6)$

Substituting equation (5) in (6), we get

$$= neA\left(\frac{e\,E}{m}\right)\tau$$

$$= nA\left(\frac{e^2\,E}{m}\right)\tau \longrightarrow (7)$$

The current density J = $\frac{I}{A}$ $\longrightarrow$ (8)

Substituting equation (7) in (8), we get

$$= \frac{nA(\frac{e^2 E}{m})\tau}{A}$$

$$= n(\frac{e^2 E}{m})\tau \longrightarrow (9)$$

The electrical conductivity $\sigma = \frac{J}{E}$ $\longrightarrow$ (10)

Substituting equation (9) in (10), we get

$$= \frac{n(\frac{e^2 E}{m})\tau}{E}$$

According to kinetic theory of gases,

Relaxation time $\tau = \frac{\lambda}{c}$ and

Root mean square velocity $c = \sqrt{\frac{3K_B T}{m}}$

$$\therefore \text{The electrical conductivity} \quad \sigma = n(\frac{e^2}{m})\tau = n(\frac{e^2}{m})\frac{\lambda}{\bar{c}} = \frac{ne^2\lambda}{\sqrt{3mK_B T}}$$

(OR)

$$\text{The electrical resistivity } \rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau} = \frac{m\,\bar{c}}{ne^2\lambda} = \frac{\sqrt{3mK_B T}}{ne^2\lambda}$$

The above equations represent the electric conductivity and resistivity of metals.

Merits of classical free electron theory:

- It verifies Ohm's law.
- It explains the electrical conductivity of metals.
- It explains the electrical resistivity of metals.
- It explains the thermal conductivities of metals.
- It derives Wiedemanns-Franz law. (i.e., the relation between electrical conductivity and thermal conductivity)

- It explains optical properties of metals.

Demerits of classical free electron theory:

- This theory fails to explain the photo electric effect, Compton Effect and the black body radiation etc.
- This theory fails to explain specific heat of metals.
- According to this theory, the electrical conductivity (σ) of a metal is inversely proportional to its square root of its absolute temperature (T).i.e.,$\sigma\ \alpha\ \frac{1}{\sqrt{T}}$.But practically, it has been observed that the electrical conductivity is inversely proportional its absolute temperature. i.e., $\sigma\ \alpha\ \frac{1}{T}$.

Thus, this theory failed to explain electrical conductivity of metals.

- This theory fails to explain mean fee path of electrons. Because theoretical and experimental values of mean free path (λ) does not coincide.

Quantum free electron theory:

- This theory was developed by Sommerfeld in 1928 based on the Pauli Exclusion Principle and Femi- Dirac Statistics.
- According to this theory, metals consist of positive ion cores and free electrons.
- The free electrons bound to move within the metal in a completely uniform potential field among these ion cores and obey quantum laws.
- Mutual attraction between electrons and positive ions and the repulsion between individual electrons may be ignored.
- The Free electrons will have discrete energy states (or) levels in a metal.
- The occupation (or) distribution of electrons among these energy levels will be governed by Pauli's exclusion principle.
- According to Pauli's exclusion principles, no two electrons can have same set of quantum numbers in a given energy state. i.e., no two electrons can occupy the same energy level and each energy can accept only two electrons one with spin up (↑) and other with spin down (↓).
- At absolute zero temperature (0K), the first electron can occupy the lowest available energy state (E_0) with spin up (↑) and the next electron can also occupy the same energy level with opposite spin down (↓)of first electron. The third electron cannot occupy in this energy level, because of Pauli's exclusion principle and it will go to the next energy (E_1).

In this way, all free electrons can be occupied in the different energy states in the one dimensional potential box as shown in figure.

- The highest energy level occupied by electrons at absolute zero temperature is known as "Fermi energy level" (E_F).Above Fermi energy level all energy levels are empty. i.e., at 0K all the energy levels below Fermi energy level are filled by free electrons. So they possess different energies and velocities or momentums.

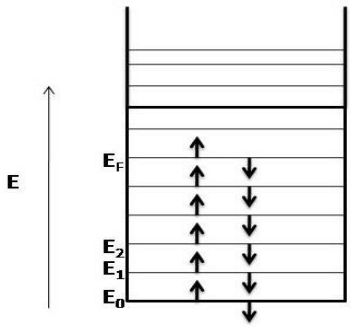


Fig: Distribution of free electrons in various energy levels at 0K.

- The different velocities of these free electrons of a metal can be represented in velocity space as shown fig.
- At 0K, free electrons present in Fermi energy level possess maximum velocity represented by V_F.
- We assume a sphere of radius V_F, at the origin of velocity space as shown in fig.
- At 0K, all points inside the Fermi sphere are occupied.
- Each point inside the sphere from the origin represents the velocity of a free electron.
- The vectors joining different points inside the sphere from the origin represent velocity vectors.
- In the absence of electric field, the velocity vectors in the Fermi sphere present from origin will be cancelled with each other and the net velocity of electrons in all directions is zero.
- If the electric field E is applied on the free electrons along positive X-direction, then electrons experience a Lorentz force F_L along negative X-direction.
- The electrons which are present near the Fermi surface take electrical energy and occupy higher vacant energy levels as a result the sphere slightly displaces as shown fig. The remaining electrons unable to occupy higher vacant energy levels because Lorentz force is less on these electrons.

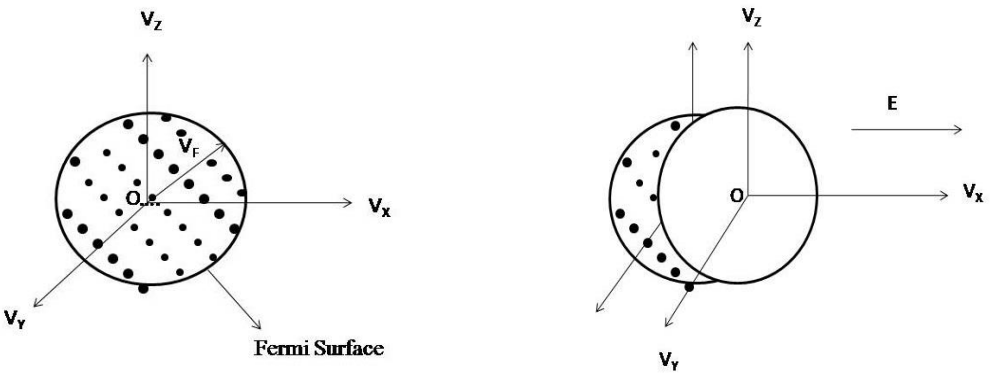


Fig: Displacement of Fermi sphere under applied electric field.

- <u>Under an electric field:</u>

Each electron experiences a Lorentz force $F_L = e\,E$ (1) ⟶

From Newton's Second law of motion,

The force on the electron $F = ma$ (2) ⟶

Equating equation (2) and equation (3), we have

$$ma = e\,E \longrightarrow (3)$$

The relation between momentum $\boldsymbol{P}$ and the wave or propagation vector K is given by

$$P = \hbar k \longrightarrow (4)$$

$$\text{But } P = mv \longrightarrow (5)$$

Equating equation (4) and equation (5), we have

$$mv = \hbar k$$

$$\boldsymbol{v} = \frac{\hbar \boldsymbol{k}}{\boldsymbol{m}} \longrightarrow (6)$$

Differentiating equation (6) with respect to time gives acceleration,

$$a = \frac{dv}{dt} = \frac{\hbar}{m}\frac{dk}{dt} \qquad (7) \longrightarrow$$

Substituting equation (7) in (3), we get

$$m\frac{\hbar}{m}\frac{dk}{dt} = e\,E \qquad (8) \longrightarrow$$

$$dk = \frac{eE}{\hbar}\,dt \qquad (9) \longrightarrow$$

Integrating the equation (9), taking limits from *0* to *t;*

$$\int_0^t dk = \int_0^t \frac{eE}{\hbar}\, dt$$

$$K(t)-K(0) = \frac{eEt}{\hbar} \qquad (10) \longrightarrow$$

Let the mean collision time and mean free path of electron present at Fermi surface is represented as τ_F and λ_F respectively then, we have:

$$\tau_F = \frac{\lambda_F}{V_F} \qquad (11) \longrightarrow$$

For an electron at Fermi surface, t= τ_F and *K* (t)-K (0) =Δk.

$$\text{Then } \Delta k = \frac{eE\tau_F}{\hbar} \qquad (12) \longrightarrow$$

Substituting equation (11) in (12), we get

$$\Delta k = \frac{eE}{\hbar}\left[\frac{\lambda_F}{V_F}\right] \qquad (13) \longrightarrow$$

If the increased velocity is Δv due to application of electric field then the current density

$$J = ne\Delta v \qquad (14) \longrightarrow$$

Where *n* is the number of free electrons per unit volume and *e* is the charge of the electron. From equations (6) and (13), analytically Δv can be expressed as

$$\Delta v = \frac{\hbar\Delta k}{m^*} = \frac{\hbar}{m^*}\frac{eE}{\hbar}\left[\frac{\lambda_F}{V_F}\right] = \frac{eE}{m^*}\left[\frac{\lambda_F}{V_F}\right] \longrightarrow (15)$$

Where m* is the effective mass of free electron.

Substituting equation (15) in (14), we get

$$J = ne \times \frac{eE}{m^*}\left[\frac{\lambda_F}{V_F}\right]$$

$$= \frac{ne^2E}{m^*}\left[\frac{\lambda_F}{V_F}\right] \qquad (16) \longrightarrow$$

The electrical conductivity $\sigma = \frac{J}{E}$ (17) $\longrightarrow$

Substituting equation (16) in (17), we get

$$\sigma = \frac{\frac{ne^2E}{m^*}\left[\frac{\lambda_F}{V_F}\right]}{E}$$

$$= \frac{ne^2}{m^*}\left[\frac{\lambda_F}{V_F}\right] \qquad \text{[From eq(11)]}$$

$$\boxed{\sigma = \frac{ne^2\tau_F}{m^*}}$$

This is the expression for electrical conductivity which is found to be in good agreement with experimental values. Thus we can say that, according to this theory, all free electrons are not responsible for electrical conduction, only those free electrons which are very near to Fermi surface.

Merits of Quantum free electron theory:

- It explains the electrical conductivity of metals.
- It explains the electrical resistivity of metals.
- It explains the thermal conductivities of metals.
- It derives Wiedemanns-Franz law. (i.e., the relation between electrical conductivity and thermal conductivity)
- It explains optical properties of metals.
- It successfully explains the photo electric effect, Compton Effect and the black body radiation etc.
- It explains the specific heat of metals.

Demerits of Quantum free electron theory:

- This theory fails to explain the differences between insulators, conductors and semiconductors.
- Positive hall coefficient of metals and
- Lower conductivities of divalent metals than monovalent metals.

Fermi-Dirac distribution:-

According to Fermi-Dirac statistics, electrons are indistinguishable and half integral spin particles which obey the Pauli's exclusion principle. They are known as fermions (or) Fermi particles.

- The Fermi-Dirac distribution describes the behaviour of free electrons in a metal, based on the quantum mechanics and Pauli's exclusion principle.
- According to quantum mechanics, the electrons will have discrete energy states (or) levels and the occupation (or) distribution of electrons among these energy levels will be governed by Pauli's exclusion principle.

- According to Pauli's exclusion principles, no two electrons can have same set of quantum numbers in a given energy state. i.e., each energy can accept only two electrons one with spin up (↑) and other with spin down (↓).
- The occupation of free electrons in one –dimensional potential well as shown in fig.

At absolute zero temperature (0K), the first electron can occupy the lowest available energy state (E_0) with spin up (↑) and the next electron can also occupy the same energy level with opposite spin down (↓)of first electron. The third electron cannot occupy in this energy level, because of Pauli's exclusion principle and it will go to the next energy (E_1). In this way, all free electrons can be occupied in the different energy states in the one dimensional potential box as shown in figure.

- *Fermi Energy Level (E_F):-*

The highest energy level occupied by electrons at absolute zero temperature is known as "Fermi energy level" (E_F) which divides (or) separates the occupied states from the unoccupied states.

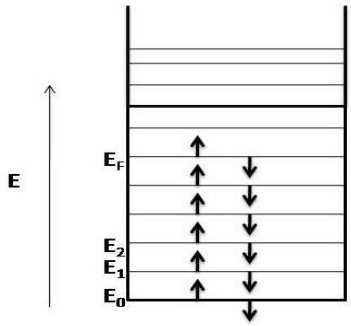


Fig: Distribution of free electrons in various energy levels at 0K.

Fermi-Dirac distribution function:-

The Fermi-Dirac distribution function gives the probability of occupation of energy levels by the electrons (fermions).

- The probability F(E) of occupation of particular energy level (E) by the electrons at temperature (T) is given by

$$F(E) = \frac{1}{1+\exp\frac{(E-E_F)}{K_B T}}$$

Where E = Particular energy level (ev)

E_F = Fermi energy level (ev)

K_B = Boltzmann constant = 1.38×10^{-23} J/K.

T = Temperature (K).

Effect of Temperature on F-D Distribution Function:

Case 1: Probability of occupation at T= 0K, When $E>E_F$

$E>E_F$ means energy levels lying below E_F, the term $E-E_F$ takes a negative value. Hence equation (1) becomes

$$\frac{(E-E_F)}{K_B T} = -\infty \text{ at T= 0K, When } E>E_F$$

$$F(E) = \frac{1}{1+\exp^{-\infty}} = \frac{1}{1+0} = 1 \qquad [\text{ Since } \exp^{-\infty} = 0]$$

$$\boxed{\therefore F(E) = 1}$$

It implies that, all the energy levels below EF are occupied by free electrons.

Case 2: Probability of occupation at T= 0K, When $E<E_F$

$E<E_F$ means energy levels lying above E_F, the term $E-E_F$ takes a postive value. Hence equation (1) becomes

$$\frac{(E-E_F)}{K_B T} = \infty \text{ at T= 0K, When } E<E_F .$$

$$F(E) = \frac{1}{1+\exp^{\infty}} = \frac{1}{1+\infty} = \frac{1}{\infty} = 0 \quad [\text{ Since } \exp^{\infty} = \infty]$$

$$\boxed{\therefore F(E) = 0}$$

It implies that, all the energy levels below EF are unoccupied by free electrons.

Case 3: Probability of occupation at T> 0K, When $E=E_F$

The term $\frac{(E-E_F)}{K_B T} = 0$ at T> 0K, When $E=E_F$. Hence equation (1) becomes

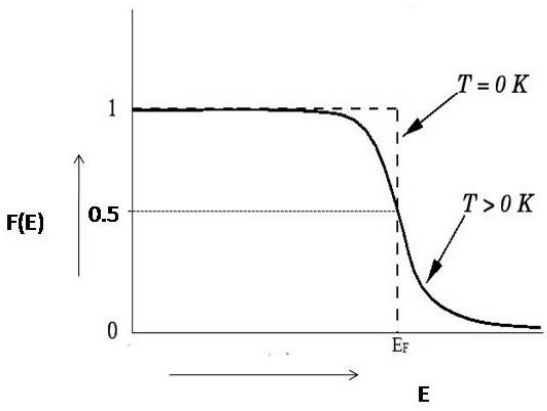
$$F(E) = \frac{1}{1+\exp^{0}} = \frac{1}{1+1} = 0.5 \qquad [\text{ Since } \exp^{0} = 1]$$

$$\boxed{\therefore F(E) = 0.5}$$

It implies that, the probability of occupancy of 50% at any temperature above 0K.

Graph:

A graph is plotted between F (E) and E at T=0K and T>0K as shown in fig. From the curve, we can observe that the curve is a step like character at T=0K When $E>E_F$ and $E<E_F$. This implies that all the energy levels below E_F are filled and above E_F are empty. Further, for T> 0K, the Fermi function F (E) smoothly falls to zero as shown in fig.



The source of electrical resistance:

We know that, the opposition offered by substances to flow of charge carriers through them is called electrical resistance.

This opposition arises due to the collision (scattering) of electrons. The cause for scattering of electrons is the non periodicity of the lattice. This non periodicity in the lattice arises due to the three reasons. They are i).Lattice vibrations

ii) Imperfections (Crystal defects) and

iii) Impurities present in the materials etc.

i) Lattice vibrations or thermal vibrations:

- Under normal condition, all atoms or molecules or ions of the lattice vibrate about their normal positions called lattice vibrations or thermal vibrations. The quantization energy of lattice or thermal vibrations is known as phonon.
- With an increase of temperature, the amplitude of thermal vibrations of lattice will be increases. As a result, non-periodicity of the lattice and scattering of electrons increases.
- Hence resistivity increases.

ii) Imperfections (Crystal defects) :

- In ideal Crystal solids, the atoms or molecules are arranged in a regular and periodically in three-dimensional manner.
- But in real crystals, there exist few defects or imperfections.
- Due to imperfections, non-periodicity of the lattice and scattering of electrons increases.
- Hence resistivity increases

iii) Impurities present in the materials etc.

- In real crystals, there exist few impurities.
- With increase of impurity concentration, non-periodicity of the lattice and scattering of electrons increases.
- Hence resistivity increases.
- The variation of impure metal with temperature as shown in fig.
- Therefore the total resistivity of metal due to impurities and imperfections and also due to thermal vibrations is given by

$$\rho_{tot} = \rho_i + \rho(T)$$

Where, ρ_i - Resistivity due to impurities and imperfections.

$\rho_{(T)}$ - Resistivity due to thermal vibrations.

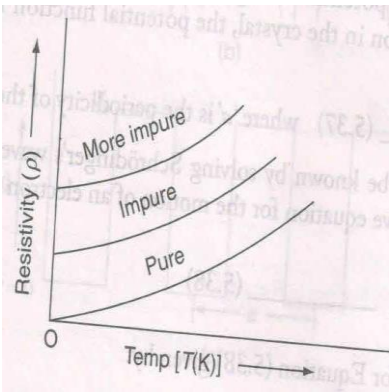
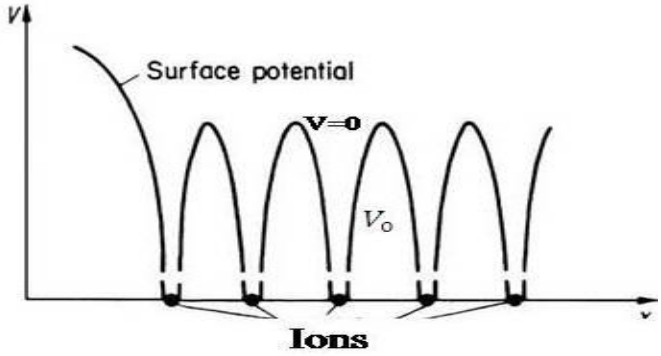


Fig: The variation of impure metal with temperature.

Band theory or zone theory of solids:

- This theory was introduced by Bloch in 1928.
- According to this theory, the free electrons move within a metal with periodic potential field provided by the positive ion cores and obey quantum laws.
- The potential of the electron varies periodically with periodicity of positive ion cores.

- The potential energy of energy is zero at the positive ion sites and maximum in between two ions as shown in fig.



The periodic potential of electron is given by V(x) =V(x+a).

The Schrödinger wave equation for periodic potential is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-V(x))\,\psi=0 \longrightarrow (1)$$

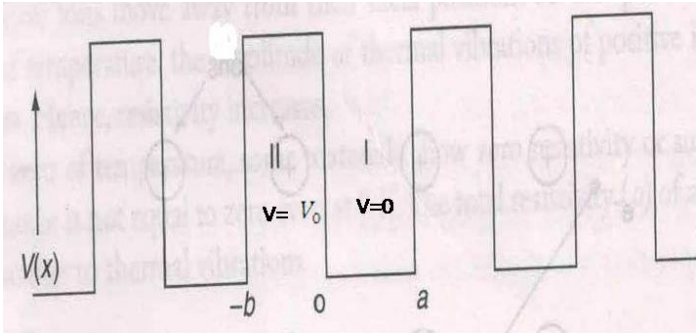
The solution of the above equation is

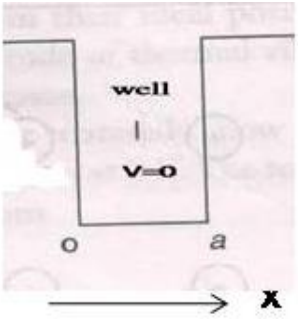
$$\psi = \exp^{\pm ikx}\ u_k(x) \longrightarrow (2)$$

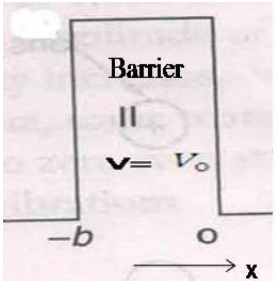
- Where $u_k(x)$ represents the periodic function and is given by $u_k(x)= u_k(x+a)$.
- But, it is very difficult to solve Schrödinger equation with sinusoidal potentials.
- So, kroning and Penny modified the above curves as rectangular wells and barriers as shown in fig. This model is known as kroning –Penny model.

Kroning –Penny model:

- According to this theory, the free electrons move within a metal with periodic potential field provided by the positive ion cores and obey quantum laws.
- The potential of the electron varies periodically with periodicity of positive ion cores.
- kroning and Penny modified the periodic potential curves as rectangular wells and barriers as shown in fig.
- The potential energy of energy is zero at the positive ion sites and maximum in between two ions.







- From fig , we consider two regions : (i) Region I (Well)
  (ii) Region II (Barrier)

Region I (Well):

In this region, the potential of electron is V(x) = 0 when 0<x<a.

The time independent Schrödinger wave equation for the region I (Well) is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 mE}{h^2}\psi = 0 \longrightarrow (1)$$

$$\text{(or)}\ \frac{d^2\psi}{dx^2} + \alpha^2\psi = 0 \longrightarrow (2)$$

$$\text{Where } \alpha^2 = \frac{8\pi^2 mE}{h^2} \longrightarrow (3)$$

Region II (Barrier):

In this region, the potential of electron is V(x) = V_0 when -b<x<a.

The time independent Schrödinger wave equation for the region II (Barrier) is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V_0)\psi = 0 \longrightarrow (4)$$

$$\text{(or)}\ \frac{d^2\psi}{dx^2} + \beta^2\psi = 0 \longrightarrow (5)$$

$$\text{Where } \beta^2 = \frac{8\pi^2 mE}{h^2} \longrightarrow (6)$$

The solution for the equations (2) and (5) can be written in the Bloch's form as

$$\psi = \exp^{\pm ikx}\ u_k(x) \longrightarrow (7)$$

Where $u_k(x)$ represents the periodic function and is given by $u_k(x)= u_k(x+a)$ and $k = \frac{2\pi}{\lambda}$ is called propagation vector or wave vector.

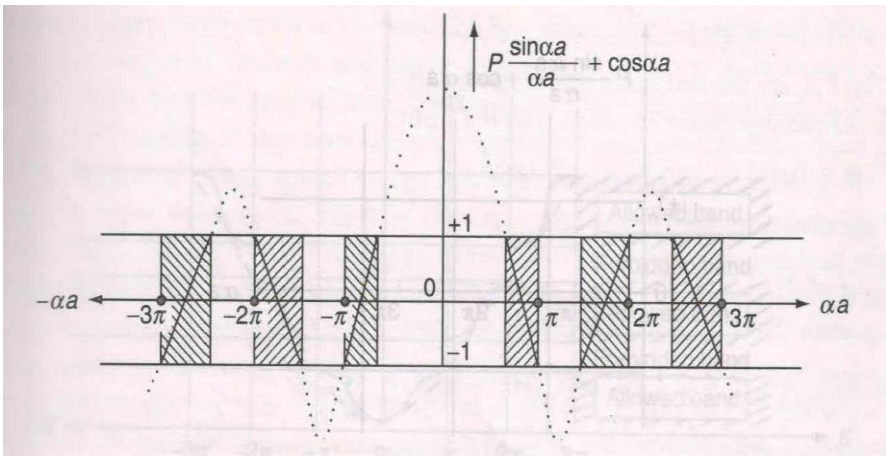
Differentiating equation (7) and substituting it in equations (2) and (5) and then further solving applying boundary conditions, finally we get

$$\left(\frac{mV_0ab}{\hbar^2}\right)\frac{\sin\alpha a}{\alpha a} + \cos\alpha a = \cos ka \qquad (8) \longrightarrow$$

$$P\frac{\sin\alpha a}{\alpha a} + \cos\alpha a = \cos ka \qquad (9) \longrightarrow$$

Where $P = \frac{mV_0ab}{\hbar^2}$ is known as scattering power of potential barrier and is measure of potential barrier strength with which the electrons are attracted by the positive ions.

In equation (9), there are only two variables $and\ k$.



From fig (2) the following conclusions can be drawn:

i. The energy spectrum of the electron consists of a number of allowed and forbidden energy bands.

ii. The width of the allowed energy band increases with increase of energy values i.e., increasing the values of αa.

iii. The width of the allowed energy band decreases with increase in the value of 'P' (potential barrier strength) i.e., with increasing in the binding energy of the electrons. As P $\rightarrow \infty$, the allowed bands are compressed into narrow lines ie., energy levels and the energy spectrum becomes a line spectrum as shown in fig If P $\rightarrow \infty$, then equation (9) has the only solution,

i.e., $\sin \alpha a = 0$

but $\sin n\pi = 0$

$\alpha a = n\pi$

$\alpha = \frac{n\pi}{a}$

And $\alpha^2 = \frac{n^2\pi^2}{a^2} \longrightarrow$ (10)

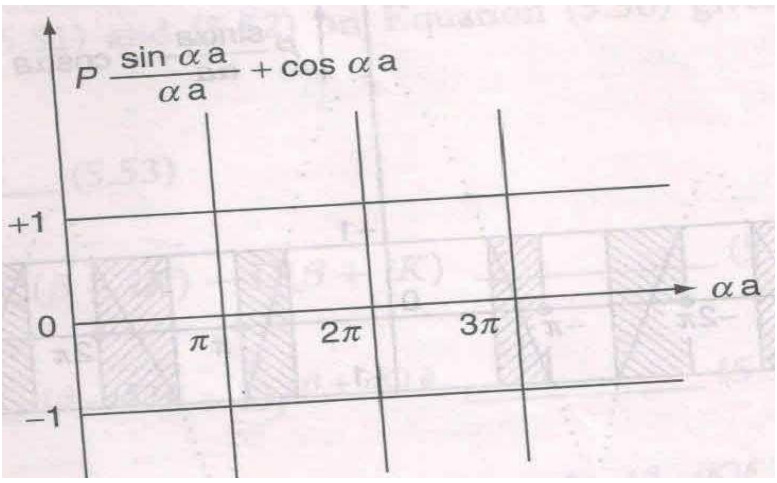
But $\alpha^2 = \frac{8\pi^2 mE}{h^2} \longrightarrow$ (3)

Equating the equations (10) and (3)

$$\frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{a^2} \qquad \text{(Since, } \hbar = \frac{h}{2\pi}\text{)}$$

$$\boxed{E = \frac{n^2h^2}{8ma^2}}$$

This expression shows that the energy spectrum of the electron contains discrete energy levels separated by forbidden regions.



iv. When P 0, then Equation (9) becomes

$$\cos \alpha a = \cos Ka$$

$\alpha = k$

$$\alpha^2 = K^2$$

But $\alpha^2 = \frac{2mE}{\hbar^2}$ (Since, $\hbar = \frac{h}{2\pi}$ *and* $k = \frac{2\pi}{\lambda}$)

$$E = \frac{\hbar^2 k^2}{2m}$$

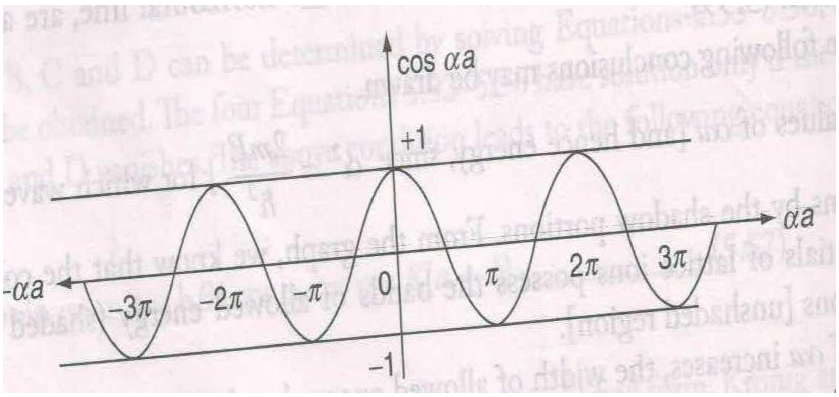
$$= \frac{h^2}{2m}\left(\frac{1}{\lambda^2}\right)$$

$$= \frac{h^2}{2m}\left(\frac{P^2}{m^2}\right)$$

$$= \frac{p^2}{2m}$$

$$E = \frac{1}{2}mv^2$$

The equation shows that all electrons are completely free to move in the crystal without ant constraints. Hence, no energy level exists i.e., all the energies are allowed to the electrons. This equation represents energy of a free particle.



Origin of energy bands in solids:

- We know that in a real solid, there are a very large number of atoms closely packed together.
- An isolated atom consists of positively charged nucleus and negatively charged electrons revolve around the nucleus in various orbits as shown in fig (1).
- In the innermost orbits, some electrons are tightly bonded with nucleus are called as bound electrons or inner electrons.
- In the outermost orbits, some electrons are loosely bounded with nucleus are called as valence electrons and the attraction force between the nucleus and these valence electrons

will be very less. So that the electrons can be easily detached from their outermost orbits and become free and hence these electrons are known as free electrons.

- Let us consider the case of sodium atom with atomic number 11 and mass number 23. There are 11 electrons. Two electrons revolve in the first orbit, eight electrons in the second orbit and one electron in the third orbit. The first, second, third orbits are also known as K, L, M orbits respectively.
- Each orbit has amount of energy associated with it. i.e., *each electron in an orbit will possess discrete energy is called energy level.*
- A convenient way of representing the different energy orbits of electron in an isolated atom is known as energy level diagram as shown in fig (2).
- The first orbit represents the first energy level; the second orbit represents the second energy level and so on.
- When large numbers of atoms are closely packed in a solid, then their corresponding energy levels are closely packed together and formed as regions called energy bands. Hence, *energy band can be defined as the range of energies possessed by an electron in a solid or the large number of discrete but closely but closely spaced energy levels.*
- A convenient way of representing the different energy bands of electron in a solid is known as energy band diagram as shown in fig (2).
- The range of energies possessed by bound electrons or inner electrons in a sold is called inner bands.
- The range of energies possessed by valence electrons or outermost electrons in a sold is called valence band. This band completely or partially filled.
- The range of energies possessed by free electrons or conduction electrons in a sold is called conduction band.
- Energy bands are separated by regions called forbidden energy bands.
- The separation between conduction band and valence band on the energy level diagram is known as forbidden energy gap.
- No electron can allow or stay in this region, because there are no energy levels in this region.

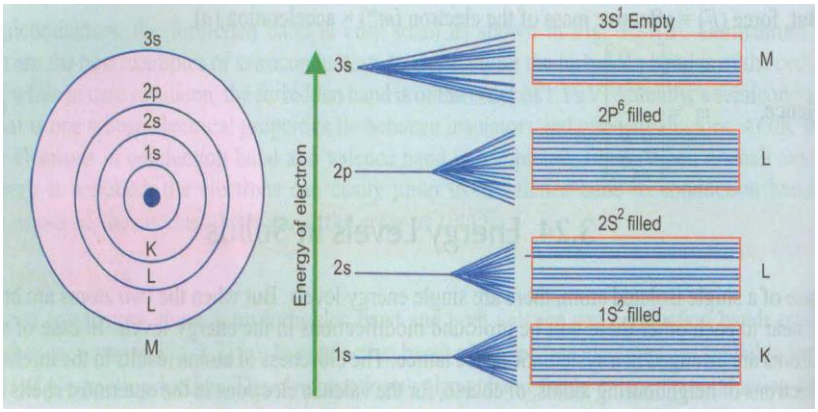
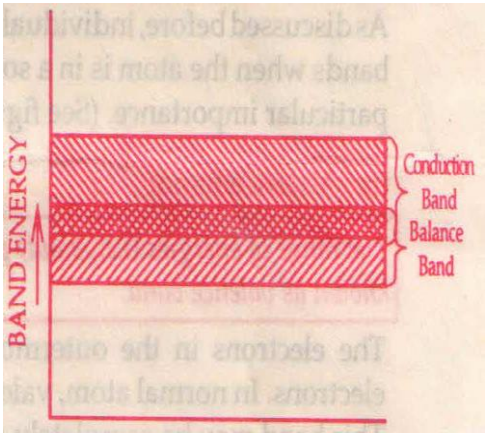


Fig: Energy levels and bands in sodium material.

Distinction between conductors, semiconductors and Insulators :

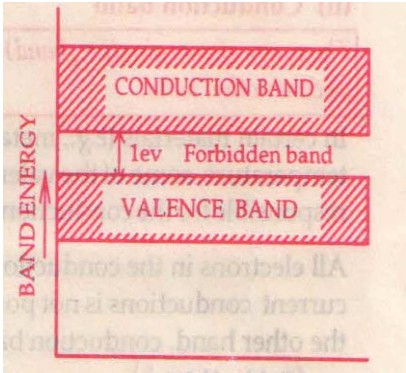
Conductors:

- Conductors are those substances which easily allow the passage of electric current through them. It is because large of free electrons available in a conductor.
- Examples: Metals (silver, copper and aluminum etc).
- In terms of band theory, the valence band and conduction band overlap each other as shown in fig.
- So no energy is required to move the electron from valence band to conduction band.
- Free electrons are available in conduction band and thus current flows through it, when a *p.d* is applied across it.
- When the temperature increases, then the electrical resistance increases due to collisions of electrons with each other.
- So, conductors have positive temperature coefficient of resistance.



Insulators:

- Insulators are those substances which do not allow the flow of electric current through them.
- Examples are dry wood, glass and diamond etc.
- In terms of band theory, the valence band is full while the conduction band is empty. Further the energy gap between valence band and conduction band is very large .i.e., greater than 5eV.
- So electrons cannot jump from the valence band into the conduction band.
- Therefore, a very high electric field is required to jump electrons from the valence band into the conduction band.
- For this reason the electrical conductivity is extremely small and may be nil under ordinary conditions.
- At room temperature, the electrons in the valance band don't have sufficient energy to cross over to the conduction band. However, when the temperature increased, some of the electrons in the valance band may acquire sufficient energy to cross over to the conduction band. Hence, the resistance of an insulator decreases with increase in temperature i.e., an insulator has negative temperature coefficient of resistance.



Semiconductors :

- Semiconductors are those substances whose electrical conductivity lies in between conductors and insulators.
- Examples: Si,Ge, GaAs , ZnS and GaP etc.
- In terms of band theory, the valence band is almost filled and conduction band is almost empty. Further the energy gap between valence band and conduction band is very small (0.78 eV for Ge and 1.21eV for Si etc)

- So electrons can easily jump from the valence band into the conduction band.
- Therefore, a very small electric field is required to jump electrons from the valence band into the conduction band.
- At 0 K, the valence band is completely full and conduction band is completely empty. Therefore, *semiconductor virtually behaves an insulator at low temperatures*. However, even at room temperature, some electrons (about one electron for 10^{10} atoms) in the valance band may acquire sufficient energy to cross over to the conduction band imparting little conductivity to the semiconductor.
- As the temperature increased, more the electrons in the valance band cross over to the conduction band and conductivity increases. Hence, the resistance of semiconductors decrease with increase in temperature i.e., semiconductors have negative temperature coefficient of resistance.

UNIT V

SEMICONDUCTORS AND SUPERCONDUCTORS

Semiconductors:

A Semiconductor is a substance whose conductivity or resistivity (10^{-4} to 0.5 ohm –m) lies in between conductors and insulators.

Examples: Si, Ge, GaAs, A*l*AS and InP etc.

Properties:

- They have crystalline structure.
- They are formed by covalent bonds.
- They have negative temperature of coefficient of resistance.
- At T=0K, they behaves as insulators.
- When the impurities (III group or V group elements) added to the pure semiconductor, then electrical conductivity increases.
- .

Intrinsic semiconductors and extrinsic semiconductors

Based on purity semiconductors are classified into types,

1. Intrinsic semiconductors
2. Extrinsic semiconductors

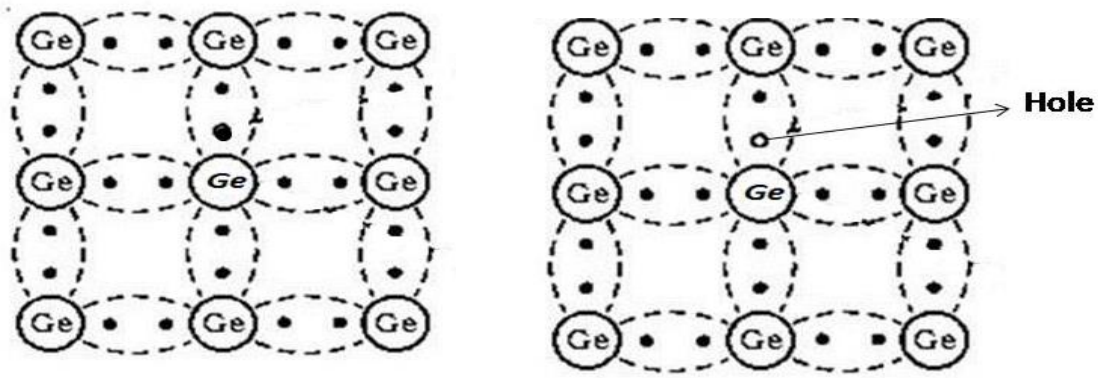
Intrinsic semiconductor:

A semiconductor in an extremely pure form is known as intrinsic semiconductor.

Examples: Si, Ge etc.

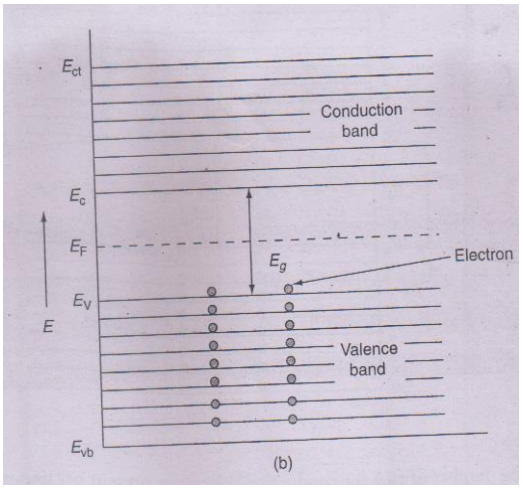
Explanation:

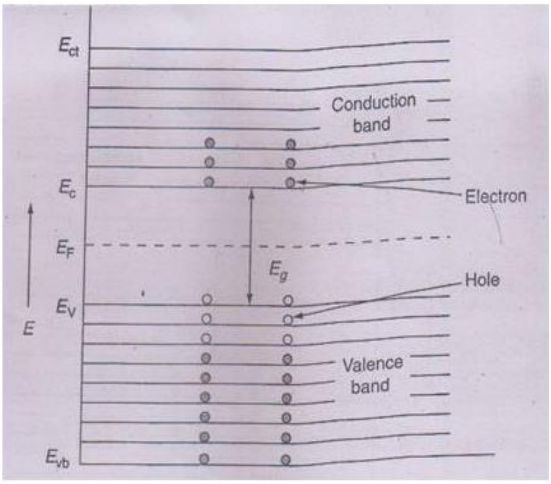
- Consider a pure (semi conductor) Si or Ge Crystal. We know that Ge is a fourth group element, i.e., it has four valence electrons; to get stability each silicon atom will make four covalent bonds with neighboring Silicon atoms as shown in fig.
- **At absolute zero temperature (0K):** all the electrons are tightly bound by the semi conductor atoms and the covalent bonds are very strong. So there are no free electrons. *Therefore, the semi conductor crystal behaves as a perfect insulator at absolute zero temperature.*



a) At T=0K b) At T >0K

- **<u>Above absolute zero (> 0 k):</u>** When the temperature is raised, some of the covalent bonds in the semi conductor break due to the thermal energy supplied.
- The breaking of bonds set those electrons free, at the same time vacant site produced in the covalent bond called "hole".
- The result is that a few free electrons exist in the semi conductor. These free electrons can constitute a tiny electric current if potential difference is applied across the semi conductor as shown in fig(c)
- **<u>Energy band description:</u>** Fig (d) shows the energy band diagram. As the temperature is raised, some of the valence electrons acquire sufficient energy to enter into the conduction band and thus become free electrons.
- Under the influence of electric field, these free electrons will constitute electric current. It may be noted that each time a valence electron enters into the conduction band; a hole is created in the valence band, which acts a positive charge.
- For one electron set free one hole is created. Therefore, thermal energy creates hole-electron pairs;
- In an intrinsic semi conductor, even at room temperature electron-hole pairs are created. Every free electron creates one hole in the valence band.
- *Therefore the number of electrons in the conduction band and the number of holes in the valence band become equal.*





a) At T=0K b) At T >0K

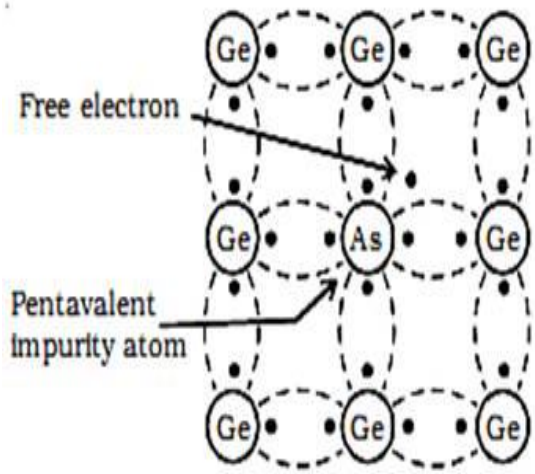
- When electric field is applied across an intrinsic semi conductor, the current conduction takes place by two processes namely: by free electrons and holes.
- Therefore the total current inside the semiconductor is the sum of currents due to free electrons and holes

$$I = I_{Electron} + I_{Hole}$$

<u>Extrinsic Semi Conductor:</u>

The electrical conductivity of a pure semiconductor is very small. To increase the conductivity, impurities are added. The impurity added semi conductor is called **"Extrinsic Semi Conductor". (OR)**

The conductivity of intrinsic semiconductor can be increased enormously by adding small amounts of impurity atoms [such as III or V group atoms]. This impure semiconductor is called **"Extrinsic Semi Conductor".**



Doping:

The process of adding impurities to a semiconductor is called doping and the impurity added is called **"Dopant".**

Depending upon the type of impurity added, Extrinsic Semiconductors are classified into two types.

1. N - Type Semi Conductor.
2. P - Type Semi Conductor

N – Type Semi Conductor:

When a small quantity of pentavalent impurity is added to a pure semi conductor, it is known as 'n – type semi conductor'.

Explanation:

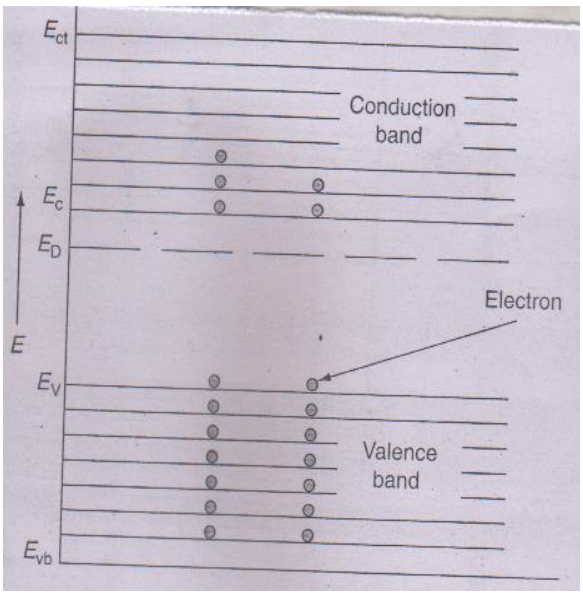
- To explain the formation of n-type semi conductor, consider a pure (semi conductor) Germanium Crystal.
- We know that germanium atom has four valence electrons, when a small amount of pentavalent impurities like arsenic (As) is added to a germanium crystal, a large number of free electrons available in the crystal.
- The reason is simple. Arsenic is pentavalent i.e., its atom has five valance electrons. The four of the five valence electrons of Arsenic will make covalent bonds with four germanium atoms. The fifth valence electron of arsenic atom finds no place in covalent bonds and is thus free as shown in fig.
- Therefore for each arsenic atom is added, one free electron will be available in the germanium crystal.
- Thus, a small amount of arsenic impurities (As) are added to a germanium crystal, a large number of free electrons available in the crystal.
- Pentavalent impurities are also known as donor impurities because they donate free electrons to semi conductor crystal.

Energy Band Description:

- Fig (2) shows the energy band description of n-type semi conductor.
- The addition of donor impurity to an intrinsic semi conductor creates extra energy level (called donor energy level) just below the bottom of conduction band.
- **At Absolute Zero:** At absolute zero temperature the fifth electrons of arsenic atoms occupy the donor level (represented by dashed lines) as shown in fig.
- **At above Absolute Zero:** Even room temperature provides enough thermal energy to push the electrons sitting at donor energy level to conduction band. (The result is that at room temperature, there are a large number of free electrons in the conduction band

as shown in fig) and also few of the covalent bonds in the semi conductor break due to the thermal energy applied.

- Due to the breaking of covalent bond, there will be a few holes formed in the valance band at this temperature. The result is that at room temperature, there are a large number of free electrons in the conduction band compared to the holes in valence band, i.e., Electrons are majority carriers and holes are minority carriers. So this material is called n-type semi conductor.



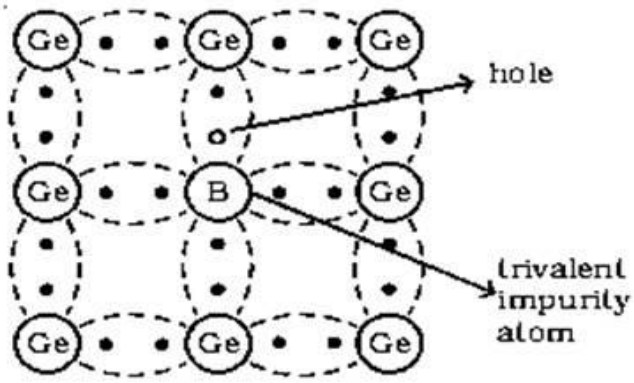
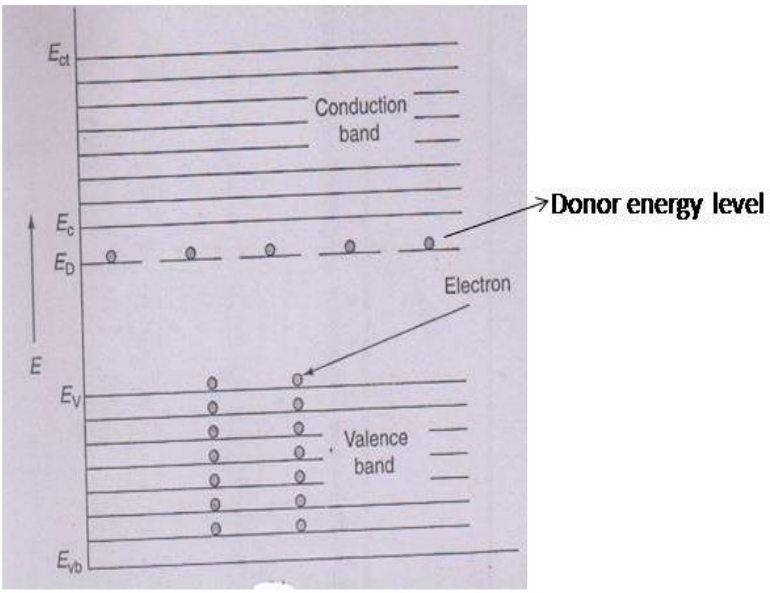


Fig: a) At T=0K b) At T >0K

P-Type Semi Conductor:

When a small quantity of trivalent impurity (such as indium (In) Boron (B), aluminum (Al) etc. is added to a pure semi conductor, it is called P-type semi conductor.

Explanation:



- To explain the formation of p-type semi conductor, consider a pure (semi conductor) Germanium Crystal.
- We know that germanium atom has four valence electrons, when a small amount of trivalent impurities like boron atoms (B) are added to a germanium crystal, a large number of holes available in the crystal.
- The reason is simple. Boron is trivalent i.e., its atom has thee valence electrons. The three valence electrons of Boron atom will make three covalent bonds with surrounding Ge atoms.

- The fourth bond is incomplete because short of one electron. This missing electron is called a “hole”.
- Therefore, for each boron atom is added, one hole will be available in the germanium crystal.
- Thus, a small amount of boron impurities (B) are added to a germanium crystal, a large number of free holes available in the crystal.
- Trivalent dopant impurities also called acceptors. Because they have need of extra electrons to complete their fourth covalent bonds formation with Ge atoms.
- **Energy Band Description:** Fig (2) shows the energy band description of p-type semi conductorThe addition of acceptor impurity to an intrinsic semi conductor creates extra energy level (celled acceptor energy level) just above the top of the valance bond [see fig].
- Since the energy difference between acceptor level and valence band energy level is very small so even at low temperature the valence band electrons gain sufficient thermal energy to occupy the acceptor energy level states as shown in fig.
- After receiving an electron the, Boron atom becomes a negative ion.
- But at room temperature provides enough thermal energy to push the electrons in the valence band to the acceptor energy level and also due to thermal agitation a few electrons jumps from V.B to C.B.
- The result is that at room temperature there are a large number of holes in the V.B compared to the electrons in the C.B. i.e., holes are majority carriers and electrons are minority carries. So, this material is called P-type semi conductors.

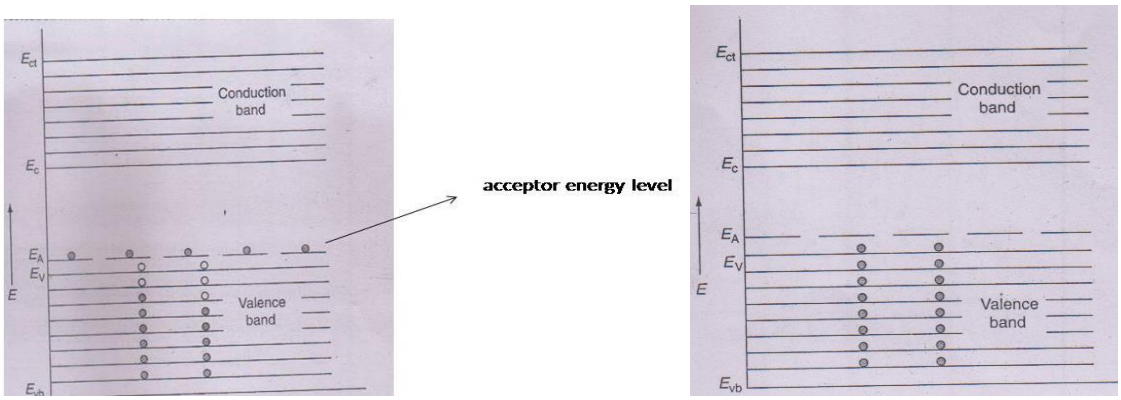


Fig: a) At T=0K b) At T >0K

Direct Band Gap And Indirect Band Semi Conductors:

Based on the type of energy emission the semiconductors can be of two groups.

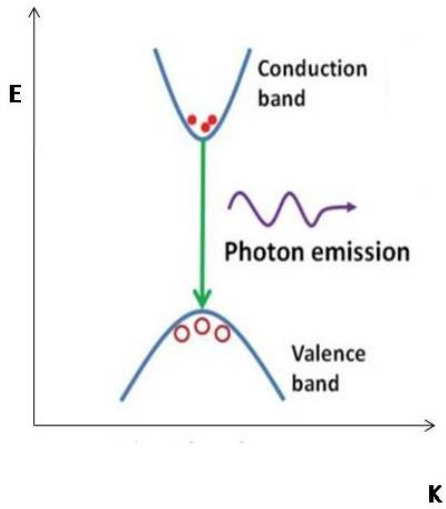
1. Direct band gap semiconductors
2. Indirect band gap semiconductors

Direct Band Gap Semi Conductors:

- The semiconductor, in which the minimum of the conduction band lies directly above the maximum of the valance band in the E-K space diagram (as shown in fig), is known as direct band gap semiconductor.
- In a direct band gap semiconductor such as GaAS, A*l*AS and InP, when an excited electron falls back into the valence band, the electrons and holes recombine to produce light energy.

i.e., ē + hole ⟶ hv (photon)

- This process is known as radiative recombination and also called spontaneous emission.
- These direct band gap semiconductors are used to make LED's and lasers of different colours.

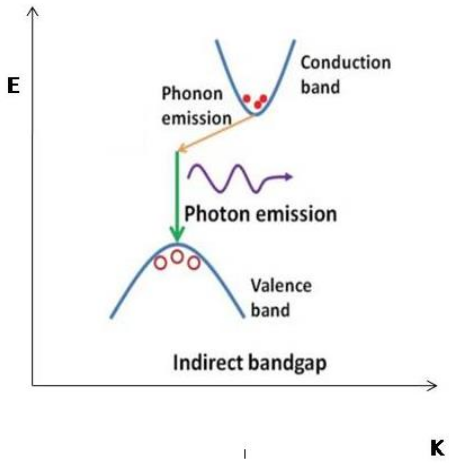


<u>Indirect Band Gap Semi Conductors:</u>

- The semiconductor, in which the minimum of the conduction band does not lies directly above the maximum of the valance band in the E-K space diagram(as shown in fig), is known as indirect band gap semiconductor.
- In an indirect gap semiconductor such as Si, Ge and Gap, when an excited electron falls back into the valance band, the electrons and holes recombine to generate heat and is dissipated within the material.

  i.e., ē + hole ⟶ phonon

This process is known as non-radiative recombination



Electrical conduction in semiconductors

In the semiconductors electrical conduction arises due to two processes namely

1. Drift and
2. Diffusion

Drift:

Definition:-

Under an applied electric field both the charge carriers i.e., electrons and holes acquire an average velocity called the drift velocity (V_d) and this process is called drift.

The drift velocity V_d is small. It is directly proportional to the electrical field E.

i.e., $V_d \propto E$

$$V_d = \mu E \longrightarrow (1)$$

Where μ is a constant and is called the mobility of charge carriers.

If μ_n and μ_p are the electron mobility and hole mobility respectively, then

$$V_n = \mu_n E \text{ and } \longrightarrow (2)$$

$$V_p = \mu_p E \longrightarrow (3)$$

1. We know that, Current $I = ne AV_d$.
But in a semi conductor, the total current is the sum of electrons current and holes current.

Let n be the electron concentration and p be the hole concentration.

Therefore $I = I_n + I_p$

$= neAV_n + pAV_p$

$= eA (nV_n + pV_p)$ [From equations (2) &(3)]

$= eA(n \mu_n E + p \mu_p E)$

$= eA(n \mu_n + p \mu_p)E$

2. Current density $J = \frac{I}{A}$

$J_n = \frac{neA\mu_n E}{A}$ and

$$J_p = \frac{peA\mu_p E}{A}$$

Therefore, total current density $J = J_n + J_p$

$$= \frac{eA(n\mu n + P\ \mu p]E}{A} = e(n\mu_n + P\ \mu_p)E$$

Conductivity $\sigma = \frac{J}{E} = \frac{e(n\mu n + P\ \mu p]E}{E} = e(n\mu_n + P\ \mu_p)$

NOTE:-

1. For an intrinsic semiconductor: $n = p = n_i$

Where n_i is the carrier concentration of the intrinsic semiconductor.

$$I_{inc} = eA(n_i\mu_n + n_i\mu_p)E$$

$$= eAn_i(\mu_n + \mu_p)E$$

$\therefore$ Current density $J_{in} = \frac{I_{in}}{A} = \frac{eAn_i(\mu n + \mu p]E}{A} = en_i(\mu n + \mu p)E$

Conductivity $\sigma_{in} = \frac{J_{in}}{E} = \frac{en_i(\mu n + \mu p]E}{E} = en_i(\mu n + \mu p)$

FOR AN EXTRINSIC SEMICONDUCTOR:-

a)**For n-type semiconductor :-**The hole current is neglected, as the holes are minority carriers and the electrons are the majority carriers.

$n \simeq N_d$ is the donor electron concentration

[$\therefore$ Where N_a can be neglected]

$$I = eA\ V_n N_d$$

$$= eA\mu_n EN_d$$

$$J = \frac{I}{A} = e\mu_n EN_d$$

$$\sigma = \frac{J}{E} = e\mu_n EN_d$$

b)**For P-type semiconductor :-**The electron current is neglected and $P = N_{ai}$, where N_a is the acceptor hole concentration. [$\therefore$ When N_d can be neglected].

$$I = eA\ V_P N_a$$

$$= eA\mu_p E N_a$$

$$J = \frac{I}{A} = e\,\mu_p\,E\,N_a$$

$$\sigma = \frac{J}{E} = e\,\mu_p\,N_a$$

DIFFUSION :-

Diffusion takes place due to existence of non-uniform concentration of charge carriers.

Definition:-

Due to non –uniform carrier concentration in a semiconductor, the charge carrier's moves from a region of higher concentration to a region of higher concentration to a region of lower concentration. This process is known as diffusion of charge carriers.

→ According to Fick's law, the diffusion flux F is directly proportional to concentration gradient <i.e. carrier concentration per unit length >of the particles and is in the opposite direction.i.e;

$$F \propto \frac{-\partial(\Delta N)}{\partial X}$$

$$F = -D\,\frac{\partial(\Delta N)}{\partial X} \longrightarrow (1)$$

Where D is diffusion constant.

And ΔN is the excess carrier concentration.

Let Δn and Δp be the excess concentration in a semiconductor material.

Then Fick's

$$F \propto \frac{-\partial(\Delta N)}{\partial X} \quad \text{(For electrons)}$$

$$F = -D_n\,\frac{\partial(\Delta N)}{\partial X} \longrightarrow (2)$$

and

$$F \propto \frac{-\partial(\Delta P)}{\partial X} \quad \text{(For electrons)}$$

$$F = -D_p\,\frac{\partial(\Delta P)}{\partial X} \longrightarrow (3)$$

→ But the current density due to diffusion depends on charge of particle and rate of diffusion.

$\therefore$ Current density (J) = Charge of particle × Rate of diffusion ⟶ (4)

→ The diffusion flux density due to electron is given by

$$J_{n(Diff)} = -e\left[-Dn\,\frac{\partial(\Delta N)}{\partial X}\right] \qquad \therefore \text{from}(4)$$

$$J_{n(Diff)} = e D_n\,\frac{\partial(\Delta N)}{\partial X} \longrightarrow (5)$$

The diffusion flux density due to hole is given by

$$J_{p(Diff)} = +\,e\left[Dp\,\frac{\partial(\Delta P)}{\partial X}\right] \longrightarrow (6)$$

Where +e and –e are charge of the hole and electron respectively.

$\therefore$ The total current density due to diffusion is given by

$J_{TOTAL,(DIFF)} = J_{n(diff)} + J_{p(diff)}$

$$J_{TOTAL(DIFF)} = eD_n \frac{\partial(\Delta N)}{\partial X} - eD_P \frac{\partial(\Delta P)}{\partial X} \longrightarrow (7)$$

NOTE:- The total current density due to excess electrons is the sum of the current densities due to drift and diffusion of electrons.

$J_n = J_{n(drift)} + J_{n(diff)}$

$$= \Delta n \mu_n E + eD_n \frac{\partial(\Delta N)}{\partial X} \longrightarrow (8)$$

Similarly,

The total current density due to excess holes is the sum of the holes densities due to drift and diffusion of holes.

$$J_P = \Delta P e \mu_P E - eD_P \frac{\partial(\Delta P)}{\partial X} \longrightarrow (9)$$

EINSTEIN'S RELATION:-

Einstein showed the direct relation between the mobility (μ) and diffusion coefficient (D) of a semiconductor. This relation is called Einstein relation.

At any disturbance in equilibrium, the drift and diffusion current densities due to carrier concentration are equal.

i.e.; $J_{n(drift)} = J_{n(diff)}$

$$(\Delta n)\, eE\mu_n = D_n e \frac{\partial(\Delta n)}{\partial X} \longrightarrow (1)$$

The force on excess electrons (Δn) to restore equilibrium is given by the product of Δn and electric forces eE on each electron.Thus,

$$F = (\Delta n) eE \longrightarrow (2)$$

Equation (1) can be written as

$$F.\mu_n = D_n e \frac{\partial(\Delta n)}{\partial X}$$

$$F = \frac{D_n}{\mu_n} e \frac{\partial(\Delta n)}{\partial X} \longrightarrow (3)$$

From the kinetic theory of gases, the force on gas molecules (charge carriers) is given by

$$F = K_B T \frac{\partial(\Delta n)}{\partial X} \longrightarrow (4)$$

Comparing (3) and (4),we get

$$K_B T \frac{\partial(\Delta n)}{\partial X} = \frac{D_n}{\mu_n} e \frac{\partial(\Delta n)}{\partial X}$$

$$K_B T = \frac{D_n}{\mu_n} e$$

$$D_n = \frac{\mu_n k_B T}{e}$$

$$\frac{D_n}{\mu_n} = \frac{k_B T}{e} \longrightarrow \quad (5)$$

Similarly for holes,

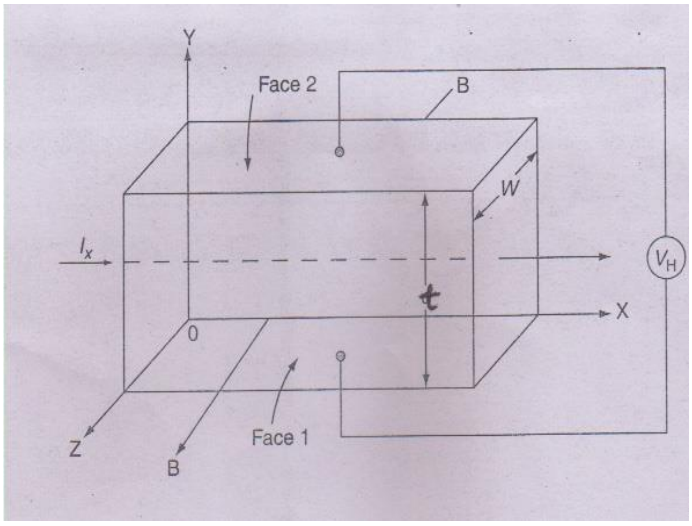
$$\frac{D_p}{\mu_p} = \frac{k_B T}{e} \longrightarrow \quad (6)$$

From equations (5) and (6)

$$\frac{D_n}{\mu_n} = \frac{D_P}{\mu_P}$$

$$\frac{D_n}{D_P} = \frac{\mu_n}{\mu_P}$$

The above relation is known as Einstein's relation.



Hall Effect:

When a current carrying semiconductor is placed in the magnetic field, then a potential difference (voltage) or electric field is developed in the material perpendicular to both the current and the applied magnetic field .this effect is known as Hall Effect and the developed potential difference (voltage) is known as hall voltage.

Explanation:

- Consider a semiconductor slab of thickness'*t*' and width '*w*' in which current (I) flowing along X-direction and magnetic field is applied along Z- direction as shown in fig.
- The charge carriers inside the semiconductor experience a force due to the applied magnetic field called Lorentz force.
- As a result, electrons move (drift) towards downward direction with velocity 'V_d' and accumulate at bottom surface (face 1) and the corresponding positive charge on upper surface (face2) of the material as shown in fig.
- Thus a potential difference and electric field developed across the surface. This developed electric field along Y-direction exerts a force ($F_e = eE_H$) on the electrons to prevent further accumulation at the bottom surface.
- Therefore, the force due to potential difference $F_e = eE_H$

  the force due to magnetic field $F_L = BeV_d$

At steady state

$$F_e = F_L$$

$$eE_H = Bev_d$$

$$E_H = Bv_d \longrightarrow (1)$$

For n-type:

If 'n' is the electrons carrier concentration in the semiconductor, then current density

$$J = -neV_d$$

$$V_d = \frac{J}{-ne} \longrightarrow (2)$$

Substituting eq (2) in eq(1)

$$E_H = \frac{BJ}{-ne}$$

$$\boxed{E_H = R_H BJ} \longrightarrow (3)$$

Where $R_H = \frac{1}{-ne}$ is known as Hall coefficient.

For p-type:

If 'p' is the electrons carrier concentration in the semiconductor, then current density

$$J = peV_d$$

$$V_d = \frac{J}{pe} \longrightarrow (2)$$

Substituting eq (2) in eq(1)

$$E_H = \frac{BJ}{pe}$$

$$\boxed{E_H = R_H BJ} \longrightarrow (3)$$

Where $R_H = \frac{1}{pe}$ is known as Hall coefficient

Hall coefficient in terms of hall voltage:

If the thickness of the sample is 't' and the developed voltage is V_H, then the Hall voltage

$$V_H = E_H.t \longrightarrow (4)$$

Substituting eq (3) in eq(4);

$$V_H = R_H BJ\, t \longrightarrow (5)$$

If 'w' is the width of the semiconductor,

then area of the sample $A = w.t$

But current density $J = \frac{I}{A} = \frac{I}{w\,t} \longrightarrow (6)$

Substituting eq (8) in eq(7); we get

$$V_H = \frac{R_H IB}{w}$$

$$R_H = \frac{V_H w}{IB}$$

$$\boxed{R_H = \frac{V_H w}{IB}}$$

This is the relation between Hall coefficient and hall voltage.

- The electrical conductivity in a semiconductor is given by

$$\sigma = ne\mu$$

$$\mu = \frac{\sigma}{ne}$$

$$\boxed{\mu = R_H\,\sigma}$$

APPLICATIONS:

- Knowing R_H and σ, the mobility of charge carriers can be measured. $i.e., \mu = R_H\,\sigma$
- It is used to fine the carrier concentration

$$\boxed{n = -\frac{1}{eR_H} \quad \text{and } p = \frac{1}{eR_H}}$$

- It is used to determine whether the material is p-type or n-type semiconductor.
  i.e.,if R_H is negative then the material is n-type.
  if R_H is positive then the material is p-type.

P-n junction:

When a p-type semiconductor is suitably joined to n-type semiconductor, the contact surface is established between them, is called p-n junction.

Formation of p-n junction:

- The formation of p-n junction as shown in fig
- Let us consider the formation of a junction when two separate semiconductors of p and n-types are brought together.
- We know that, in a p-type semiconductor holes are the majority carriers and electrons are the minority carriers and in a n-type semiconductor free electrons are the majority carriers and holes are minority carriers.
- When they are joined ,due to non-uniform carrier concentration ,free electrons diffuse from n-region to p-region and recombine with holes and holes diffuse from p-region to n-region and recombine with electrons.
- As result, a net negative charge is created on the p-side and a net positive charge is created on the n-side. These charges produce the potential difference, is called potential barrier and this region is called depletion region because it opposes the further flow of electrons from n-region to p-region.
- The potential barrier is the order of 0.1 to 0.3 volt.

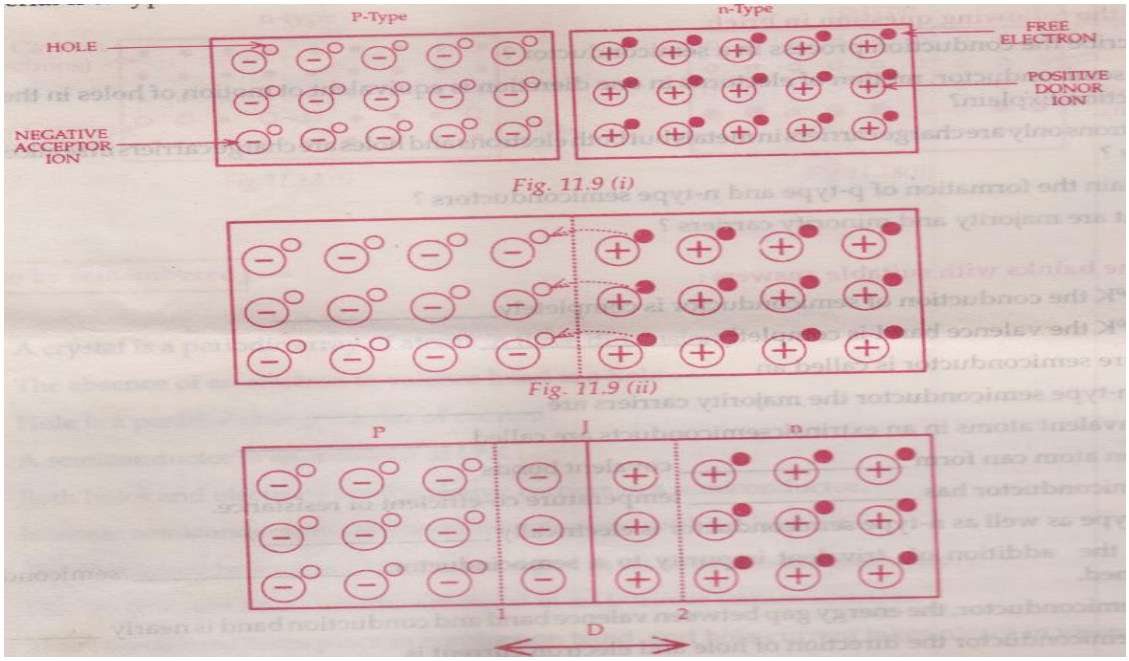


Fig: Formation of p-n junction

P-n junction diode:

- A two electrode device is called diode.
- In the p-n junction, p-type semiconductor and n-type semiconductors behaves as electrodes.
- The symbol for the p-n junction diode is shown in fig.
- In fig, arrow of head indicates a p-type semiconductor and shows the direction of conventional current flow when the junction is forward biased, and the bar indicates a n-type semiconductor.

Bias:

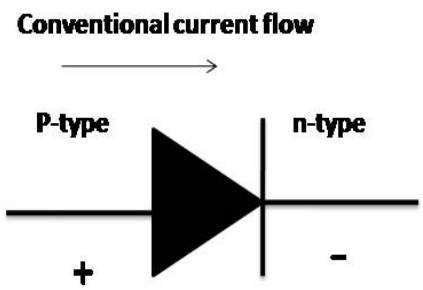
To apply an external voltage to the p-n junction is called bias.

Bias can be divided into two types

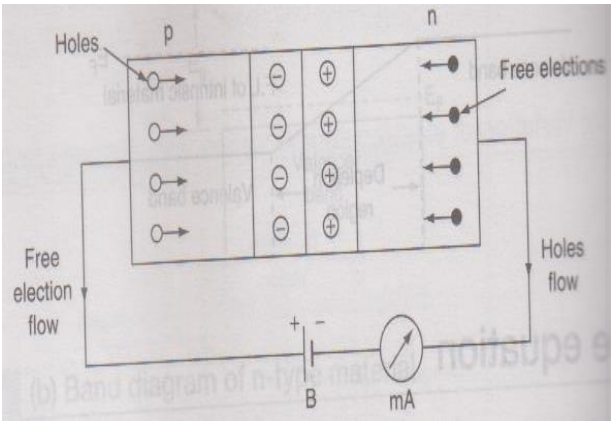
1. Forward bias
2. Reverse bias

Forward bias:

- When a p-type semiconductor is connected to the positive terminal of the battery and n-type semiconductor is connected to the negative terminal of the battery, then it is known as forward bias.
- When the junction is forward biased
  a) The width of depletion region is decreased
  b) The potential barrier is reduced
  c) The Junction offers a very low resistance.
  d) Electrons flows from n-region to p-region through the junction and this gives rise to forward current



e) Current is in the order of mA



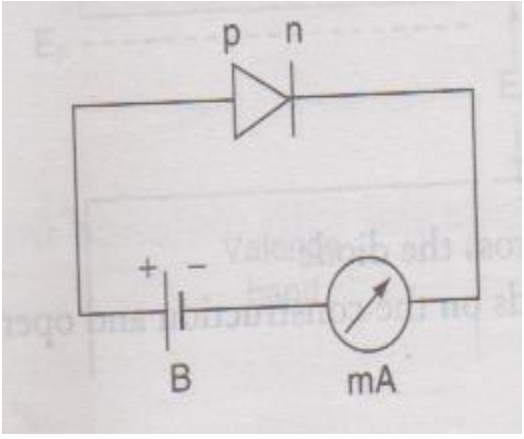


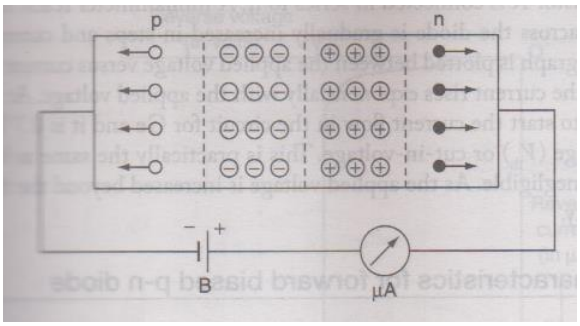
Fig: Forward bias p-n junction

Reverse bias:

When a p-type semiconductor is connected to the negative terminal of the battery and n-type semiconductor is connected to the positive terminal of the battery, then it is known as reverse bias.

When the junction is reverse biased

a) The width of depletion region is increased
b) The potential barrier is increased
c) The Junction offers a very high resistance.
d) Current is in the order of μA



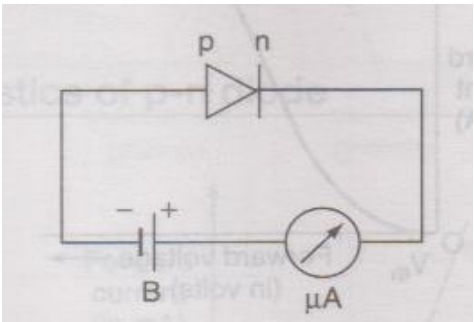


Fig: Reverse bias p-n junction

SUPER CONDUCTIVITY

Introduction:

Super conductivity is the phenomenon in which the electrical resistance of certain materials becomes zero at very low temperatures.

For example, the electrical resistance of pure mercury suddenly drops to zero, when it is cooled below 4.2 Kelvin and becomes a superconducter.This was first observed by the Dutch physicist, Heike Kammerlingh Onnes on April 8, 1911. Further, the theory of super conductivity was developed in 1957 by three American physicists-John Bardeen, Leon Cooper, and John Schrieffer, through their Theories of Superconductivity, known as the **BCS Theory.**

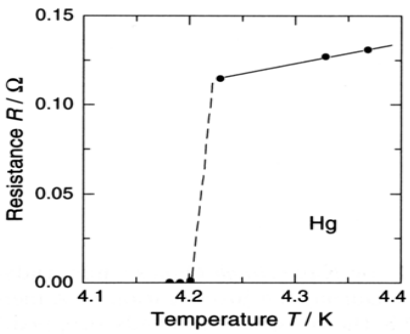


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Figure: The resistance of mercury measured by Onnnes.

Super conductors:

A Super conductor is a material that loses all its resistance to flow of electric current, when it is cooled below a certain low temperature.

Examples:

Material	Type
Tungsten	Metal
Zinc	Metal
Aluminum	Metal
Tin	Metal
Mercury	Metal
Lead	Metal
NbTi	Inter metallic compound
Nb_3Sn	Inter metallic compound
Nb_3Ge	Inter metallic compound
$YBa_2Cu_3O_7$	Ceramic
TlBaCaCuO	Ceramic

General properties of super conductor:

- Superconductivity is the physics of the Cooper Pairs.It occurs due to the movement of electron pairs called cooper pairs through the lattice points.These cooper pairs are formed due to the **electron –lattice –electron interaction.**
- Super conductivity is a low-temperature phenomenon.
- The electrical resistivity of a super conducting material is very low and is the order of 10^{-5} Ωcm.
- The temperature at which a normal conductor loses its resistivity and becomes a super conductor is known as transition temperature or critical temperature (T_C).Different materials will have different critical temperatures.
- When impurities are added to superconducting elements, the super conducting property is not lost, but the T_c value is lowered.
- The critical temperature(Tc) decreases with increasing isotopic mass **M**.This effect is known as isotope effect.

i.e.,Tc $\alpha M^{\frac{-1}{2}}$

- The magnetic field at which super conductor loses its super conductivity and becomes normal conductor is known as critical magnetic field H_c. Different materials will have different Hc values.
- Super conductors do not allow magnetic flux through them and behave as a diamagnetic. This property of expulsion of magnetic flux is known as meissner effect.
- Ferromagnetic materials like Fe,Co,Ni do not show superconductivity.
- The magnetic flux (Φ) lines passing through a super conducting ring due to persistent current is quantized in terms of integral multiples of $\frac{h}{2e}$

$$\Phi = \frac{nh}{2e}$$

Where n = 1, 2, 3……

Where $\frac{h}{2e}$ is known as fluxion (or) fluxiod. = 2.07 X 10^{-15} Weber's.

<u>Effect of temperature-Critical temperature:</u>

In the year 1911, kammerligh Onnes observed that the electrical resistance of pure mercury suddenly drops to zero, when it is cooled below 4.2 Kelvin and becomes a superconductor.

The temperature at which a normal conductor loses its resistivity and becomes a super conductor is known as transition temperature or critical temperature (T_C) as show in fig.

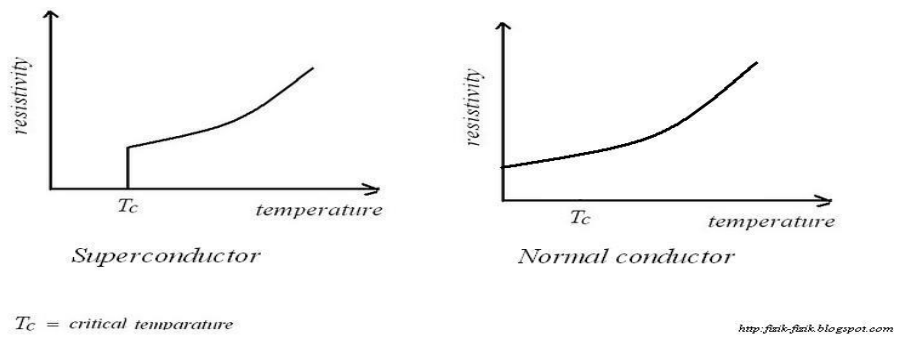


Fig: The variation of electrical resistance with temperature

Figure shows the variation of electrical resistivity with temperature. Below T_C the material is said to be in the superconducting state and above T_C the material is said to be in non-superconducting state (i.e., normal state).

The value of this critical temperature varies from material to material.

Material	**Type**	**T_c(K)**
Tungsten	Metal	0.01
Zinc	metal	0.88
Aluminum	metal	1.19
Tin	metal	3.72

Mercury	metal	4.15
Lead	metal	7.2
NbTi	Inter metallic compound	9.5
Nb_3Sn	Inter metallic compound	21
Nb_3Ge	Inter metallic compound	23.2
$YBa_2Cu_3O_7$	ceramic	90
TlBaCaCuO	ceramic	125

High temperature superconductors:

Super conductors are divided into two types based on their transition temperatures.

a) Low T_C super conductors

b) High T_C super conductors

If a transition temperature is low (below 30 K), then the superconductors are known as low temperature superconductors.

If a transition temperature is high (above 30 K), then the superconductors are known as high temperature superconductors.

The first high-T_c superconductor was discovered in 1986 by Georg Bednorz and Muller, in ceramics. They found that the mixed metallic oxide of lanthanum-barium-copper (La_1Ba_2 Cu_3 O_7) exhibited superconductivity at about 30 K.

Further it has been developing by many scientists and co-research scholars.

Some examples are:

S.No	Material	T_C K
1	Ba-pbBi-O_3	38
2	$YBa_2Cu_3O_7$	92
3	$Bi_2Sr_2CaCu_2O_8$	85
4	$Bi_2Sr_2Ca_2Cu_3O_6$	110
5	$Tl_2Ba_2CuO_6$	80
6	$Tl_2Ba_2CaCu_2O_8$	108
7	$Tl_2Ba_2Ca_2Cu_3O_{10}$	125
8	$TlBa_2Ca_3Cu_4O_{11}$	122
9	$HgBa_2CuO_4$	94
10	$HgBa_2CaCu_2O_6$	128
11	$HgBa_2Ca_2Cu_3O_8$	134

Properties:

Some of the properties of high T_C superconductors as follows:

1. They have high transition temperature.
2. They are brittle in nature.
3. They are highly anisotropic.
4. They are reactive, brittle and can't be easily formed or joined.
5. They are oxides of copper in combination with other elements.
6. The hall coefficient is positive indicating that the charge carriers are holes.
7. Their behavior can't be explained by BCS theory.

Effect of magnetic field - Critical magnetic field:

Kammerlingh Onnes observed in 1913 that superconductivity vanishes if a suffiently strong magnetic field is increased.

When a magnetic field is applied to a super conductor, then particular value of applied field and below its critical temperature, it loses super conductivity and becomes a normal conductor. This minimum magnetic field required to destroy the super conducting state is called the critical magnetic field H_C.

The critical magnetic field of a superconductor is a function of temperature.

The variation of Hc with temperature is given by

$$\mathbf{H_C = H_0} \left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

Where H_C = critical magnetic field,

H_0= critical magnetic field at T=ok, and

T_C= critical temperature.

Figure shows the variation of critical magnetic field H_C as a function of temperature. The material is said to be in the superconducting state within the curve and is non super conducting (i.e., normal state) in the region the outside the curve.

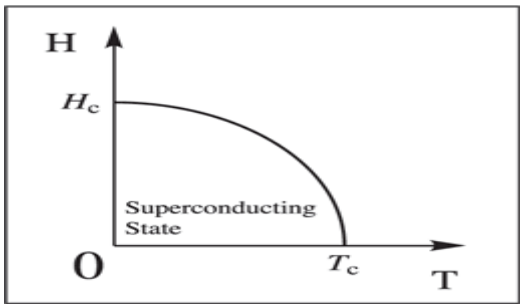


Figure: Effects of temperature and magnetic field on the superconducting state.

Meissner effect:

When the supper conducting material is placed in magnetic field, under the condition when $T \leq T_C$ and with $H \leq H_C$ the flux lines are expelled from the material. Thus the material exhibits perfect diamagnetism. This phenomenon is called as Meissner effect.

Explanation:

Consider a normal conducting material at room temperature. When a magnetic field H is applied to it, then it allows the magnetic lines of force to pass through it. Thus we have a magnetic tic field B in a conductor as shown in fig (1).

Now, when the material is cooled below its critical temperature ($T \leq T_C$) and with $H \leq H_C$, then the magnetic lines of forces are expelled or ejected out from the material as shown in fig(2).

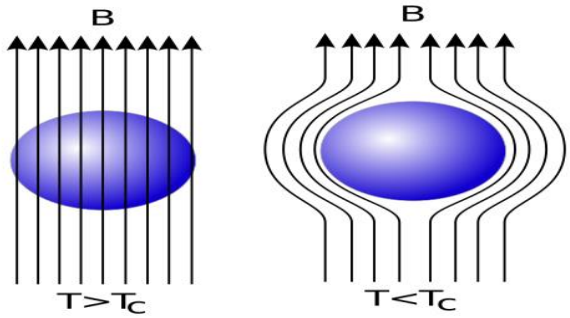


Fig (1): Normal state (B ≠ 0 at T > Tc). **Fig (2): Superconducting state**. (B=0 at T < Tc)

Proof:

We know that, the total magnetic flux density in a normal conductor is given by

$$B = \mu_0 (M+H) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where μ_0 is the permeability of free space=$4\pi x10^{-7}$ H/ m

M is the intensity of magnetization

and, H is he4 applied magnetic field strength.

But, for superconductor B=0

Therefore, equation (1) can be written as

$$0 = \mu_0 (M+H)$$

$$[\because \ \mu_0 \neq 0]$$

$$M+H=0$$

or $M=-H$

or $\frac{M}{H} = -1$

Hence, $\chi_m = \frac{M}{H} = -1$ is called the magnetic susceptibility. Thus this means that, for a superconductor the susceptibility is negative .i.e., a super conductor exhibits perfect diamagnetism.

Types of superconductors:

Depending upon their behavior in an external magnetic field, superconductors are divided into two types:

1) Type I superconductors and 2) Type II superconductors

Let us discuss them one by one:

1) **Type I superconductors**:

   Those superconductors which loose their superconductivity very easily or abruptly when placed in the external magnetic field are known as Type I superconductors.

   Explanation:

   When the super conductor is kept in the magnetic field and if the field is increased the super conductor becomes a normal conductor abruptly at critical magnetic field as shown in fig. These types of materials are termed as Type – I super conductors. After Hc, the Type I superconductor will become conductor.

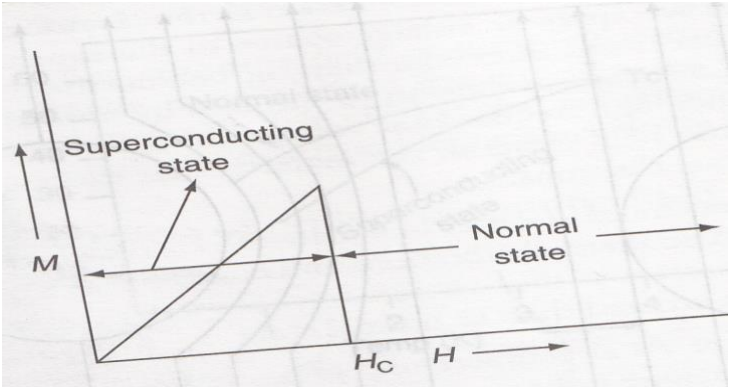


Fig: The variation of magnetization (M) with external magnetic field H in a type I Superconductor.

Properties:

- These superconductors are also known as **soft superconductors** because of this reason that is they loose their superconductivity easily and relatively small magnetic field requirement to bring them back to their normal state. These superconductors exhibit perfect and complete Meissner effect.
- Only one critical field exists for these superconductors.
- The critical magnetic field value is very low and highest value is 0.1Tesla.
- No mixed states exist in these superconductors.
- Materials with pure form are Type I superconductors.
- Examples: :

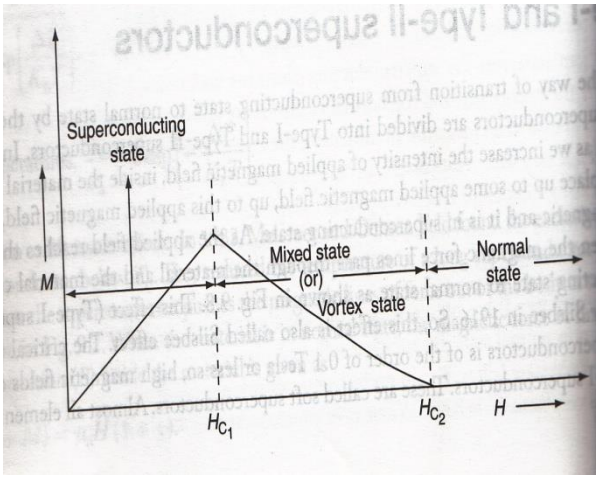
Material	Hc (Tesla)
Zinc	0.0054
Aluminum	0.0105
Mercury	0.014
Stransiam	0.03
Lead	0.08

2) **Type II superconductors**:
Those superconductors which lose their superconductivity gradually but not easily or abruptly when placed in the external magnetic field are known as Type II superconductors.

Explanation:

When the super conductor is kept in the magnetic field and if the field is increased, below the lower critical field Hc_1, the material exhibits perfect diamagnetism i.e., it behaves as a super conductor and above Hc_1, the magnetization decreases and hence the magnetic flux starts penetrating through the material. The specimen is said to be in a mixed state (or) vortex state between Hc_1 and Hc_2, above Hc_2 (upper critical field) it becomes a normal conductor as shown in fig.

Type – II Super conductors are also called as hard super conductors because of relatively sma ll magnetic field requirement to bring them back to their normal state.



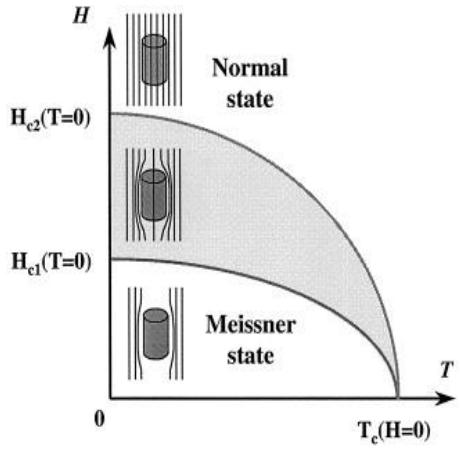


Fig: The variation of magnetization (M) with external magnetic field H in a type II Superconductor.

Properties:

- These superconductors are also known as **hard superconductors** because of this reason that is they lose their superconductivity gradually but not easily and relatively large magnetic field requirement to bring them back to their normal state.
- These superconductors exhibit Meissner effect but not completely.
- Two critical fields Hc_1 (lower critical magnetic field) and Hc_2 (upper critical magnetic field) exist for these superconductors.
- The critical magnetic field value is very high.
- Mixed states exist in these superconductors.
- Materials with impurities or alloys are of Type II superconductors.
- Examples:

Material	**Hc (Tesla)**
NbN	8×10^6
$Babi_3$	59×10^3
Nb_3Sn	24.5
Nb_3Ge	38
$Y_1Ba_2Cu_3O_7$	300

Penetration depth:

According Meissner effect, when the material is cooled below its critical temperature ($T \leq T_C$) and with $H \leq H_C$, then the magnetic lines of forces are expelled or ejected out from the super conducting material .i.e., magnetic field drop to zero at the surface of the super conducting material.

But**,** in 1935, F. London and H.London observed that the applied magnetic field does not drop to zero at the surface of the super conductor but decreases exponentially as given by the equation.

$$H = H_0 \exp^{-x/\lambda} \ldots\ldots\ldots\ldots (1)$$

Where H is the intensity of magnetic field at a depth x from the surface,
Ho is the intensity of magnetic field at the surface,
and λ is called London Penetration depth.
To define the penetration depth, in the equation (1), let x = λ. then

$$H = H_0 \exp^{-1}$$

$$H = \frac{H}{e} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Hence penetration depth can be defined as the distance from the surface of the superconductor to a point inside the material at which the intensity of magnetic field is (1/e) of the magnetic field at the surface [i.e., Ho/e].

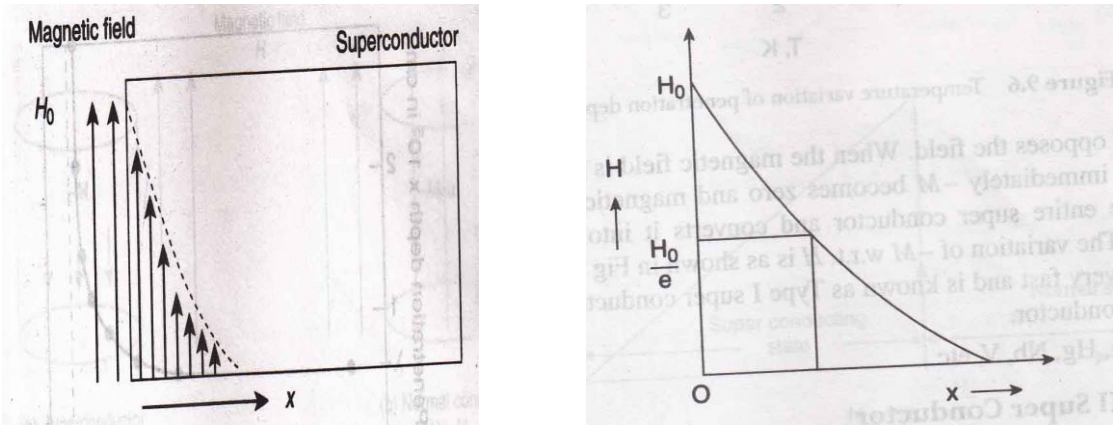


Fig: London penetration depth

The magnetic field is likely to penetrate to a depth of 10 to 100nm from the surface of a superconductor. The value of for some materials is given below:

Material	**λ (in nm)**
Mercury	70
Aluminum	50
Tin	50
Lead	39
Indium	64

<u>Flux Quantization:</u>

The quantization of magnetic flux is a special property of superconductors.
We know that electric charge is quantized in terms of integral multiples of charge on an electron.

$$\boxed{Q = \pm ne}$$

Where n = 1, 2, 3 ……

Similarly, the magnetic flux (Φ) lines passing through a super conducting ring due to persistent current is quantized in terms of integral multiples of $\frac{h}{2e}$

$$\boxed{\Phi = \frac{nh}{2e}}$$

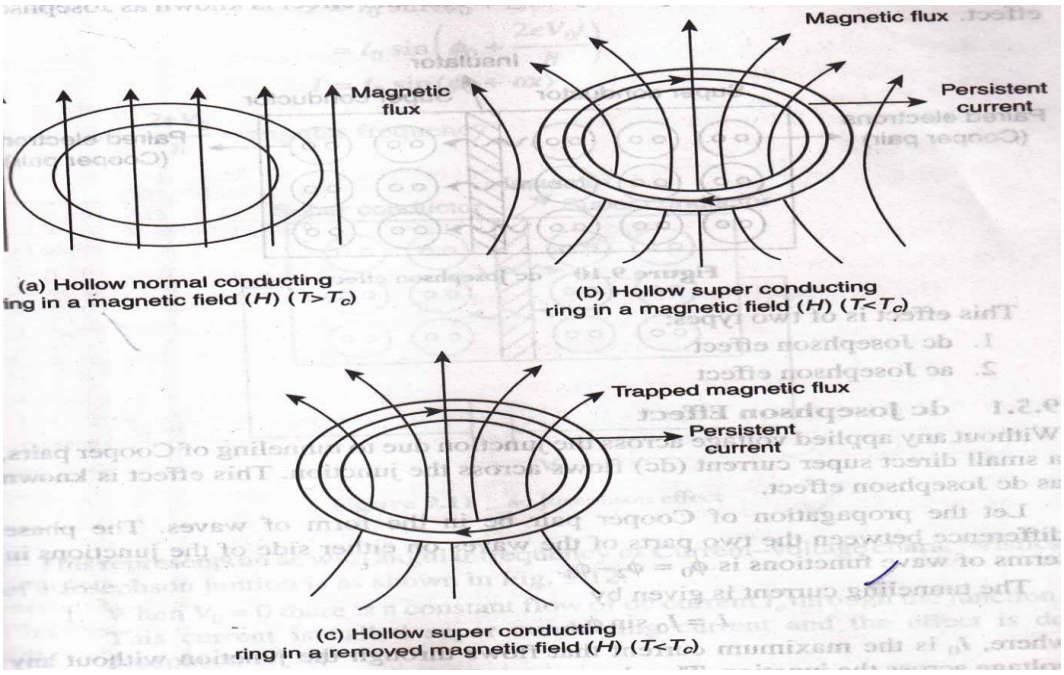
Where n = 1, 2, 3……

Where $\frac{h}{2e}$ is known as fluxion (or) fluxiod. = 2.07 X 10 $^{-15}$ Weber's.

This phenomenon is known as flux quantization of super conductor.

Explanation:

Consider a hollow normal conducting ring in a magnetic field. It allows the magnetic flux due to this field. So, we observe magnetic flux outside and inner hollow space and also on the ring as shown in fig.



When the temperature of the ring is lowered to its critical temperature (T_C); the ring expels the magnetic flux and it becomes a super conductor.

As a result, persistent current is set up in the material; this current will remain in the material even if we remove the applied magnetic field afterwards. This persistent current sets up magnetic force lines (i.e., flux) in the ring. This magnetic flux adjusts itself such that the total flux through the cylinder is quantized in integral multiples of $\frac{h}{2e}$.

$$\Phi = \frac{nh}{2e}$$

Where $\frac{h}{2e}$ is known as fluxion (or) fluxiod. = 2.07 X 10 $^{-15}$ Weber's.

This phenomenon is known as flux quantization of super conductor.

Occurrence of superconductivity:

Bardeen, cooper and Schrieffer (BCS) theory: This is the first microscopic theory, based on quantum mechanics. In 1957 John Bardeen, Leon Cooper, and John Robert Schrieffer wrote a paper called 'Theories of Superconductivity'.

Principle:

According to this theory, superconductivity occurs due to the movement of electron pairs called cooper pairs through the lattice points. These cooper pairs are formed due to electron-lattice –electron interaction.

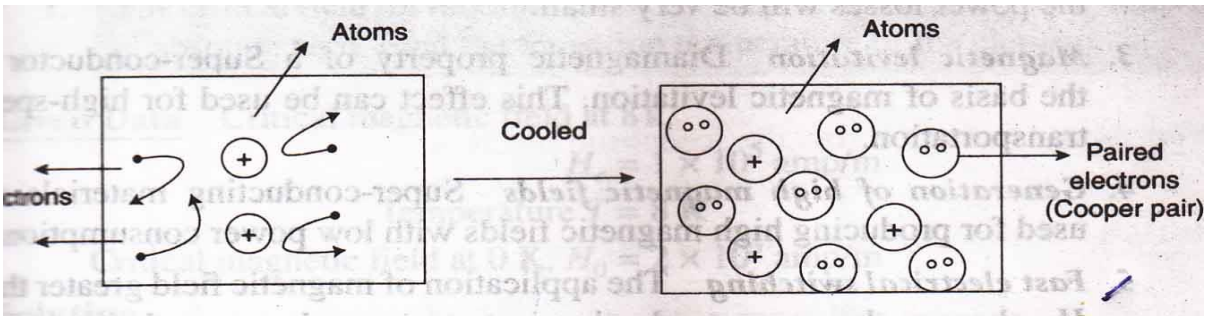
In this interaction, electrons experience a special kind of attractive interaction, overcoming the coulomb forces of repulsion between them. As result cooper pairs (electron pairs) are formed. At low temperature, these pairs move without scattering through the lattice points and as result resistance or resistivity decreases (i.e., conductivity increases).

Explanation:

Under normal condition, the ions in the lattice vibrate about their equilibrium positions due to thermal energy. These vibrations are lattice vibrations.

When electrons pass through the lattice ions in the normal state, they collide and scatter with the lattice and with each other. As a result, resistance arises in the material.

When the temperature decreases below its critical temperature, due to decrease in energy the scattering of electrons by vibrating lattice ions also decreases. As a result, electron pairs (cooper pairs) are formed. These cooper pairs move without scattering through the lattice ions and as result resistance or resistivity decreases (i.e., conductivity increases) and material becomes superconductor. **These cooper pairs are formed due to electron-lattice –electron.**



$\rho \neq 0$ when T > Tc $\qquad$ $\rho = 0$ when $T < Tc$

Fig: (a) scattering of electrons in normal state (T > Tc), b) movement of cooper pairs without scattering in superconducting sate (T < Tc).

Formation of cooper pairs:

Electron-Lattice (phonon) - Electron interaction:

According to BCS theory,

Suppose an electron (1st electron) moves through the lattice, it will be attracted by the positive ion core. It suffers attractive coulomb interaction. . Due to this attraction, positive ion core is disturbed and it is called as lattice distortion. This is shown in the figure below. The lattice vibrations are quantized in terms of phonons.

At that instant, if another electron (2nd electron) moves through the distorted lattice, it will be attracted by the greater concentration of positive ion core. It also suffers attractive coulomb interaction.
Therefore, the two electrons interact via lattice distortion or the phonon field, lowering the energy of electrons. This lowering of energy implies that the force between the two electrons is attractive. This type of interaction is called electron-lattice-electron interaction. This interaction can also be interpreted as electron –electron interaction through phonons.

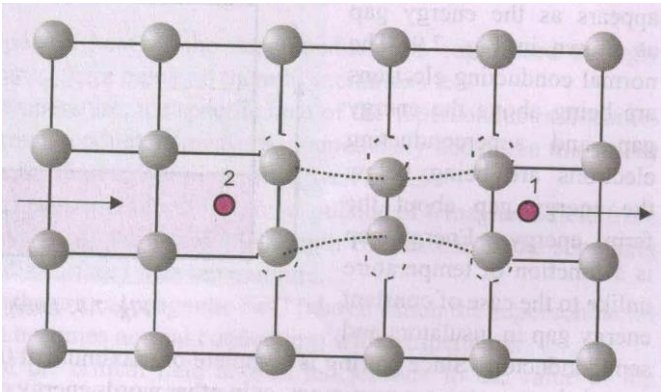
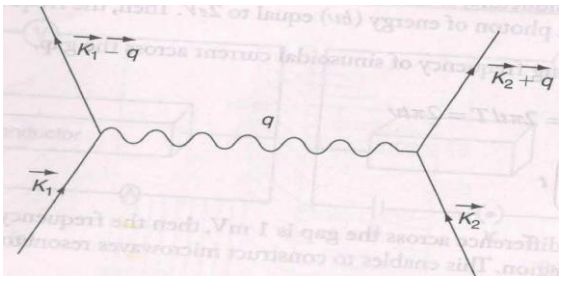


Fig:Electron-Lattice (phonon) - Electron interaction

Copper Pairs:
Cooper showed that the pair of electrons formed by the interaction between the electrons with opposite spins and momenta are known as cooper pairs. This interaction can be represented in terms of the wave vector of electrons as shown in fig.



Consider the 1st electron with wave vector **k** distorts the lattice, there by emitting a phonon of wave vector **q**.This results in the wave vector for k-q for the 1st electron.
Now, if the 2nd electron with wave vector k^I, seeks the lattice, it takes up the energy from the lattice and wave vector changes to k^I +q as shown in fig.
Two electrons with wave vectors k-q and k^I +q form a pair of electrons known as cooper pairs.
Therefore, the pair of electrons formed due to electron-lattice-electron (force of attraction) by overcoming the electron-electron (force of repulsion), with equal and opposite momentum and spins with wave vectors k-q and k^I +q, and are called cooper .
Josephson Effect:

When two super conductors are separated by a very thin insulator (oxide layer of about $20A^{O}$), forms a Joseph junction and then cooper pairs can tunnel or penetrates through the thin insulator and constitute a small super current .This effect is called Josephson Effect.

Explanation:

Consider two superconducters which are joined together with help of a thin insulating layer and forms a junction called josephson junction.These super conductors consist of paired electrons known as cooper pairs in the superconducting state.These cooper pairs will try to penetrate or tunnel through the thin insulator and constitute a small super current as shown in fig.

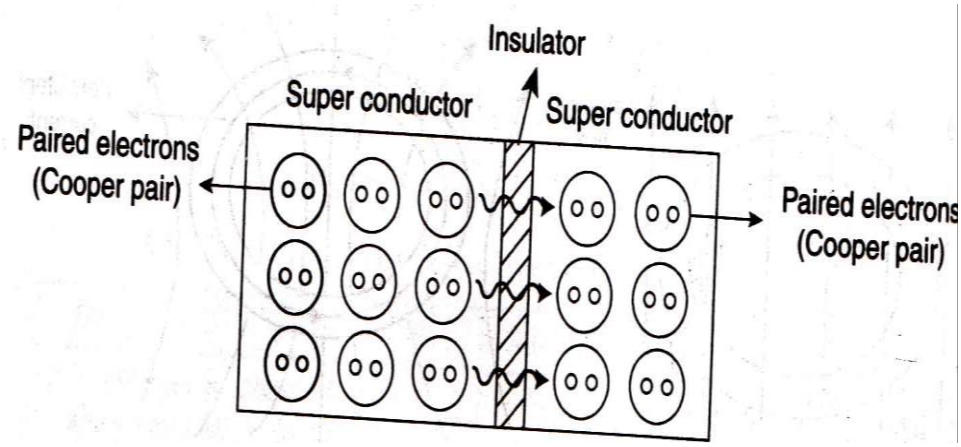


Fig: D.C. Josephson effect

Josephson Effect can be dived into two types .They are:

a) D.C. Josephson Effect
b) A.C. Josephson Effect.

D.C. Josephson Effect:

Josephson observed that the cooper pairs can tunnel from one film of superconductor into another through the thin insulator and a small direct super current flows across the junction without applied voltage across the junction. This effect is known as D.C. Josephson Effect.Josephson showed that the dc current through the in junction is given by

$$I=I_O \sin\Phi_o \text{.....}(1)$$

Where $\Phi_o = \Phi_2 - \Phi_1$ is the phase difference the wave functions describing cooper pair4s on both sides of the barrier, and I_O is the critical current which the junction can support and is dependent on the thickness and width of the insulating layer.

A.C. Josephson Effect:

If we apply the voltage across the junction, then ac current is produced and is given by

$$I=I_O \sin (wt+\Phi) \text{.....} (2)$$

Where w = angular frequency $=\frac{4\pi eV}{h}$

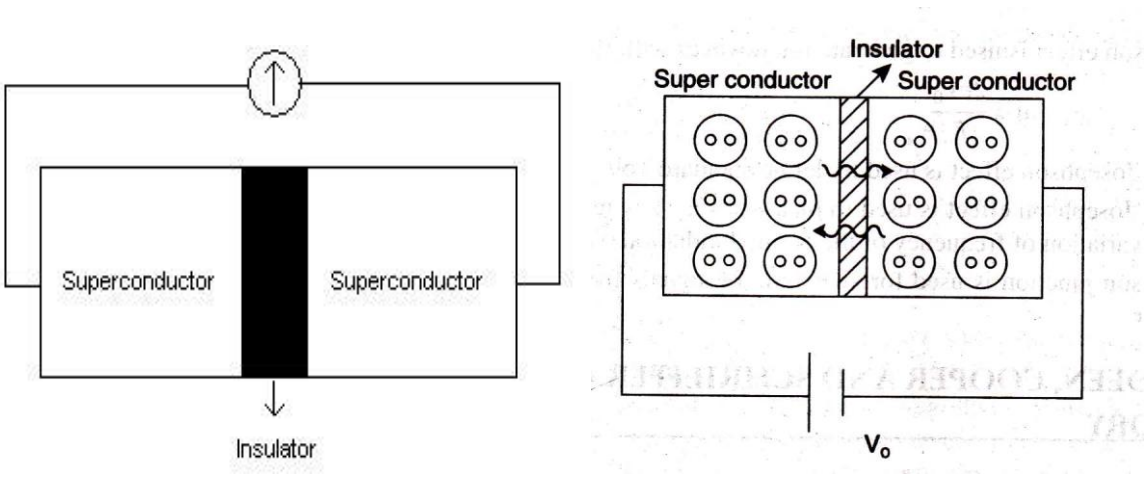


Fig: A.C. Josephson effect

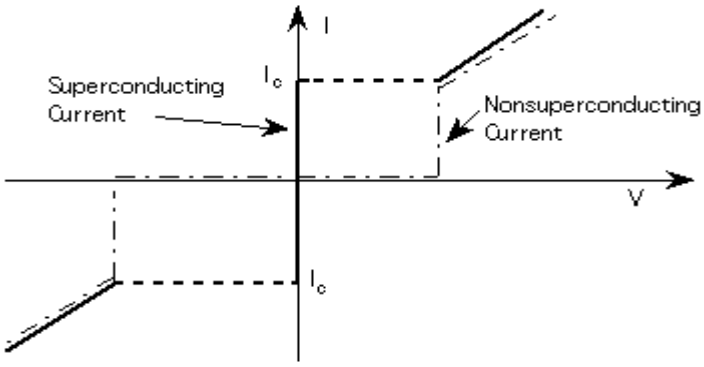


Fig: I-V characteristics of a Josephson junction

Applications:

Superconductors are used in the following applications:

1. Magnetic Levitation

Magnetic levitation, **maglev**, or **magnetic suspension** is a method by which an object is suspended with no support other than magnetic fields.

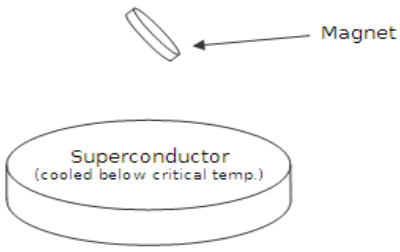


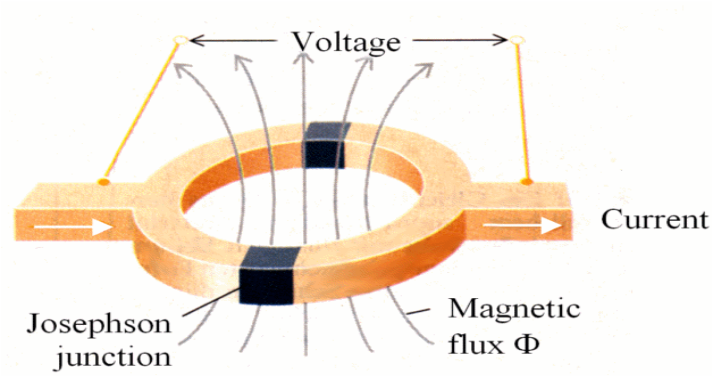
Fig: Maglev train

Magnetic levitation is used for high speed transportation.

For example, Maglev (magnetic levitation) trains travel 500 km/h. These work because a superconductor repels a magnetic field so a magnet will float above a superconductor (Messiner effect)– this virtually eliminates the friction between the train and the track.

2.SQUID :

The superconducting quantum interference device (SQUID) consists of two superconductors separated by thin insulating layers to form two parallel Josephson junctions as shown fig.



Uses:

- This device used as sensitive magnetometer to detect small magnetic fields from brain and heart.

  Threshold for SQUID: 10^{-14}
  Magnetic field of heart: 10^{-10} T
  Magnetic field of brain: 10^{-13} T. .
- They are used in mine detection equipment to help in the removal of land mines.

3. **Large hadron collider or particle accelerator:**

This use of superconductors was developed at the Rutherford Appleton Laboratory in Oxford shire, UK in the 1960s. The latest and biggest large hadron collider is currently being built in Switzerland by a coalition of scientific organizations from several countries. Superconductors are used to make extremely powerful electromagnets to accelerate charged particles very fast (to near the speed of light).

4. Magnetic Resonance Imaging (MRI)

MRI is a technique developed in the 1940s that allows doctors to see what is happening inside the body without directly performing surgery.

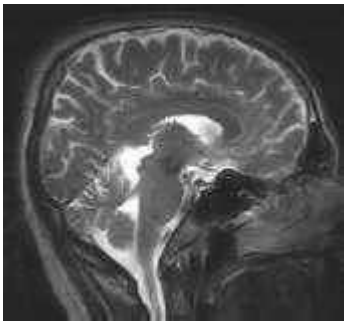


Fig:MRI scan of a human skull

5.Efficient Electricity Transportation:

Superconducting magnets are also more efficient in generating electricity than conventional copper wire generators - in fact, a superconducting generator about half the size of a copper wire generator is about 99% efficient; typical generators are around 50% efficient.