(20A01201T) STRENGTH OF MATERIALS

- **PREPARED BY**

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COURSE OUTCOMES

CO 1	Understand the basic concepts of forces, Draw Free body Diagrams for forces and Determine the centroid and moment of inertia for different cross section areas
CO 2	Understand concepts of stresses, strains, elastic moduli and strain energy and Evaluate relations between different moduli
CO 3	Draw the shear force and bending moment diagrams for cantilevers, simply supported beams and Overhanging beams with different loads and Understand the relationship between shear force and bending moments
CO 4	Compute the flexural stresses for different cross sections and Design beam sections for flexure
CO 5	Determine shear stresses for different shapes and analyze trusses

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UNIT-I: Introduction to Mechanics:

Basic Concepts, system of Forces Coplanar Concurrent Forces -Components in Space Resultant -Moment of Forces and its Application - Couples and Resultant of Force Systems. Equilibrium of system of Forces: Free body diagrams, Equations of Equilibrium of Coplanar Systems and Spatial systems-

Center of Gravity and moment of inertia: Introduction – Centroids of rectangular, circular, I, L and T sections - Centroids of built up sections. Area moment of Inertia: Introduction – Definition of Moment of Inertia of rectangular, circular, I, L and T sections - Radius of gyration. Moments of Inertia of Composite sections.

UNIT - II :Simple Stresses and Strains:

Types of stresses and strains – Hooke's law – Stress – strain diagram for mild steel – working stress – Factor of safety – lateral strain, Poisson's ratio and volumetric strain – Elastic moduli and the relationship between them – Bars of Varying section – Composite bars – Temperature stresses. Strain energy – Resilience – Gradual, Sudden, impact and shock loadings – simple applications.

UNIT – III Shear Force and Bending Moment:

Definition of beam – types of beams – Concept of Shear force and bending moment – S.F and B.M diagrams for cantilever, simply supported and over changing beams subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – point of contra flexure – Relation between S.F, B.M and rate of loading at section of a beam.

UNIT - IV Flexural Stresses:

Theory of simple bending – Assumptions – Derivation of bending equation: $M/I = f/Y = E/R$ – Neutral axis – Determination of bending stresses – Section modulus of rectangular and circular sections (Solid and Hallow), I, T, Angle and Channel Sections – Design of simple beam sections.

UNIT - V: Shear Stresses:

Derivation of formula-Shear stress distribution across various beam sections like rectangular, circular, triangular, I, T and angle sections. Combined bending and shear.

Analysis of trusses by Method of Joints & Sections.

Textbooks:

1. S. Timoshenko, D.H. Young and J.V. Rao, "Engineering Mechanics", Tata McGraw-HillCompany.
2. Sadhu Singh, "Strength of Materials", 11th edition 2015, Khanna Publishers.

Reference Books:

1. S.S.Bhavikatti, "Strength of materials", Vikas publishing house Pvt. Ltd.
2. R. Subramanian, "Strength of Materials", Oxford University Press.
3. R. K. Bansal, "Strength of Materials", Lakshmi Publications House Pvt. Ltd.
4. Advanced Mechanics of Materials – Seely F.B and Smith J.O. John wiley & Sons inc.,New York.

ME101: Engineering Mechanics

<u>Mechanics</u>: Oldest of the Physical Sciences

Archimedes (287-212 BC): Principles of Lever and Buoyancy!

Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies subjected to the action of forces

Rigid-body Mechanics € ME101

Statics

Dynamics

Deformable-Body Mechanics, and

Fluid Mechanics

Engineering Mechanics

Rigid-body Mechanics

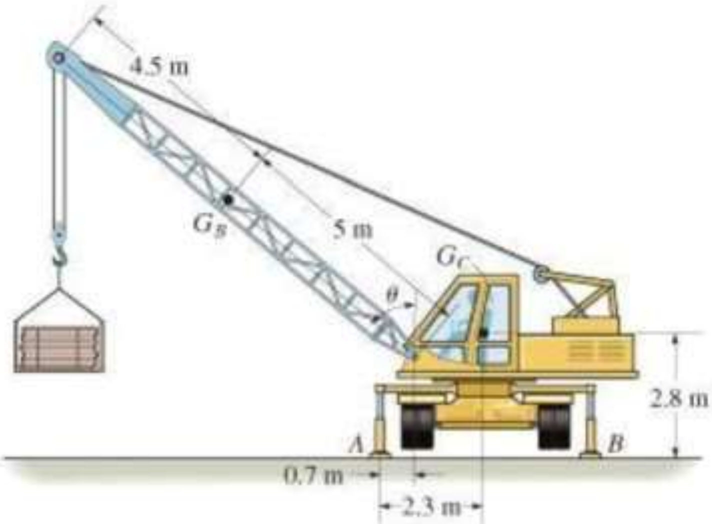
- a basic requirement for the study of the mechanics of deformable bodies and the mechanics of fluids (advanced courses).
- essential for the design and analysis of many types of structural members, mechanical components, electrical devices, etc, encountered in engineering.

A rigid body does not deform under load!

Engineering Mechanics

Rigid-body Mechanics

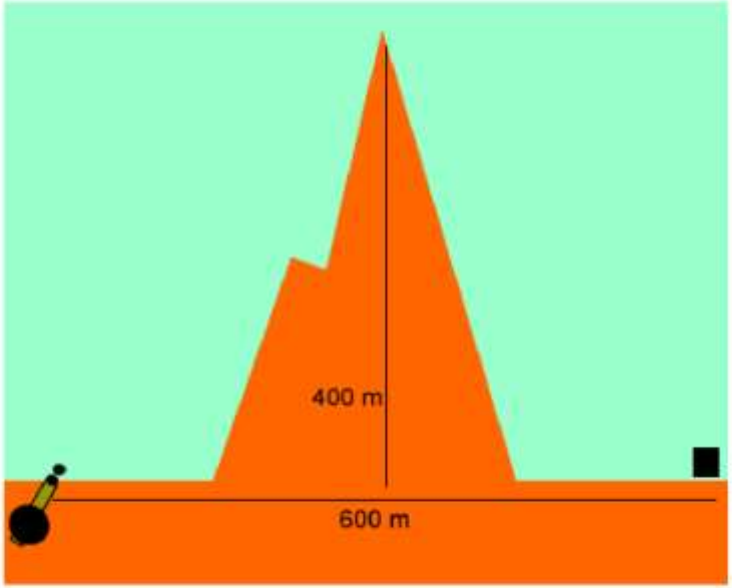
Statics: deals with equilibrium of bodies under action of forces (bodies may be either at rest or move with a constant velocity).



Engineering Mechanics

<u>Rigid-body Mechanics</u>

- Dynamics: deals with motion of bodies (accelerated motion)



Mechanics: Fundamental Concepts

Length (Space): needed to locate position of a point in space, & describe size of the physical system € Distances, Geometric Properties

Time: measure of succession of events € basic quantity in Dynamics

Mass: quantity of matter in a body € measure of inertia of a body (its resistance to change in velocity)

Force: represents the action of one body on another € characterized by its magnitude, direction of its action, and its point of application

€ Force is a Vector quantity.

Mechanics: Fundamental Concepts

Newtonian Mechanics

Length, Time, and Mass are absolute concepts
independent of each other

Force is a derived concept
not independent of the other fundamental concepts.
Force acting on a body is related to the mass of the body
and the variation of its velocity with time.

Force can also occur between bodies that are physically separated (Ex: gravitational, electrical, and magnetic forces)

Mechanics: Fundamental Concepts

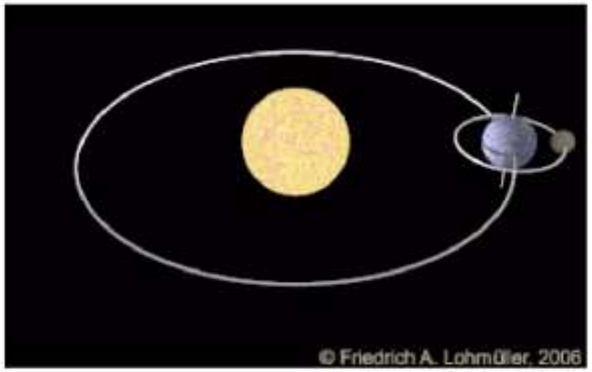
Remember:

- Mass is a property of matter that does not change from one location to another.
- Weight refers to the gravitational attraction of the earth on a body or quantity of mass. Its magnitude depends upon the elevation at which the mass is located
- Weight of a body is the gravitational force acting on it.

Mechanics: Idealizations

To simplify application of the theory

Particle: A body with mass but with dimensions that can be neglected



Size of earth is insignificant compared to the size of its orbit. Earth can be modeled as a particle when studying its orbital motion

Mechanics: Idealizations

Rigid Body: A combination of large number of particles in which all particles remain at a fixed distance (practically) from one another before and after applying a load.

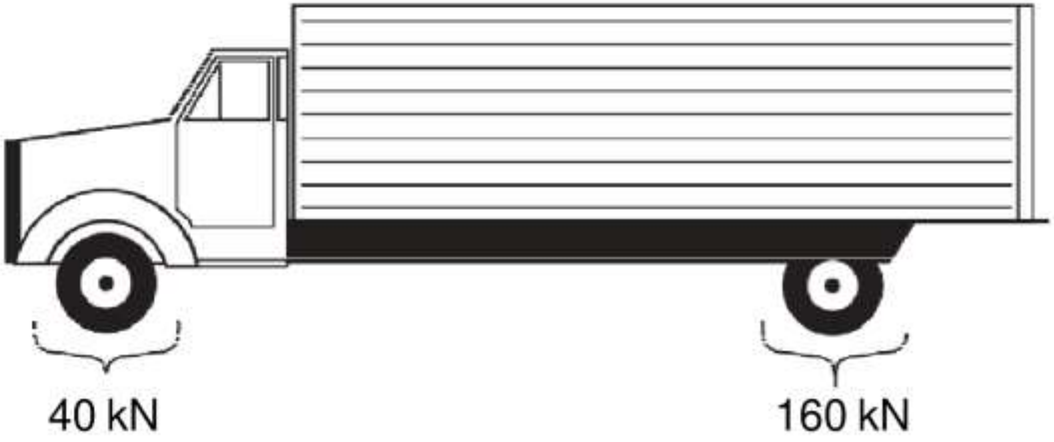
Material properties of a rigid body are not required to be considered when analyzing the forces acting on the body.

In most cases, actual deformations occurring in structures, machines, mechanisms, etc. are relatively small, and rigid body assumption is suitable for analysis

Mechanics: Idealizations

Concentrated Force: Effect of a loading which is assumed to act at a point (CG) on a body.

•Provided the area over which the load is applied is very small compared to the overall size of the body.



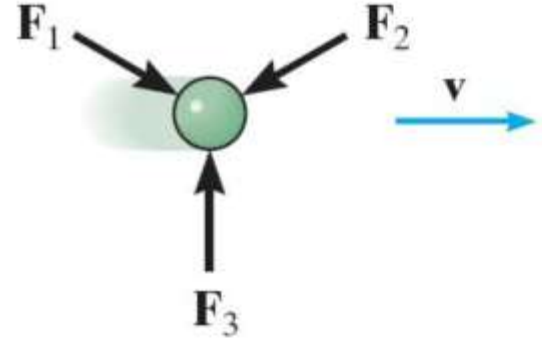
Ex: Contact Force between a wheel and ground.

Mechanics: Newton's Three Laws of Motion

Basis of formulation of rigid body mechanics.

First Law: A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is not subjected to an unbalanced force.

First law contains the principle of the equilibrium of forces € main topic of concern in Statics



Equilibrium

Mechanics: Newton's Three Laws of Motion

Second Law: A particle of mass "m" acted upon by an unbalanced force "F" experiences an acceleration "a" that has the same direction as the force and a magnitude that is directly proportional to the force.

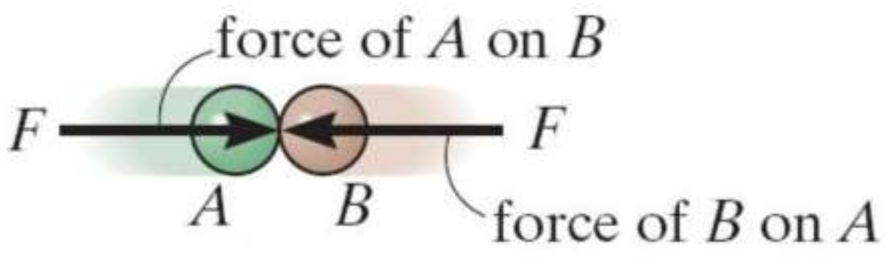
$F = ma$

Accelerated motion

Second Law forms the basis for most of the analysis in Dynamics

Mechanics: Newton's Three Laws of Motion

Third Law: The mutual forces of action and reaction between two particles are equal, opposite, and collinear.



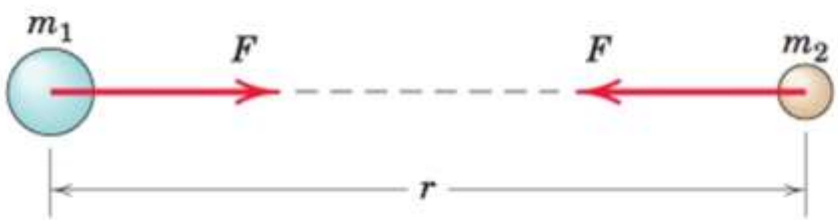
Action – reaction

Third law is basic to our understanding of Force € Forces always occur in pairs of equal and opposite forces.

Mechanics: Newton's Law of Gravitational Attraction

Weight of a body (gravitational force acting on a body) is required to be computed in Statics as well as Dynamics.
This law governs the gravitational attraction between any two particles.

$$F = G\frac{m_1 m_2}{r^2}$$



F = mutual force of attraction between two particles

G = universal constant of gravitation

Experiments € $G = 6.673 \times 10^{-11}\ m^3/(kg.s^2)$

Rotation of Earth is not taken into account

m_1, m_2 = masses of two particles

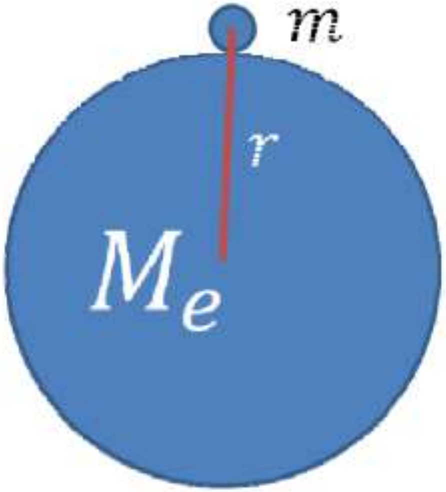
r = distance between two particles

Gravitational Attraction of the Earth

Weight of a Body: If a particle is located at or near the surface of the earth, the only significant gravitational force is that between the earth and the particle

Weight of a particle having mass $\boldsymbol{m_1 = m}$:

Assuming earth to be a non-rotating sphere of constant density and having mass $\boldsymbol{m_2 = M_e}$



$$W = G\frac{mM_e}{r^2}$$

$\boldsymbol{r}$ = distance between the earth's center and the particle

$$W = mg$$

Let $\boldsymbol{g = G\, M_e / r^2}$ = acceleration due to gravity ($9.81 m/s^2$)

Mechanics: Units

Four Fundamental Quantities

Quantity	Dimensional Symbol	SI UNIT	
		Unit	Symbol
Mass	M	Kilogram	Kg
Length	L	Meter	M
Time	T	Second	s
Force	F	Newton	N

Basic Unit

$$\boxed{F = ma}$$ € N = kg.m/s²

$$\boxed{W = mg}$$ € N = kg.m/s²

1 Newton is the force required to give a mass of 1 kg an acceleration of 1 m/s^2

Mechanics: Units Prefixes

	Exponential Form	Prefix	SI Symbol
Multiple			
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	k
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

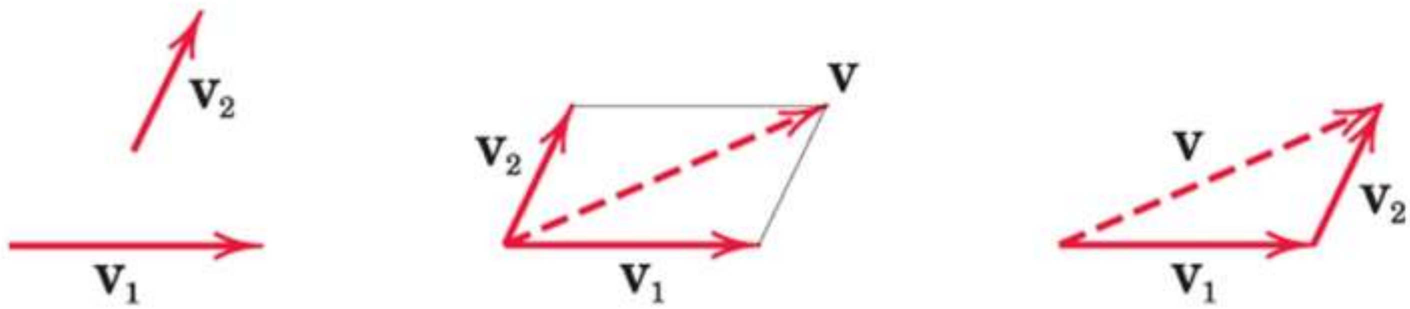
Scalars and Vectors

Scalars: only magnitude is associated.
Ex: time, volume, density, speed, energy, mass

Vectors: possess direction as well as magnitude, and must obey the parallelogram law of addition (and the triangle law).

Ex: displacement, velocity, acceleration, force, moment, momentum

Equivalent Vector: $V = V_1 + V_2$ (Vector Sum)



Speed is the magnitude of velocity.

Vectors

A Vector V can be written as: V = *V*n

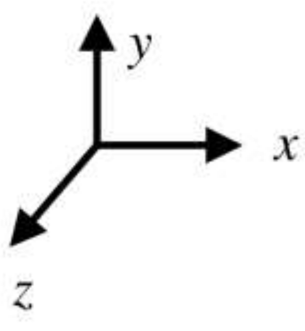
V = magnitude of V

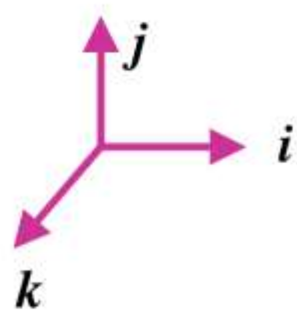
n = unit vector whose magnitude is one and whose direction coincides with that of V

Unit vector can be formed by dividing any vector, such as the geometric position vector, by its length or magnitude

Vectors represented by Bold and Non-Italic letters (V)

Magnitude of vectors represented by Non-Bold, Italic letters (*V*)

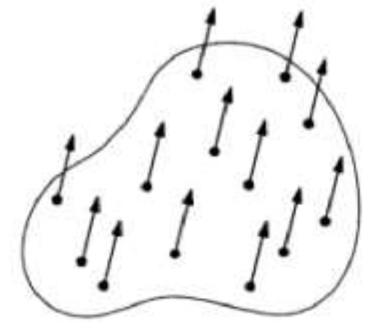




$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ – unit vectors

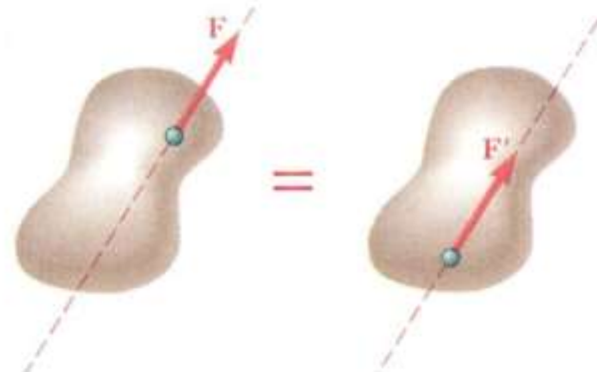
Vectors

Free Vector: whose action is not confined to or associated with a unique line in space
Ex: Movement of a body without rotation.

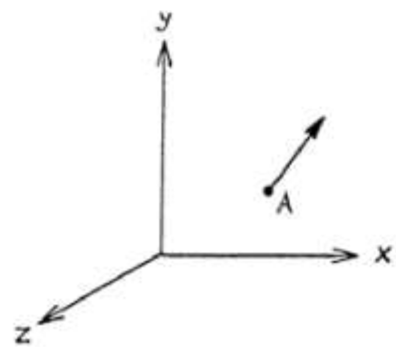


Sliding Vector: has a unique line of action in space but not a unique point of application
Ex: External force on a rigid body

€ Principle of Transmissibility
€ Imp in Rigid Body Mechanics



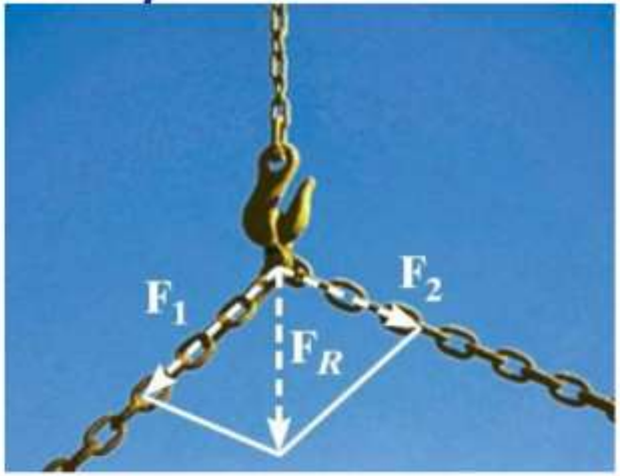
Fixed Vector: for which a unique point of application is specified
Ex: Action of a force on deformable body



Vector Addition: Procedure for Analysis

Parallelogram Law (Graphical)

Resultant Force (diagonal)
Components (sides of parallelogram)



Algebraic Solution

Using the coordinate system

Cosine law:

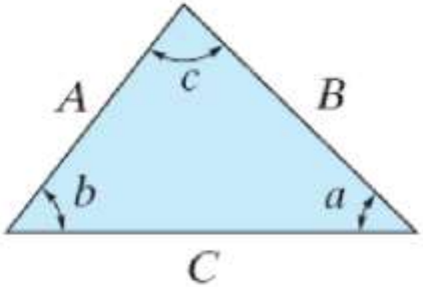
$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Trigonometry (Geometry)

Resultant Force and Components from Law of Cosines and Law of Sines



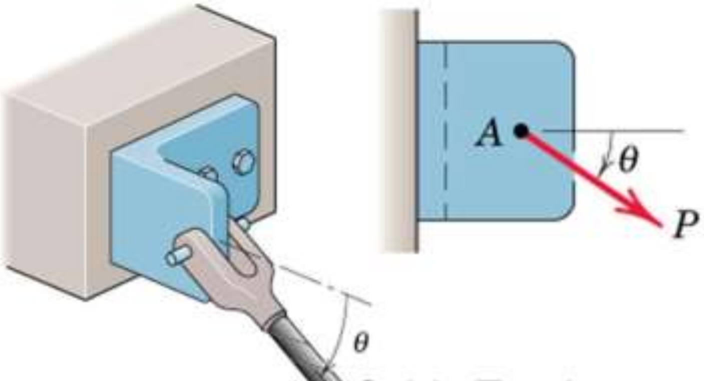
Force Systems

Force: Magnitude (*P*), direction (arrow) and point of application (point A) is important

Change in any of the three specifications will alter the effect on the bracket.

Force is a Fixed Vector

In case of rigid bodies, line of action of force is important (not its point of application if we are interested in only the resultant external effects of the force), we will treat most forces as



Cable Tension *P*

External effect: Forces applied (applied force); Forces exerted by bracket, bolts, Foundation (reactive force)

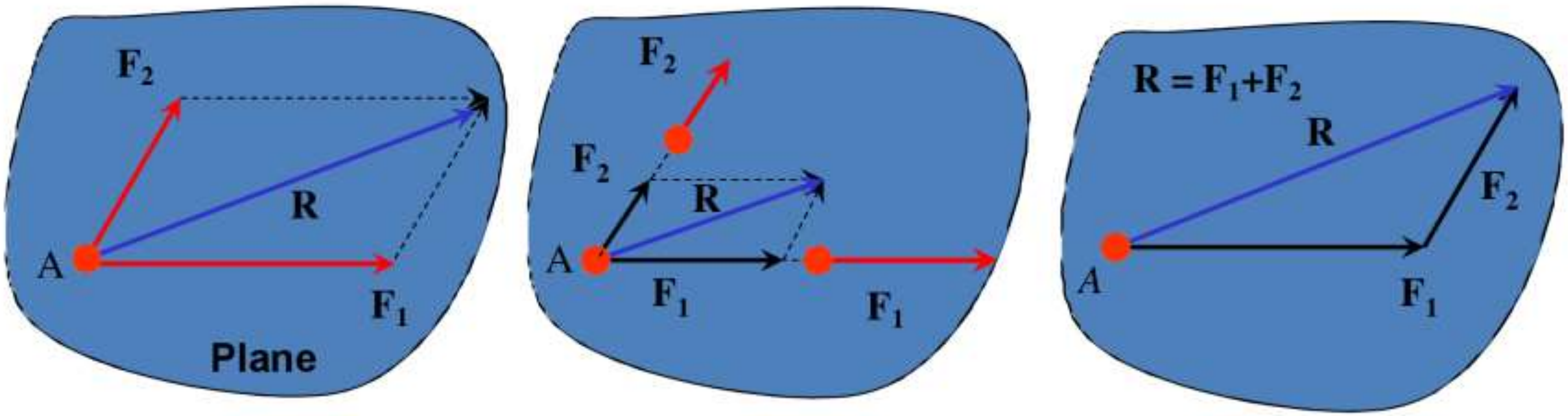
Internal effect: Deformation, strain pattern – permanent strain; depends on material properties of bracket, bolts, etc.

Force Systems

Concurrent force:

Forces are said to be concurrent at a point if their lines of action intersect at that point

F_1, F_2 are concurrent forces; R will be on same plane; $R = F_1 + F_2$



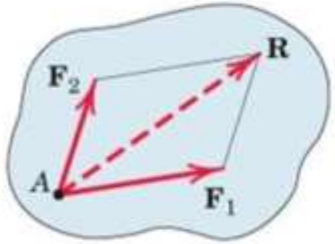
Forces act at same point Forces act at different point Triangle Law

(Apply Principle of Transmissibility)

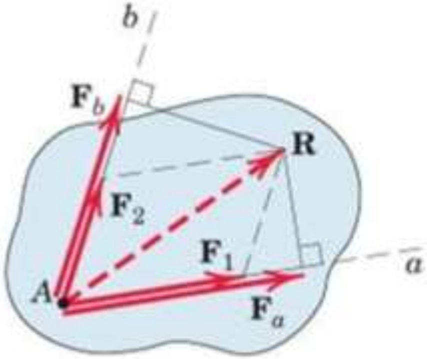
Components and Projections of Force

Components of a Force are not necessarily equal to the Projections of the Force unless the axes on which the forces are projected are orthogonal (perpendicular to each other).

F_1 and F_2 are components of R. $R = F_1 + F_2$



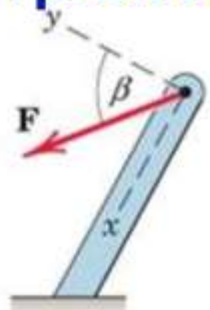
F_a and F_b are perpendicular projections on axes a and b, respectively.



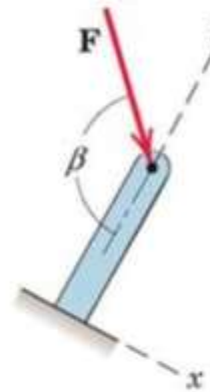
$R \neq F_a + F_b$ unless a and b are perpendicular to each other

Components of Force

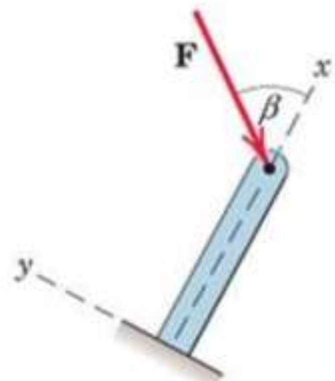
Examples



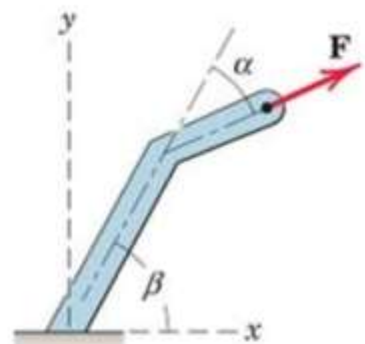
$F_x = F \sin \beta$
$F_y = F \cos \beta$



$F_x = F \sin(\pi - \beta)$
$F_y = -F \cos(\pi - \beta)$

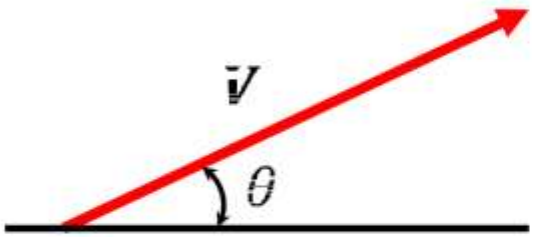


$F_x = -F \cos \beta$
$F_y = -F \sin \beta$

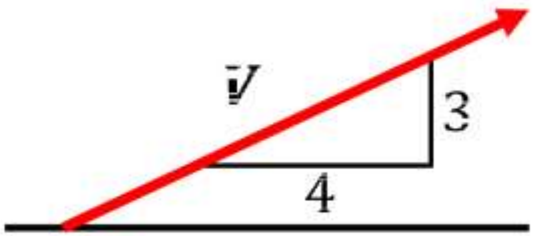


$F_x = F \cos(\beta - \alpha)$
$F_y = F \sin(\beta - \alpha)$

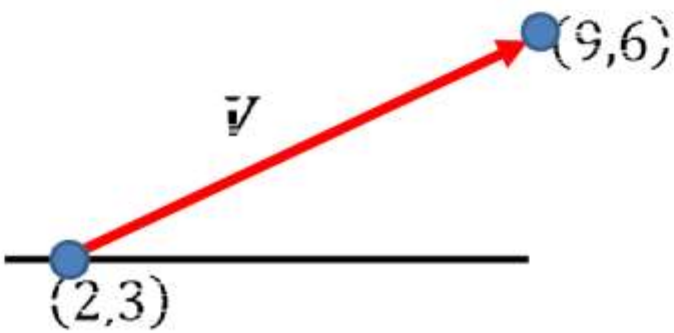
Vector



$$\boldsymbol{V} = V(cos\theta\boldsymbol{i} + sin\theta\boldsymbol{j})$$



$$\boldsymbol{V} = V\left(\frac{4\boldsymbol{i} + 3\boldsymbol{j}}{\sqrt{4^2 + 3^2}}\right)$$

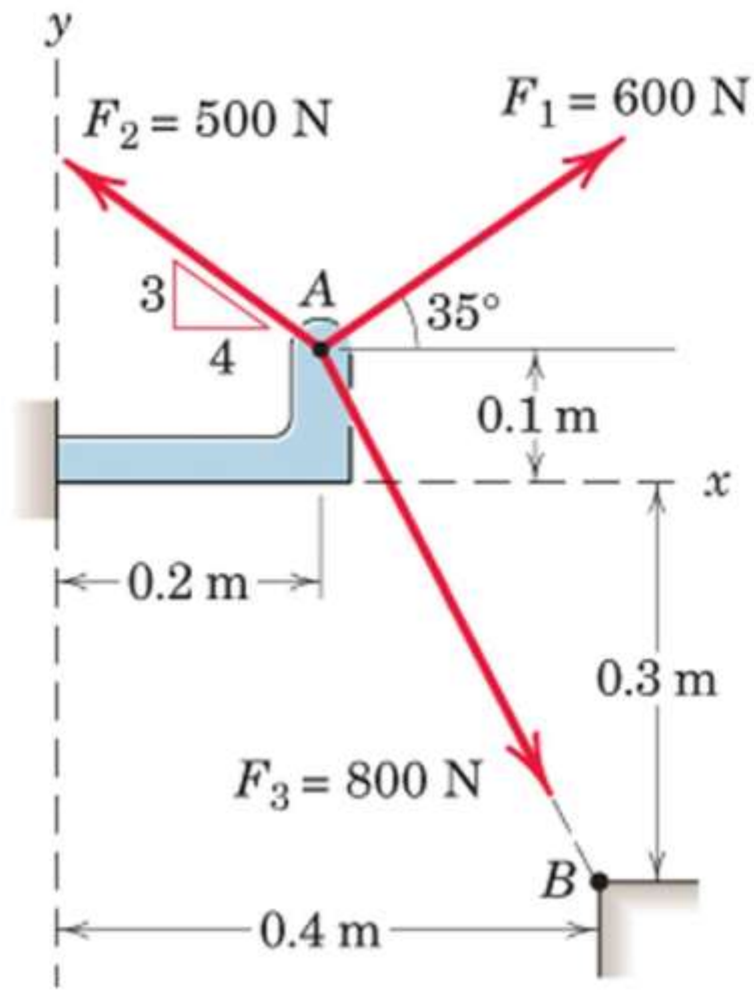


$$\boldsymbol{V} = V\left(\frac{(9-2)\boldsymbol{i} + (6-3)\boldsymbol{j}}{\sqrt{(9-2)^2 + (6-3)^2}}\right)$$

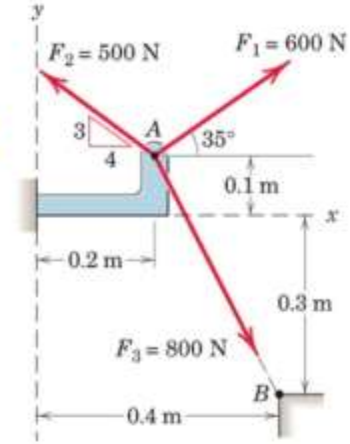
Components of Force

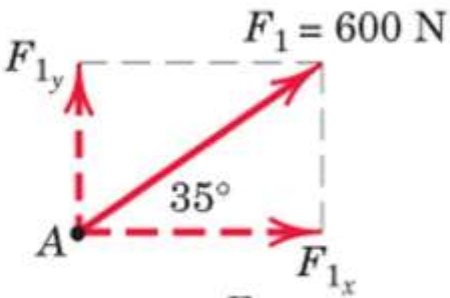
Example 1:

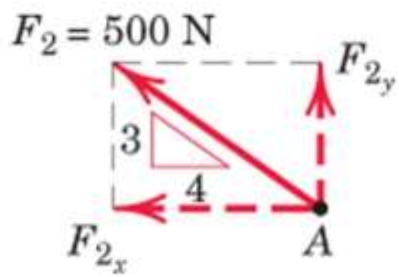
Determine the x and y scalar components of F_1, F_2, and F_3 acting at point A of the bracket

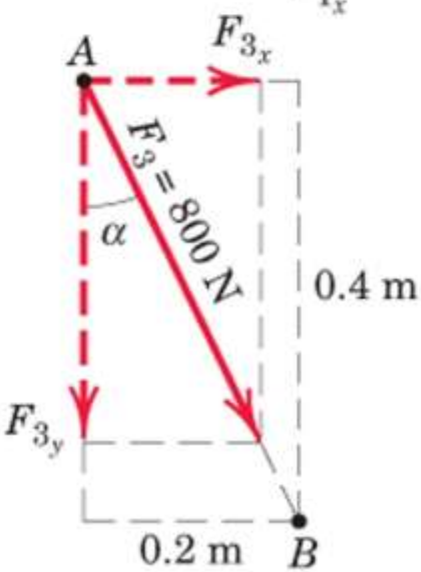


Components of Force









$$F_{1_x} = 600 \cos 35^\circ = 491 \text{ N}$$

$$F_{1_y} = 600 \sin 35^\circ = 344 \text{ N}$$

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N}$$

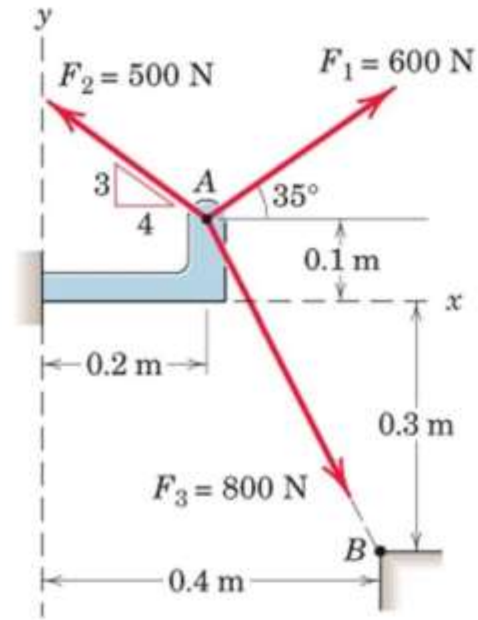
$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N}$$

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6^\circ$$

$$F_{3_x} = F_3 \sin \alpha = 800 \sin 26.6^\circ = 358 \text{ N}$$

$$F_{3_y} = -F_3 \cos \alpha = -800 \cos 26.6^\circ = -716 \text{ N}$$

Components of Force



Alternative Solution

$$\boldsymbol{F}_1 = F_1\boldsymbol{n}_1 = F_1\frac{cos(35^o)\boldsymbol{i} + sin(35^o)\boldsymbol{j}}{\sqrt{\left(cos(35^o)\right)^2 + \left(sin(35^o)\right)^2}}$$

$$= 600[0.819\boldsymbol{i} - 0.5735\boldsymbol{j}]$$

$$= 491\boldsymbol{i} - 344\boldsymbol{j}$$

$$F_{1x} = 491\,N \qquad F_{1y} = 344\,N$$

$$\boldsymbol{F}_2 = F_2\boldsymbol{n}_2 = F_2\frac{-4\boldsymbol{i} + 3\boldsymbol{j}}{\sqrt{(-4)^2 + (3)^2}}$$

$$= 500[-0.8\boldsymbol{i} + 0.6\boldsymbol{j}] \quad = 400\boldsymbol{i} + 300\boldsymbol{j}$$

$$F_{2x} = 400\,N \qquad F_{2y} = 300\,N$$

Components of Force

Alternative Solution

$$\overrightarrow{AB} = 0.2\boldsymbol{i} - 0.4\boldsymbol{j}$$

$$\overline{AB} = \sqrt{(0.2)^2 + (-0.4)^2}$$

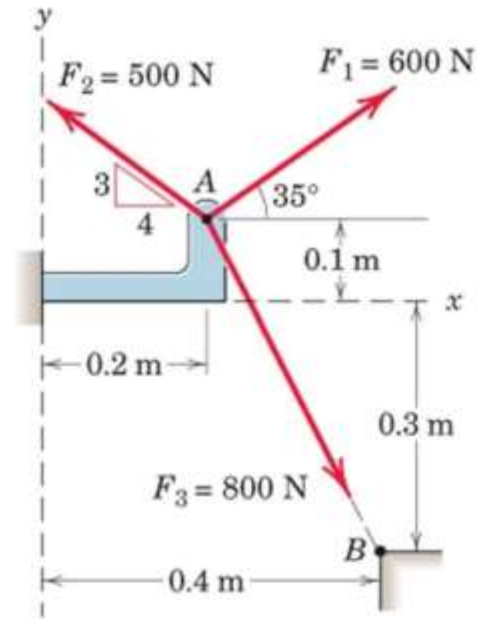
$$\boldsymbol{F}_3 = F_3\boldsymbol{n}_3 = F_3\frac{\overrightarrow{AB}}{\overline{AB}}$$

$$= 800\frac{0.2\boldsymbol{i} - 0.4\boldsymbol{j}}{\sqrt{(0.2)^2 - (-0.4)^2}}$$

$$= 800[0.447\boldsymbol{i} - 0.894\boldsymbol{j}]$$

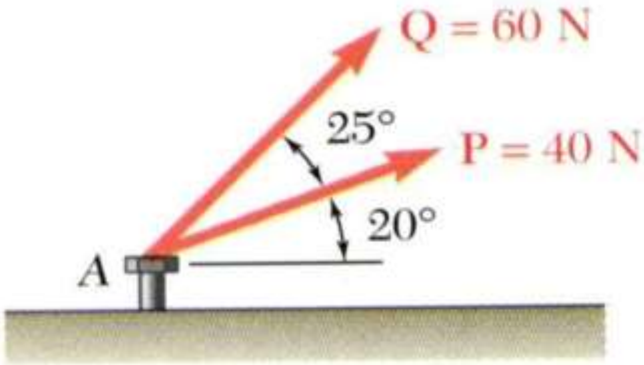
$$= 358\boldsymbol{i} - 716\boldsymbol{j}$$

$F_{3x} = 358\ N$ $\qquad$ $F_{3y} = 716\ N$



Components of Force

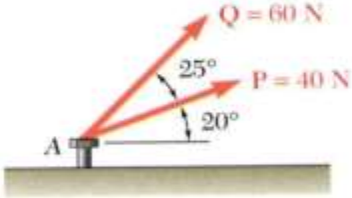
Example 2: The two forces act on a bolt at A. Determine their resultant.



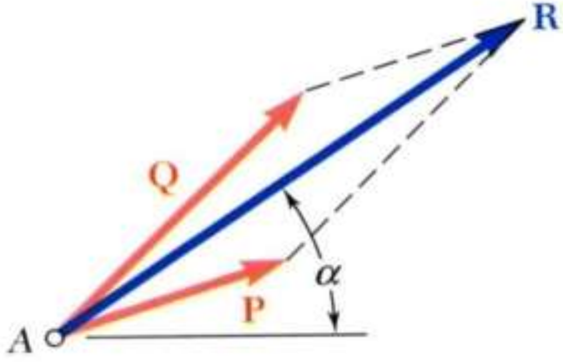
Graphical solution - construct a parallelogram with sides in the same direction as P and Q and lengths in proportion. Graphically evaluate the resultant which is equivalent in direction and proportional in magnitude to the diagonal.

Trigonometric solution - use the triangle rule for vector addition in conjunction with the law of cosines and law of sines to find the resultant.

Components of Force

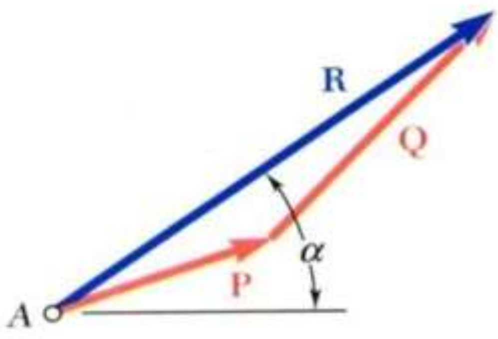


Solution:



- Graphical solution - A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

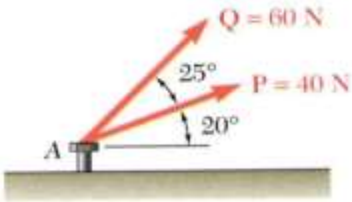
$$\boxed{\mathbf{R} = 98\ \text{N} \quad \alpha = 35°}$$



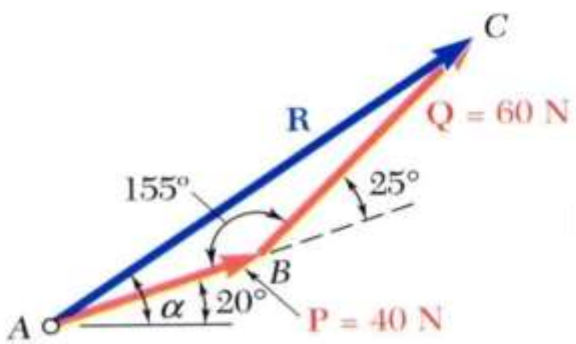
- Graphical solution - A triangle is drawn with **P** and **Q** head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

$$\boxed{\mathbf{R} = 98\ \text{N} \quad \alpha = 35°}$$

Components of Force



Trigonometric Solution: Apply the triangle rule.



From the Law of Cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$

$$= (40\,\mathrm{N})^2 + (60\,\mathrm{N})^2 - 2(40\,\mathrm{N})(60\,\mathrm{N})\cos 155°$$

$$\boxed{R = 97.73\,\mathrm{N}}$$

From the Law of Sines,

$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$

$$\sin A = \sin B\,\frac{Q}{R}$$

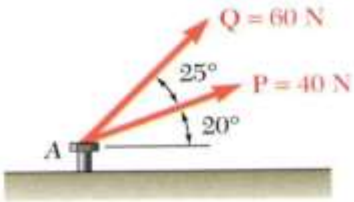
$$= \sin 155°\,\frac{60\,\mathrm{N}}{97.73\,\mathrm{N}}$$

$$A = 15.04°$$

$$\alpha = 20° + A$$

$$\boxed{\alpha = 35.04°}$$

Components of Force



$$\boldsymbol{R} = \boldsymbol{P} - \boldsymbol{Q}$$

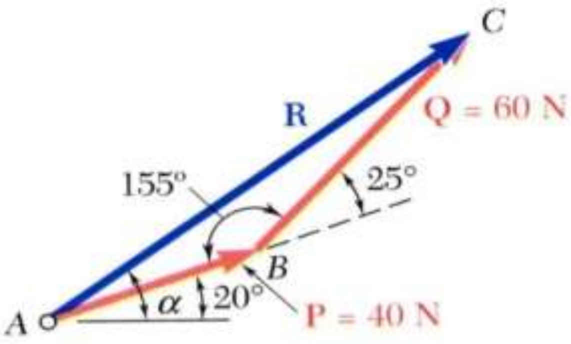
$$\boldsymbol{P} = 40[\cos(20)\boldsymbol{i} + \sin(20)\boldsymbol{j}]$$
$$= 37.58\mathbf{i} + 13.68\mathbf{j}$$

$$\boldsymbol{Q} = 60[\cos(45)\,\boldsymbol{i} + \sin(45)\boldsymbol{j}]$$
$$= 42.43\mathbf{i} + 42.43\mathbf{j}$$

$$\boldsymbol{R} = 80.01\boldsymbol{i} - 56.10\boldsymbol{j}$$

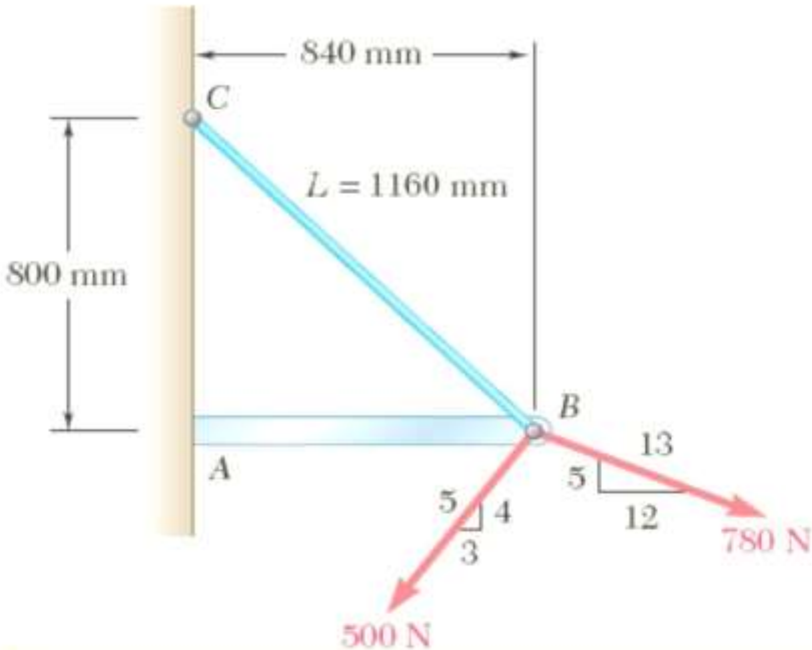
$$R = 97.72$$

$$\alpha = 35.03^o$$



Components of Force

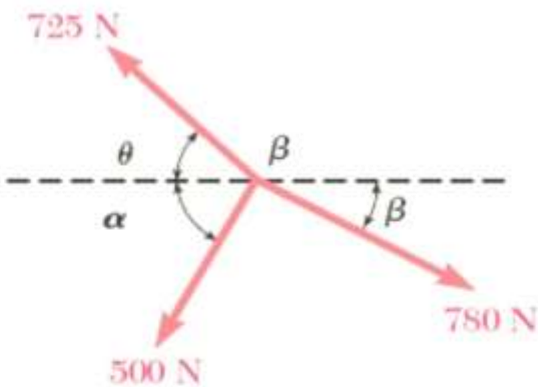
Example 3:Tension in cable *BC* is 725-N, determine the resultant of the three forces exerted at point *B* of beam *AB*.



Solution:

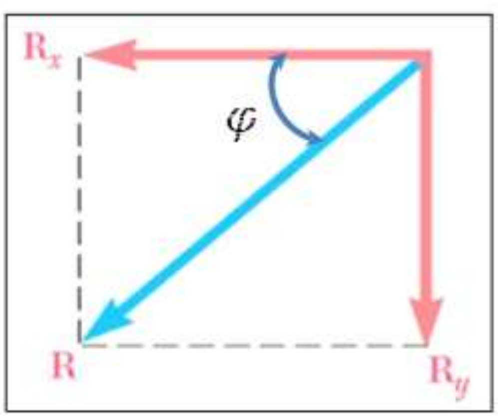
- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.

Components of Force



Resolve each force into rectangular components

Magnitude (N)	X-component (N)	Y-component (N)
725	-525	500
500	-300	-400
780	720	-300
	$R_x = -105$	$R_y = -200$



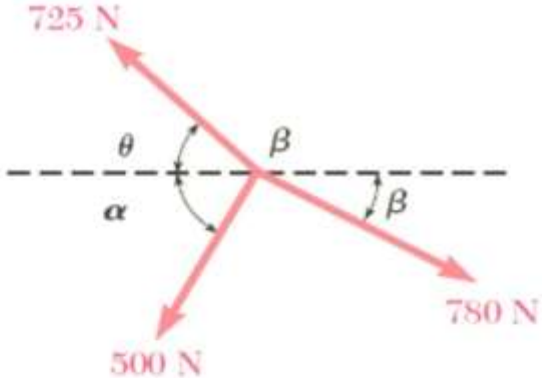
$\Re = R_x i + R_y j$ $\qquad$ $\mathbf{R} = (-105)i + (-200)j$

Calculate the magnitude and direction

$$tan\varphi = \frac{R_x}{R_y} = \frac{105}{200} \qquad \varphi = 62\ 3^o$$

$$\Re = \sqrt{R_x^2 + R_y^2} = 225.9N$$

Components of Force



Alternate solution

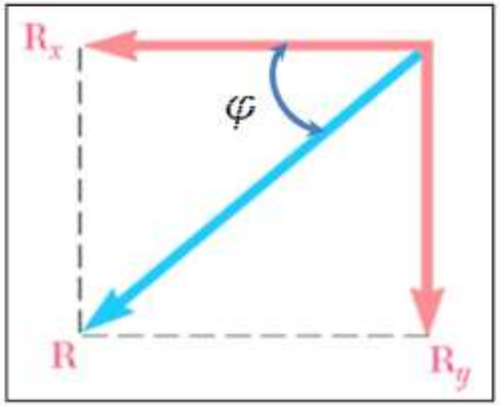
$$\boldsymbol{R} = \boldsymbol{F_1} + \boldsymbol{F_2} + \boldsymbol{F_3}$$

$$\boldsymbol{F_1} = 725[-0.724\boldsymbol{i} + 0.689\boldsymbol{j}]$$

$$\boldsymbol{F_2} = 500[-0.6\boldsymbol{i} - 0.8\boldsymbol{j}]$$

$$\boldsymbol{F_2} = 780[0.923\boldsymbol{i} - 0.384\boldsymbol{j}]$$

$$\boldsymbol{R} = -105\boldsymbol{i} - 200\boldsymbol{j}$$

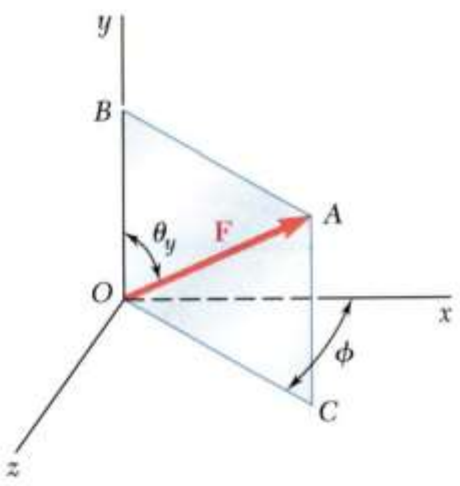


Calculate the magnitude and direction

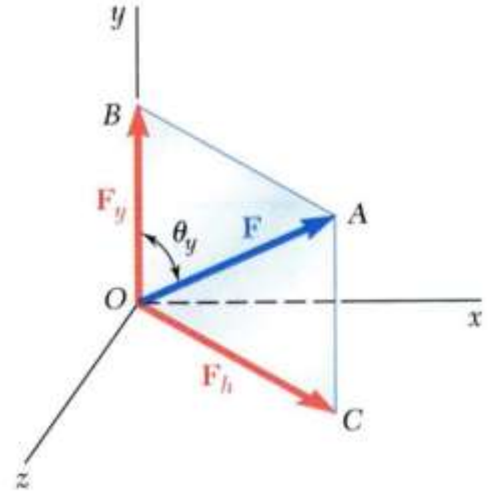
$$tan\varphi = \frac{R_x}{R_y} = \frac{105}{200} \qquad \varphi = 62\,3^o$$

$$R = \sqrt{R_x^2 + R_y^2} = 225.9N$$

Rectangular Components in Space



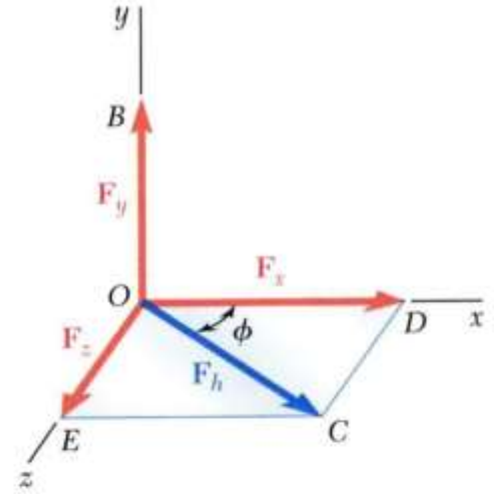
- The vector F is contained in the plane $OBAC$.



- Resolve F into horizontal and vertical components.

$$F_y = F\cos\theta_y$$

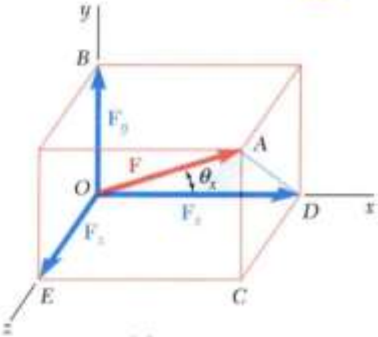
$$F_h = F\sin\theta_y$$

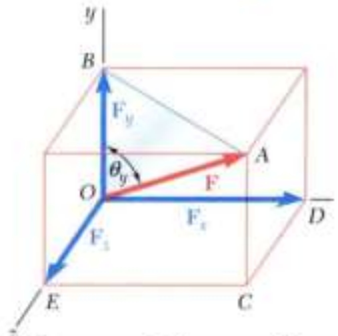


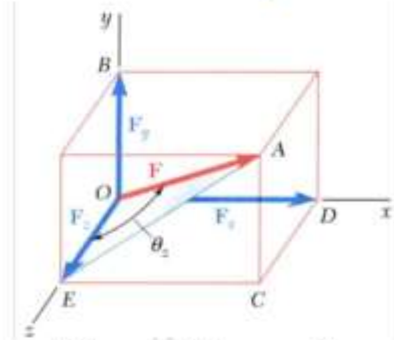
- Resolve F_h into rectangular components

$$F_x = F_h\cos\phi$$
$$= F\sin\theta_y\cos\phi$$
$$F_z = F_h\sin\phi$$
$$= F\sin\theta_y\sin\phi$$

Rectangular Components in Space







$F_x = F cos\theta_x$ $\quad$ $F_y = F cos\theta_y$ $\quad$ $F_z = F cos\theta_z$

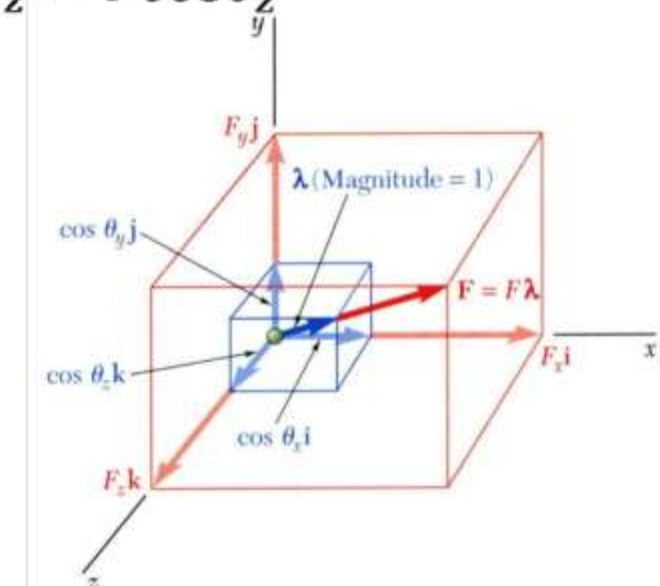
$\boldsymbol{F} = F_x \boldsymbol{i} + F_y \boldsymbol{j} + F_z \boldsymbol{k}$

$\boldsymbol{F} = F cos\theta_x \boldsymbol{i} + F cos\theta_y \boldsymbol{j} + F cos\theta_z \boldsymbol{k}$

$\boldsymbol{F} = F\left(cos\theta_x \boldsymbol{i} + cos\theta_y \boldsymbol{j} + cos\theta_z \boldsymbol{k}\right)$

$\boldsymbol{F} = F\boldsymbol{\lambda}$

Where $\boldsymbol{\lambda} = cos\theta_x \boldsymbol{i} + cos\theta_y \boldsymbol{j} + cos\theta_z \boldsymbol{k}$

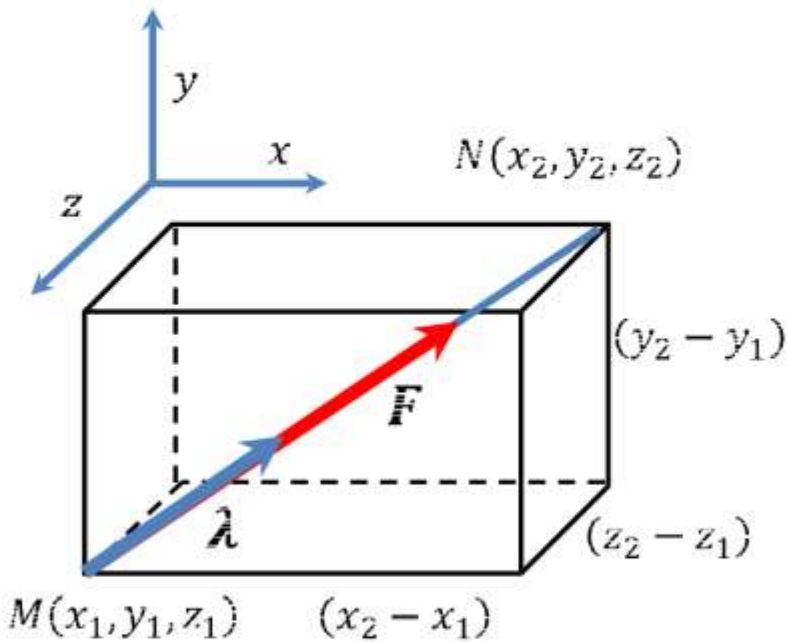


$\boldsymbol{\lambda}$ is a unit vector along the line of action of $\boldsymbol{F}$ and $cos\theta_x$, $cos\theta_y$ and $cos\theta_z$ are the direction cosine for $\boldsymbol{F}$

Rectangular Components in Space

Direction of the force is defined by the location of two points

$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



$\boldsymbol{d}$ is the vector joining M and N

$$\boldsymbol{d} = d_x\boldsymbol{i} + d_y\boldsymbol{j} + d_z\boldsymbol{k}$$

$$d_x = (x_2 - x_1) \qquad d_y = (y_2 - y_1)$$

$$d_z = (z_2 - z_1)$$

$$\boldsymbol{F} = F\boldsymbol{\lambda}$$

$$= F\left(\frac{d_x\boldsymbol{i} - d_y\boldsymbol{j} + d_z\boldsymbol{k}}{d}\right)$$

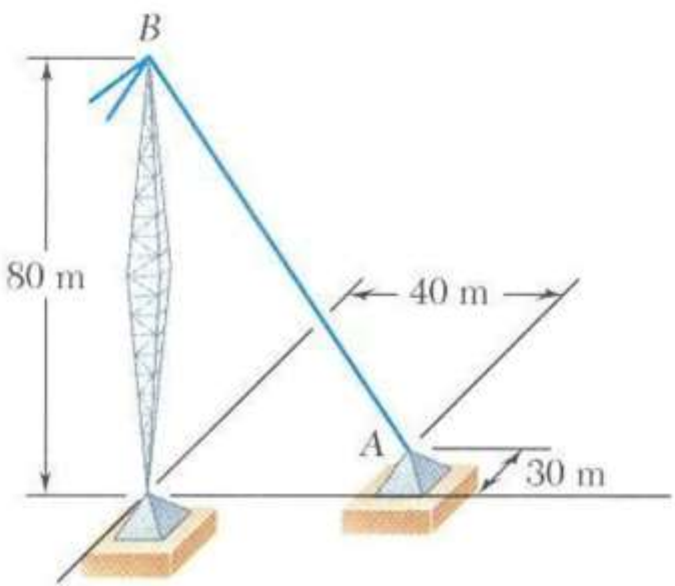
$$F_x = F\frac{d_x}{d} \qquad F_y = F\frac{d_y}{d} \qquad F_z = F\frac{d_z}{d}$$

Rectangular Components in Space

Example: The tension in the guy wire is 2500 N. Determine:

a) components F_x, F_y, F_z of the force acting on the bolt at A,

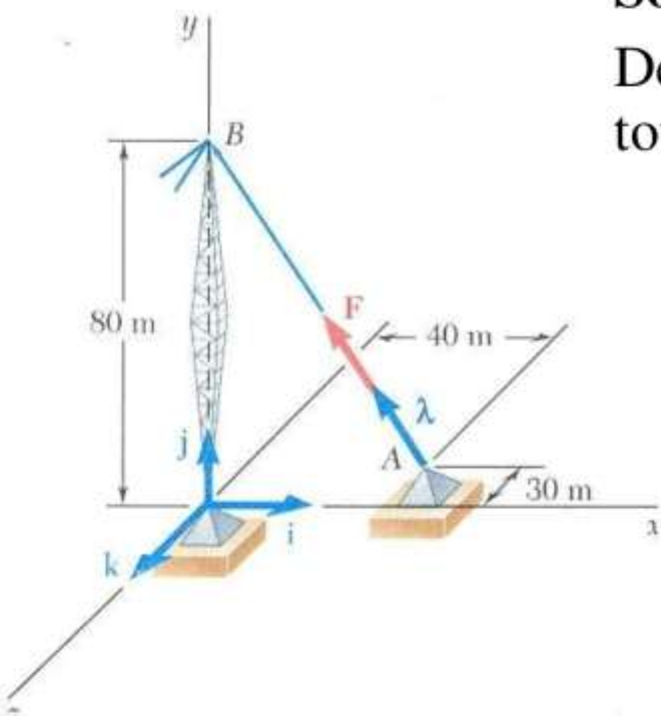
b) the angles q_x, q_y, q_z defining the direction of the force



SOLUTION:

- Based on the relative locations of the points A and B, determine the unit vector pointing from A towards B.
- Apply the unit vector to determine the components of the force acting on A.
- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

Rectangular Components in Space



Solution

Determine the unit vector pointing from *A* towards *B*.

$$\boldsymbol{AB} = -40\boldsymbol{i} - 80\boldsymbol{j} - 30\boldsymbol{k}$$

$$AB = \sqrt{(-40)^2 - (80)^2 - (30)^2} = 94.3$$

$$\boldsymbol{\lambda} = \frac{\boldsymbol{AB}}{AB} = \frac{-40\boldsymbol{i} - 80\boldsymbol{j} - 30\boldsymbol{k}}{94.3}$$

$$= -0.424\boldsymbol{i} - 0.848\boldsymbol{j} - 0.318\boldsymbol{k}$$

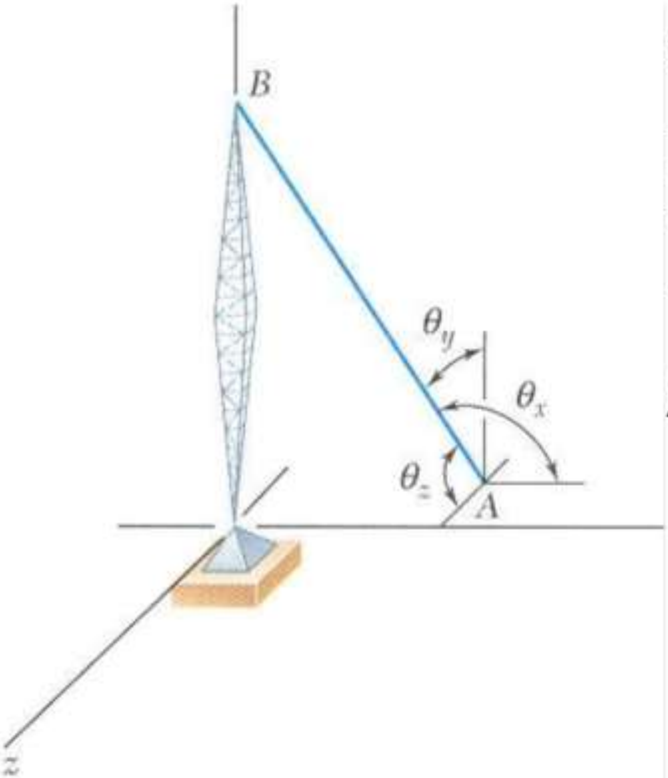
Determine the components of the force.

$$\boldsymbol{F} = F\boldsymbol{\lambda} \quad = 2500(-0.424\boldsymbol{i} - 0.848\boldsymbol{j} - 0.318\boldsymbol{k})$$

$$= -1060\boldsymbol{i} + 2120\boldsymbol{j} + 795\boldsymbol{k}$$

$$\boxed{\begin{aligned} F_x &= -1060\ N \\ F_y &= 2120\ N \\ F_z &= 795\ N \end{aligned}}$$

Rectangular Components in Space



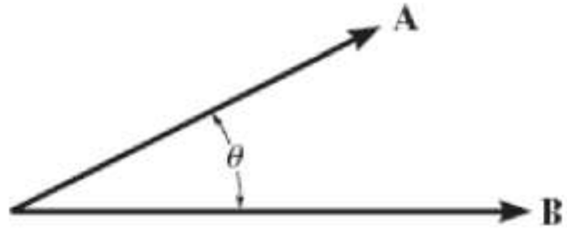
Solution

Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

$$\boldsymbol{\lambda} = cos\theta_x \boldsymbol{i} + cos\theta_y \boldsymbol{j} + cos\theta_z \boldsymbol{k}$$
$$= -0.424\boldsymbol{i} + 0.848\boldsymbol{j} + 0.318\boldsymbol{k}$$

$$\boxed{\begin{aligned} \theta_x &= 115.1^0 \\ \theta_y &= 32.0^0 \\ \theta_z &= 71.5^0 \end{aligned}}$$

Vector Products



Dot Product $\quad \boldsymbol{A}.\boldsymbol{B} = AB\cos\theta$

Applications:

to determine the angle between two vectors

to determine the projection of a vector in a specified direction

A.B = B.A (commutative)

A.(B+C) = A.B+A.C (distributive operation)

$$\boldsymbol{A}.\boldsymbol{B} = \left(A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k}\right).\left(B_x\boldsymbol{i} + B_y\boldsymbol{j} + B_z\boldsymbol{k}\right)$$

$$= A_xB_x + A_yB_y + A_zB_z$$

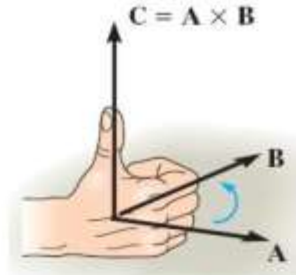
$$\boldsymbol{i}\ \boldsymbol{i} = 1$$

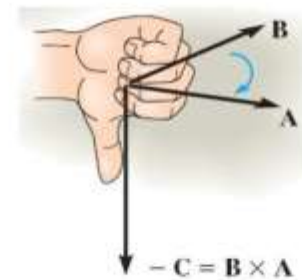
$$\boldsymbol{i}.\boldsymbol{j} = 0$$

Vector Products

Cross Product: $\boldsymbol{A} \times \boldsymbol{B} = \boldsymbol{C} = AB\sin\theta$

$\boldsymbol{A} \times \boldsymbol{B} = -(\boldsymbol{B} \times \boldsymbol{A})$

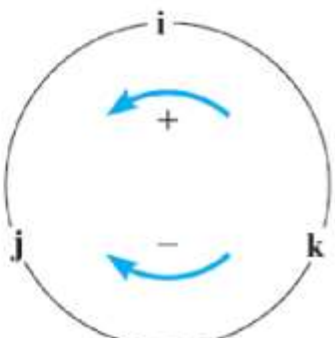


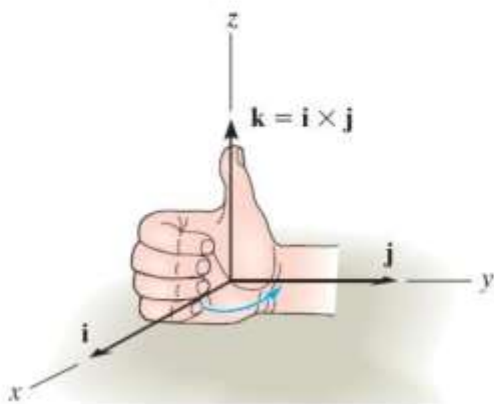


$$\boldsymbol{A} \times \boldsymbol{B} = (A_x\boldsymbol{i} - A_y\boldsymbol{j} - A_z\boldsymbol{k}) \times (B_x\boldsymbol{i} - B_y\boldsymbol{j} - B_z\boldsymbol{k})$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} (A_yB_z - A_zB_y)\boldsymbol{i} + (A_zB_x - A_xB_z)\boldsymbol{j} + (A_xB_y - A_yB_x)\boldsymbol{k}$$

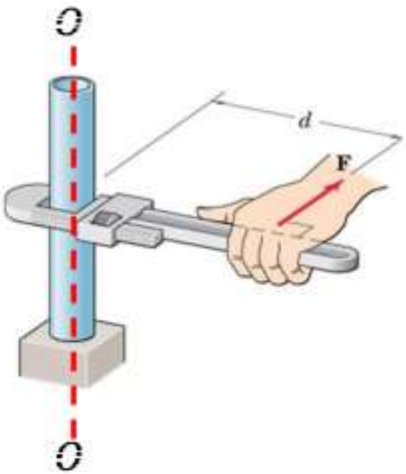
Cartesian Vector





$$\mathbf{i} \times \mathbf{j} = \mathbf{k} \quad \mathbf{i} \times \mathbf{k} = -\mathbf{j} \quad \mathbf{i} \times \mathbf{i} = \mathbf{0}$$
$$\mathbf{j} \times \mathbf{k} = \mathbf{i} \quad \mathbf{j} \times \mathbf{i} = -\mathbf{k} \quad \mathbf{j} \times \mathbf{j} = \mathbf{0}$$
$$\mathbf{k} \times \mathbf{i} = \mathbf{j} \quad \mathbf{k} \times \mathbf{j} = -\mathbf{i} \quad \mathbf{k} \times \mathbf{k} = \mathbf{0}$$

Moment of a Force (Torque)

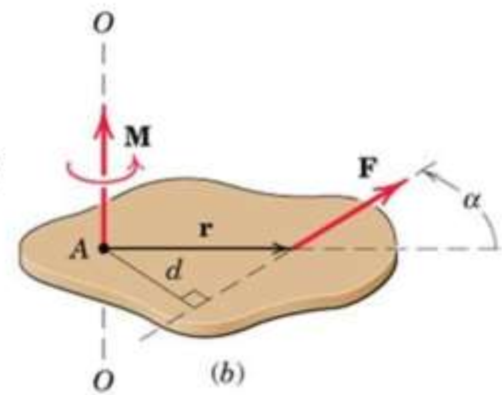


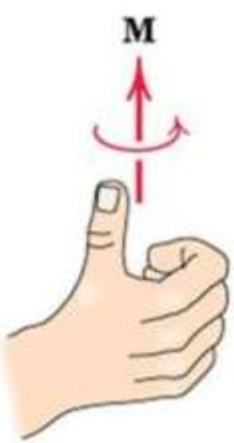
Moment about axis O-O is $\boldsymbol{M_o} = Fd$

Magnitude of $\boldsymbol{M_o}$ measures tendency of **F** to cause rotation of the body about an axis along $\boldsymbol{M_o}$.

Moment about axis O-O is $\boldsymbol{M_o} = Fr\sin\alpha$

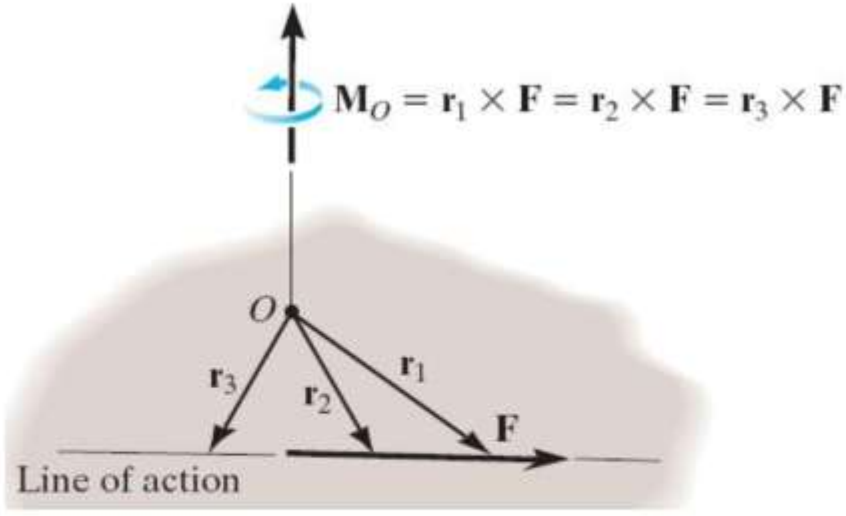
$$\boldsymbol{M_o} = \boldsymbol{r} \times \boldsymbol{F}$$



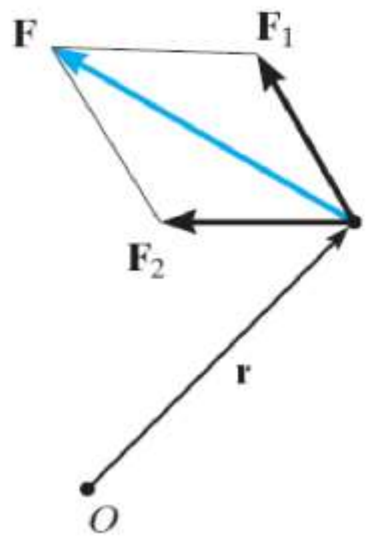


Sense of the moment may be determined by the right-hand rule

Moment of a Force



Principle of Transmissibility
Any force that has the same magnitude and direction as **F**, is *equivalent* if it also has the same line of action and therefore, produces the same moment.



Varignon's Theorem (Principle of Moments)

Moment of a Force about a point is equal to the sum of the moments of the force's components about the point.

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

Rectangular Components of a Moment

The moment of $\boldsymbol{F}$ about O,

$$\boldsymbol{M_o} = \boldsymbol{r} \times \boldsymbol{F}$$

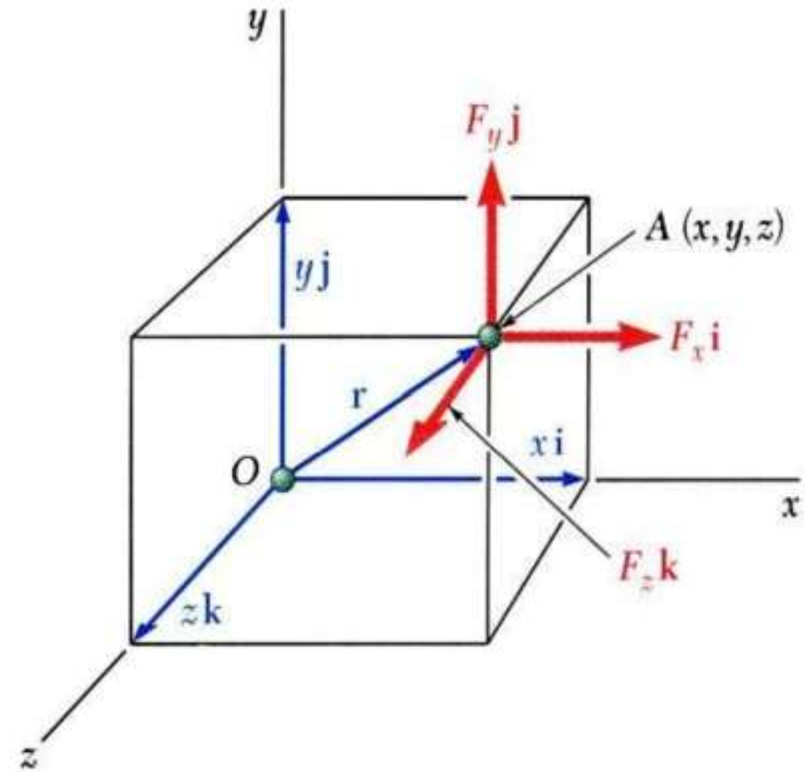
$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$

$$\boldsymbol{M_o} = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= (yF_z - zF_y)\boldsymbol{i} + (zF_x - xF_z)\boldsymbol{j} + (xF_y - yF_x)\boldsymbol{k}$$



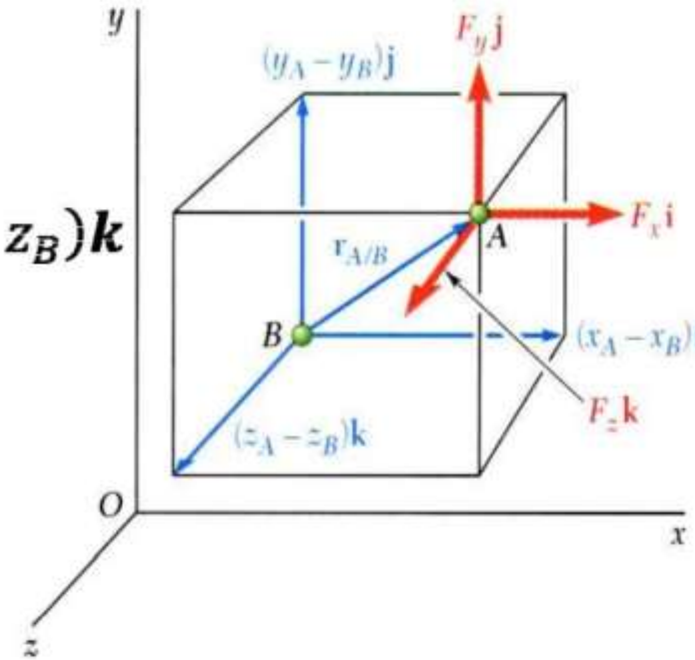
Rectangular Components of the Moment

The moment of $\boldsymbol{F}$ about B,

$$\boldsymbol{M}_B = \boldsymbol{r}_{AB} \times \boldsymbol{F}$$

$$\boldsymbol{r}_{AB} = (x_A - x_B)\boldsymbol{i} + (y_A - y_B)\boldsymbol{j} + (z_A - z_B)\boldsymbol{k}$$

$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

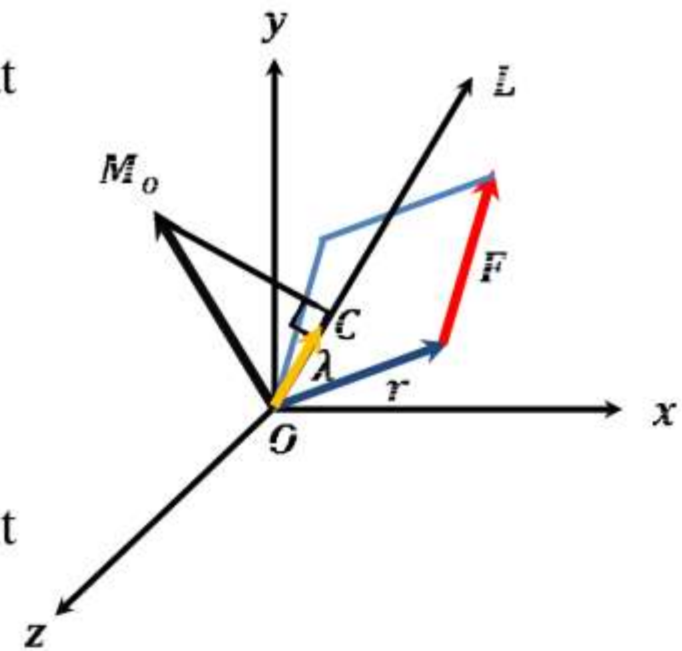


$$\boldsymbol{M}_B = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x_A - x_B & y_A - y_B & z_A - z_B \\ F_x & F_y & F_z \end{vmatrix}$$

Moment of a Force About a Given Axis

Moment $\mathbf{M_O}$ of a force $\mathbf{F}$ applied at the point $\mathbf{A}$ about a point $\mathbf{O}$



$$\boldsymbol{M}_o = \boldsymbol{r} \times \boldsymbol{F}$$

Scalar moment M_{OL} about an axis **OL** is the projection of the moment vector $\mathbf{M_O}$ onto the axis,

$$M_{OL} = \boldsymbol{\lambda}.\boldsymbol{M_o} = \boldsymbol{\lambda}.(\boldsymbol{r} \times \boldsymbol{F})$$

Moments of **F** about the coordinate axes (using previous slide)

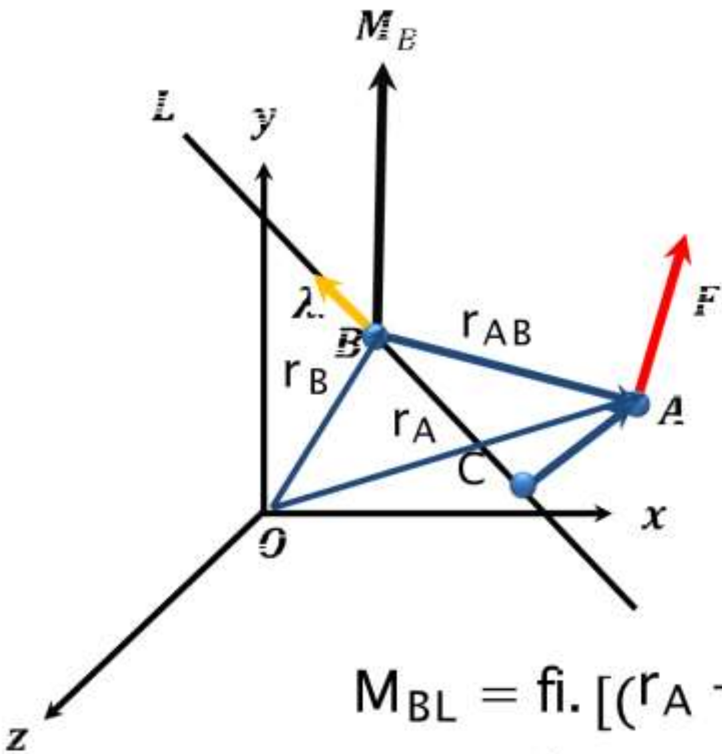
$$M_x = (yF_z - zF_y)$$

$$M_y = (zF_x - xF_z)$$

$$M_z = (xF_y - yF_x)$$

Moment of a Force About a Given Axis

Moment of a force about an arbitrary axis



$$\boldsymbol{M}_B = \boldsymbol{r}_{AB} \times \boldsymbol{F}$$

$$\boldsymbol{M}_{BL} = \boldsymbol{\lambda}.\boldsymbol{M}_B = \boldsymbol{\lambda}.(\boldsymbol{r}_{AB} \times \boldsymbol{F})$$

$$r_{AB} = r_A - r_B$$

If we take point C in place of point B

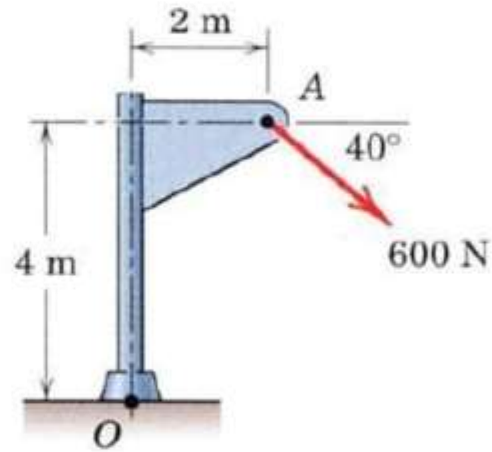
$$M_{BL} = \text{fi}.[(r_A - r_C) \times F]$$

$$= \text{fi}.[(r_A - r_B) \times F] + \boxed{\cancelto{0}{\text{fi}.[(r_B - r_C) \times F]}}$$

$(r_B - r_C)$ and fi are in the same line

Moment: Example

Calculate the magnitude of the moment about the base point O of the 600 N force in different ways

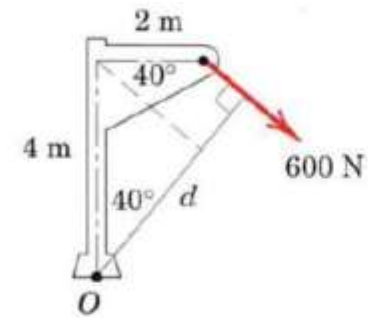


Solution 1.

Moment about O is

$$M_o = dF \qquad d = 4\cos 40^o + 2\sin 40^o = 4.35m$$

$$M_o = 600(4.35) = 2610\ N.m\ \textbf{Ans}$$

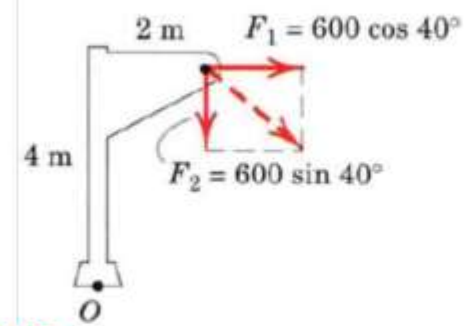


Solution 2.

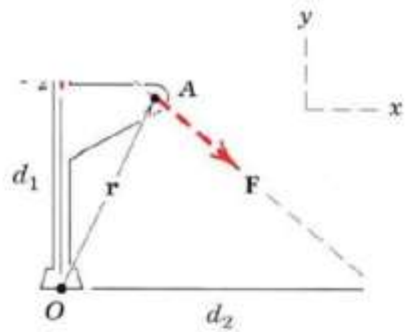
$$F_x = 600\cos 40^o = 460\ N$$

$$F_x = 600\sin 40^o = 386\ N$$

$$M_o = 460(4.00) + 386(2.00) = 2610\ N.m\ \textbf{Ans}$$



Moment: Example



Solution 3.

$$d_1 = 4 + 2tan40^o = 5.68m$$

$$M_O = 460(5.68) = 2610\,N.m$$ ***Ans***

Solution 4.

$$d_2 = 2 + 4cot40^o = 6.77m$$

$$M_O = 386(6.77) = 2610\,N.m$$ ***Ans***

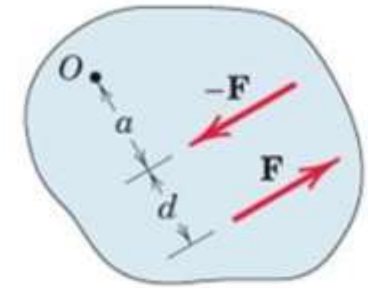
Solution 5.

$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F} = (2\boldsymbol{i} + 4\boldsymbol{j}\,) \times 600(cos40^o\boldsymbol{i} - sin40^o\boldsymbol{j})$$

$$\boldsymbol{M_0} = -2610\,N.m$$ ***Ans***

The minus sign indicates that the vector is in the negative z-direction

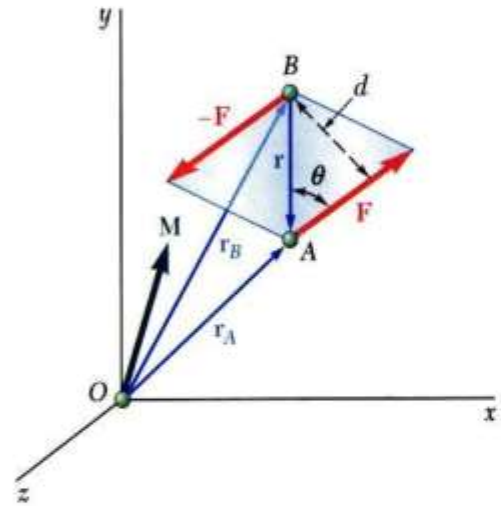
Moment of a Couple



Moment produced by two equal, opposite and non-collinear forces is called a *couple*.

Magnitude of the combined moment of the two forces about O:

$$M = F(a + d) - Fa = Fd$$

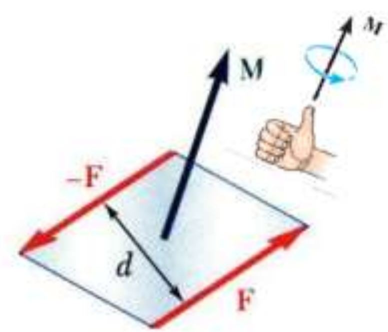


$$M = r_A \times F + r_B \times (-F)$$

$$= (r_A - r_B) \times F$$

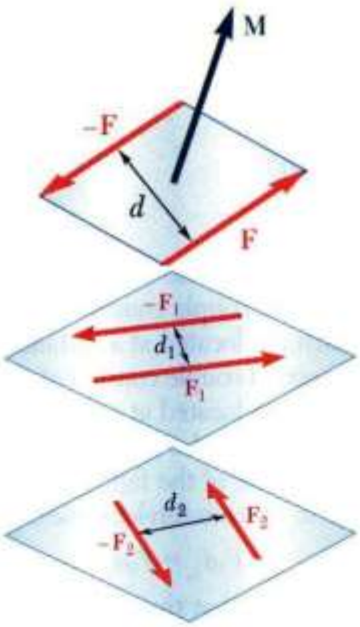
$$= r \times F$$

$$M = rF\sin 8 = Fd$$



The moment vector of the couple is independent of the choice of the origin of the coordinate axes, i.e., it is a *free vector* that can be applied at any point with the same effect.

Moment of a Couple

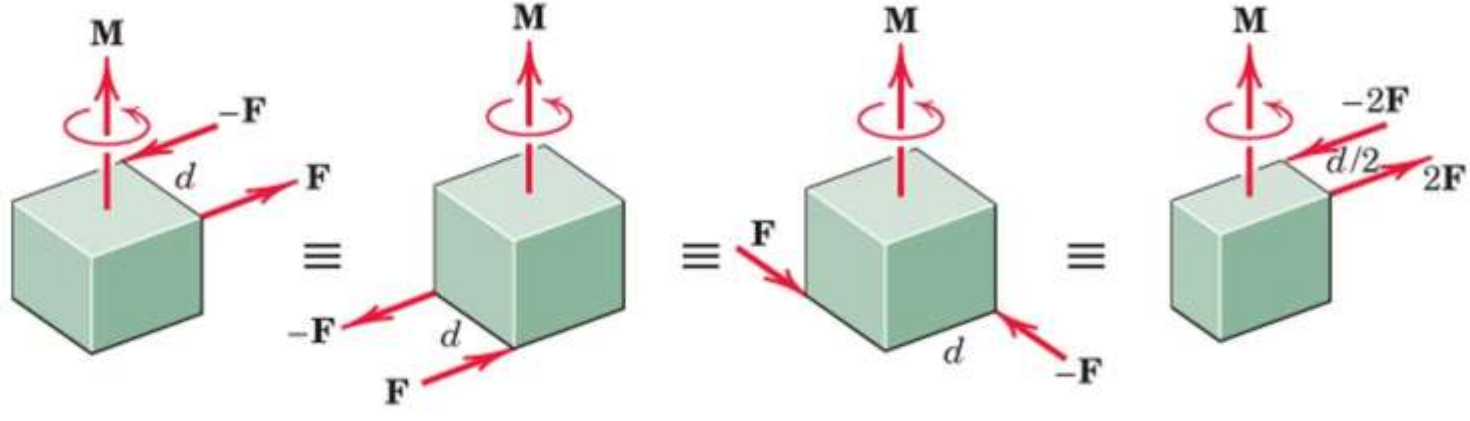


Two couples will have equal moments if $F_1 d_1 = F_2 d_2$

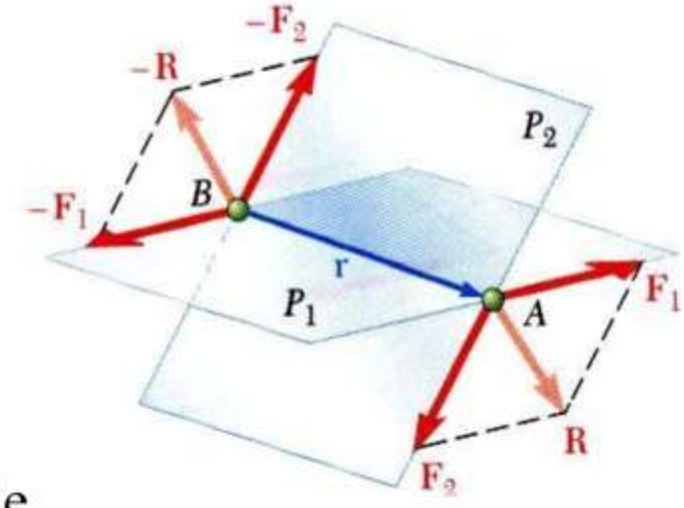
The two couples lie in parallel planes

The two couples have the same sense or the tendency to cause rotation in the same direction.

Examples:



Addition of Couples



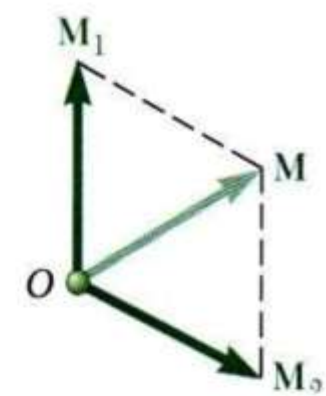
Consider two intersecting planes P_1 and P_2 with each containing a couple

$M_1 = r \times F_1$ in plane P_1

$M_2 = r \times F_2$ in plane P_2

Resultants of the vectors also form a couple

$$M = r \times R = r \times (F_1 + F_2)$$

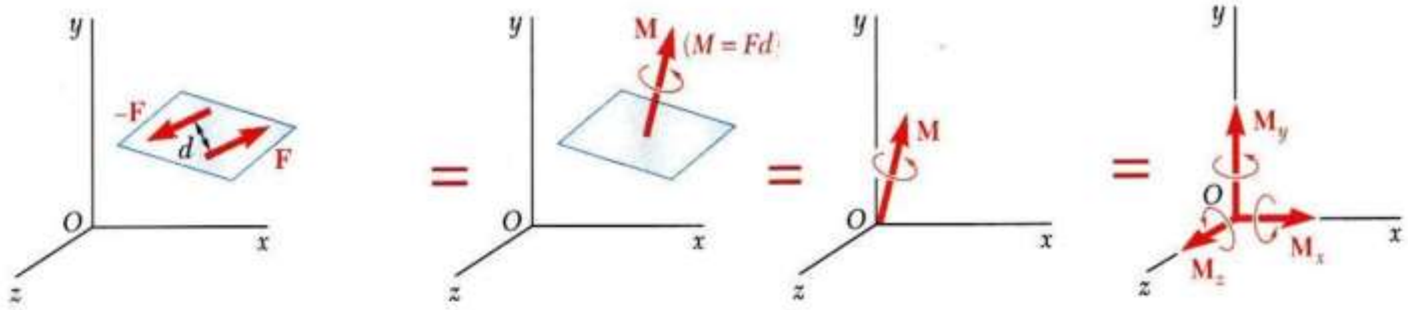


By Varigon's theorem

$$M = r \times F_1 + r \times F_2$$
$$= M_1 + M_2$$

Sum of two couples is also a couple that is equal to the vector sum of the two couples

Couples Vectors



A couple can be represented by a vector with magnitude and direction equal to the moment of the couple.

Couple vectors obey the law of addition of vectors.

Couple vectors are free vectors, i.e., the point of application is not significant.

Couple vectors may be resolved into component vectors.

Couple: Example

Moment required to turn the shaft connected at center of the wheel = 12 Nm

Case I: Couple Moment produced by 40 N forces = 12 Nm

Case II: Couple Moment produced by 30 N forces = 12 Nm

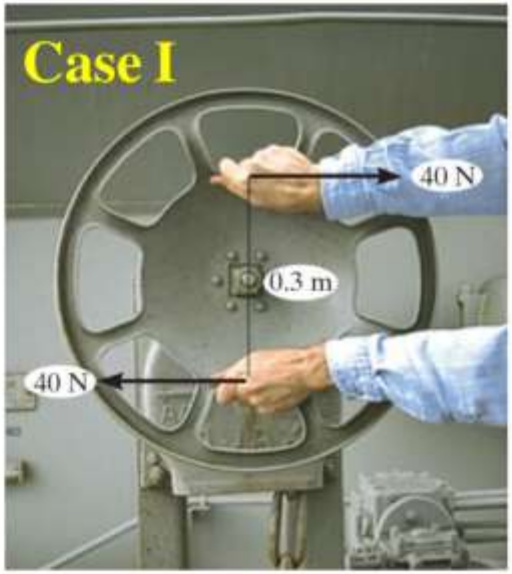
If only one hand is used?

Force required for case I is 80N

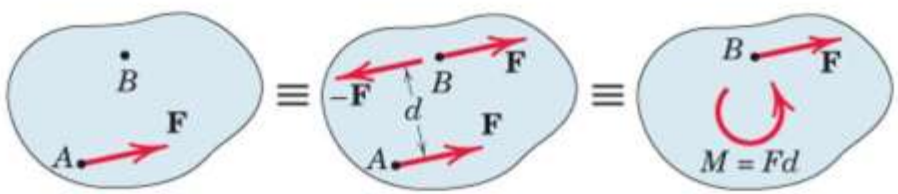
Force required for case II is 60N

What if the shaft is not connected at the center of the wheel?

Is it a Free Vector?



Equivalent Systems

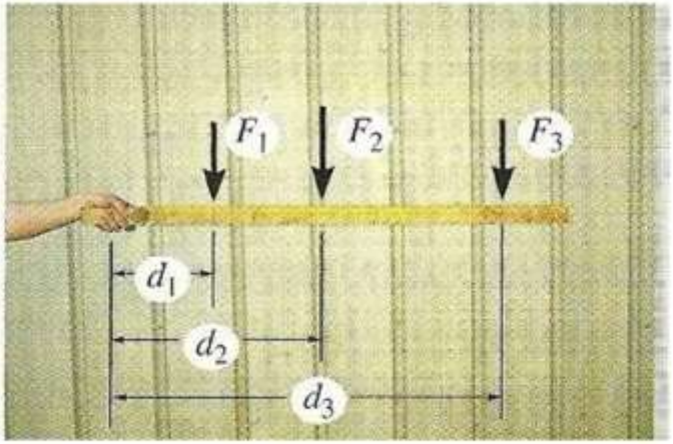


At support O

$W_r = W_1 + W_2$

$M_o = W_1 d_1 + W_2 d_2$

Equivalent Systems: Resultants



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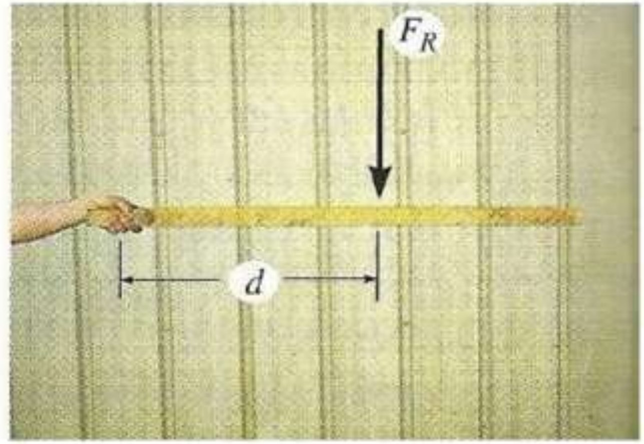


$$F_R = F_1 + F_2 + F_3$$

What is the value of d?

Moment of the Resultant force about the grip must be equal to the moment of the forces about the grip

$$\boxed{F_R d = F_1 d_1 + F_2 d_2 + F_3 d_3}$$

Equilibrium Conditions

Equivalent Systems: Resultants

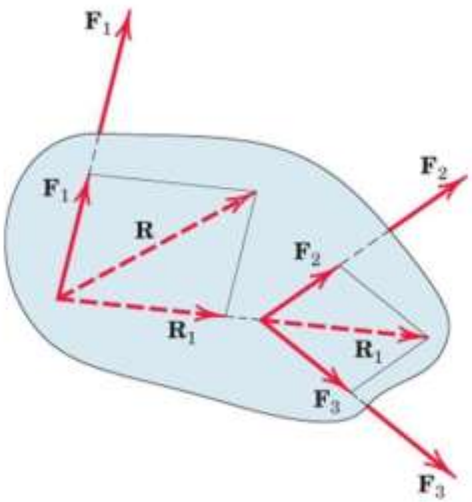
Equilibrium

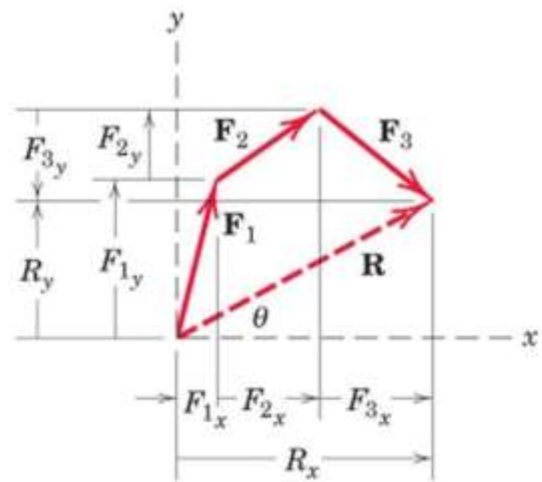
Equilibrium of a body is a condition in which the resultants of all forces acting on the body is zero.

Condition studied in Statics

Equivalent Systems: Resultants

Vector Approach: Principle of Transmissibility can be used





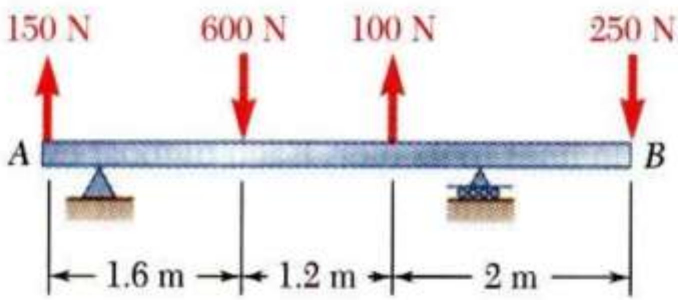
Magnitude and direction of the resultant force R is obtained by forming the force polygon where the forces are added head to tail in any sequence

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

Equivalent Systems: Example



For the beam, reduce the system of forces shown to (a) an equivalent force-couple system at *A*, (b) an equivalent force couple system at *B*, and (c) a single force or resultant.

Note: Since the support reactions are not included, the given system will not maintain the beam in equilibrium.

<u>Solution:</u>

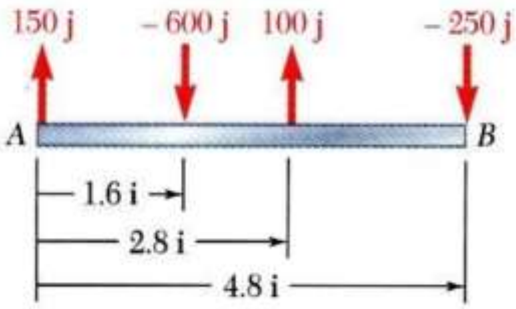
a) Compute the resultant force for the forces shown and the resultant couple for the moments of the forces about *A*.

b) Find an equivalent force-couple system at *B* based on the force-couple system at *A*.

c) Determine the point of application for the resultant force such that its moment about *A* is equal to the resultant couple at *A*.

Equivalent Systems: Example

SOLUTION



(a) Compute the resultant force and the resultant couple at A.

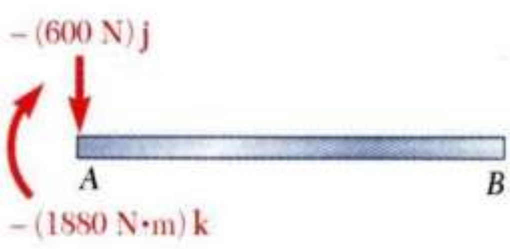
$$R = \Sigma\ F = 150j - 600j + 100j - 250j$$

$$R = -(600N)j$$

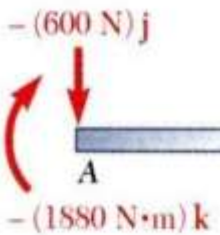
$$M_A^R = \Sigma\ r \times F$$

$$= 1.6i \times (-600j) + 2.8i \times (100j) + 4.8i \times (-250j)$$

$$M_A^R = -(1880 N.n)k$$



Equivalent Systems: Example

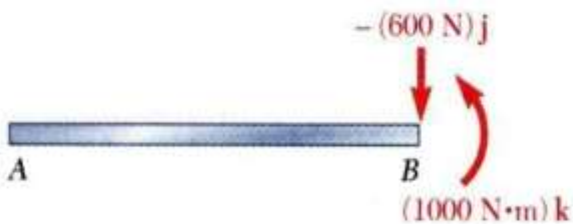


b) Find an equivalent force-couple system at *B* based on the force-couple system at *A*.

The force is unchanged by the movement of the force-couple system from *A* to *B*.

$$R = -(600N)j$$

The couple at *B* is equal to the moment about *B* of the force-couple system found at *A*.



$$M_B^R = M_A^R + r_{BA} \times R$$

$$= -1800k + (-4.8i) \times (-600j)$$

$$= (1000N.n)k$$

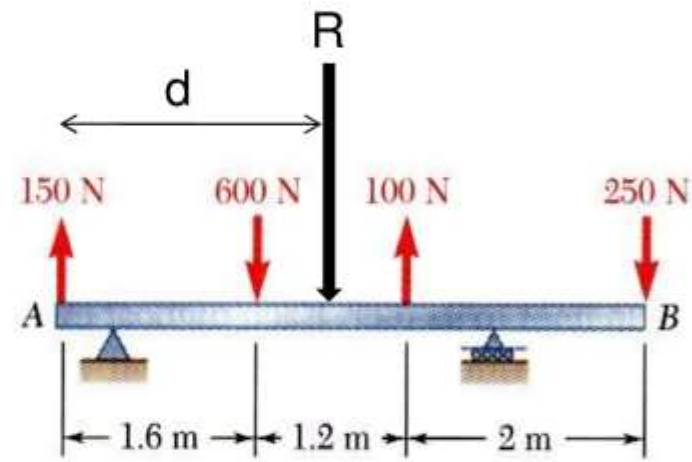
Equivalent Systems: Example

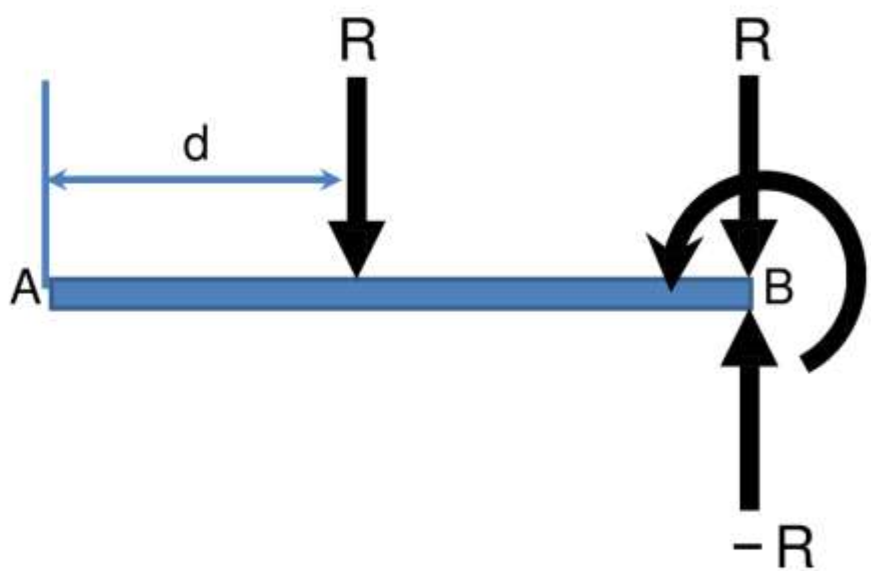
$R = F_1 + F_2 + F_3 + F_4$

$R = 150 - 600 + 100 - 250 = -600\ N$

$Rd = F_1 d_1 + F_2 d_2 + F_3 d_3 + F_4 d_4$

$d = 3.13\ n$





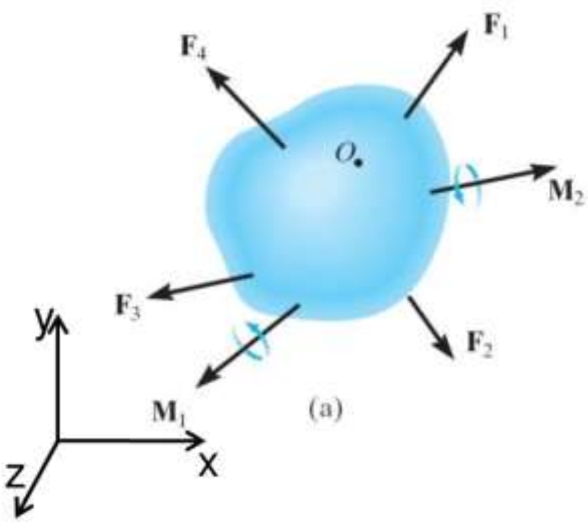
Rigid Body Equilibrium

A rigid body will remain in equilibrium provided

- sum of all the external forces acting on the body is equal to zero, and
- Sum of the moments of the external forces about a point is equal to zero

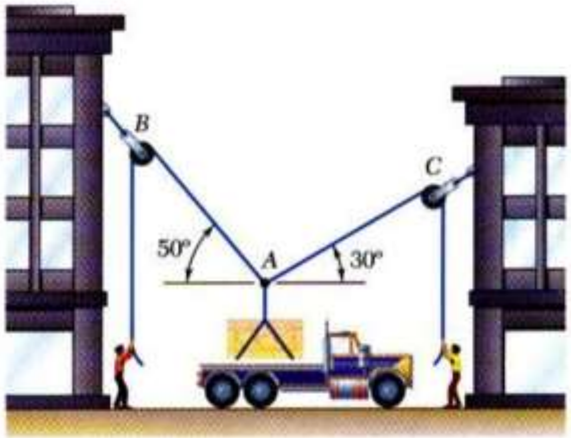
$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$$

$$\Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0$$

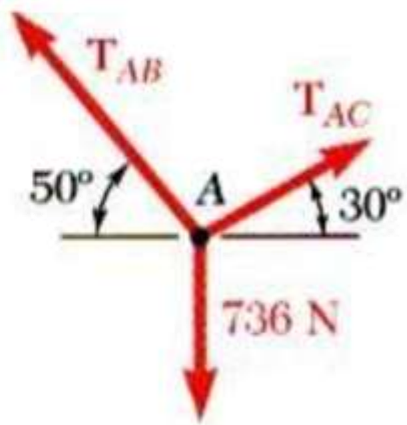


Rigid Body Equilibrium

Free-Body Diagrams



Space Diagram: A sketch showing the physical conditions of the problem.



Free-Body Diagram: A sketch showing only the forces on the selected particle.

Rigid Body Equilibrium

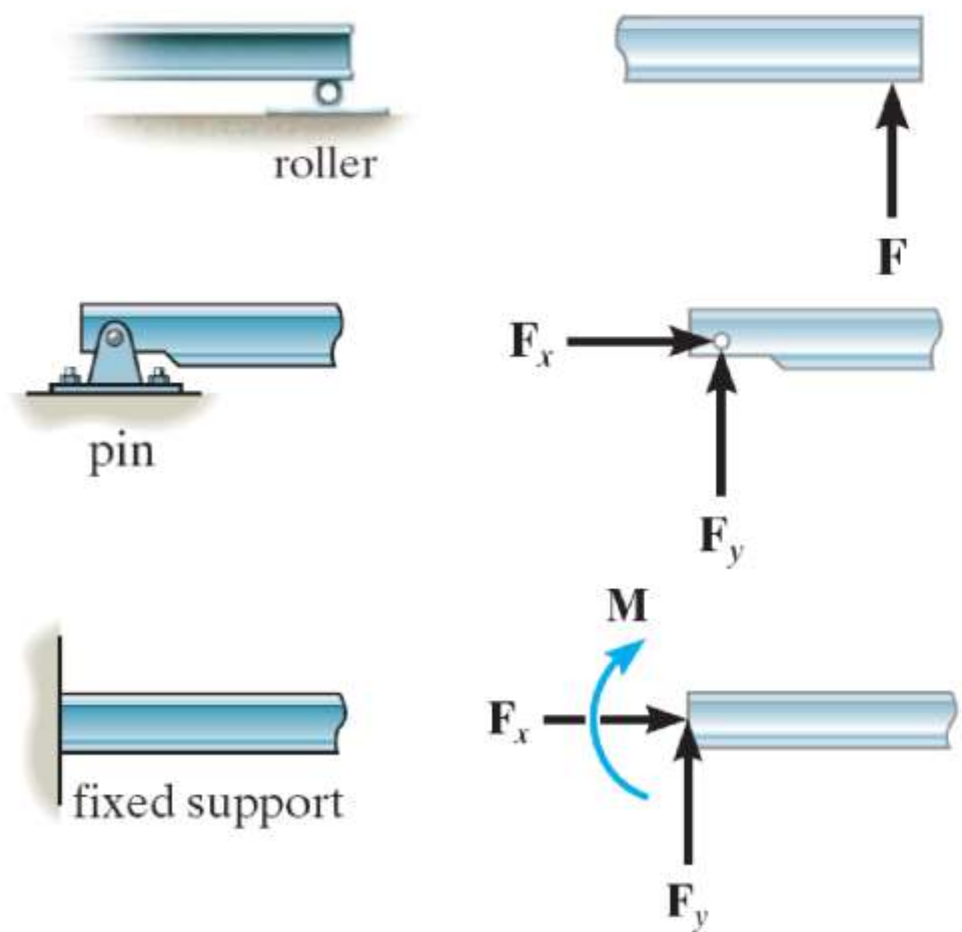
Support Reactions

Prevention of

Translation or

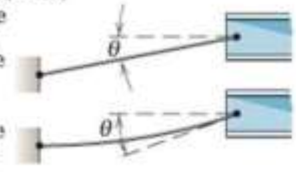
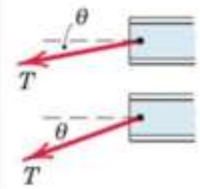
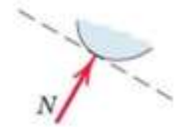
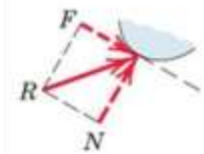
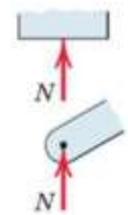
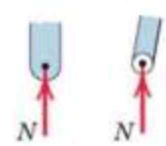
Rotation of a body

Restraints



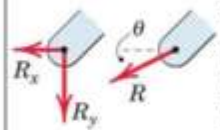
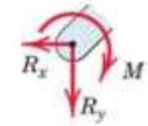
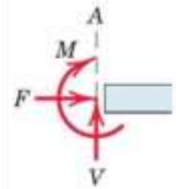
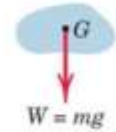
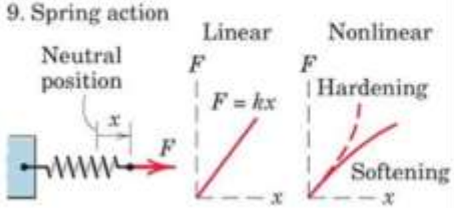
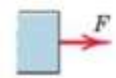
Rigid Body Equilibrium

Various Supports 2-D Force Systems

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	**Action on Body to Be Isolated**
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential compo-nent F (frictional force) as well as a normal component N of the resultant
4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.

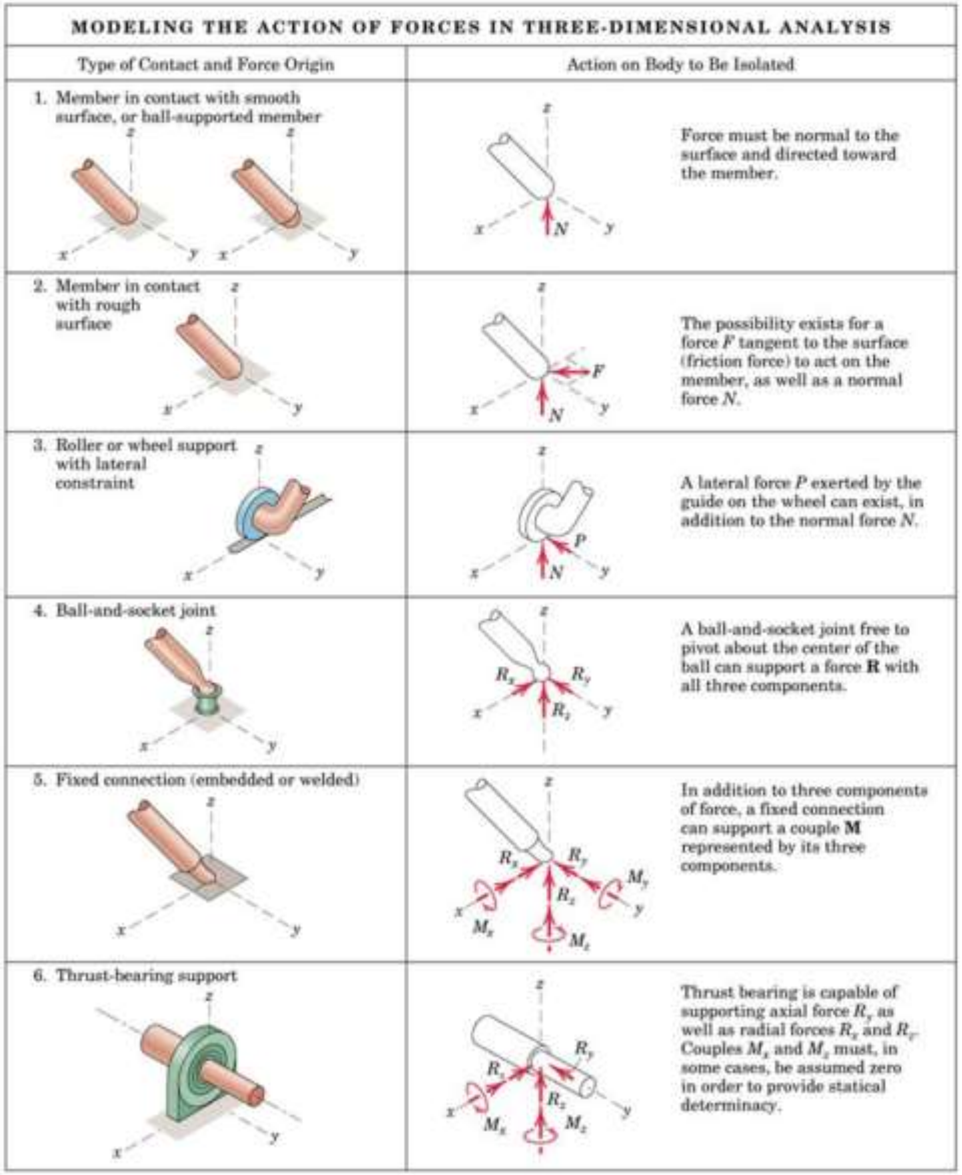
Rigid Body Equilibrium

Various Supports
2-D Force
Systems

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)		
Type of Contact and Force Origin	**Action on Body to Be Isolated**	
6. Pin connection 	Pin free to turn  Pin not free to turn 	A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support 		A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 		The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action 		Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

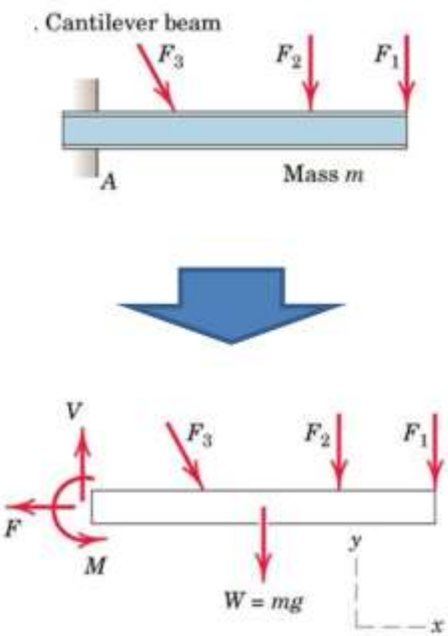
Rigid Body Equilibrium

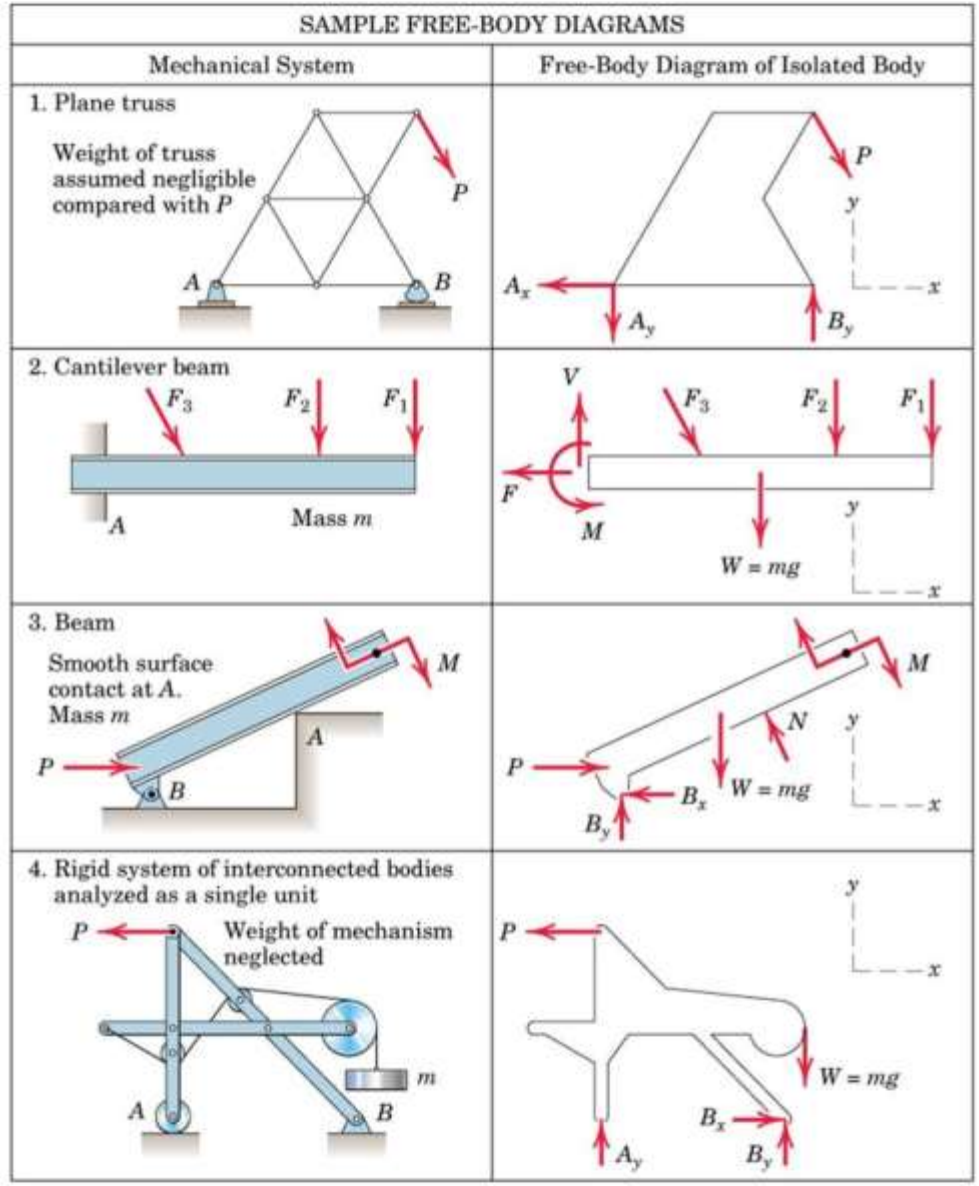
Various Supports
3-D Force Systems



MODELING THE ACTION OF FORCES IN THREE-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Member in contact with smooth surface, or ball-supported member	Force must be normal to the surface and directed toward the member.
2. Member in contact with rough surface	The possibility exists for a force F tangent to the surface (friction force) to act on the member, as well as a normal force N.
3. Roller or wheel support with lateral constraint	A lateral force P exerted by the guide on the wheel can exist, in addition to the normal force N.
4. Ball-and-socket joint	A ball-and-socket joint free to pivot about the center of the ball can support a force **R** with all three components.
5. Fixed connection (embedded or welded)	In addition to three components of force, a fixed connection can support a couple **M** represented by its three components.
6. Thrust-bearing support	Thrust bearing is capable of supporting axial force R_y as well as radial forces R_x and R_z. Couples M_x and M_z must, in some cases, be assumed zero in order to provide statical determinacy.

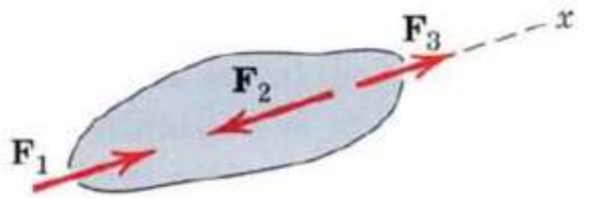
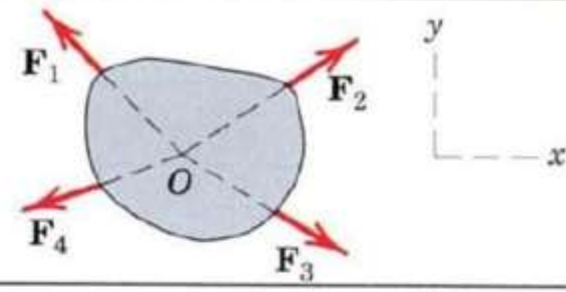
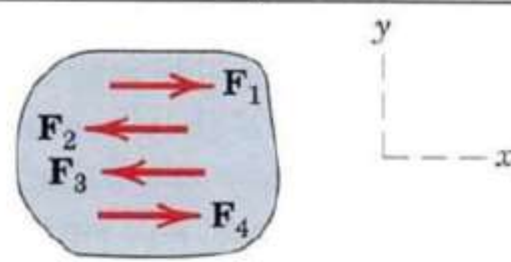
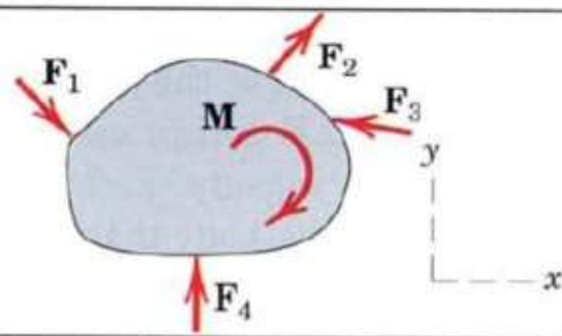
Free body diagram





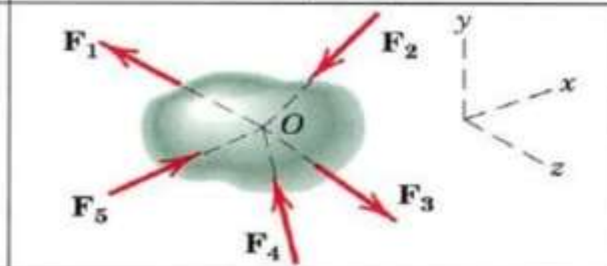
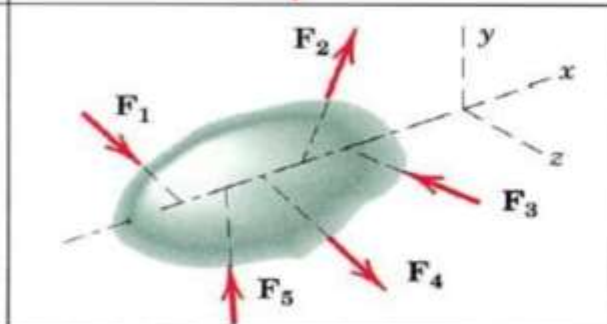
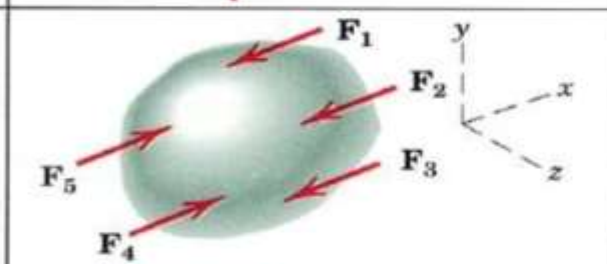
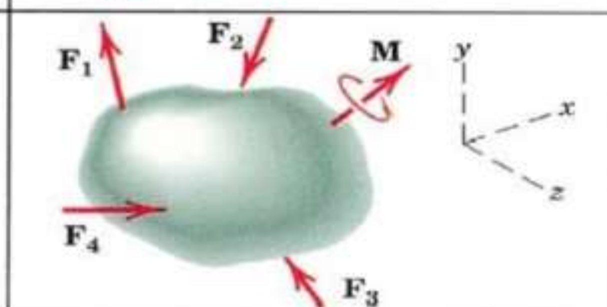
Rigid Body
Equilibrium

Categories
in 2-D

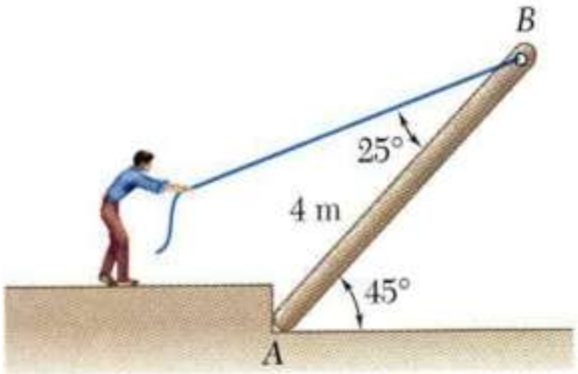
<table>
<tr><th colspan="3">CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS</th></tr>
<tr><th>Force System</th><th>Free-Body Diagram</th><th>Independent Equations</th></tr>
<tr><td>1. Collinear</td><td></td><td>$\Sigma F_x = 0$</td></tr>
<tr><td>2. Concurrent at a point</td><td></td><td>$\Sigma F_x = 0$
$\Sigma F_y = 0$</td></tr>
<tr><td>3. Parallel</td><td></td><td>$\Sigma F_x = 0 \quad \Sigma M_z = 0$</td></tr>
<tr><td>4. General</td><td></td><td>$\Sigma F_x = 0 \quad \Sigma M_z = 0$
$\Sigma F_y = 0$</td></tr>
</table>

Rigid Body Equilibrium

Categories in 3-D

CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS		
Force System	**Free-Body Diagram**	**Independent Equations**
1. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
2. Concurrent with a line		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma F_y = 0$ $\Sigma M_z = 0$ $\Sigma F_z = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_x = 0$ $\Sigma F_y = 0$ $\Sigma M_y = 0$ $\Sigma F_z = 0$ $\Sigma M_z = 0$

Rigid Body Equilibrium: Example



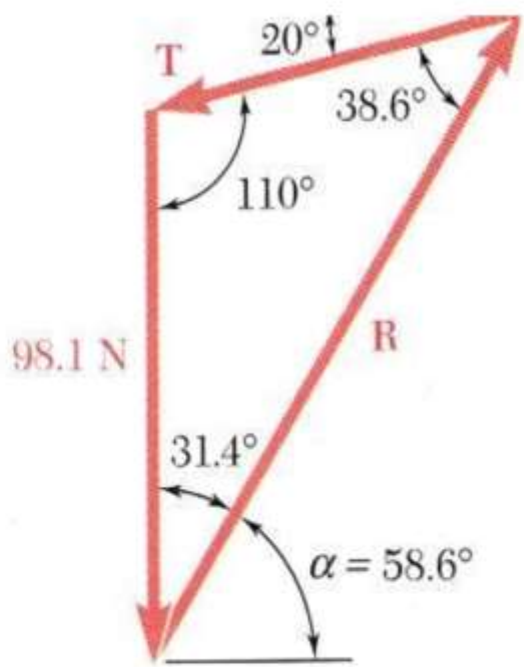
A man raises a 10 kg joist, of length 4 m, by pulling on a rope.

Find the tension in the rope and the reaction at A.

Solution:

- Create a free-body diagram of the joist. Note that the joist is a 3 force body acted upon by the rope, its weight, and the reaction at A.
- The three forces must be concurrent for static equilibrium. Therefore, the reaction ***R*** must pass through the intersection of the lines of action of the weight and rope forces. Determine the direction of the reaction force ***R***.
- Utilize a force triangle to determine the magnitude of the reaction force ***R***.

Rigid Body Equilibrium: Example



- Determine the magnitude of the reaction force R.

$$\frac{T}{\sin 31.4^{\circ}} = \frac{R}{\sin 110^{\circ}} = \frac{98.1\text{N}}{\sin 38.6^{\circ}}$$

$$\boxed{\begin{aligned} T &= 81.9\text{N} \\ R &= 147.8\text{N} \end{aligned}}$$

2.1 CENTRE OF GRAVITY

- Centre of gravity(C.G) of a body is defined as the point through which the entire weight of the body acts. A body has only one centre of gravity for all its positions.
- Centroid is defined as the point at which the total area of a plane figure is assumed to be concentrated. It is the CG of a plane figure
- CG of a rectangle is the point where the diagonals meet
- CG of a triangle is the point where the medians meet
- CG of a circle is at its centre

2.1 CENTRE OF GRAVITY

- Consider a plane figure of total area A composed of a number of small areas $a_1, a_2, a_3, \ldots, a_n$

  Let x_1= Distance of CG of a_1 from axis of reference OY

  x_2= Distance of CG of a_2 from axis of reference OY

  x_3= Distance of CG of a_1 from axis of reference OY

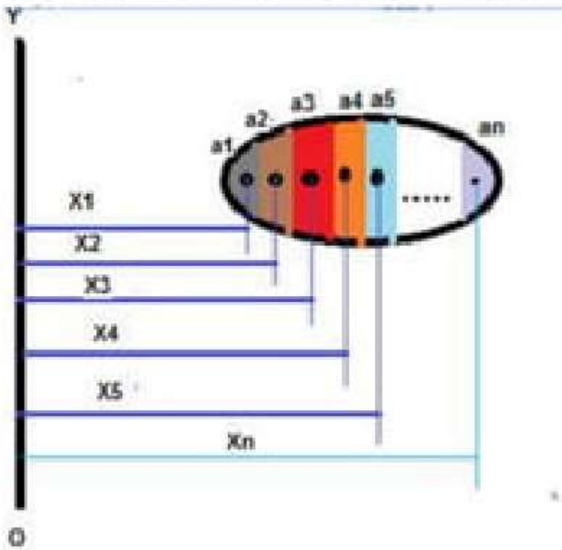
  ;

  ;

  x_n= Distance of CG of a_n from axis of reference OY

2.1 CENTRE OF GRAVITY

- The moments of all areas about axis OY= a1x1 + a2x2 + a3x3+ .... + anxn



- Let G be the centre of gravity of the total area about the axis OY whose distance from OY is Xc

2.2 CENTRE OF GRAVITY OF PLANE FIGURES BY METHOD OF MOMENTS

- Moment of total area about OY= AXc
- Sum of moments of small areas about axis OY must be equal to the moment of total area about OY.
- Hence a1x1 + a2x2 + a3x3+ .... + anxn= AXc
- $X_C = \sum_{i=1}^{n} a_i x_i / \Sigma a_i$ 1x1 + a2x2 + a3x3+ ...... + anxn)/A
- $Y_C = \sum_{i=1}^{n} a_i y_i / \Sigma a_i$

2.2 CENTRE OF GRAVITY OF PLANE FIGURES BY METHOD OF INTEGRATION

- If the areas are large in number (that is i tends to infinity), then the summations in the above equations can be replaced by integration. And when the number of split up areas are large, the size of component areas will be small, hence a can be replaced by dA in the above equation.
- Hence

  $X_c = \int x\,dA / \int dA$

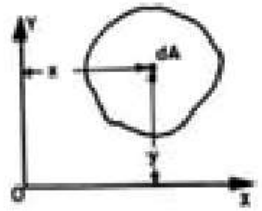
  $Y_c = \int y\,dA / \int dA$

2.3 IMPORTANT NOTES TO CALCULATE CENTRE OF GRAVITY

- Axes about which moments of areas are taken, is known as axis of reference.
- Axis of reference of plane figures is generally taken as the lowest line of the figure for determining Axis of reference of plane figures is generally taken as the lowest line of the figure for determining c and left line of the figure for calculation Xc
- If the given section is symmetrical about X-X or Y-Y, CG will lie in the line of symmetry

2.2 AREA MOMENT OF INERTIA

- Consider an area A as shown. Let dA be an elemental area of the area A with coordinates x and y. The term $\Sigma x^2 dA$ is called the moment of inertia of area A about y axis and term $\Sigma y^2 dA$ is called the moment of inertia of area A about x-axis



- $I_{xx} = \Sigma y^2 dA$ & $I_{yy} = \Sigma x^2 dA$

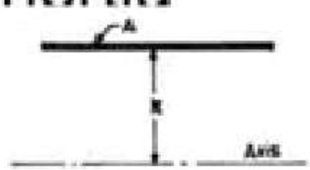
- When dA is very small, mathematically $\Sigma y^2 dA = \int y^2 dA$
  $\Sigma x^2 dA = \int x^2 dA$

2.4 AREA MOMENT OF INERTIA

- $I_{xx} = \int y^2 dA$ & $I_{yy} = \int x^2 dA$
- If r is the distance of an area dA (which is a part of area A) from an axis AB, then the sum of terms $r^2 dA$ (ie $\Sigma r^2 dA$) to cover the entire area is called the moment of inertia of the area A about axis AB or second moment of area of area A about axis AB.
- Moment of inertia is a fourth dimensional term as it is obtaine ying area by distance squared. He unit is m^4

2.5 RADIUS OF GYRATION

- Mathematical term defined by the expression, $k=\sqrt{(I/A)}$ is called radius of gyration.
- $I=Ak^2$
- Hence the radius of gyration can be considered a that distance at which the complete area is squeezed and kept as a strip of negligible width such that there is no change in the moment of inertia



2.6 THEOREMS OF MOMENT OF INERTIA

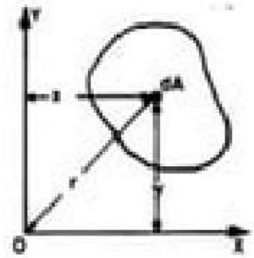
- **1) Perpendicular axis theorem**:-

  The area moment of inertia about an axis perpendicular to its plane at any point is equal to the sum of moment of inertia about two mutually perpendicular axes through the point O and lying in the same plane of area.

Polar moment of inertia is defined as the product of area and square of distance between CG from the axis of reference perpendicular to the area.

2.6 THEOREMS OF MOMENT OF INERTIA

- Proof:- Consider an infinitesimal elemental area dA with co-ordinates (x,y).

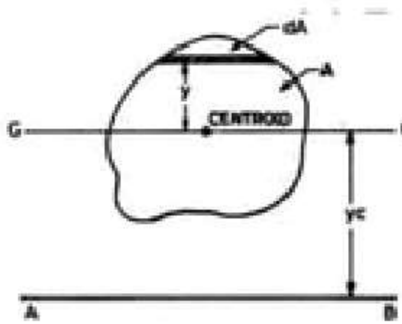


- $Ixx=\int y^2 dA$; $Iyy=\int x^2 dA$; $Izz=\int r^2 dA$
- $r^2= x^2 + y^2$

  Hence $r^2 \times dA= x^2\, dA + y^2 \times dA$
- Hence $Izz= Ixx + Iyy$

2.6 THEOREMS OF MOMENT OF INERTIA

- **Parallel axis theorem**:- Moment of inertia about any axis in the plane of an area is equal to sum of moment of inertia about a parallel centroidal axis and the product of area and square of the distance between the two parallel axes
- Referring to the figure given below, the theorem



IGG + Ayc^2

2.6 THEOREMS OF MOMENT OF INERTIA

- IAB→ Moment of inertia about axis AB
- IGG→ Moment of inertia about centroidal axis GG parallel to AB
- A→ The area of plane figure given.
- Yc→ The distance between the axis AB and the parallel centroidal axis GG
- Proof:- Consider an elemental strip dA whose CG is at a distance y from centroidal axis G-G

2.6 THEOREMS OF MOMENT OF INERTIA

- $I_{GG} = \int y^2 \, dA$

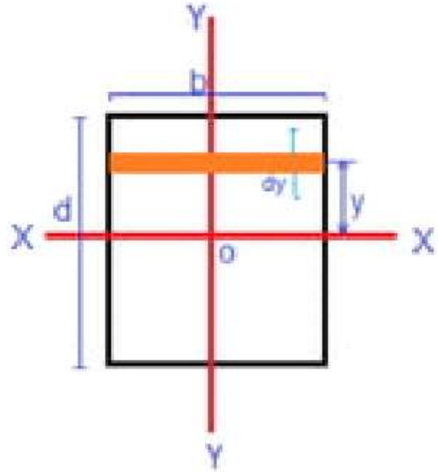
  $I_{AB} = \int (y + y_c)^2 \, dA$

  $I_{AB} = \int (y^2 + y_c^2 + 2yy_c) \, dA$

- y is a variable and yc is a constant; hence $\int y^2 \, dA = I_{GG}$ and $\int y_c^2 \, dA = Ay_c^2$.
- $\int y \, dA / A$ =distance of centroid from the axis of reference= 0 as GG is passing through the centroid.
- Hence $I_{AB} = I_{GG} + Ay_c^2$

2.7 DETERMINATION OF MOMENT OF INERTIA

- **1) Moment of inertia of a rectangular section about the centroidal axis in the plane of section**
- Consider a rectangular section of length b and depth d. Let … ntal axis passing through the … a.

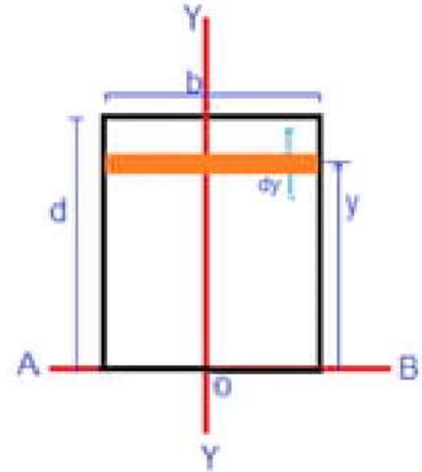


2.7 DETERMINATION OF MOMENT OF INERTIA

- Consider a rectangular elemental strip whose CG is at a distance y from the horizontal centroidal axis.
- Area of the strip dA= b x dy
- $I_{xx} = \int y^2 dA = \int_{-d/2}^{d/2} y^2 \times b \times dy$ nt of inertia of strip about the centroidal axis,
- $I_{xx} = bd^3/12$
- Similarly $I_{yy} = db^3/12$

2.7 DETERMINATION OF MOMENT OF INERTIA

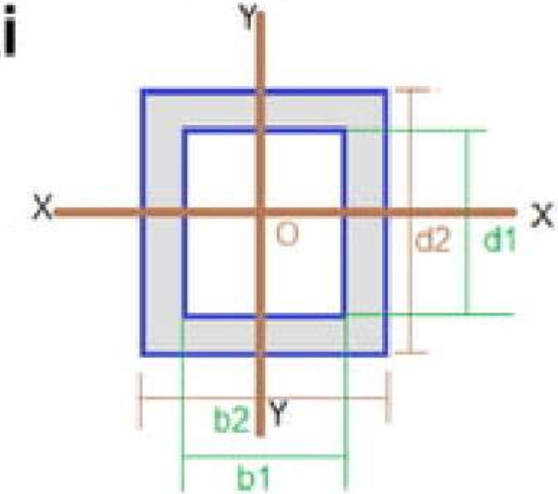
- **2) Moment of inertia of a rectangular section about an axis passing through the base of the rectangle**
- Consider a rectangular section of length b and depth d. Let AB be the horizontal ax through the base of the rectangle.



- IAB= IGG + A x Yc²= bd³/12 + bd x (
- IAB= bd³/3

2.7 DETERMINATION OF MOMENT OF INERTIA

- **3) Moment of inertia of a hollow rectangular section about the centroidal axi of section**



- Moment of inertia of main section about X-X axis= $b2d2^3/12$
- Moment of inertia of the cut out section about X-X axis= $b1d1^3/12$

2.7 DETERMINATION OF MOMENT OF INERTIA

- Moment of inertia of the hollow rectangular section about X-X axis= $(b2d2^3/12 - b1d1^3/12)$
- Similarly Moment of inertia of the hollow rectangular section about Y-Y axis= $(d2b2^3/12 - d1b1^3/12)$
- Moment of inertia of the hollow rectangular section about any axis= (MI of outer rectangular section about the axis - MI of cut-out rectangular section about the axis)

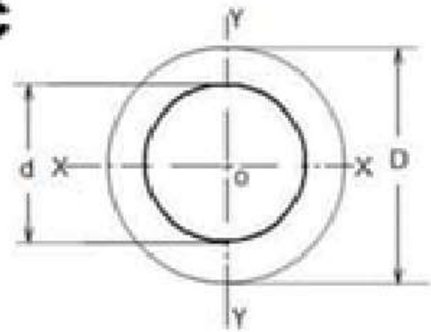
2.7 DETERMINATION OF MOMENT OF INERTIA

- **4) Moment of inertia of a circular section passing through the centre and lying in the plane of the figure**
- Consider an elementary circular ring of radius r and thickness dr. Area of the circular ring =2πrdr
- The moment of inertia about an axis passing through the centre O of the circle and p $Izz=\int r^2 dA=\int_0^R r^2 \times 2\pi r \times dr$ lar to the plane of area,

- Izz= πR^4/2= πD^4/32
- According to perpendicular axis theorem, Ixx=Iyy= ½ Izz= πD^4/64

2.7 DETERMINATION OF MOMENT OF INERTIA

- **5) Moment of inertia of a hollow circular sec**

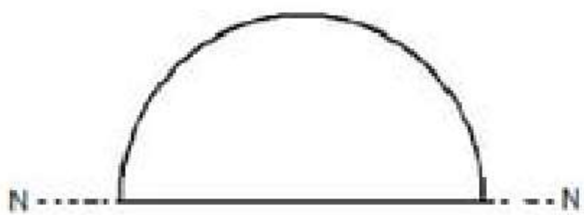


- Moment of inertia of outer circle about X-X axis= $\pi D^4/64$

  Moment of inertia of the cut-out circle= $\pi d^4/64$
- Moment of inertia of the hollow circular section about X-X axis $I_{xx} = \pi \times (D^4 - d^4)/64$
- Similarly $I_{yy} = \pi \times (D^4 - d^4)/64$

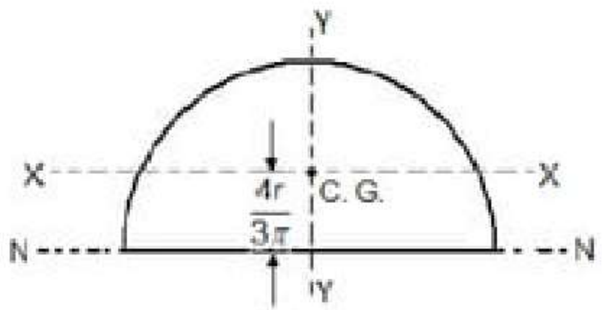
2.7 DETERMINATION OF MOMENT OF INERTIA

- **6) Moment of inertia of a semicircular area**
- I_{NN}= Moment of inertia of the semicircular lamina about an axis passing through the centre of the semicircle=1/2 x Moment of inertia of a circular lamina about an axis passing through the centre and lying in the plane of the figure= $\pi D^4/128$



2.7 DETERMINATION OF MOMENT OF INERTIA

- Moment of inertia of the semicircle about an axis passing through the CG of the semicircle= Inn + A

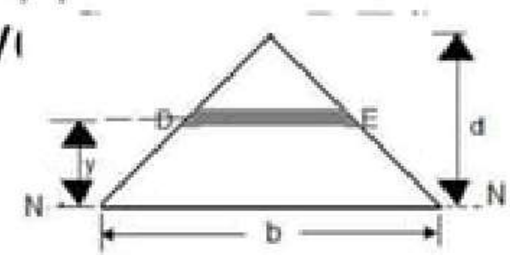


- Ixx= $\pi D^4/128 + \pi D^2/8 \times (2D/3\pi)^2 = 0.11R^4$

2.7 DETERMINATION OF MOMENT OF INERTIA

- **7) Moment of inertia of a triangular section about its base**
- Consider an elemental strip DE of the triangle at a distance y from the vertex opposite to the base of the triangle. From the abov (d-y)/d

  DE=b (1- y/d)



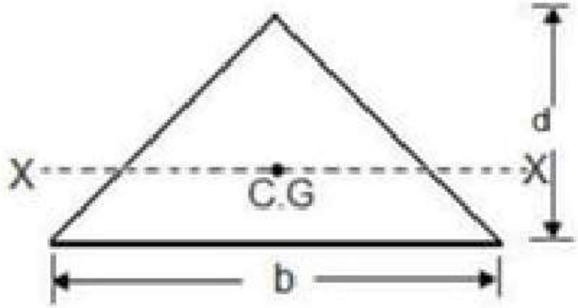
- Area of the elemental strip dA= DE x dy = b (1-

$$I_{nn}=\int_0^d y^2 \times b\,(1-y/d) \times dy$$

- Moment of inertia of the area about N-N,
- $I_{NN} = bd^3/12$

2.7 DETERMINATION OF MOMENT OF INERTIA

- **8) Moment of inertia of a triangle about the centroidal axis parallel to the base of the triangle**
- $I_{xx} = bd^3/12 = I_{gg} + Ay_c^2$



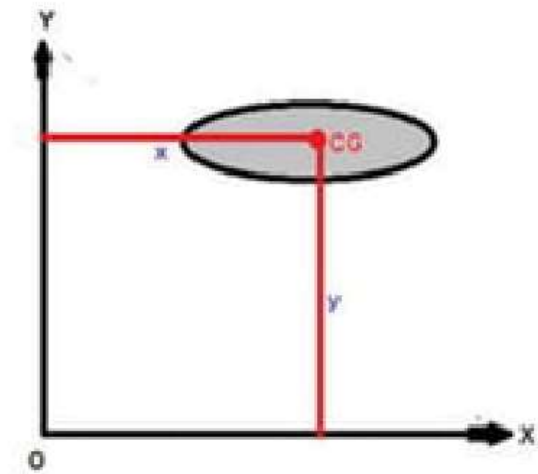
- $bd^3/12 - bd/2 \times d^2/9 = I_{gg}$
- $I_{gg} = bd^3/36$

2.8 MASS MOMENT OF INERTIA

- Consider a body of mass M lying in the XY plane. Let

X= distance of CG of the body from OY axis

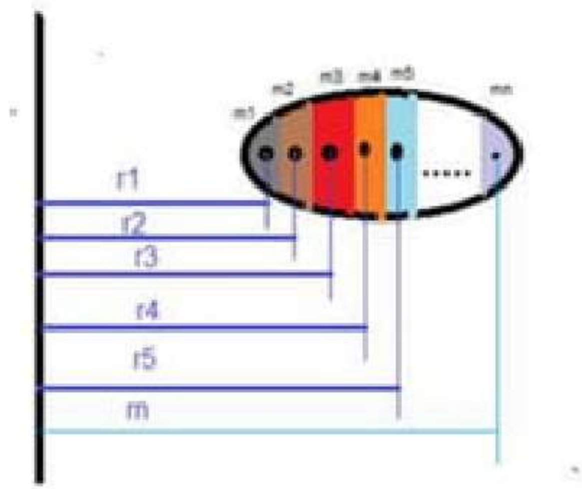
Y f the body from OX axis



- Moment of mass M about OY axis= Mx

2.8 MASS MOMENT OF INERTIA

- Second moment of mass M about OY axis= Mx^2
- Second Moment of mass is known as mass moment of inertia
- Mass moment of inertia(IM) about an axis is hence defined as the product of mass of a body
  s perpendicular distance from
  e.

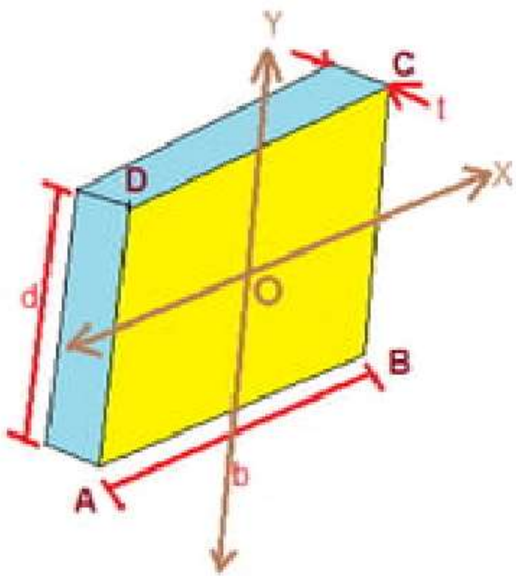


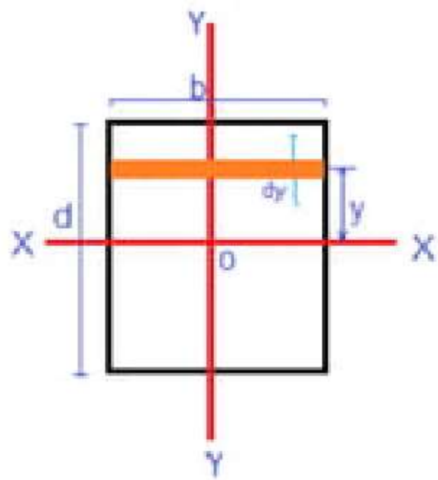
2.8 MASS MOMENT OF INERTIA

- Suppose the body is split up into small masses m1, m2, m3,....., mn. Let the distance of CGs of masses be at distances r1, r2, r3,.....rn from an axis of reference. Then mass moment of inertia about that axis is given by
- $IM = \Sigma mi \times ri^2$
- If the small masses are large in number, then the summation in above equation can be replaced by integral,
- $Im = \int r^2 dm$
- Physical meaning of mass moment of inertia:- It is the resistance of a rotating body against the change in angular velocity

2.9 MASS MOMENT OF INERTIA OF A RECTANGULAR PLATE ABOUT CENTROIDAL AXIS PARALLEL TO THE BASE OF THE PLATE

- Consider a rectangular plate of width b, depth d and thickness t composed of a material of density ρ.





RECTANGULAR PLATE ABOUT CENTROIDAL AXIS PARALLEL TO THE BASE OF THE PLATE

- Mass of the plate= $\rho \times b \times d \times t$
- Consider an elementary rectangular strip of width b, depth dy and thickness t at a distance y from the centroidal axis X-X . The area of the strip is, $dA = b \times dy$
- Mass of the strip, dm= Volume of the strip x density= Thickness x Area of the strip x density=$t \times b \times dy \times \rho$
- Mass moment of inertia of the strip = $y^2\,dm = y^2 \times (t \times b \times dy \times \rho)$

RECTANGULAR PLATE ABOUT CENTROIDAL AXIS PARALLEL TO THE BASE OF THE PLATE

- Mass moment of inertia of the entire mass about XX axis= $\int v^2\, dm = \int v^2 x\ (t \times b \times dy \times \rho)$

$$= \int_{-d/2}^{d/2} y^2 x\ (t \times b \times dy \times \rho)$$

$=bt\ \rho \times d^3/12= \rho \times t \times b\ d^3/12=$ Density x thickness x Moment of inertia of the rectangular section about the centroidal axis parallel to the base

- $bt\ \rho \times d=$ Mass of the rectangular plate= M

RECTANGULAR PLATE ABOUT CENTROIDAL AXIS PARALLEL TO THE BASE OF THE PLATE

- Hence moment of inertia of a rectangular plate about the centroidal axis parallel to its base $I_{mxx} = Md^2/12$
- Moment of inertia of a rectangular plate about the centroidal axis perpendicular to its base, $I_{myy} = Mb^2/12$
- Moment of inertia of a hollow rectangular plate of outer section dimensions B, D and inner section dimensions b, d is given by the equation, $I_{mxx} = 1/12\,(MD^2 - md^2)$; where M is the mass of outer section and m is the mass of cut-out section.

2.9 MASS MOMENT OF INERTIA OF A RECTANGULAR PLATE ABOUT AN AXIS PASSING THROUGH ITS BASE

- Consider a rectangular plate of width b, depth d and thickness t composed of a material of density ρ.
- Mass of the plate= $\rho \times b \times d \times t$
- Consider an elementary rectangular strip of width b, depth dy and thickness t at a distance y from the base AB . The area of the strip is, $dA = b \times dy$
- Mass of the strip, dm= Volume of the strip x density= Thickness x Area of the strip x density= $t \times b \times dy \times \rho$

2.9 MASS MOMENT OF INERTIA OF A RECTANGULAR PLATE ABOUT AN AXIS PASSING THROUGH ITS BASE

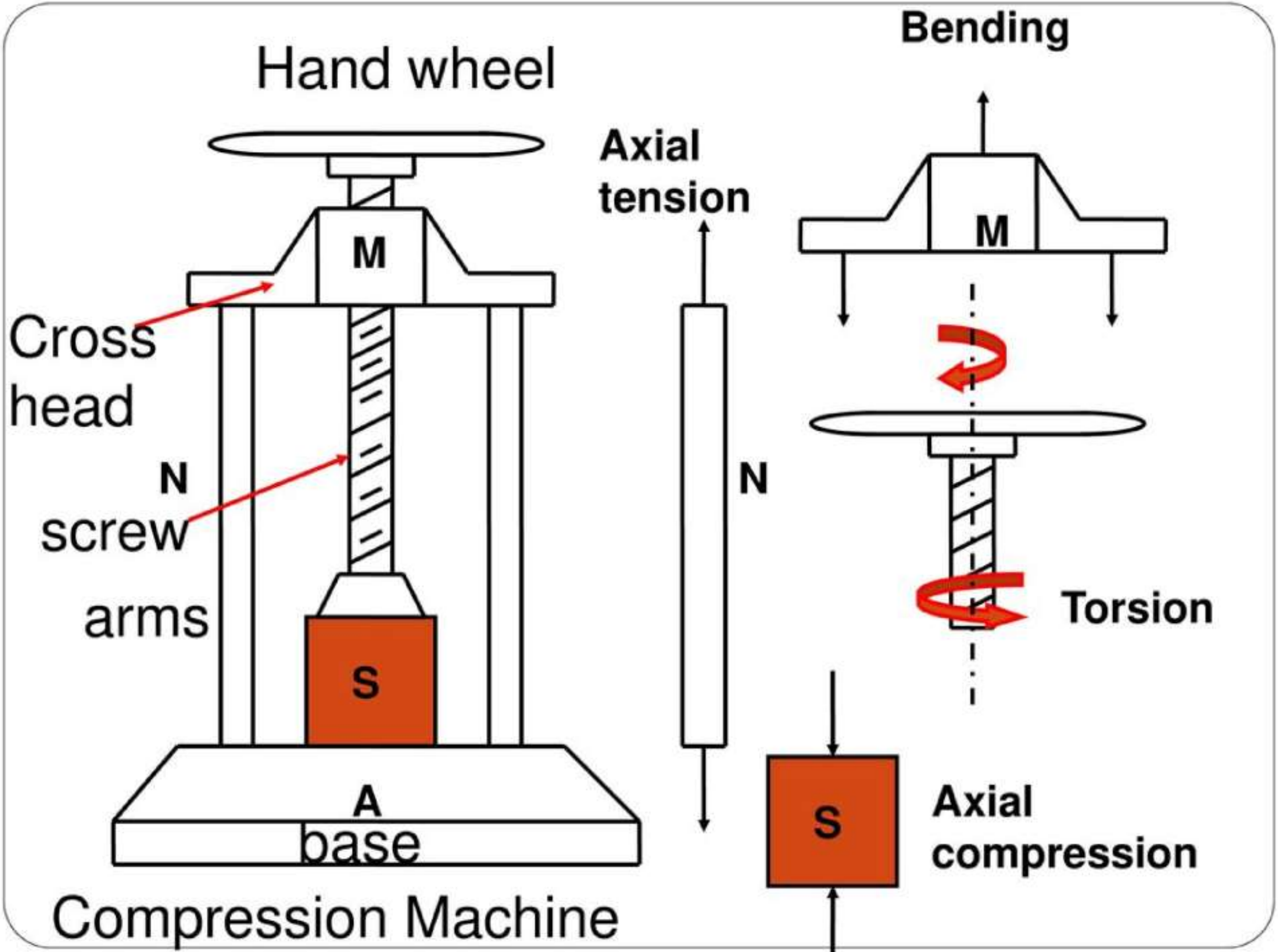
- Mass moment of inertia of the strip = y^2 dm= y^2x (t x b x dy x ρ)
- Mass moment of inertia of the entire mass about XX axis= $\int y^2\, dm = \int y^2 x\, (t \times b \times dy \times \rho)$

$$= \int_0^d y^2 x\, (t \times b \times dy \times \rho)$$

=bt ρ x $d^3/3$= ρ x t x b $d^3/3$= Density x thickness x Moment of inertia of the rectangular section about the centroidal axis parallel to the base

2.9 MASS MOMENT OF INERTIA OF A RECTANGULAR PLATE ABOUT AN AXIS PASSING THROUGH ITS BASE

- bt ρ x d= Mass of the rectangular plate= M
- Hence moment of inertia of a rectangular plate about a horizontal axis passing through to its base $I_{mxx} = Md^2/3$
- Moment of inertia of a rectangular plate about the centroidal axis perpendicular to its base and passing through the vertical side, $I_{myy} = Mb^2/3$


Hand wheel
Bending
Axial
tension
M
M
Cross
head
N
N
screw
arms
Torsion
S
S
A
base
Axial
compression
Compression Machine

LOAD

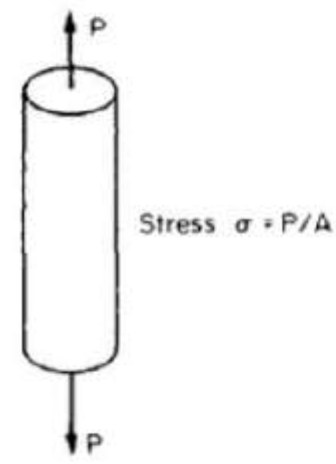
- Load is defined as the set of external forces acting on a mechanism or engineering structure which arise from service conditions in which the components work
- Common loads in engineering applications are tension and compression
- Tension:- Direct pull. Eg:Force present in lifting hoist
- Compression:- Direct push. Eg:- Force acting on the pillar of a building
- Sign convention followed: Tensile forces are positive and compressive negative

TYPES OF LOAD

- There are a number of different ways in which load can be applied to a member. Typical loading types are:
- A) **Dead/ Static load**- Non fluctuating forces generally caused by gravity
- B) **Live load**- Load due to dynamic effect. Load exerted by a lorry on a bridge
- C) **Impact load or shock load**- Due to sudden blows
- D) **Fatigue or fluctuating or alternating loads**: Magnitude and sign of the forces changing with time

STRESS

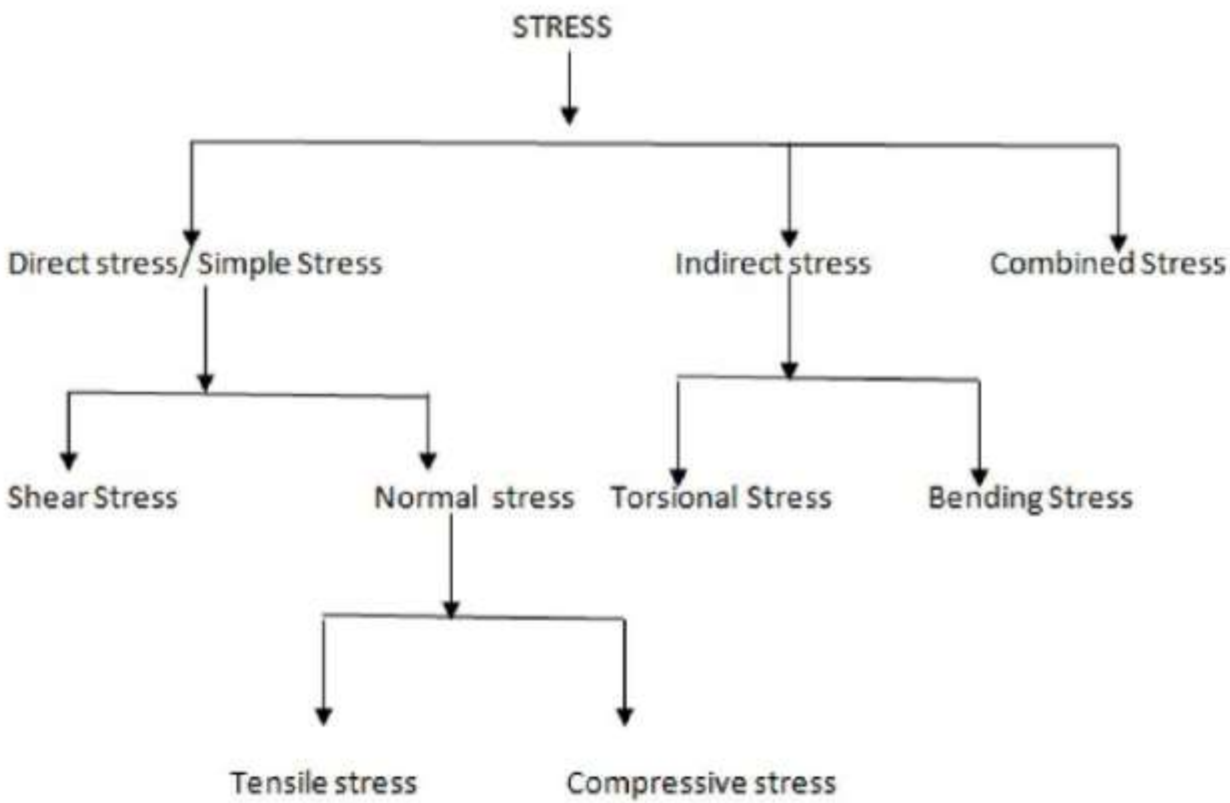
- When a material is subjected to an external force, a resisting force is set up within the component, this internal resistance force per unit area is called stress. SI unit is N/m^2(Pa). 1kPa=1000Pa, 1MPa=10^6 Pa, 1 Gpa=10^9Pa, 1 Terra Pascal=10^12 Pa
- In engineering applications, we use the the original cross section area of the specimen and it is known as conventional stress or Engineering stress



STRAIN

- When a body is subjected to some external force, there is some change of dimension of the body. The ratio of change of dimension of the body to its original dimension is known as strain
- Strain is a dimensionless quantity
- Strain may be:- a) Tensile strain b) Compressive strain c) Volumetric strain d) Shear strain
- **Tensile strain-** Ratio of increase in length to original length of the body when it is subjected to a pull force
- **Compressive strain**- Ratio of decrease in length to original length of the body when it is subjected to a push force
- **Volumetric strain-** Ratio of change of volume of the body to the original volume
- **Shear strain-**Strain due to shear stress

TYPE OF STRESSES



TYPES OF DIRECT STRESS

- Direct stress may be normal stress or shear stress
- **Normal stress (σ)** is the stress which acts in direction perpendicular to the area. Normal stress is further classified into tensile stress
- **Tensile stress** is the stress induced in a body, when it is subjected to two equal and opposite pulls (tensile forces) as a result of which there is a tendency in increase in length
- It acts normal to the area and pulls on the area

TYPES OF DIRECT STRESS (Tensile stress)

- Consider a bar subjected to a tensile force P at its ends. Let

  A= Cross sectional area of the body

  L=Original length of the body

  dL= Increase in length of the body due to its pull P

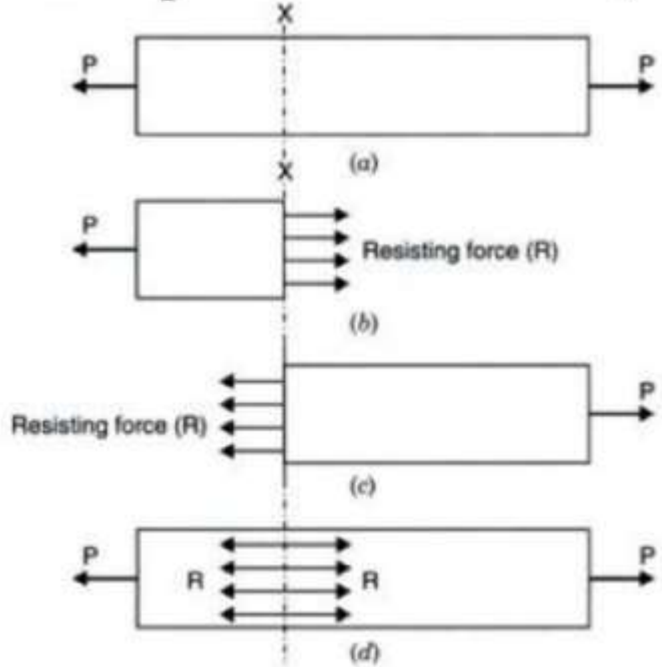
  σ= Stress induced in the body

  e=Tensile strain

Consider a section X-X which divides the body into two halves

TYPES OF DIRECT STRESS (Tensile stress)

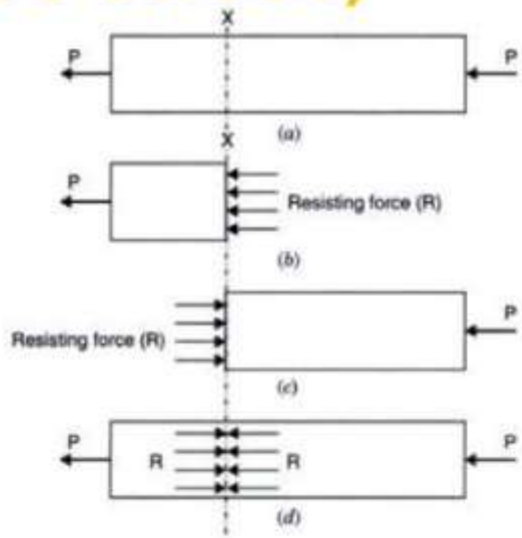
- The left part of the section x-x, will be in equilibrium if P=R (Resisting force). Similarly the right part of the section x-x will be in equilibrium if P=R (Resisting force)



TYPES OF DIRECT STRESS (Tensile stress)

- Tensile stress (σ)= Resisting force/ Cross sectional area= Applied force/Cross sectional area=P/A
- Tensile strain= Increase in length/Original length= dL/L
- Compressive stress:- Stress induced in a body, when subjected to two equal and opposite pushes as a result of which there is a tendency of decrease in length of the body
- It acts normal to the area and it pushes on the area
- In some cases the loading situation is such that the stress will vary across any given section. In such cases the stress at any given point is given by
- $\sigma = \text{Lt}_{\Delta A \to 0} \Delta P / \Delta A = dP/dA$ = derivative of force w.r.t area

TYPES OF DIRECT STRESS (Compressive stress)



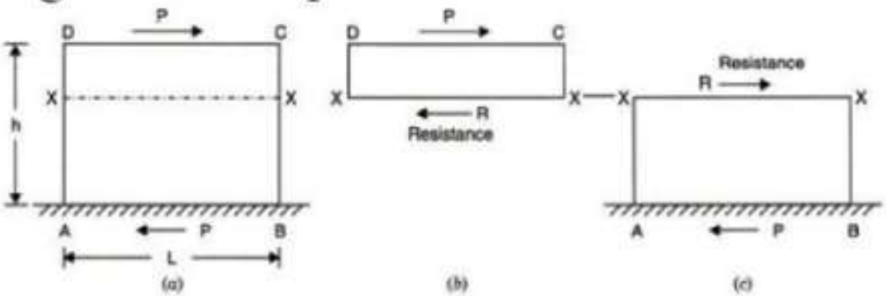
- **Compressive stress**=Resisting force/ cross sectional area= Applied force/ cross sectional area
- Compressive strain= Decrease in length/ Original length= -dL/L
- Sign convention for direct stress and strain:- Tensile stresses and strains are considered positive in sense producing an increase in length. Compressive stresses and strains are considered negative in sense producing decrease in length

TYPES OF DIRECT STRESS (Shear stress)

- **Shear stress** :- Stress Induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting section as a result of which the body tends to shear off across that section
- Consider a rectangular block of height h, length L and width unity. Let the bottom face AB of the block be fixed to the surface as shown. Let P be the tangential force applied along top face CD of the block. For the equilibrium of the block, the surface AB will offer a tangential reaction force R which is equal in magnitude and opposite in direction to the applied tangential force P

TYPES OF DIRECT STRESS (Shear stress)

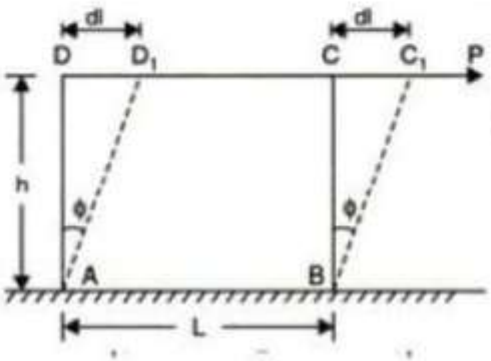
- Consider a section X-X cut parallel to the applied force which splits rectangle into two parts



- For the upper part to be in equilibrium; Applied force P=Resisting force R
- For the lower part to be in equilibrium; Applied force P=Resisting force R
- Hence, shear stress τ= Resisting force/Resisting area=P/L x 1=P/L
- Shear stress is tangential to the area on which it acts

TYPES OF DIRECT STRESS (Shear stress)

- As the face AB is fixed, the rectangular section ABCD will be distorted to ABC1D1, such that new vertical face AD1 makes an angle φ with the initial face AD



(a)

- Angle φ is called shear strain. As φ is very small,
- $\varphi = \tan\varphi = DD1/AD = dl/h$
- Hence shear strain=dl/h

ELASTICITY & ELASTIC LIMIT

- The property of a body by virtue of which it undergoes deformation when subjected to an external force and regains its original configuration (size and shape) upon the removal of the deforming external force is called **elasticity.**
- The stress corresponding to the limiting value of external force upto and within which the deformation disappears completely upon the removal of external force is called **elastic limit**
- **A material is said to be elastic** if it returns to its original, unloaded dimensions when load is removed.
- If the external force is so large that the stress exceeds the elastic limit, the material loses to some extent its property of elasticity. If now the force is removed, the material will not return to its original shape and size and there will be a residual deformation in the material

HOOKE'S LAW & ELASTIC MODULI

- Hooke's law states that: "When a body is loaded within elastic limit, the stress is proportional to strain developed" or "Within the elastic limit the ratio of stress applied to strain developed is a constant"
- The constant is known as **Modulus of elasticity or Elastic modulus or Young's modulus**
- Mathematically within elastic limit

  Stress/Strain=σ/e=E

  σ= P/A; e =ΔL/L

  E=PL/A Δ L

HOOKE'S LAW & ELASTIC MODULI

- Young's modulus (E) is generally assumed to be the same in tension or compression and for most of engineering applications has a high numerical value. Typically, E=210 x 10^9 N/m^2 (=210 GPa) for steel
- **Modulus of rigidity**, G= τ/φ= Shear stress/ shear strain
- **Factor of safety**= Ultimate stress/Permissible stress
- In most engineering applications strains donot often exceed 0.003 so that the assumption that deformations are small in relation to orinal dimensions is generally valid

STRESS-STRAIN CURVE (TENSILE TEST)

- Standard tensile test involves subjecting a circular bar of uniform cross section to a gradually increasing tensile load until the failure occurs
- Tensile test is carried out to compare the strengths of various materials
- Change in length of a selected gauge length of bar is recorded by extensometers
- A graph is plotted with load vs extension or stress vs strain

1.8 STRESS-STRAIN CURVE (TENSILE TEST)

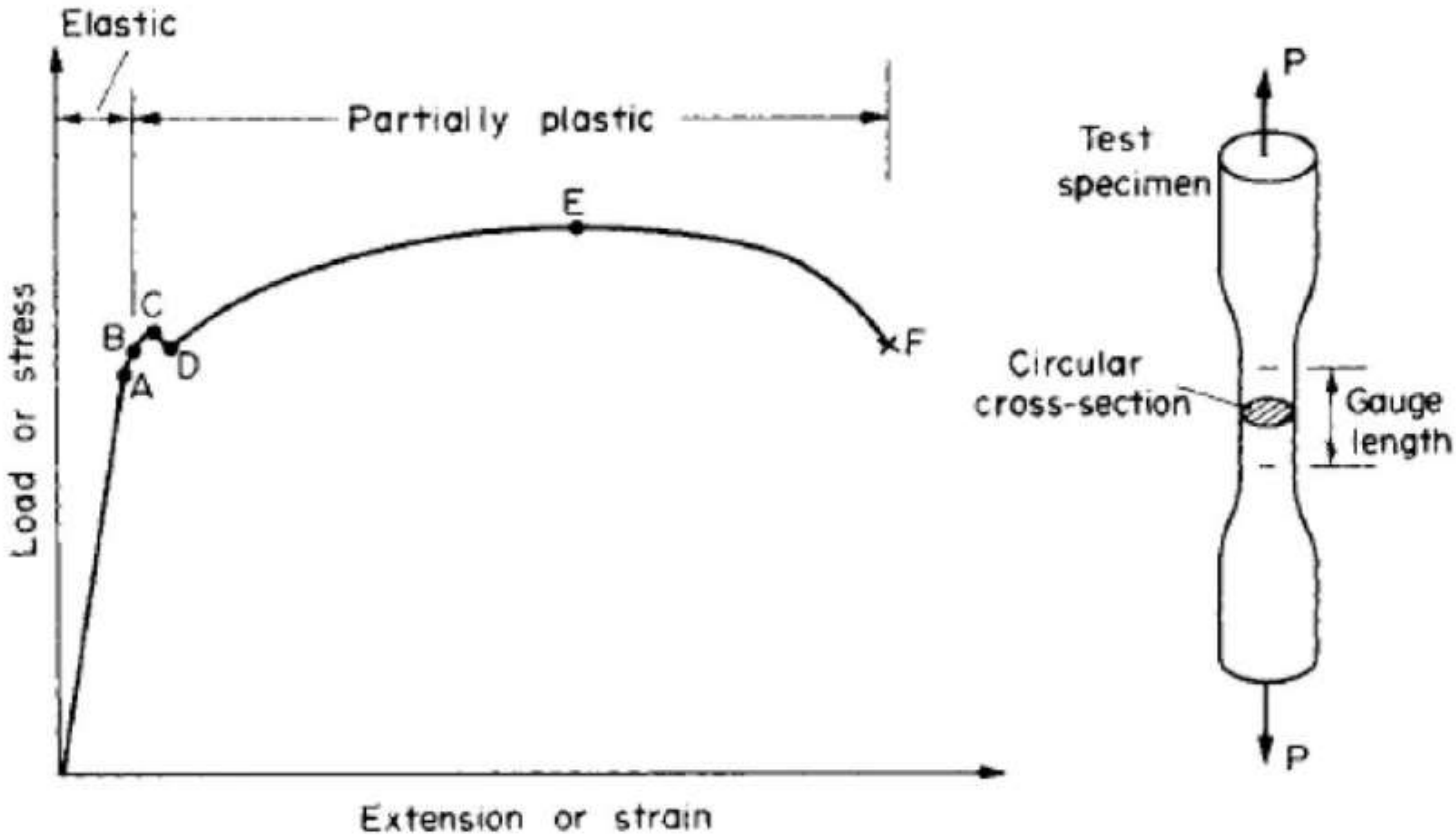


Fig. 1.3. Typical tensile test curve for mild steel.

STRESS-STRAIN CURVE (TENSILE TEST DIAGRAM)

- A→ Limit of proportionality; It is the point where the linear nature of the stress strain graph ceases
- B→ Elastic limit; It is the limiting point for the condition that material behaves elastically, but hooke's law does not apply . For most practical purposes it can be often assumed that limit of proportionality and elastic limits are the same
- Beyond the elastic limits, there will be some permanent deformation or permanent set when the load is removed
- C (Upper Yield point), D (Lower yield point)→ Points after which strain increases without correspondingly high increase in load or stress
- E→ Ultimate or maximum tensile stress; Point where the necking starts
- F→ Fracture point

CONSTITUTIVE RELATIONSHIPS BETWEEN STRESS & STRAIN

- **A) 1-Dimensional case** (due to pull or push or shear force)

  $\sigma = Ee$

- **B) 2-Dimensional case**
- Consider a body of length L, width B and height H. Let the body be subjected to an axial load. Due to this axial load, there is a deformation along the length of the body. This strain corresponding to this deformation is called longitudinal strain.
- Similarly there are deformations along directions perpendicular to line of application of fore. The strains corresponding to these deformations are called lateral strains

CONSTITUTIVE RELATIONSHIPS BETWEEN STRESS & STRAIN

δL = Increase in length,
δb = Decrease in breadth, and
δd = Decrease in depth.

Then longitudinal strain $= \dfrac{\delta L}{L}$

and lateral strain $= \dfrac{\delta b}{b}$ or $\dfrac{\delta d}{d}$

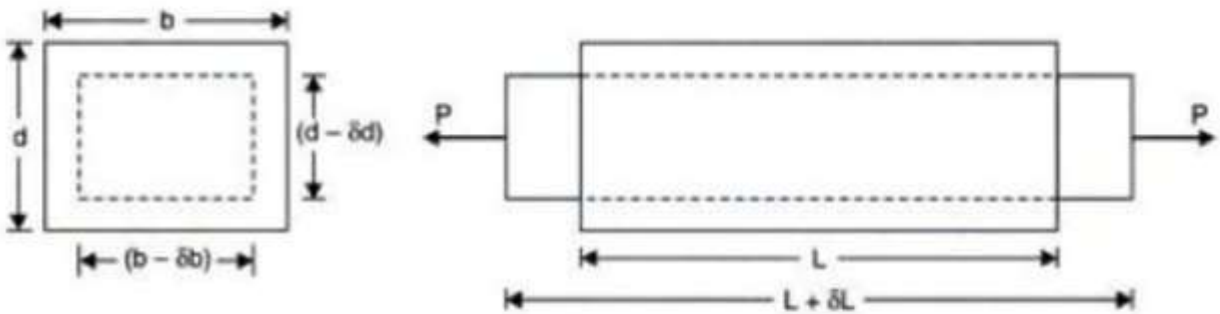


Fig. 1.3. Typical tensile test curve for mild steel.

CONSTITUTIVE RELATIONSHIPS BETWEEN STRESS & STRAIN

- Longitudinal strain is always of opposite sign of that of lateral strain. Ie if the longitudinal strain is tensile, lateral strains are compressive and vice versa
- Every longitudinal strain is accompanied by lateral strains in orthogonal directions
- Ratio of lateral strain to longitudinal strain is called **Poisson's ratio (μ)**; Mathematically,
- μ=-Lateral strain/Longitudinal strain
- Consider a rectangular figure ABCD subjected a stress in σx direction and in σ y direction

YOUNG'S MODULUS (E):--

Young's Modulus (E) is defined as the Ratio of Stress (σ) to strain (ε).

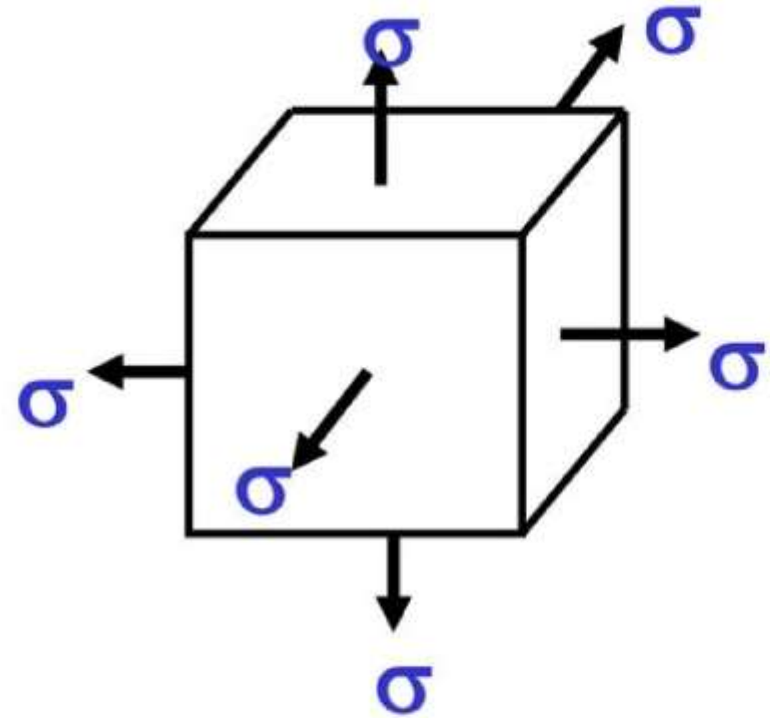
$$\boxed{E = \sigma / \varepsilon} \qquad \text{------------}(5)$$

BULK MODULUS (K):--

- When a body is subjected to the identical stress σ in three mutually perpendicular directions, the body undergoes uniform changes in three directions without the distortion of the shape.
- The ratio of change in volume to original volume has been defined as volumetric strain(ε_v)

•Then the bulk modulus, K is defined as $K = \sigma / \varepsilon_v$

BULK MODULUS (K):--



$$\boxed{K = \sigma / \varepsilon_v} \quad \text{------------(6)}$$

Where, $\varepsilon_v = \Delta V/V$

$$= \frac{\text{Change in volume}}{\text{Original volume}}$$

= Volumetric Strain

MODULUS OF RIGIDITY (N): OR

MODULUS OF TRANSVERSE ELASTICITY OR

SHEARING MODULUS

Up to the elastic limit,

shear stress (τ) $\propto$ shearing strain(ϕ)

$$\tau = N\,\phi$$

Expresses relation between shear stress and shear strain.

where

Modulus of Rigidity = N = τ / ϕ ------------(7)

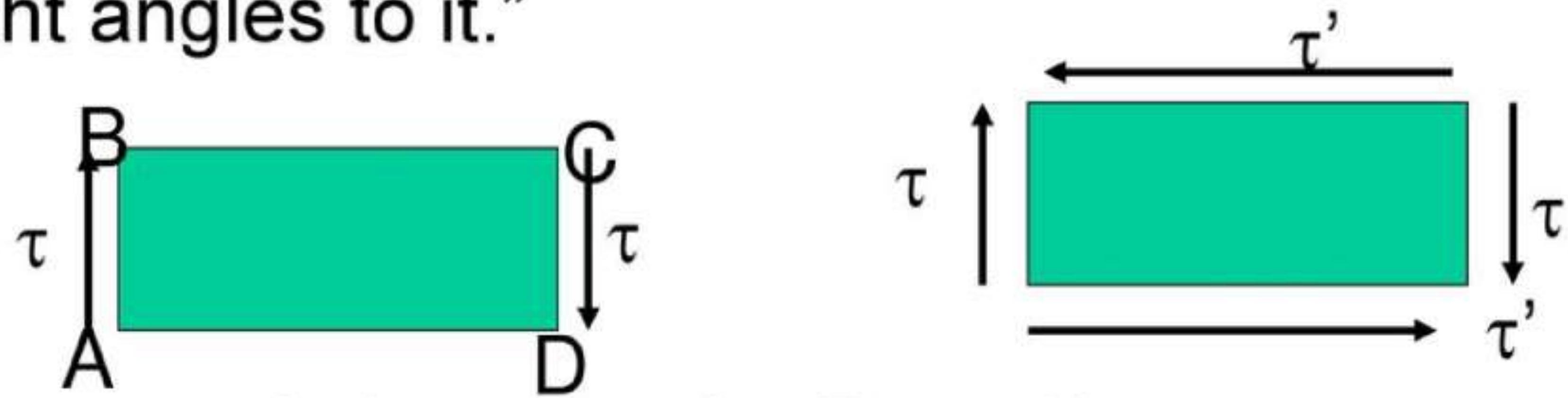
ELASTIC CONSTANTS

YOUNG'S MODULUS $E = \sigma / \varepsilon$

BULK MODULUS $K = \sigma / \varepsilon_v$

MODULUS OF RIGIDITY $N = \tau / \phi$

COMPLEMENTRY STRESSES:"A stress in a given direction cannot exist without a balancing shear stress of equal intensity in a direction at right angles to it."



Moment of given couple=Force *Lever arm

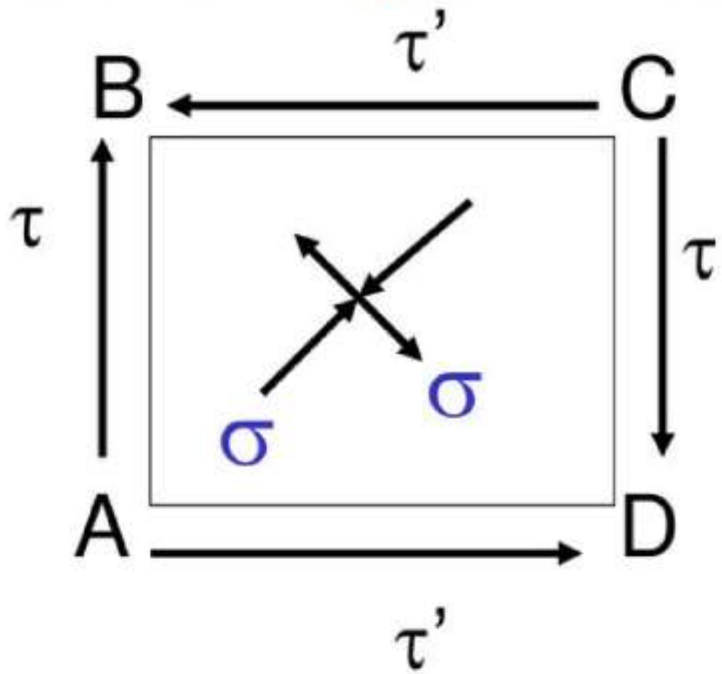
$= (\tau.AB)*AD$

Moment of balancing couple= $(\tau'.AD)*AB$

so $(\tau.AB)*AD=(\tau'.AD)*AB \Rightarrow \tau= \tau'$

Where τ=shear stress & τ'=Complementary shear stress

State of simple shear: Here no other stress is acting - only simple shear.



Let side of square = b

length of diagonal AC =$\sqrt{2}$ .b

consider unit thickness perpendicular to block.

Equilibrium of piece ABC

the resolved sum of τ perpendicular to the diagonal = $2^*(\tau^*b^*1)\cos 45^0 = \sqrt{2}\ \tau.b$

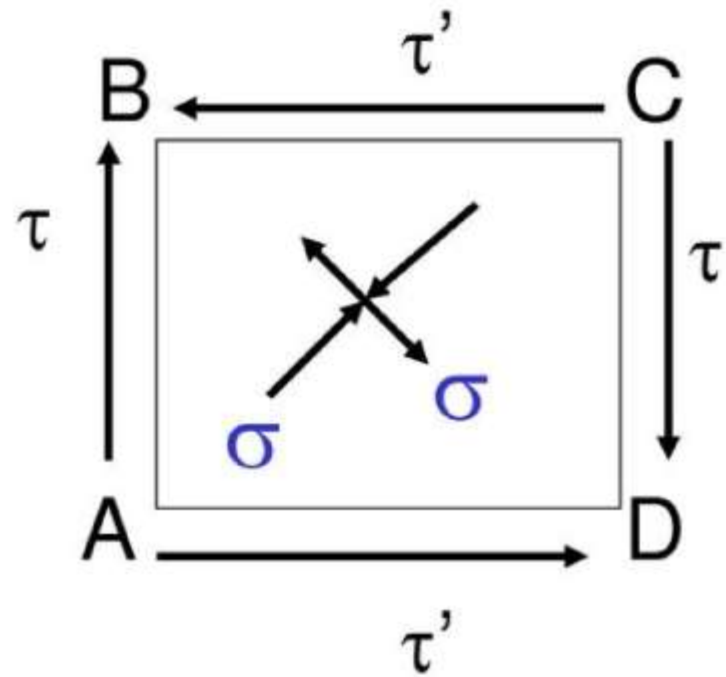
if σ is the tensile stress so produced on the diagonal

$\sigma(AC^*1)=\sqrt{2}\ \tau.b$

$\sigma(\sqrt{2}\ .b)=\sqrt{2}\ \tau.b$

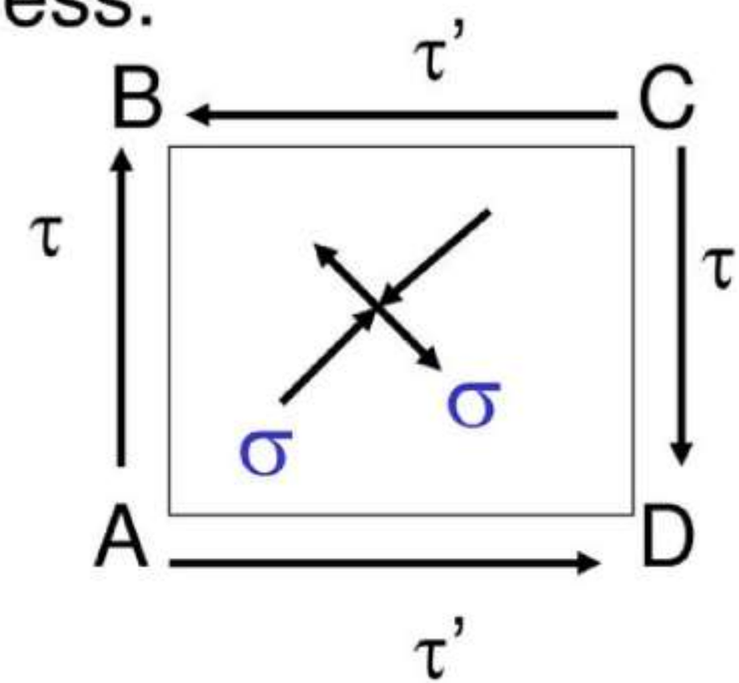
so

$\sigma = \tau$



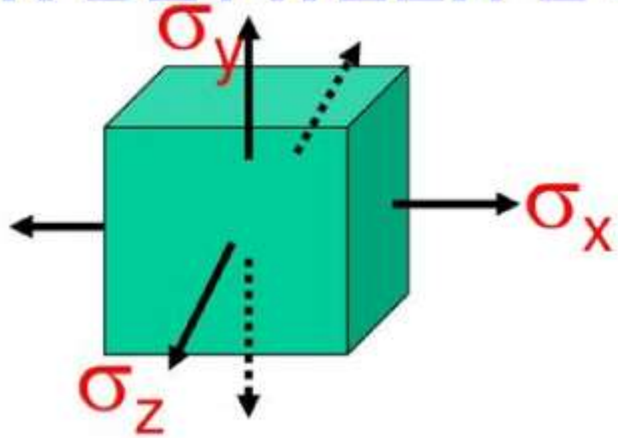
Similarly the intensity of compressive stress on plane BD is numerically equal to τ.

"Hence a state of simple shear produces pure tensile and compressive stresses across planes inclined at 45 ^{0} to those of pure shear, and intensities of these direct stresses are each equal to pure shear stress."



RELATION BETWEEN ELASTIC CONSTANTS

(A) RELATION BETWEEN E and K



Let a cube having a side L be subjected to three mutually perpendicular stresses of intensity σ

By definition of bulk modulus

$K = \sigma / \boldsymbol{\varepsilon_v}$

Now $\boldsymbol{\varepsilon_v} = \delta_v / V = \sigma / K$

The total linear strain for each side

$\varepsilon = \sigma/E - \sigma/(mE) - \sigma/(mE)$

so $\delta L / L = \varepsilon = (\sigma/E) * (1-2/m)$

now $V = L^3$

$\delta V = 3 L^2 \delta L$

$\delta V/V = 3 L^2 \delta L / L^3 = 3 \delta L/L$

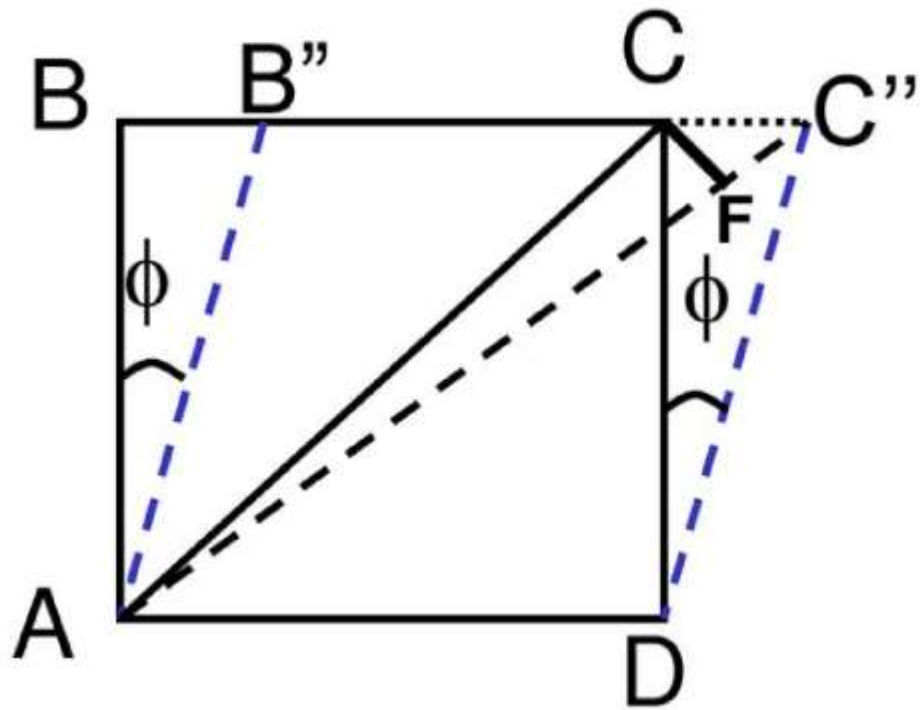
$= 3 (\sigma/E) * (1-2/m)$

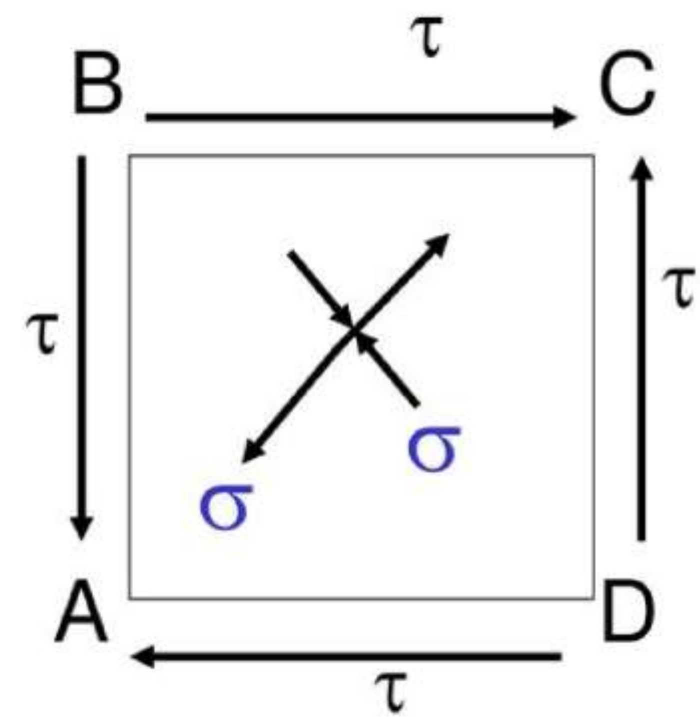
Equating (i) and (iii)

$\sigma/K = 3(\sigma /E)(1\text{-}2 /m)$

$$\boxed{E = 3\,K(1\text{-}2 /m)}$$

(B) Relation between E and N





Linear strain of diagonal AC,

$\varepsilon = \phi/2 = \tau/2N$

State of simple shear produces tensile and compressive stresses along diagonal planes and

$\sigma = \tau$

Strain ε of diagonal AC, due to these two mutually perpendicular direct stresses

$\varepsilon = \sigma/E - (- \sigma/mE) = (\sigma/E)*(1+1/m)$

But $\sigma = \tau$

so $\varepsilon = (\tau /E)*(1+1/m)$

From equation (i) and (iii)

$\tau /2N = (\tau /E)(1+1/m)$

OR

$E = 2N(1+1/m)$

But $E = 3K(1-2/m)$

Eliminating E from --(9) & --(10)

$\mu = 1/m = (3K - 2N) / (6K + 2N)$

Eliminating m from –(9) & --(10)

$\boxed{E = 9KN / (N+3K)}$

(C) Relation between E ,K and N:--

$E = 2N(1+1/m)$

$E = 3K\,(1-2\,/m)$

$E = 9KN\,/\,(N+3K)$

(D) Relation between μ,K and N:--

$\mu = 1/m = (3K-2N)/(6K+2N)$

CONSTITUTIVE RELATIONSHIPS BETWEEN STRESS & STRAIN

- Strain along x direction due to $\sigma x = \sigma x/E$

  Strain along x direction due to $\sigma y = -\mu \times \sigma y/E$

  Total strain in x direction $ex = \sigma x/E - \mu \times \sigma y/E$

  Similarly total strain in y direction, $ey = \sigma y/E - \mu \times \sigma x/E$
- In the above equation tensile stresses are considered as positive and compressive stresses as negative
- **C) 3 Dimensional case:-**

  Consider a 3 D body subjected to 3 orthogonal normal stresses in x,y and z directions respectively

CONSTITUTIVE RELATIONSHIPS BETWEEN STRESS & STRAIN

- Strain along x direction due to $\sigma x = \sigma x/E$

  Strain along x direction due to $\sigma y = -\mu_x \sigma y/E$

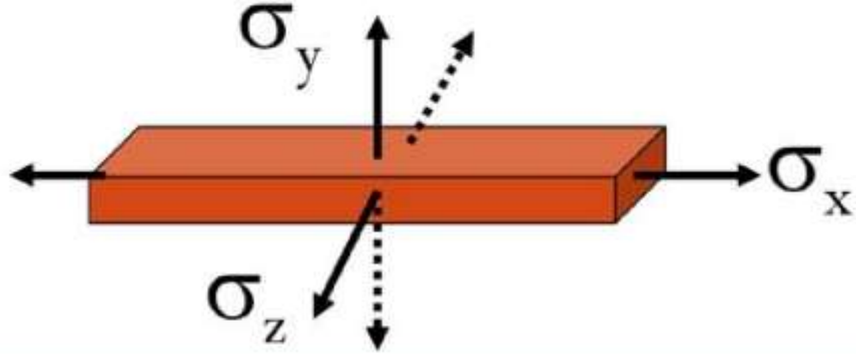
  Strain along x direction due to $\sigma z = -\mu_x \sigma z/E$

  Total strain in x direction $ex = \sigma x/E - \mu_x (\sigma y/E + \sigma z/E)$

  Similarly total strain in y direction, $ey = \sigma y/E - \mu_x (\sigma xE + \sigma z/E)$

  Similarly total strain in z direction, $ez = \sigma z/E - \mu_x (\sigma xE + \sigma y/E)$

Stress σ_x along the axis and σ_y and σ_z perpendicular to it.



$\varepsilon_x = \sigma_x/E - \sigma_y/mE - \sigma_z/mE$-----(i)

$\varepsilon_y = \sigma_y/E - \sigma_z/mE - \sigma_x/mE$-----(ii) -------(3)

$\varepsilon_z = \sigma_z/E - \sigma_x/mE - \sigma_y/mE$-----(iii)

Note:- If some of the stresses have opposite sign necessary changes in algebraic signs of the above expressions will have to be made.

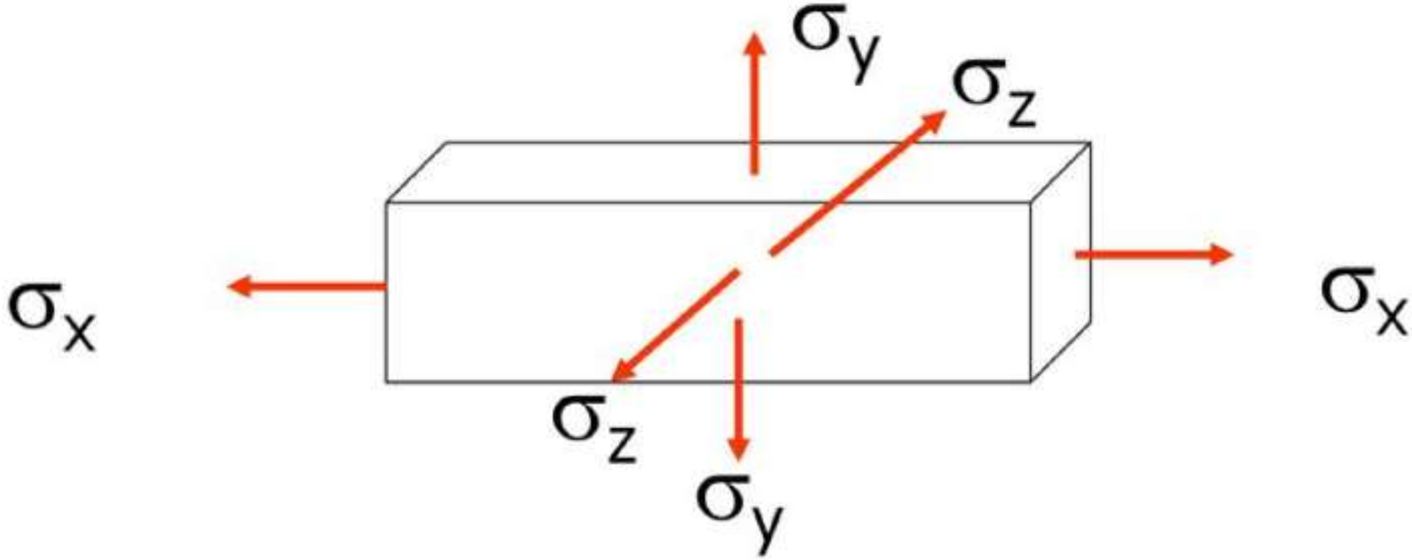
Upper limit of Poisson's Ratio:

adding (i),(ii) and (iii)

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = (1 - 2/m)(\sigma_x + \sigma_y + \sigma_z)/ E_{-} \quad \text{--------(4)}$$

known as DILATATION

For small strains represents the change in volume /unit volume.

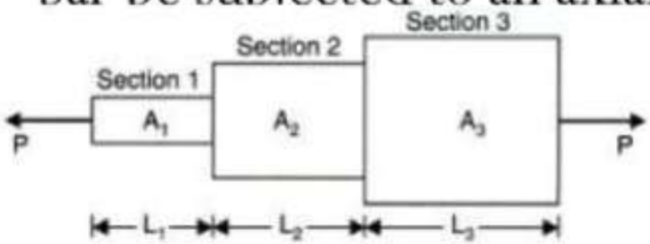


	ε_x	ε_y	ε_z
σ_x	σ_x/E	$-\mu\ \sigma_x/E$	$-\mu\ \sigma_x/E$
σ_y	$-\mu\ \sigma_y/E$	σ_y/E	$-\mu\ \sigma_y/E$
σ_z	$-\mu\ \sigma_z/E$	$-\mu\ \sigma_z/E$	σ_z/E

Sum all

ANALYSIS OF BARS OF VARYING CROSS SECTION

- Consider a bar of different lengths and of different diameters (and hence of different cross sectional areas) as shown below. Let this bar be subiected to an axial load P.



- The total change in length will be obtained by adding the changes in length of individual sections
- Total stress in section 1: $\sigma 1 = E1 \times \Delta L1 / L1$

  $\sigma 1 \times L1 / E1 = \Delta L1$

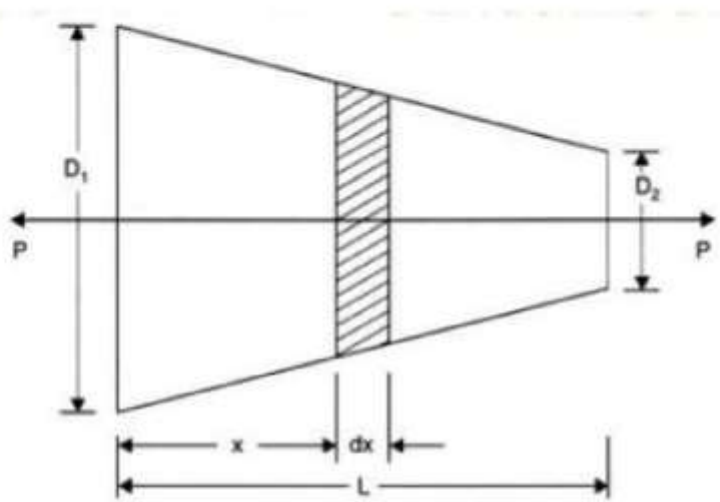
  $\sigma 1 = P / A1$; Hence $\Delta L1 = PL1 / A1E1$
- Similarly, $\Delta L2 = PL2 / A2E2$; $\Delta L3 = PL3 / A3E3$

ANALYSIS OF BARS OF VARYING CROSS SECTION

- Hence total elongation $\Delta L = Px\ (L1/A1E1 + L2/A2E2 + L3/A3E3)$
- If the Young's modulus of different sections are the same, $E1=E2=E3=E$; Hence $\Delta L = P/Ex\ (L1/A1 + L2/A2 + L3/A3)$
- When a number of loads are acting on a body, the resulting strain, according to principle of superposition, will be the algebraic sum of strains caused by individual loads
- While using this principle for an elastic body which is subjected to a number of direct forces (tensile or compressive) at different sections along the length of the body, first the free body diagram of individual section is drawn. Then the deformation of each section is calculated and the total deformation is equal to the algebraic sum of deformations of individual sections

ANALYSIS OF UNIFORMLY TAPERING CIRCULAR ROD

- Consider a bar uniformly tapering from a diameter D1 at one end to a diameter D2 at the other end
- Let
- P→ Axial load acting on the bar
- L→ Length of bar
- E→Young's modulus of the material



ANALYSIS OF UNIFORMLY TAPERING CIRCULAR ROD

- Consider an infinitesimal element of thickness dx, diameter Dx at a distance x from face with diameter D1.

  Deformation of the element $d(\Delta x) = P \times dx / (Ax\ E)$

  $Ax = \pi/4 \times Dx^2$; $Dx = D1 - (D1 - D2)/L \times x$

  Let $(D1-D2)/L = k$; Then $Dx = D1 - kx$

  $d(\Delta Lx) = 4 \times P \times dx / (\pi \times (D1-kx)^2 \times E)$

  Integrating from x=0 to x=L $4PL/(\pi ED1D2)$

$$\int_0^L d(\Delta x) = \int_0^L 4 \times P \times dx / (\pi \times (D1-kx)^2 \times E)$$

  Let $D1-kx = \lambda$; then $dx = -(d\lambda/k)$

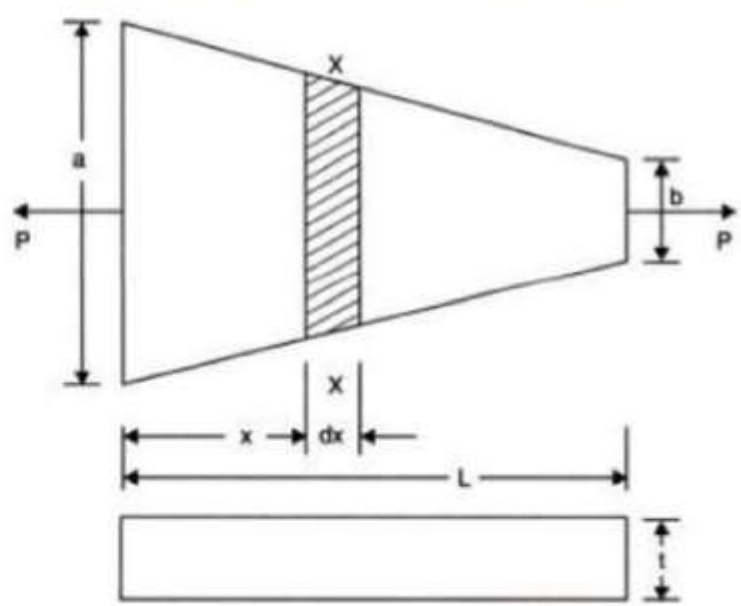
  When $x=0$, $\lambda = D1$; When $x=L$, $\lambda = D2$

$$\int_0^L d(\Delta Lx) = \int_{D1}^{D2} -4 \times P \times dx / (\pi \times \lambda^2\ k \times E)$$

$$\Delta Lx = 4PL/(\pi ED1D2)$$

ANALYSIS OF UNIFORMLY TAPERING RECTANGULAR BAR

A bar of constant thickness and uniformly tapering in width from one end to the other end is shown in Fig. 1.14.

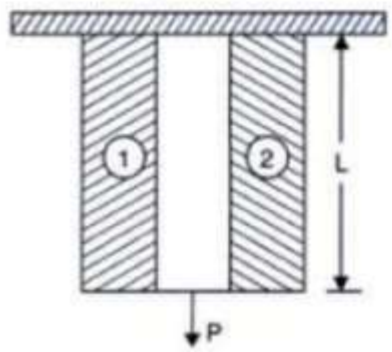


Let P = Axial load on the bar
L = Length of bar
a = Width at bigger end
b = Width at smaller end
E = Young's modulus
t = Thickness of bar

$$dL = \frac{PL}{Et(a-b)} \log_e \frac{a}{b}.$$

ANALYSIS OF BARS OF COMPOSITE SECTIONS

- A bar, made up of two or more bars of equal lengths but of different materials rigidly fixed with each other and behaving as one unit for elongation and shortening when subjected to axial loads is called composite bar.
- Consider a composite bar as shown below



- Let

P→ Applied load

L→ Length of bar

A1→ Area of cross section of Inner member

A2→ Cross sectional area of Outer member

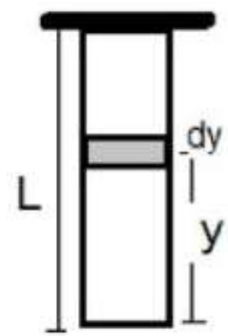
ANALYSIS OF BARS OF COMPOSITE SECTIONS

- Strain developed in the outer member= Strain developed in the inner member

  $\sigma 1/E1 = \sigma 2/E2$
- Total load (P)= Load in the inner member (P1) + Load in the outer member (P2)
- $\sigma 1 \times A1 + \sigma 2 \times A2 = P$
- Solving above two equations, we get the values of $\sigma 1$, $\sigma 2$ & e1 and e2

STRESS & ELONGN. PRODUCED IN A BAR DUE TO ITS SELF WEIGHT

- Consider a bar of length L, area of cross section A rigidly fixed at one end. Let ρ be the density of the material. Consider an infinitesimal element of thickness dy at a distance y from the bottom of th



- The force acting on the element considered= weight of the portion below it=ρAgy

STRESS & ELONGN. PRODUCED IN A BAR DUE TO ITS SELF WEIGHT

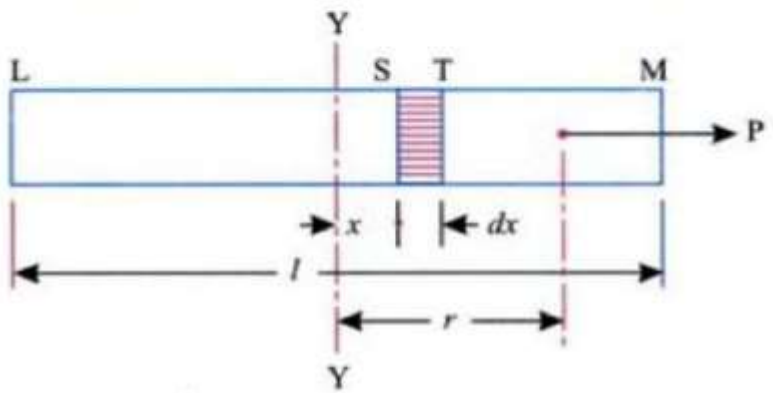
- Tensile stress developed= Force acting on the element/Area of cross section= ρgy.

- From the above equation, it is clear that the maximum stress at the section where $y=L$, ie at the fixed end (ρgL) and minimum stress is at the free end($=0$)
- Elongation due to self weight

$$\Delta Ly = \int_0^L \rho gy\,dy/AE = \rho gL^2/2AE$$

STRESS IN BAR DUE TO ROTATION

Consider a bar of length l rotating about the axis y at a constant angular velocity ω. Consider an infinitesimal element of thickness dx at a distance x from the axis of rotation.



Tensile force on element ST= Centrifugal force on element TM

Centrifugal force on element TM= Mass of element TM x r x ω^2= {l/2 – (x+dx)} x A x ρ x r x ω^2

r= x + ½ x (l/2- (x+dx)

As dx is numerically very small, x + dx≈x

Hence tensile force on element ST=(l/2 – x) x A x{x + ½ x (l/2-x)} x ρ x ω^2

= A x ρ x ω^2 x (l^2/4 - x^2)/2

STRESS IN BAR DUE TO ROTATION

Tensile stress developed= Tensile force/cross sectional area= $A \times \rho \times \omega^2 \times (l^2/4 - x^2)/2A$

$\sigma rod = \rho \times \omega^2 \times (l^2/4 - x^2)/2$

$\sigma rod = 0$, when $x = l/2$

σrod =Maximum when $d(\sigma rod)/dx=0$; ie when $x=0$

$\sigma rodmax == \rho \times \omega^2 \times l^2/8$

Extension of element= $\sigma rod \times dx/E$

Extension of entire bar= $\int_0^l \rho \times \omega^2 \times (l^2/4 - x^2)dx/2 = \rho \times \omega^2 \times l^3/12E$

Extension of entire bar= $\rho \times \omega^2 \times l^3/12E$

THERMAL STRESS

- Thermal stresses are the stresses induced in a body due to change in temperature. Thermal stresses are set up in a body, when the temperature of the body is raised or lowered and the body is restricted from expanding or contracting
- Consider a body which is heated to a certain temperature

  Let

  L= Original length of the body

  ΔT=Rise in temp

  E=Young's modulus

  α=Coefficient of linear expansion

  dL= Extension of rod due to rise of temp
- If the rod is free to expand, Thermal strain developed

  et= Δ L/L=α x ΔT

THERMAL STRESS

- The extension of the rod, $\Delta L = L \times \alpha \times \Delta T$
- If the body is restricted from expanding freely, Thermal stress developed is $\sigma t/et = E$
- $\sigma t = E \times \alpha \times \Delta T$
- Stress and strain when the support yields:-

  If the supports yield by an amount equal to δ, then the actual expansion is given by the difference between the thermal strain and δ

  Actual strain, $e = (L \times \alpha \times \Delta T - \delta)/L$

  Actual stress= Actual strain x E= $(L \times \alpha \times \Delta T - \delta)/L \times E$

Temperature stresses:-

Material

↓

Change in temp.

↓

Expands/ Shortens

↓

no constraint is present

Material

↓

Constrained

↓

No Expansion/ contraction

↓

Temperature stresses Induced in material

Expansion $\Delta = L\alpha\, t$

but $\Delta = PL/AE = P/A \ast L/E = \sigma_{tp}\, L/E$

so $\sigma_{tp} = \Delta \ast E/L = L\alpha\, t \ast E / L = \alpha\, tE$

σ_{tp}= compressive , if temp. increases

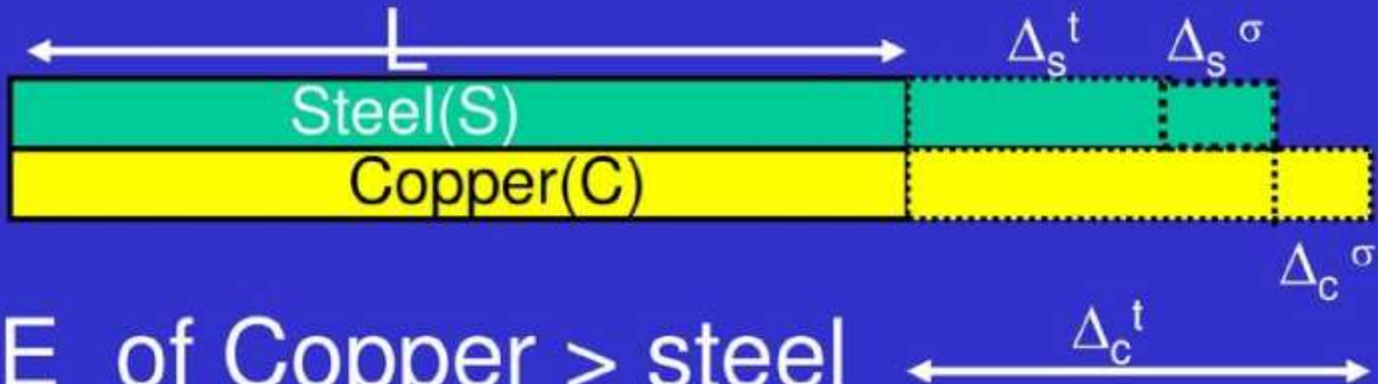
σ_{tp}= tensile, if temp. decreases

Suppose the support yield by an amount δ

$\sigma_{tp} = (\Delta - \delta) \ast E/L = (L\alpha\, t - \delta) \ast E/L$

Composite Section:- (Temp. stresses .)

Extension in steel = Contraction in copper



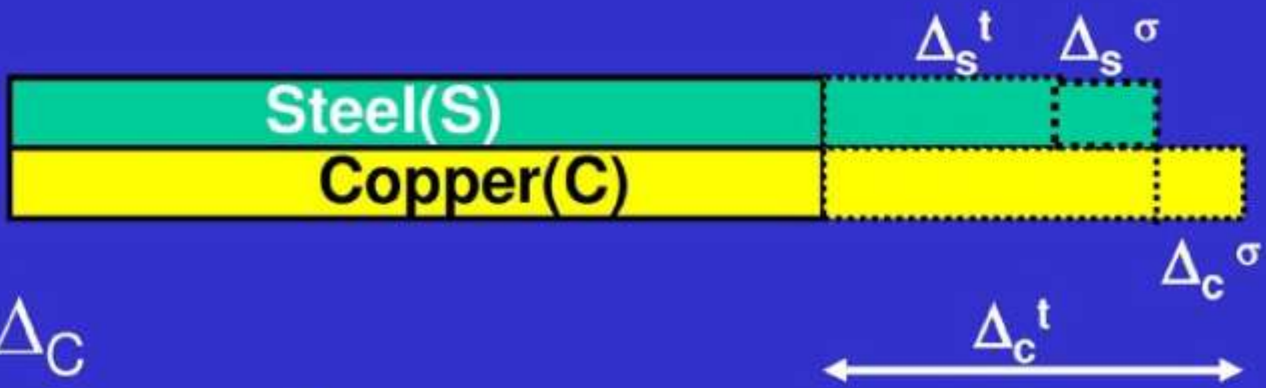
E of Copper > steel

Δ_s^t =Free expansion of steel due to rise in temp.

Δ_c^t =Free expansion of copper due to rise in temp.

Δ_s^σ =Additional extension in steel to behave as composite section

Δ_c^σ =contraction in copper to behave as composite section



$$\Delta_S = \Delta_C$$

$$\Delta_s^{\,t} + \Delta_s^{\,\sigma} = \Delta_c^{\,t} - \Delta_c^{\,\sigma}$$

$$\Delta_s^{\,\sigma} + \Delta_c^{\,\sigma} = \Delta_c^{\,t} - \Delta_s^{\,t}$$

$$PL(1/A_sE_s + 1/A_cE_c) = Lt(\alpha_c - \alpha_s) \qquad \text{----(1)}$$

$$P = t(\alpha_c - \alpha_s)/\ (1/A_sE_s + 1/A_cE_c)$$

Substituting in eq.(1)

$$\sigma_s = P/A_s \text{ and } \sigma_c = P/A_c$$

$$\sigma_s/E_s + \sigma_c/E_c = t(\alpha_c - \alpha_s)$$

$$\varepsilon_s + \varepsilon_c = t\ (\alpha_c - \alpha_s)$$ strain relation

APPLIED AND REACTIVE FORCES

- Forces that act on a Body can be divided into two Primary types: applied and reactive.
- In common Engineering usage, applied forces are forces that act directly on a structure like, dead, live load etc.)
- Reactive forces are forces generated by the action of one body on another and hence typically occur at connections or supports.
- The existence of reactive forces follows from Newton's third law, which state that to every action , there is an equal and opposite reaction.

SUPPORTS

To bear or hold up (a load, mass, structure, part, etc.); serve as a foundation or base for any structure.

To sustain or withstand (weight, pressure, strain, etc.) without giving way

It is a aid or assistance to any structure by preserve its load

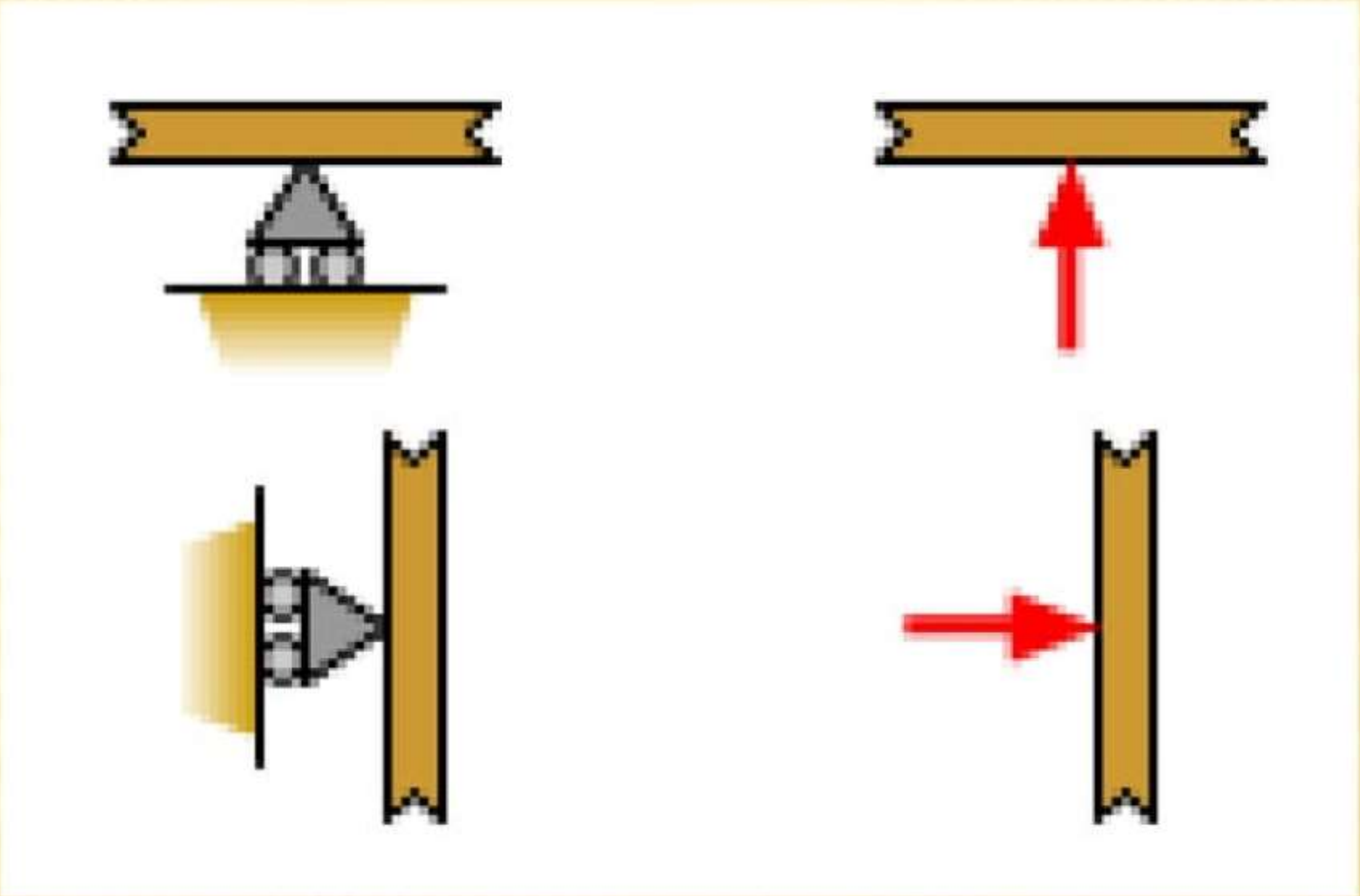
Supports are used to connect structures to the ground or other bodies in order to restrict (confine) their movements under the applied loads. The loads tend to move the structures, but supports prevent the movements by exerting opposing forces, or reactions, to neutralize the effects of loads thereby keeping the structures in equilibrium.

TYPES OF SUPPORTS

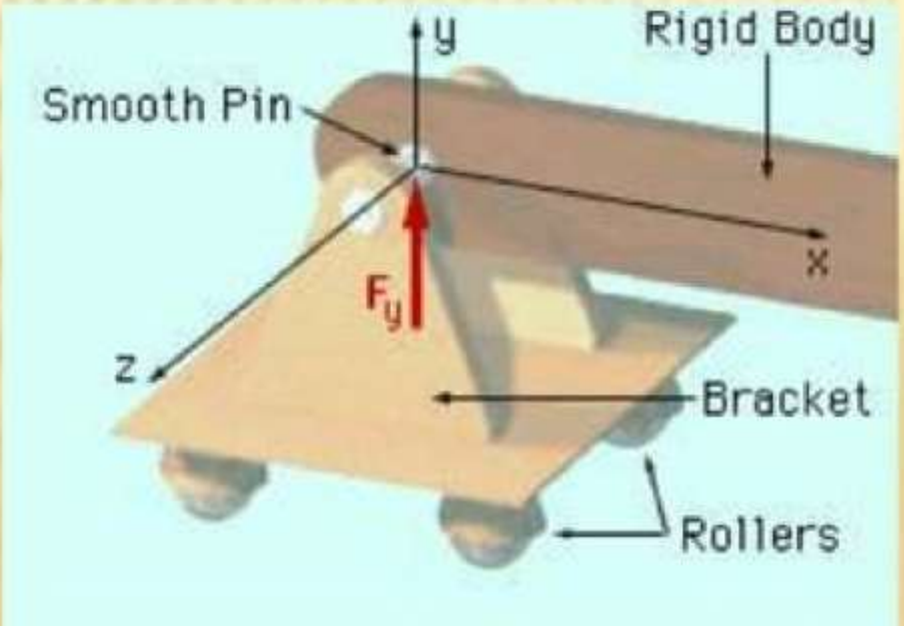
- Supports are grouped into three categories, depending on the number of reactions (1,2,or3) they exert on the structures.
- 1) Roller support
- 2) Hinge support
- 3) fixed support

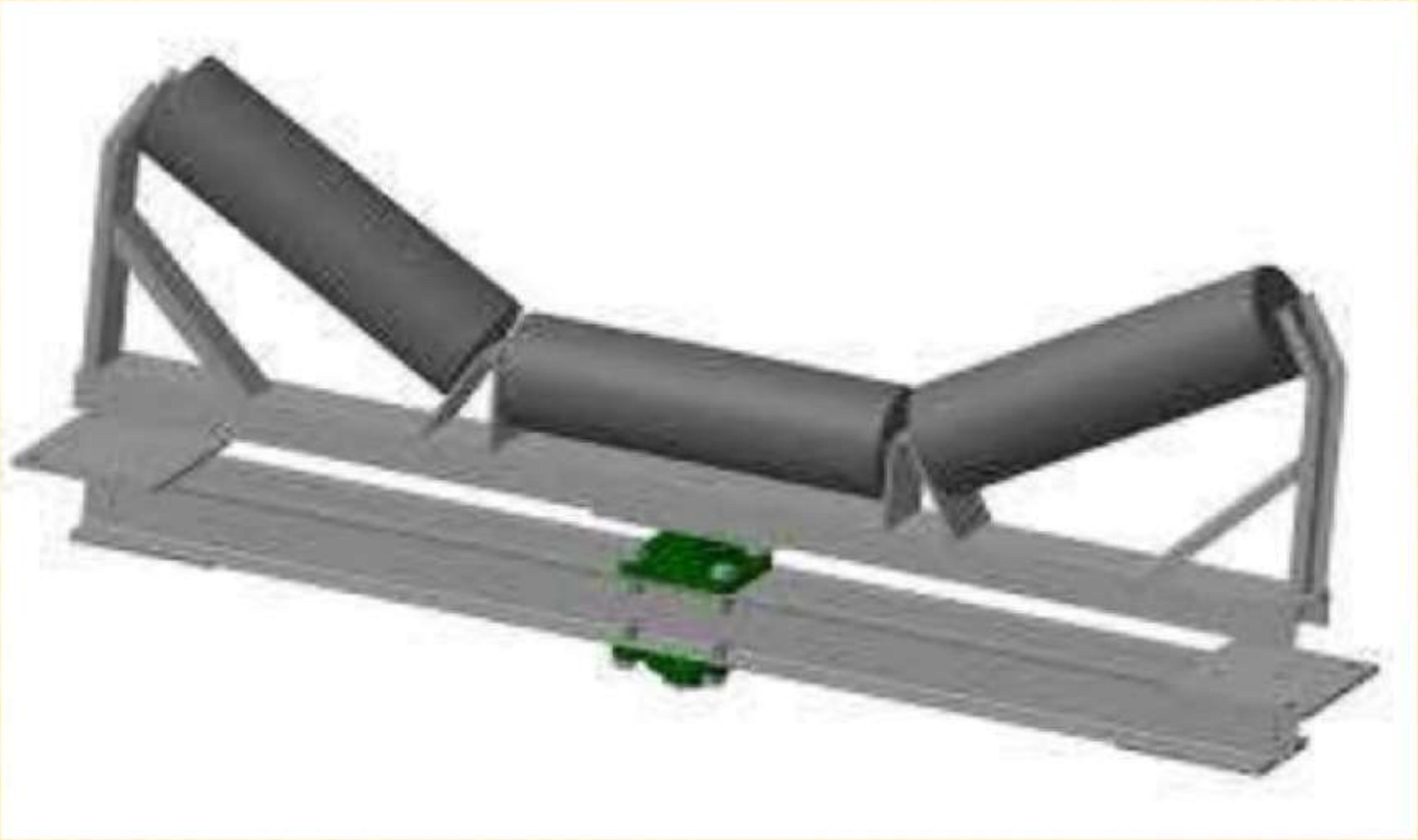
ROLLER SUPPORT

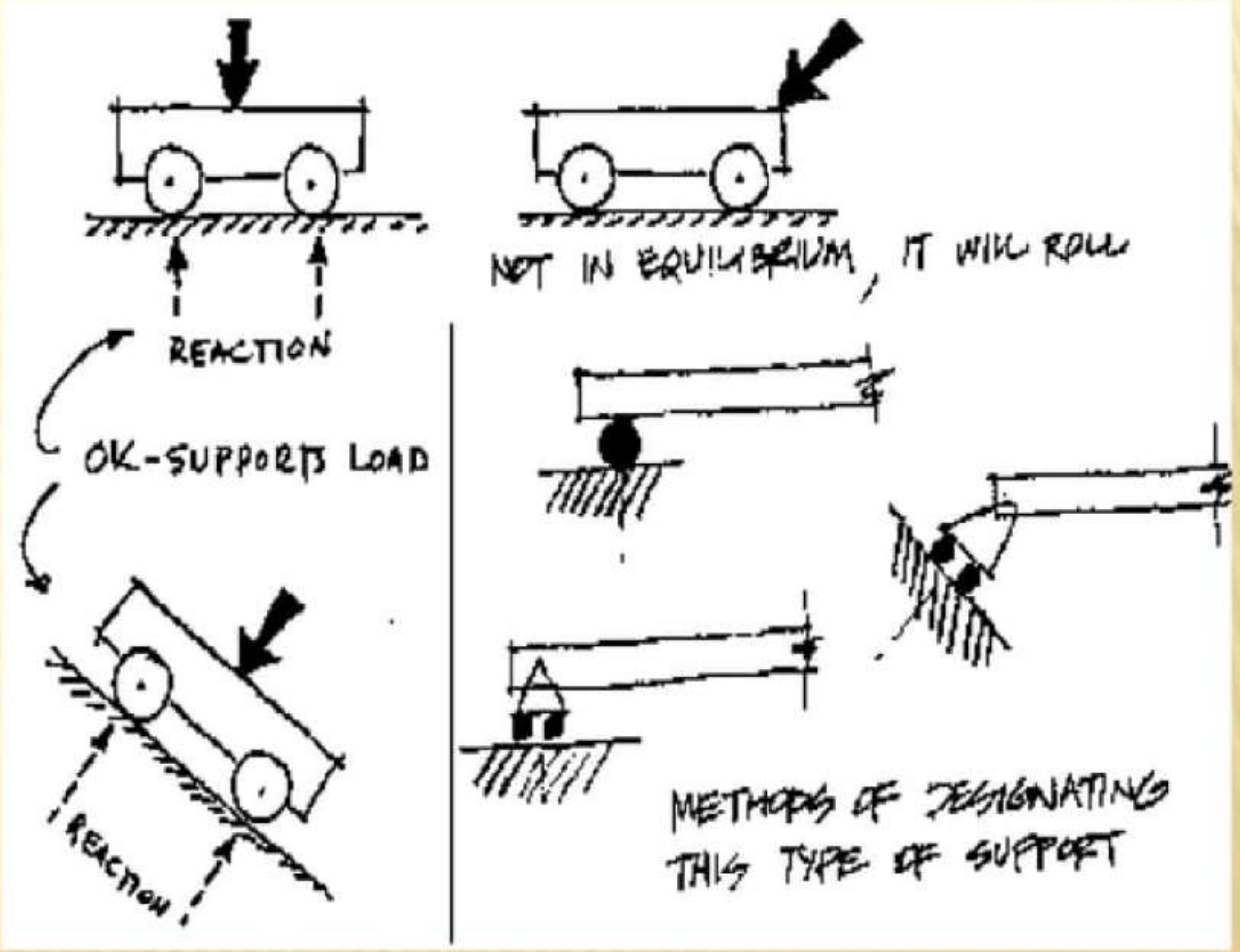
- Roller supports are free to rotate and translate along the surface upon which the roller rests.
- The surface can be horizontal, vertical, or sloped at any angle.
- The resulting reaction force is always a single force that is perpendicular to, and away from, the surface



Restrains the structure from moving in one or two perpendicular directions.


y
Rigid Body
Smooth Pin
x
F_y
z
Bracket
Rollers



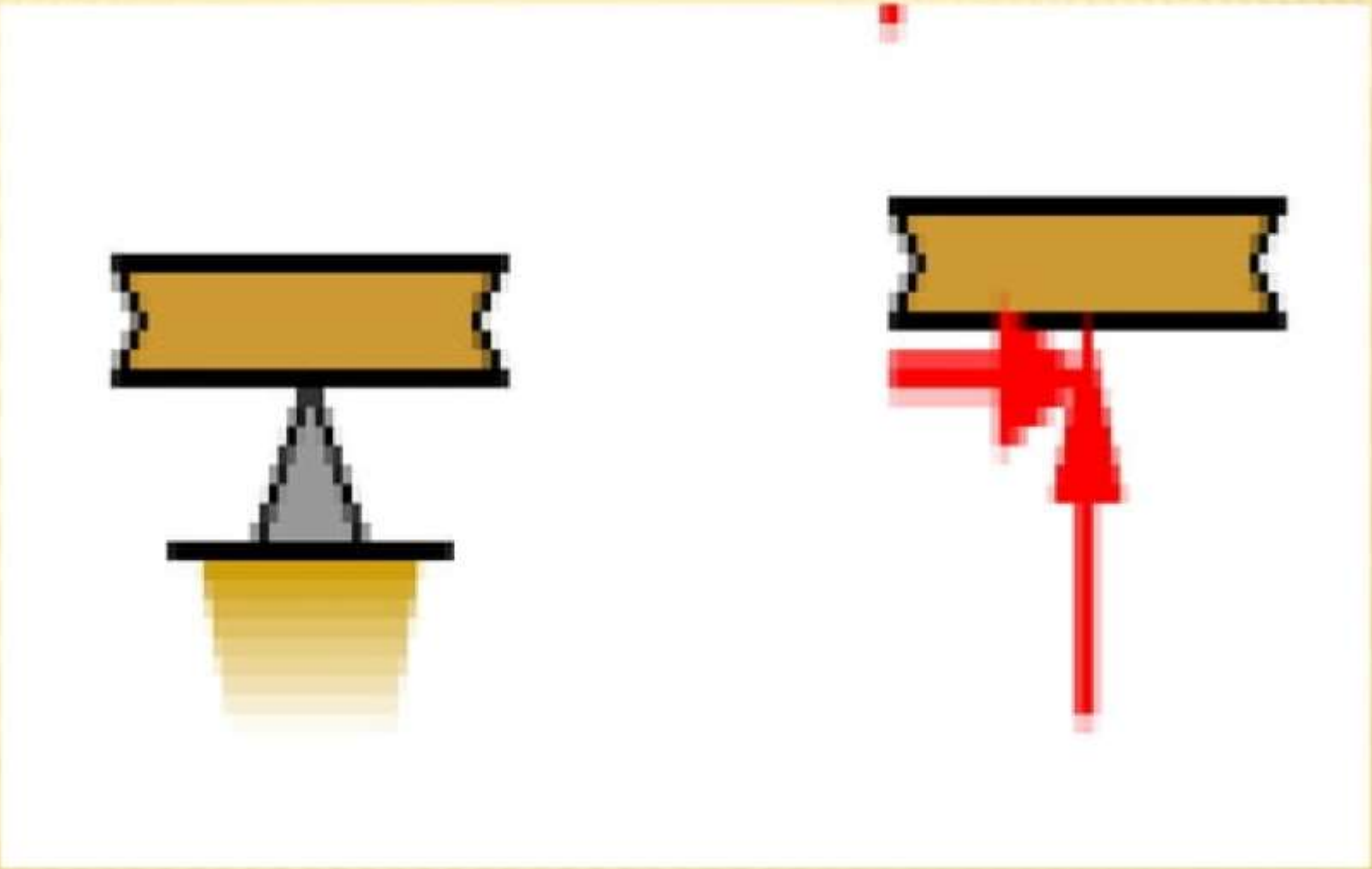

REACTION
OK-SUPPORTS LOAD
REACTION
NOT IN EQUILIBRIUM, IT WILL ROLL
METHODS OF DESIGNATING
THIS TYPE OF SUPPORT

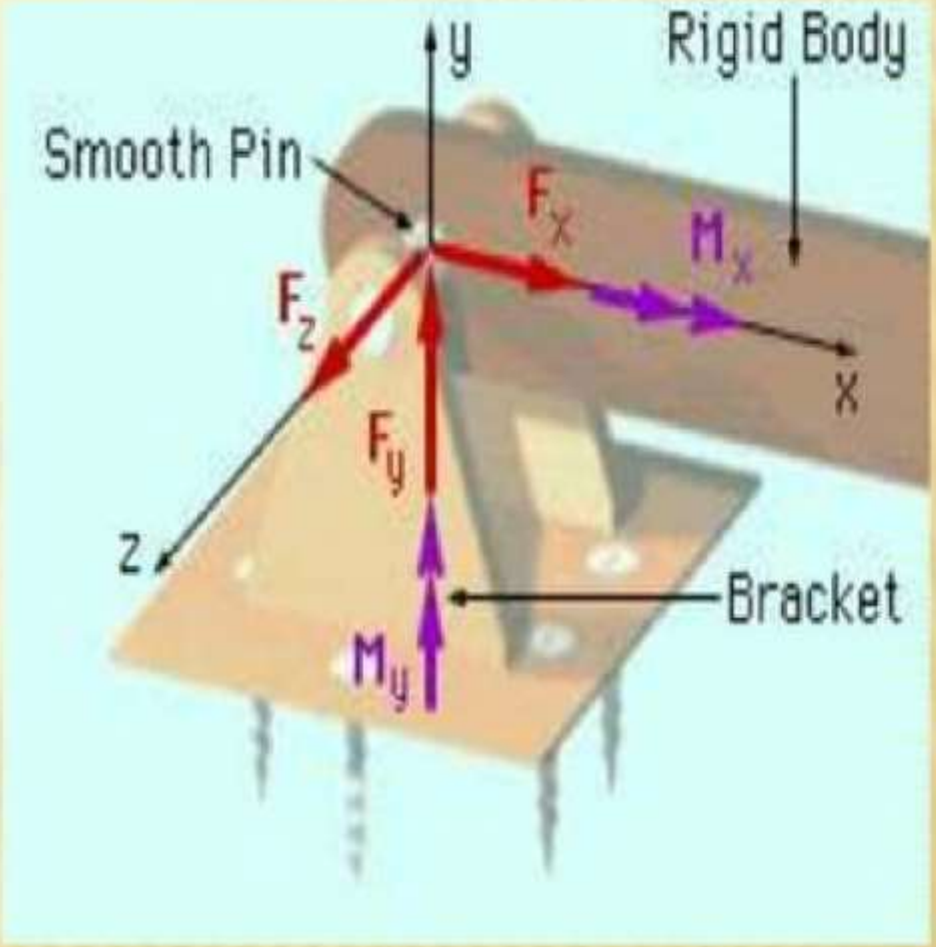
HINGE SUPPORT

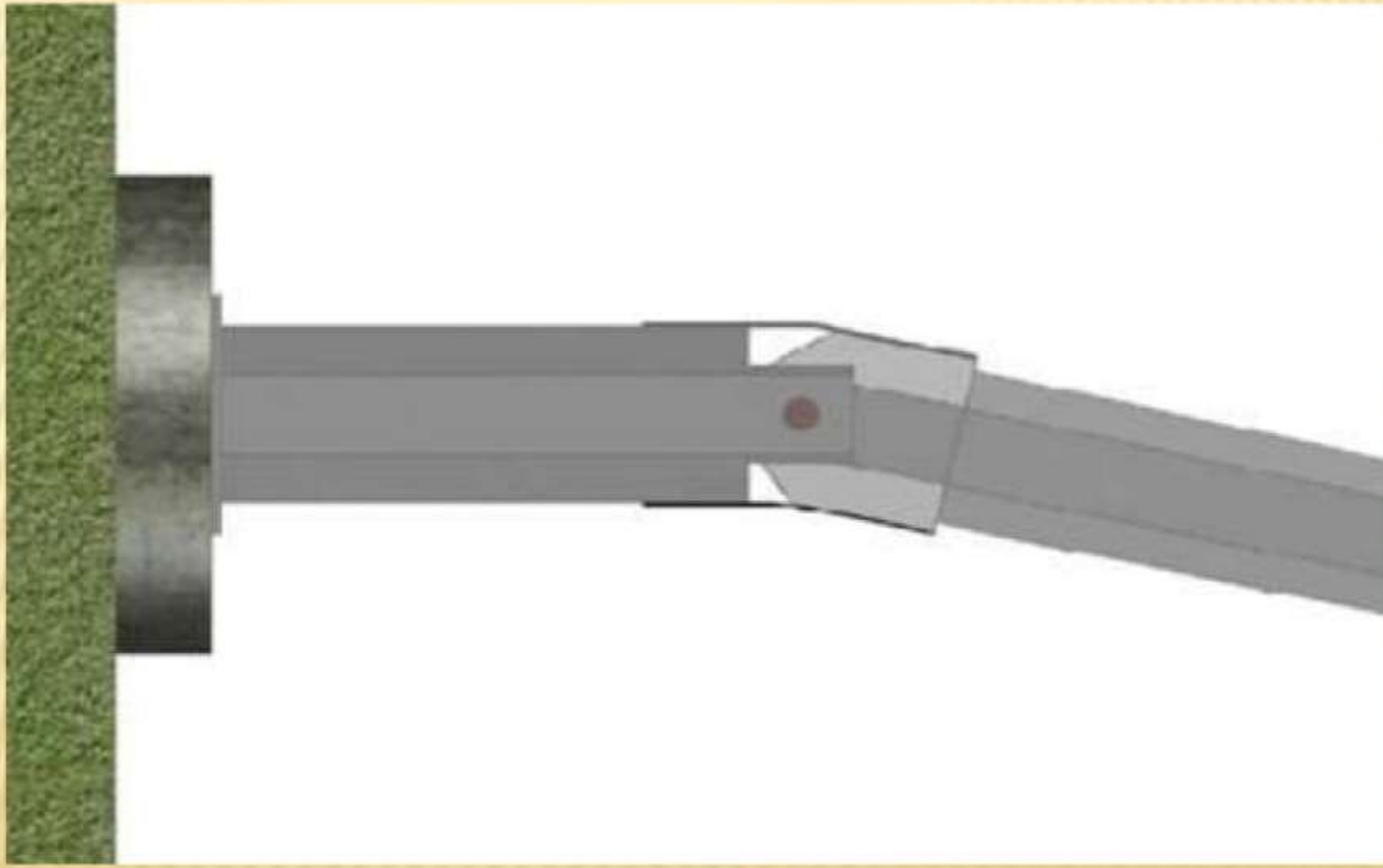
- A Hinge support can resist both vertical and horizontal forces but not a moment. They will allow the structural member to rotate, but not to translate in any direction

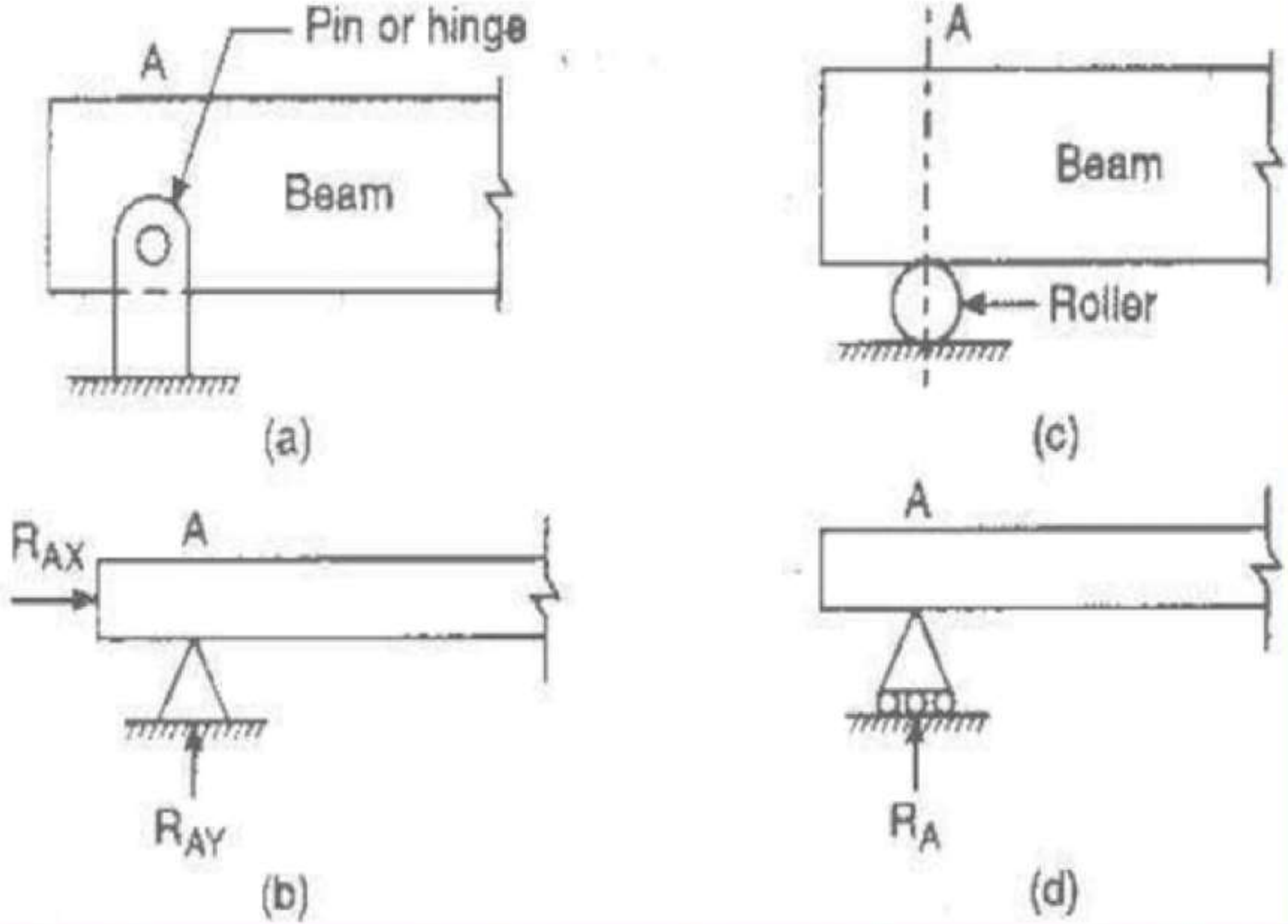
- Pin or hinge support is used when we need to prevent the structure from moving or restrain its translational degrees of freedom.

- A **hinge** is a type of bearing that connects two solid objects, typically allowing only a limited angle of rotation between them. Two objects connected by an ideal hinge rotate relative to each other about a fixed axis of rotation.



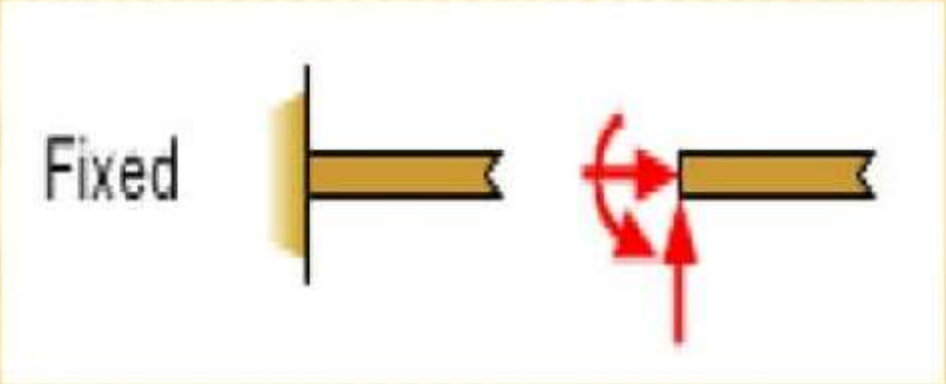

y
Rigid Body
Smooth Pin
F_x
M_x
F_z
x
F_y
z
Bracket
M_y

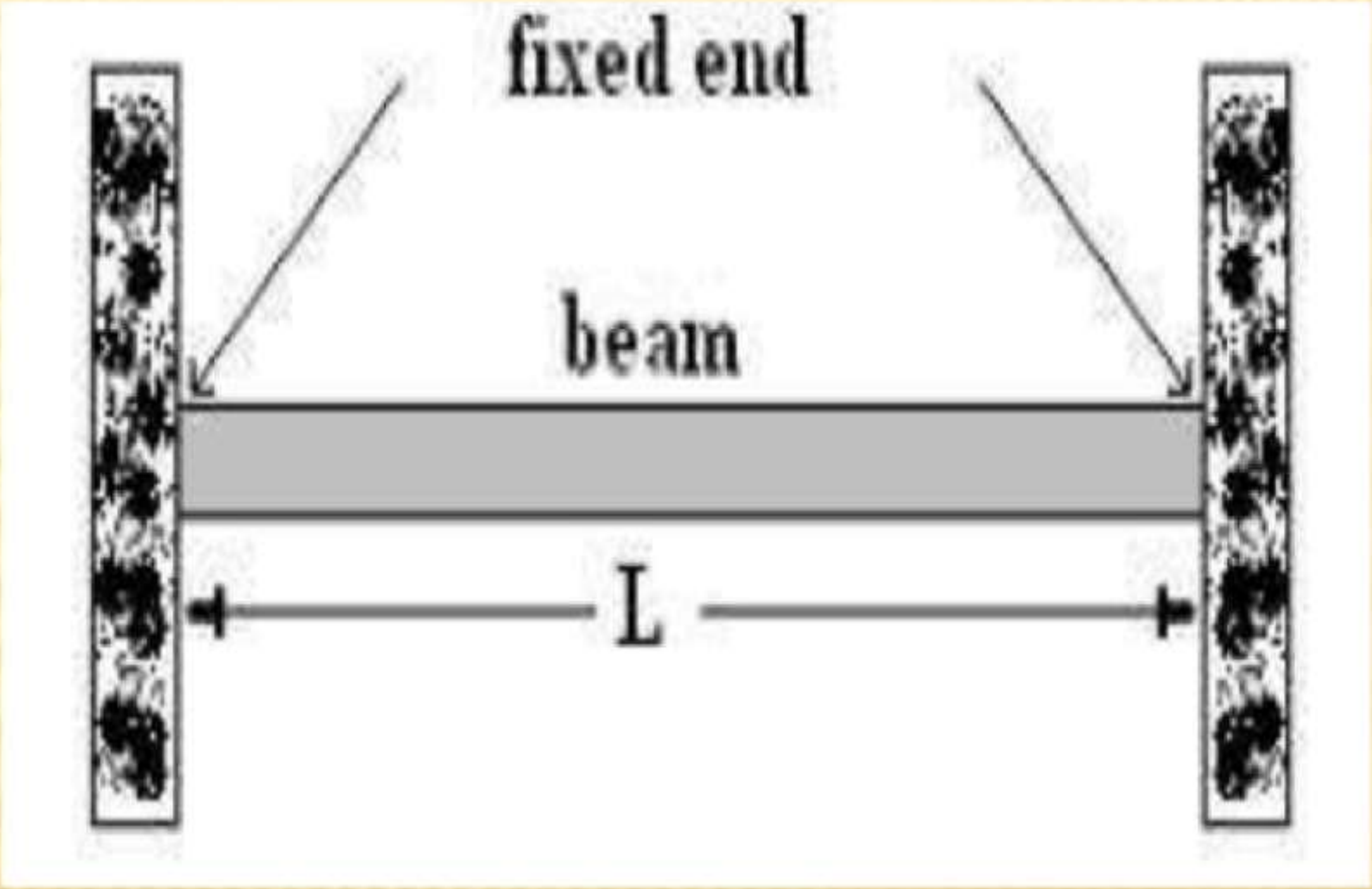



Pin or hinge
A
Beam
(a)
A
Beam
Roller
(c)
R_{AX}
A
R_{AY}
(b)
A
R_A
(d)

FIXED SUPPORT

- Fixed supports can resist vertical and horizontal forces as well as a moment. Since they restrain both rotation and translation, they are also known as rigid supports.




fixed end
beam
L

BEAM

- A beam is a structural member (horizontal) that is design to support the applied load (vertical). It resists the applied loading by a combination of internal transverse **shear force** and bending **moment**.
- It is perhaps the most important and widely used structural members and can be classified according to its support conditions.

Beams

- Extremely common structural element
- In buildings majority of loads are vertical and majority of useable surfaces are horizontal

Beams

devices for transferring vertical loads horizontally

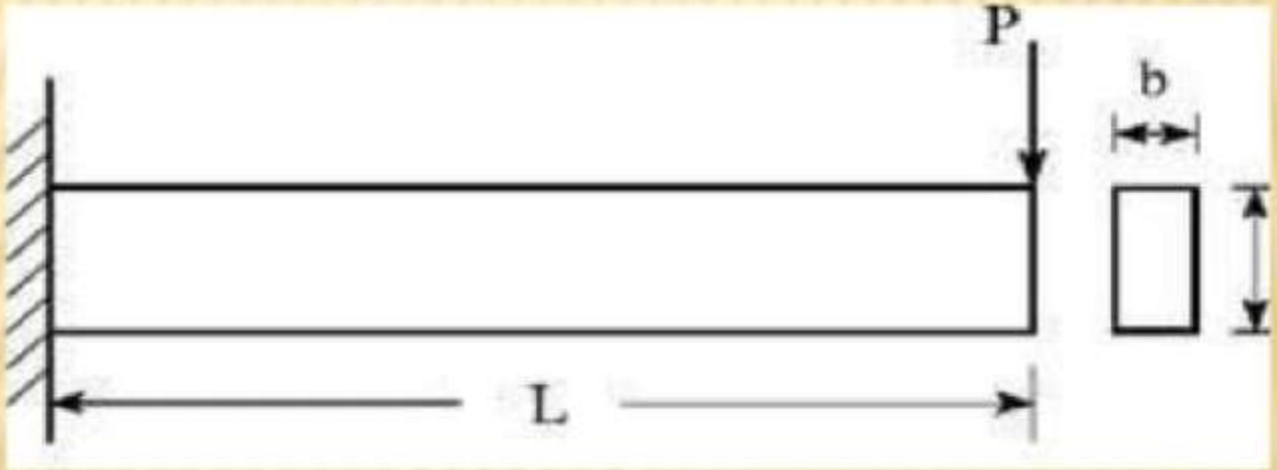
action of beams involves combination of **bending and shear**

TYPES OF BEAMS

- The following are the important types of beams:
- 1. Cantilever
- 2. simply supported
- 3. overhanging
- 4. Fixed beams
- 5. Continuous beam

CANTILEVER BEAM

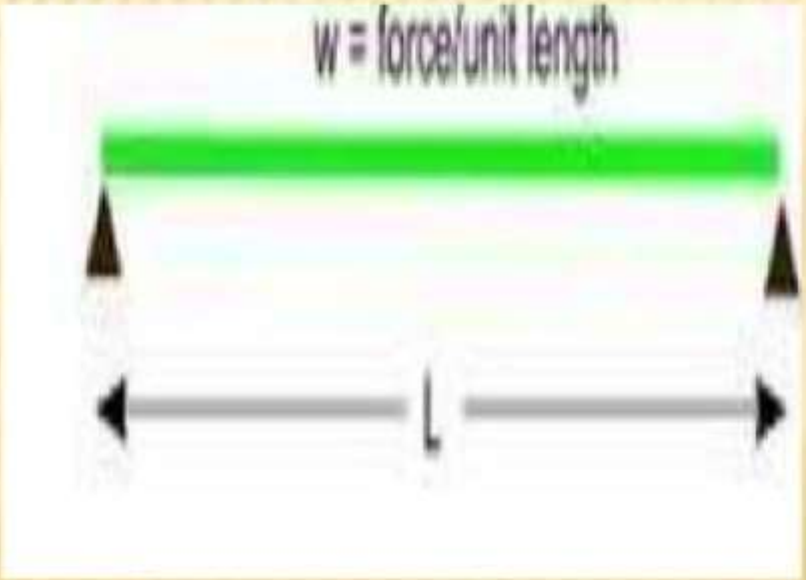
- A beam which is fixed at one end and free at the other end is known as cantilever beam.




25

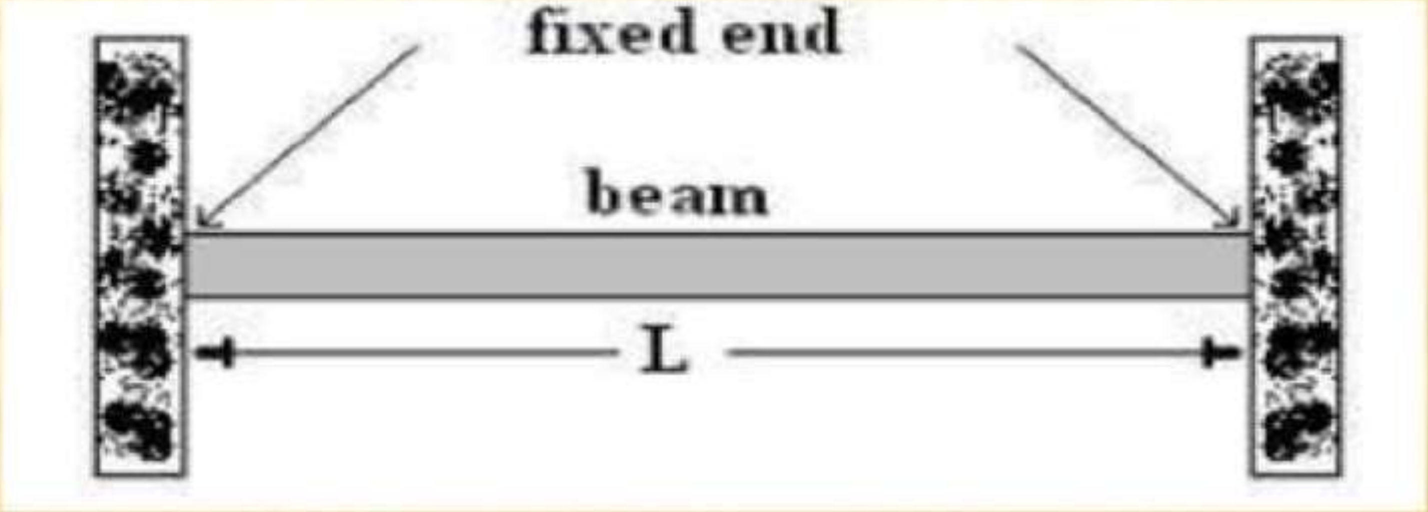
SIMPLY SUPPORTED BEAMS

- A beam supported or resting freely on the supports at its both ends,



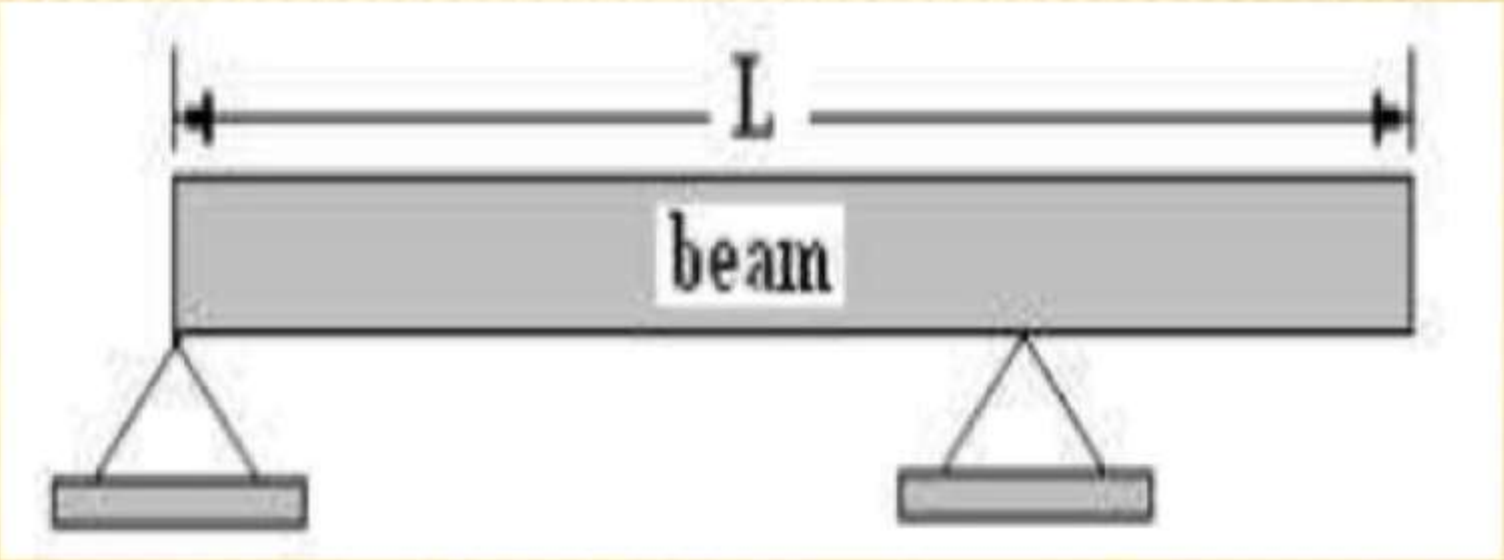
FIXED BEAMS

- A beam whose both ends are fixed and is restrained against rotation and vertical movement. Also known as built-in beam or encastred beam.



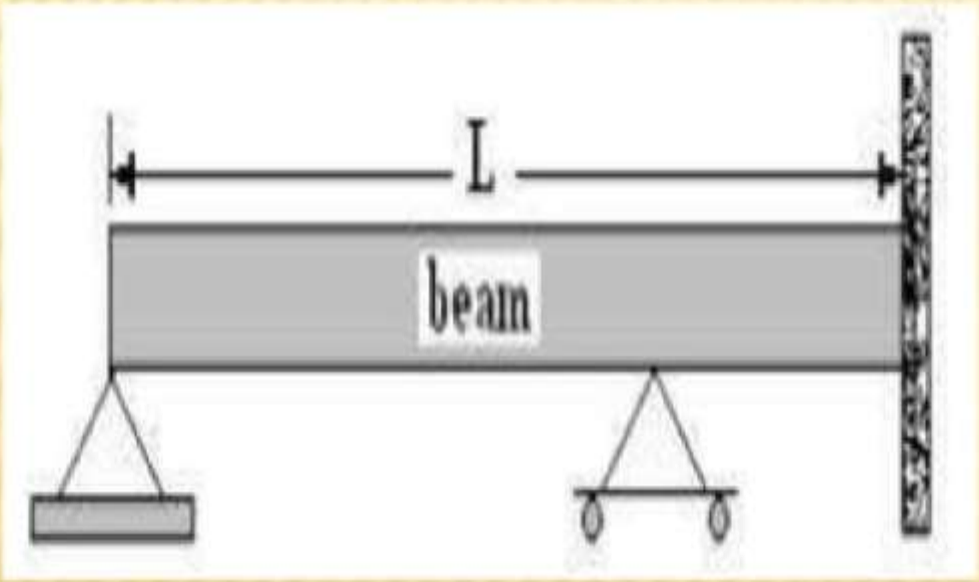
OVERHANGING BEAM

- If the end portion of a beam is extended outside the supports.



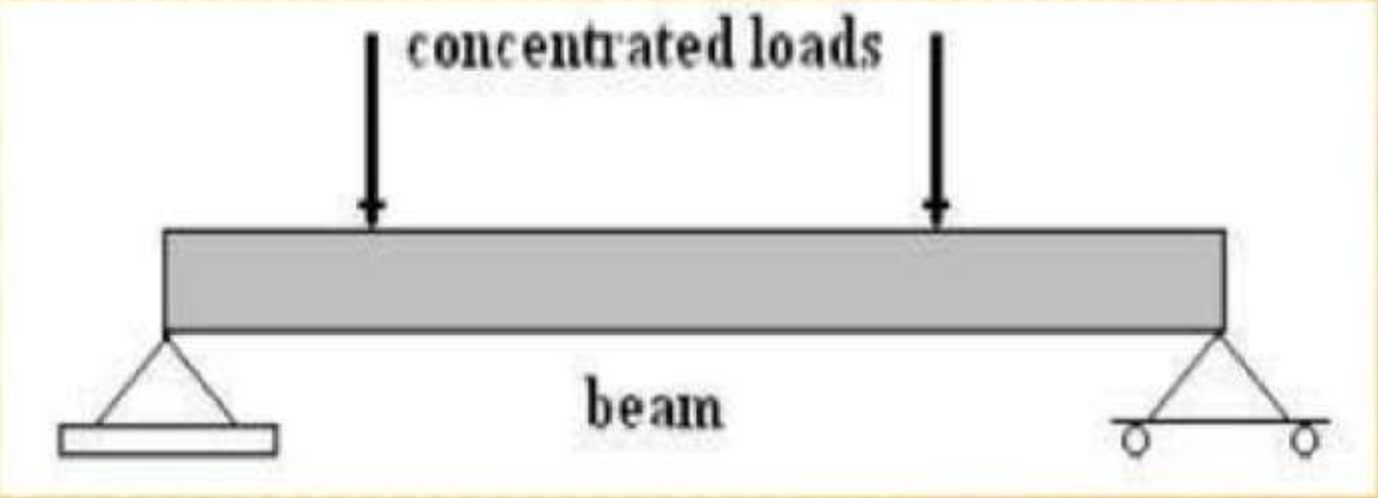
CONTINUOUS BEAMS

- A beam which is provided with more than two supports.



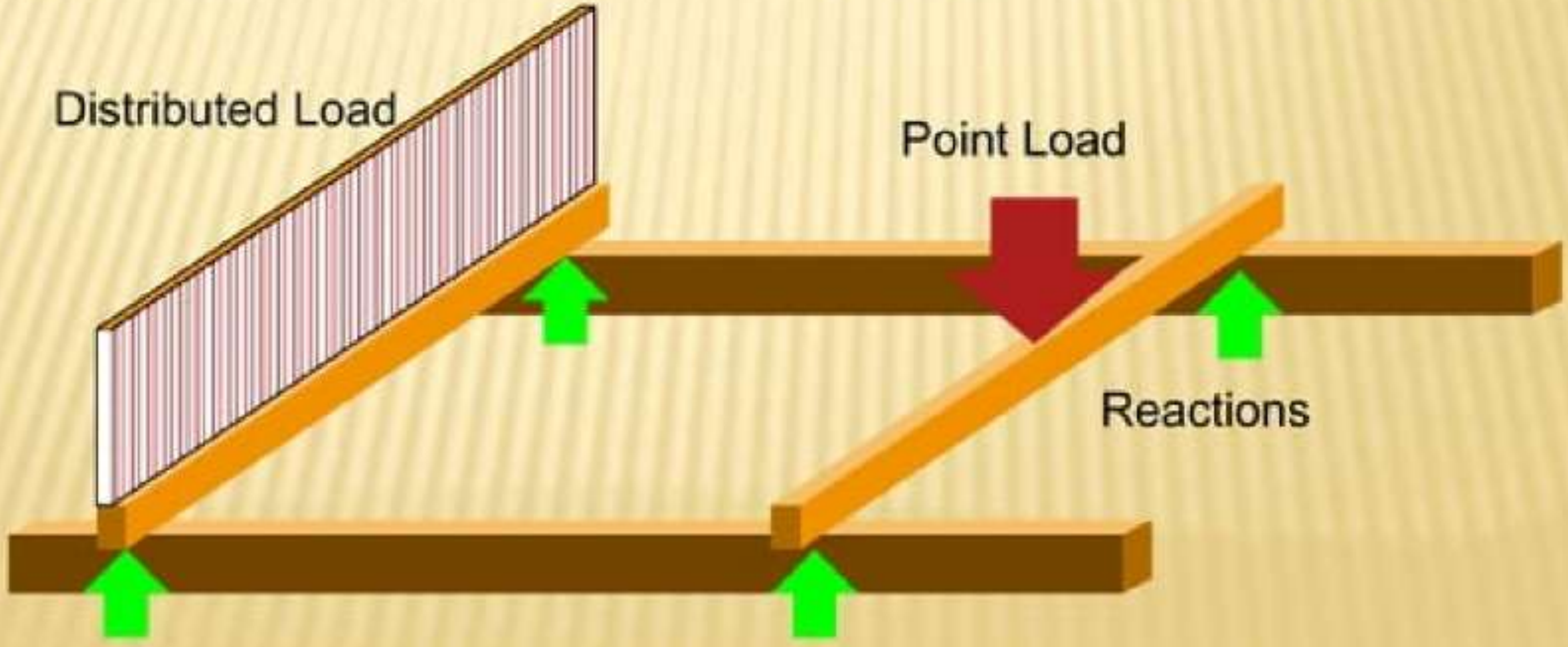
TYPES OF LOADS

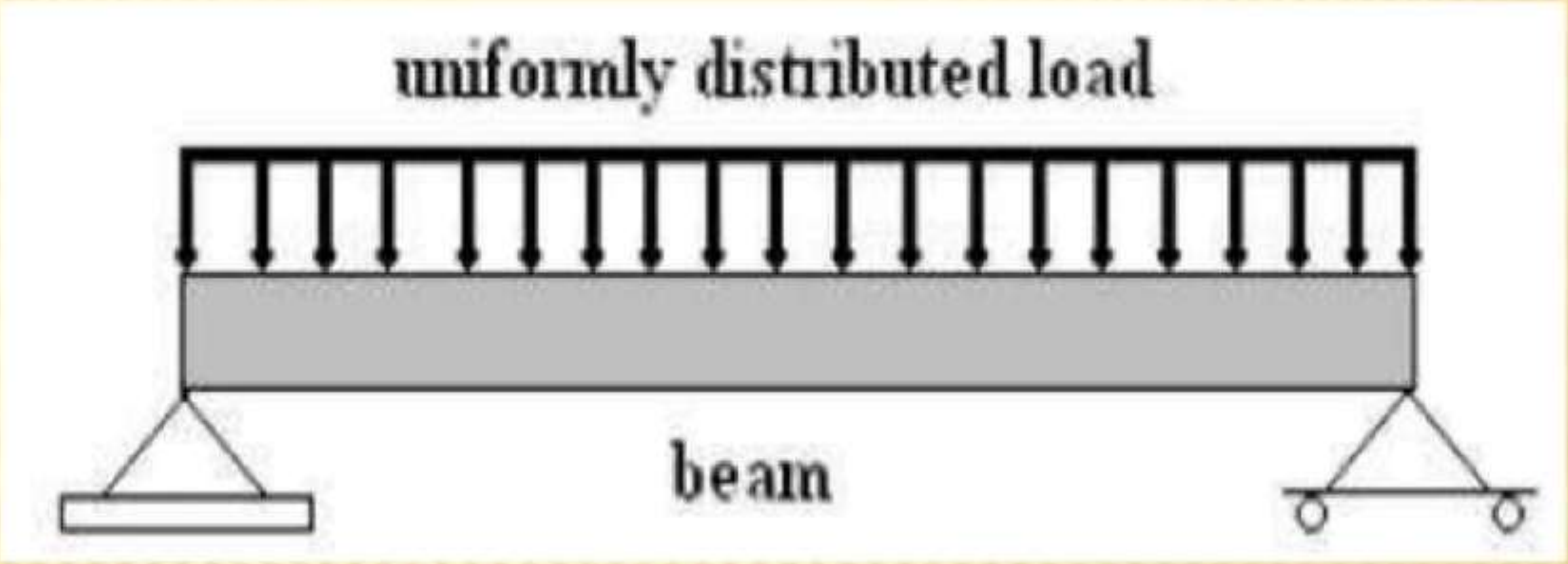
- Concentrated load assumed to act at a point and immediately introduce an oversimplification since all practical loading system must be applied over a finite area.

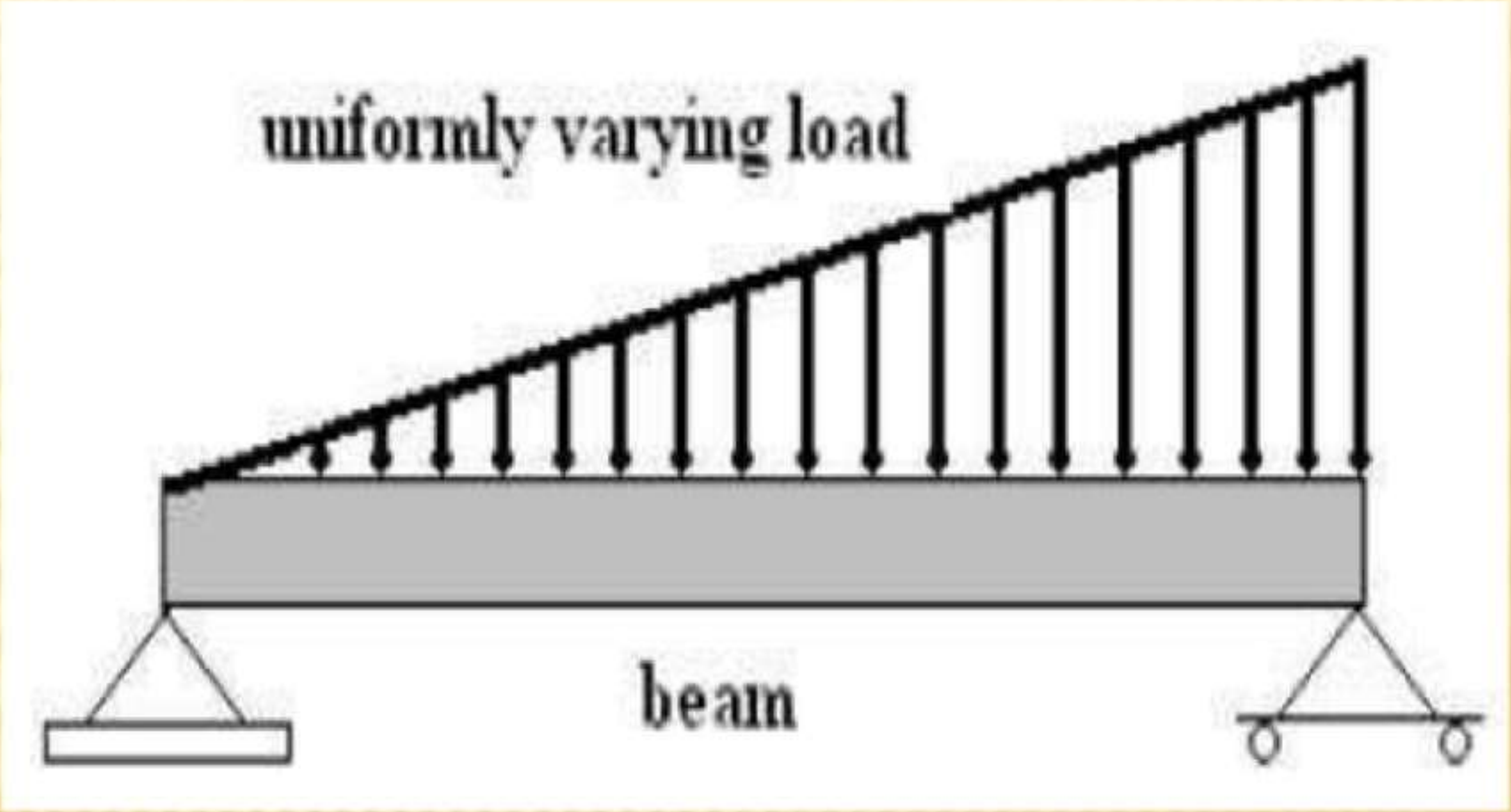


Loads on Beams

- Point loads, from concentrated loads or other beams
- Distributed loads, from anything continuous

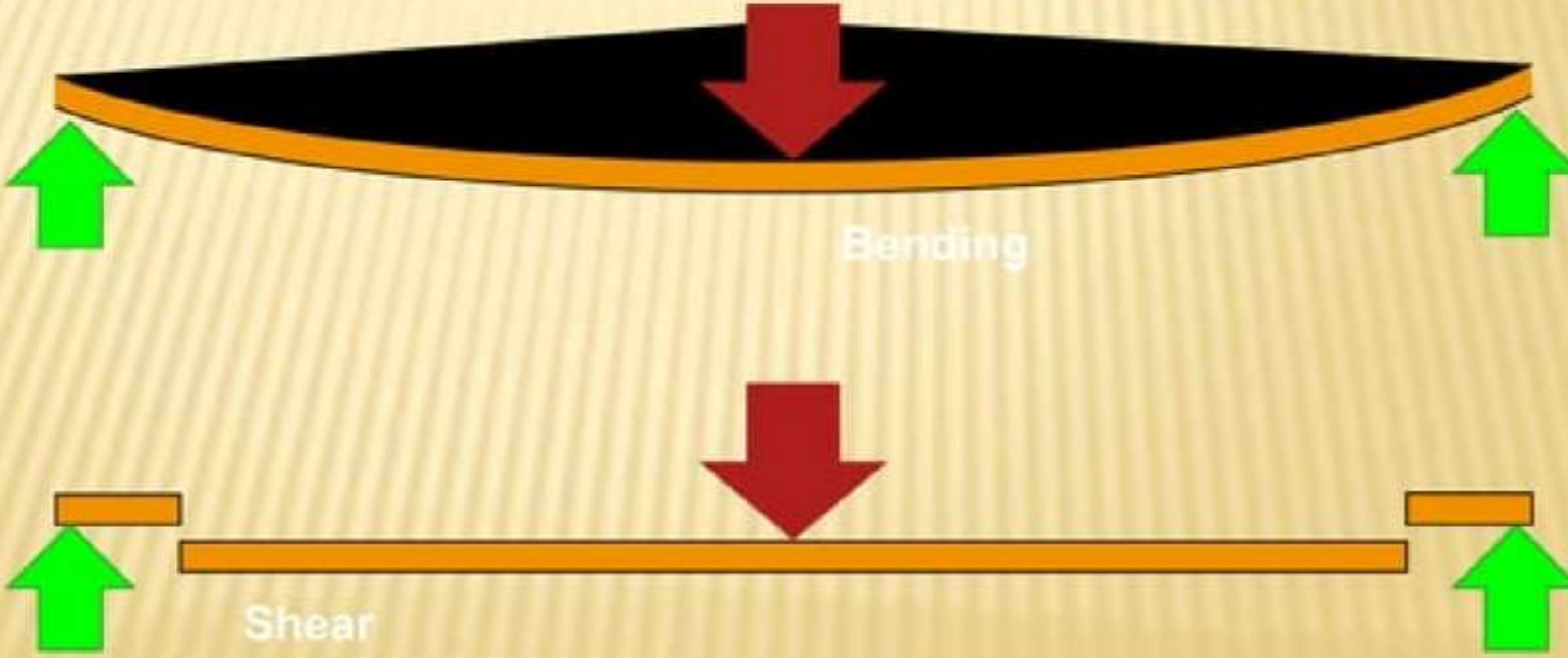



uniformly distributed load
beam

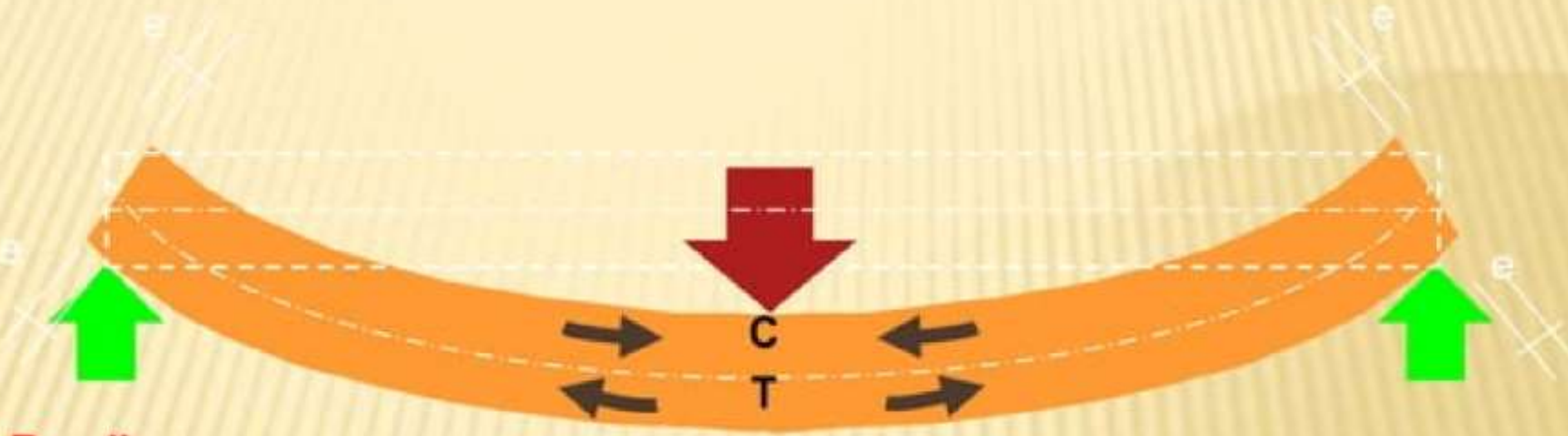

uniformly varying load
beam

What the Loads Do

- The loads (& reactions) bend the beam, and try to shear through it



What the Loads Do



Bending

Shear

Designing Beams

- in architectural structures, bending moment more important
  - importance increases as span increases
- short span structures with heavy loads, shear dominant
  - e.g. pin connecting engine parts

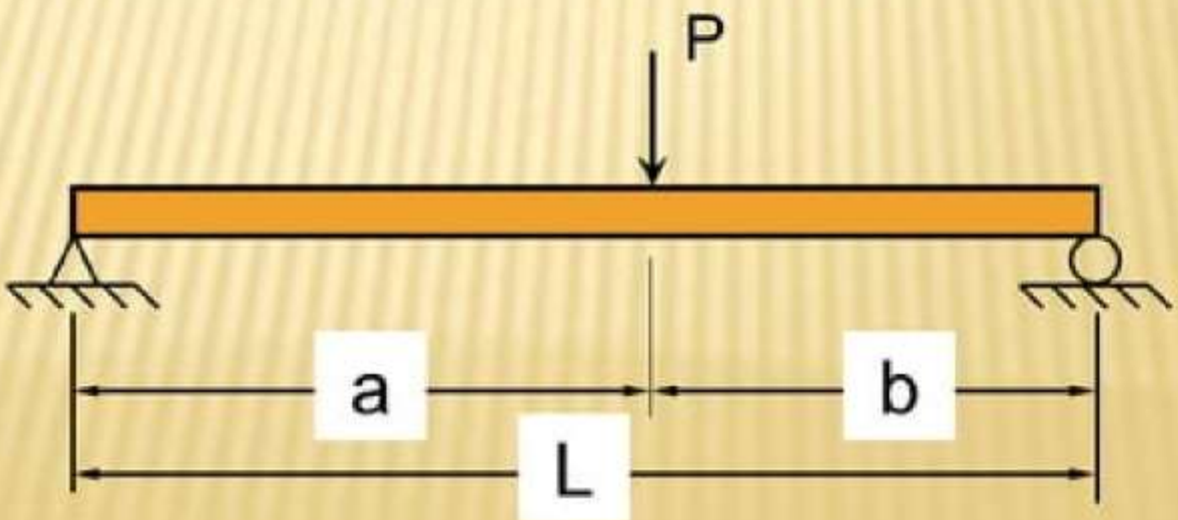
beams in building designed for bending checked for shear

How we calculate the Effects

- First, find ALL the forces (loads and reactions)
- Make the beam into a free body (cut it out and artificially support it)
- Find the reactions, using the conditions of equilibrium

INTERNAL REACTIONS IN BEAMS

- At any cut in a beam, there are 3 possible internal reactions required for equilibrium:
  - normal force,
  - shear force,
  - bending moment.



INTERNAL REACTIONS IN BEAMS

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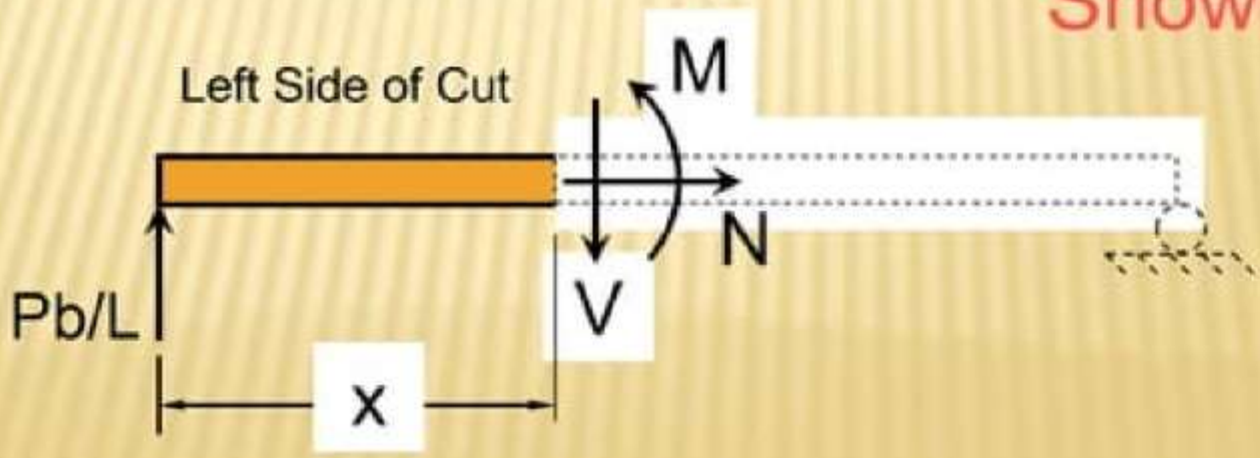
Positive Directions Shown!!!



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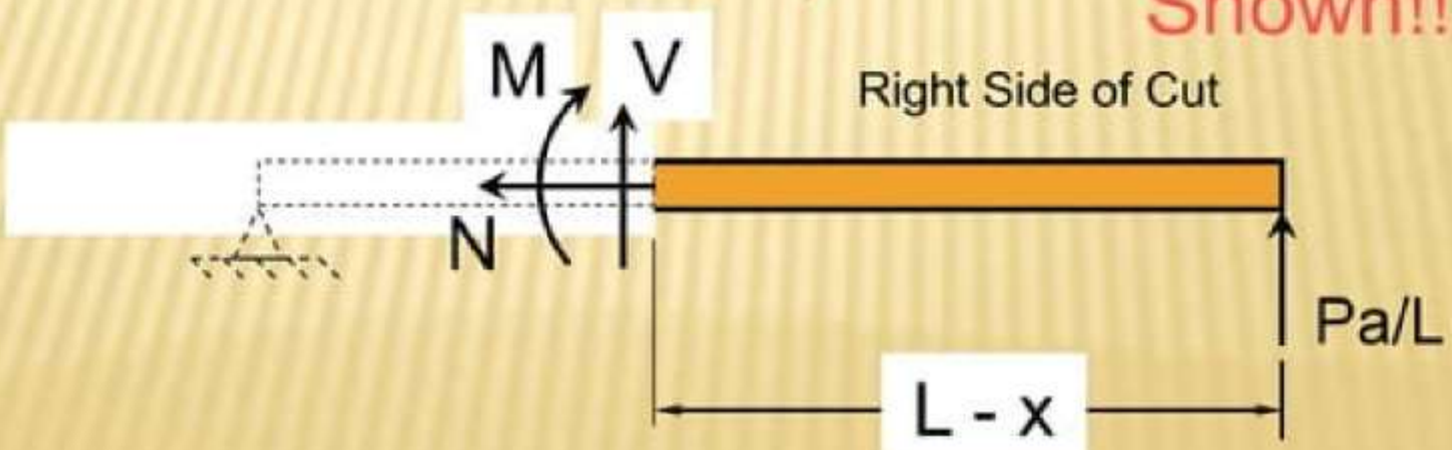
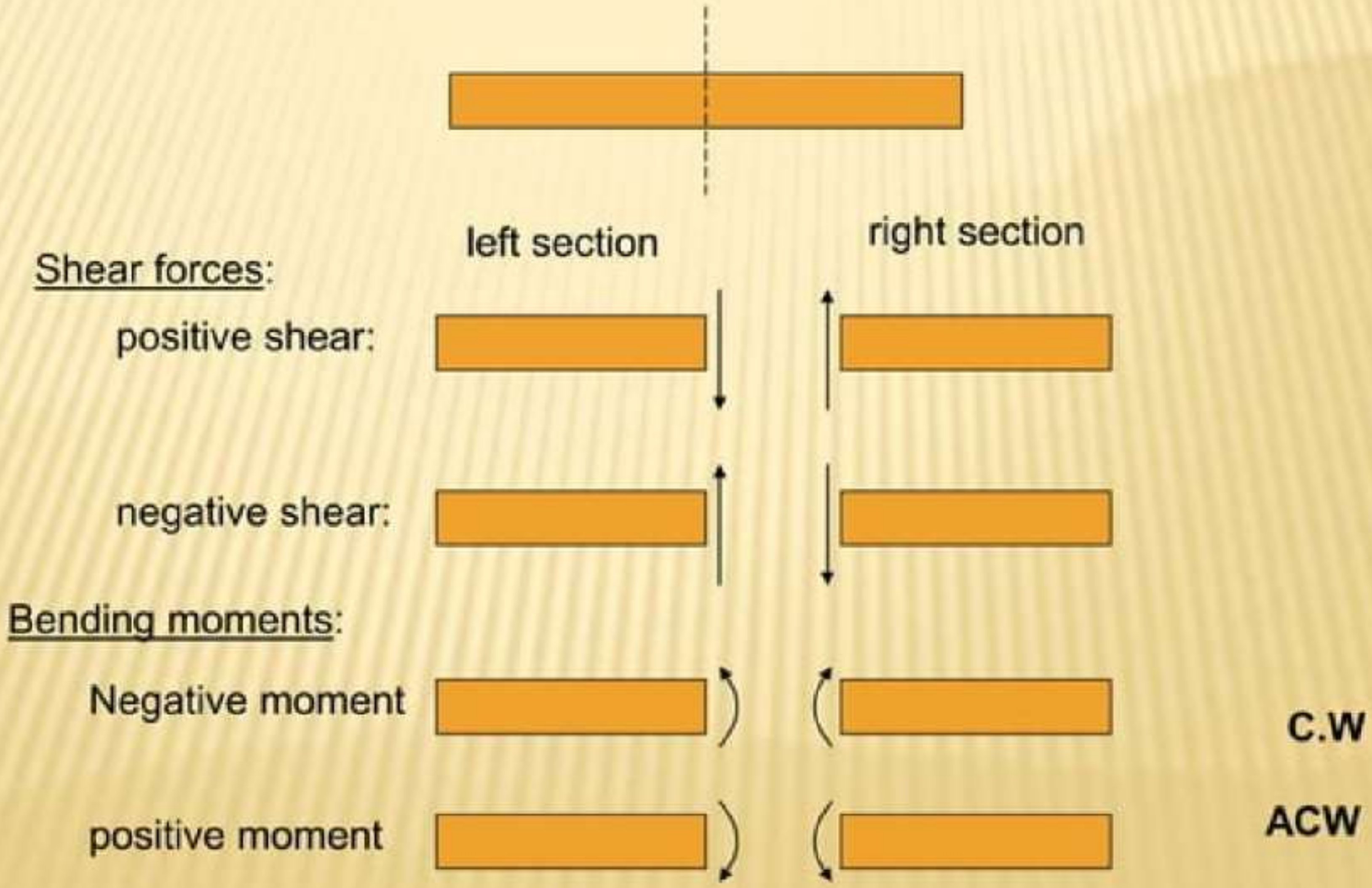
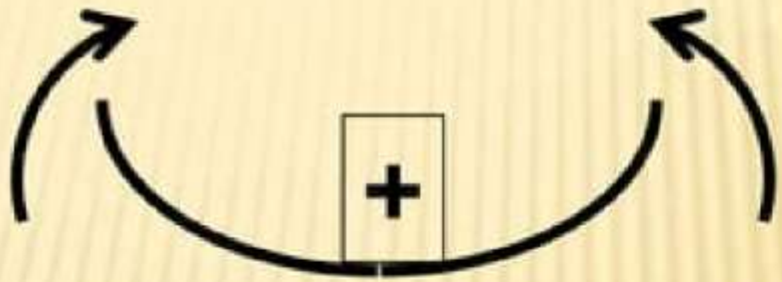


SHEAR FORCES, BENDING MOMENTS - SIGN CONVENTIONS



Sign Conventions Bending Moment Diagrams (cont.)

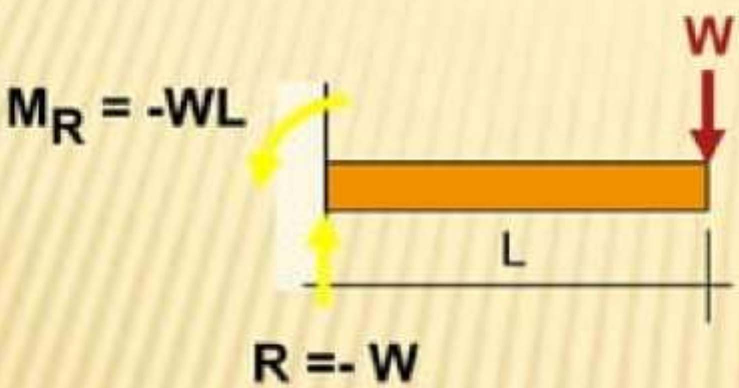
Sagging bending moment is POSITIVE (happy)



Hogging bending moment is NEGATIVE (sad)

Cantilever Beam Point Load at End

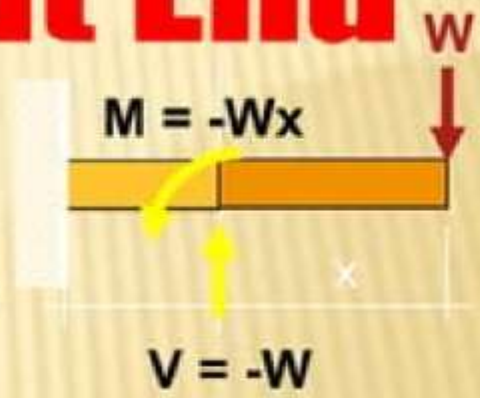
- Consider cantilever beam with point load on end



vertical reaction, R = -W
and moment reaction M_R = - WL

- **Use the free body idea to isolate part of the beam**
- **Add in forces required for equilibrium**

Cantilever Beam
Point Load at End



Take section anywhere at distance, x from end

Add in forces, V = -W and moment M = - Wx

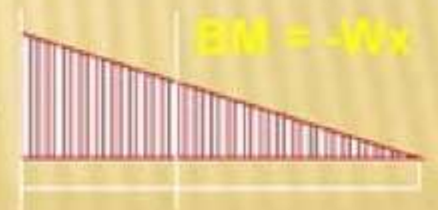
Shear V =- W constant along length

V = -W

Shear Force Diagram

Bending Moment BM = -W.x
when x = L BM = -WL
when x = 0 BM = 0

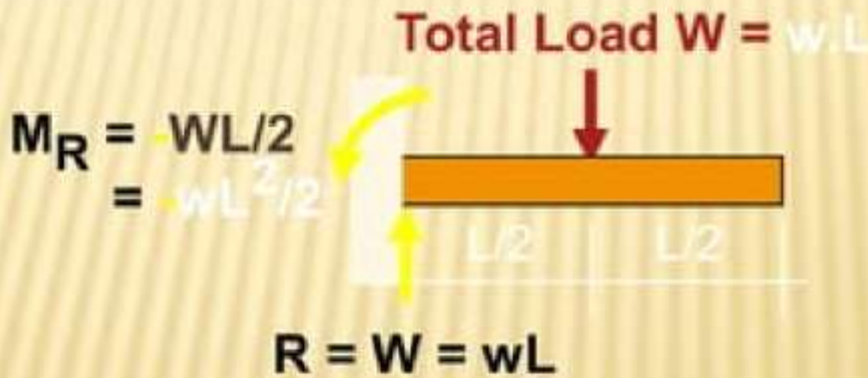
BM = WL



Bending Moment Diagram

Cantilever Beam
Uniformly Distributed Load

For maximum shear V and bending moment BM



vertical reaction, $R = W = wL$

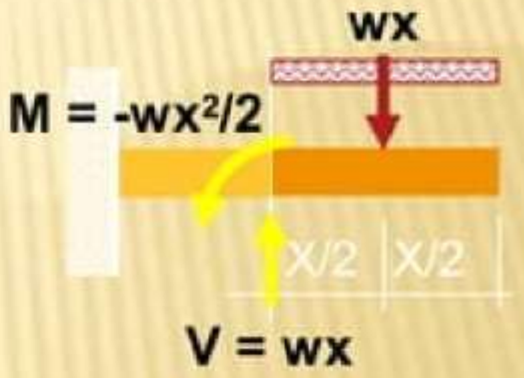
and moment reaction $M_R = -WL/2 = -wL^2/2$

Example 2 - Cantilever Beam Uniformly Distributed Load (cont.)

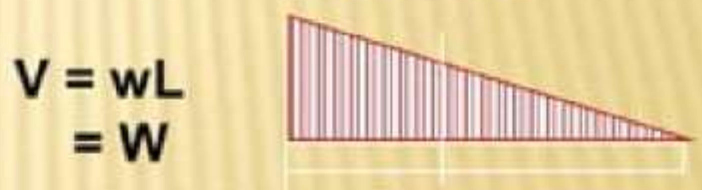
For distributed V and BM

Take section anywhere at distance, x from end

Add in forces, V = w.x and moment M = - wx.x/2

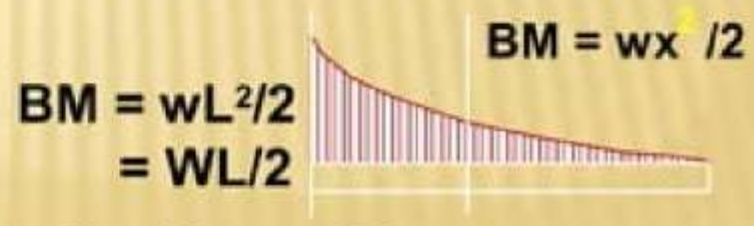


Shear $V = wx$
when $x = L$ $V = W = wL$
when $x = 0$ $V = 0$



Shear Force Diagram

Bending Moment $BM = w.x^2/2$
when $x = L$ $BM = wL^2/2 = WL/2$
when $x = 0$ $BM = 0$
(parabolic)



Bending Moment Diagram

Assumptions made in Pure bending theory

1) The beam is initially straight and every layer is free to expand or contract.
2) The material is homogenous and isotropic.
3) Young's modulus (E) is same in both tension and compression.
4) Stresses are within the elastic limit.
5) The radius of curvature of the beam is very large in comparison to the depth of the beam.

6) A transverse section of the beam which is plane before bending will remain plane even after bending.

7) Stress is purely longitudinal.

DERIVATION OF PURE BENDING EQUATION

PART I:

Relationship between bending stress and radius of curvature.

Consider the beam section of length "dx" subjected to pure bending. After bending the fibre AB is shortened in length, whereas the fibre CD is increased in length.

In b/w there is a fibre (EF) which is neither shortened in length nor increased in length (Neutral Layer).

Let the radius of the fibre E'F′ be **R** . Let us select one more fibre GH at a distance of 'y' from the fibre EF as shown in the fig.

$EF = E'F' = dx = R\, d\theta$

The initial length of fibre GH equals $R\, d\theta$

After bending the new length of GH equals

$$G'H' = (R+y)\, d\theta$$

$$= R\, d\theta + y\, d\theta$$

Change in length of fibre GH = $(\mathbf{R\, d\theta + y\, d\theta}) - \mathbf{R\, d\theta} = \mathbf{y\, d\theta}$

Therefore the strain in fibre GH

ϵ= change in length / original length= $\mathbf{y\, d\theta / R\, d\theta}$

$$\epsilon = y/R$$

If σ_b is the bending stress and E is the Young's modulus of the material, then strain

$$\epsilon = \sigma_b/E$$

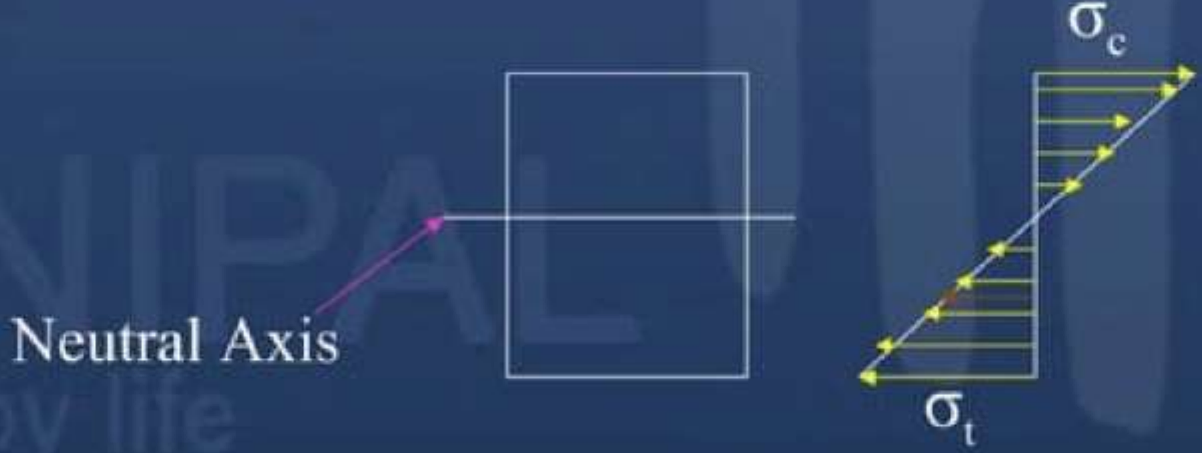
$$\sigma_b/E = y/R \Rightarrow \sigma_b = (E/R)\, y \text{---------}(1)$$

$\sigma_b = (E/R)\, y \Rightarrow$ i.e. bending stress in any fibre is proportional to the distance of the fibre (y) from the neutral axis and hence maximum bending stress occurs at the farthest fibre from the neutral axis.

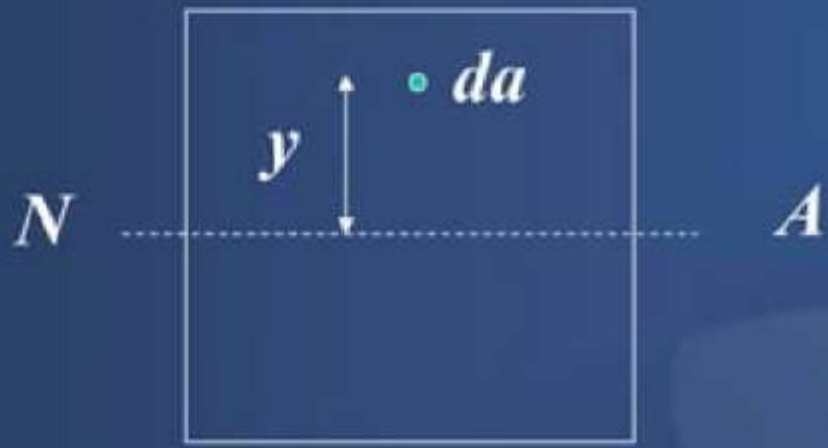
Note: Neutral axis coincides with the horizontal centroidal axis of the cross section

Moment of resistance

on one side of the neutral axis there are compressive stresses and on the other there are tensile stresses. These stresses form a couple, whose moment must be equal to the external moment M. The moment of this couple, which resists the external bending moment, is known as moment of resistance.



Moment of resistance



Consider an elemental area 'da' at a distance 'y' from the neutral axis.

The force on this elemental area = $\sigma_b \times da$

$$= (E/R)\, y \times da \qquad \{\text{from } (1)\}$$

The moment of this resisting force about neutral axis =

$$(E/R)\, y\, da \times y \quad = (E/R)\, y^2\, da$$

Total moment of resistance offered by the beam section,

$$M' = \int (E/R)\, y^2\, da$$

$$= E/R \int y^2\, da$$

$\int y^2\, da$ =second moment of the area =moment of inertia about the neutral axis.

$$\therefore M' = (E/R)\, I_{NA}$$

For equilibrium moment of resistance (M') should be equal to applied moment M

i.e. $M' = M$

Hence. We get $M = (E/R)\, I_{NA}$

$$(E/R) = (M/I_{NA}) \text{--------} (2)$$

From equation 1 & 2, $\boxed{(M/I_{NA}) = (E/R) = (\sigma_b/y)}$ ----

BENDING EQUATION.

(Bernoulli-Euler bending equation)

Where E= Young's modulus, R= Radius of curvature,
M= Bending moment at the section,

I_{NA}= Moment of inertia about neutral axis,

σ_b= Bending stress

y = distance of the fibre from the neutral axis

SECTION MODULUS:

$$(M/I)=(\sigma_b/y)$$

or $\sigma_b = (M/I)\,y$

Its shows maximum bending stress occurs at the greatest distance from the neutral axis.

Let y_{max} = distance of the extreme fibre from the N.A.

$\sigma_{b(max)}$ = maximum bending stress at distance y_{max}

$\sigma_{b(max)} = (M/I)\,y_{max}$

where M is the maximum moment carrying capacity of the section,

$M = \sigma_{b(max)}\,(I/y_{max})$

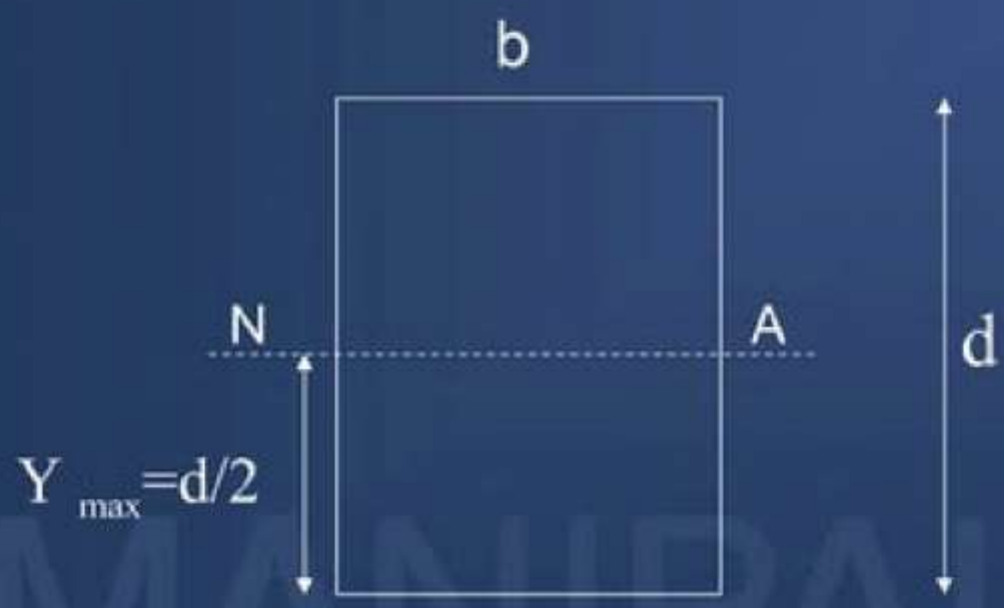
$$M = \sigma_{b(max)}\,(I/y_{max}) = \sigma_{b(max)}\,Z$$

Where $Z = I/y_{max}$ = section modulus (property of the section)

Unit ----- mm^3, m^3

section modulus

(1) Rectangular cross section

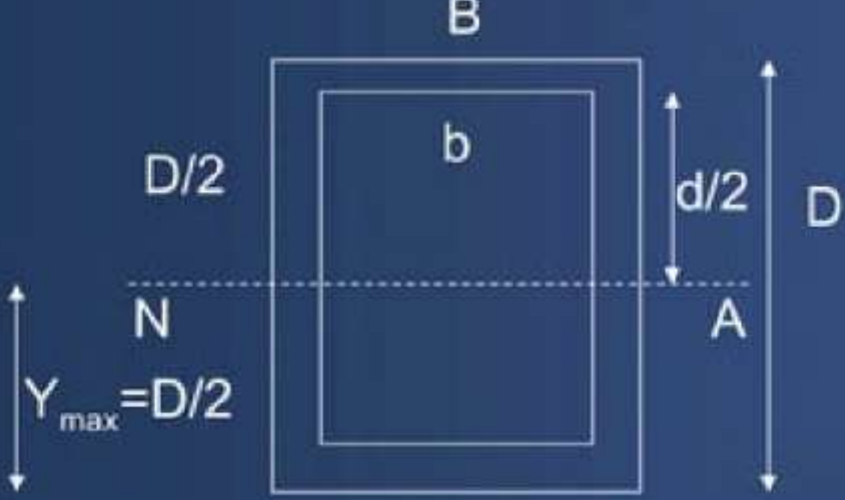


$Z = I_{NA} / y_{max}$

$= (bd^3/12) / d/2$

$= bd^2/6$

(2) Hollow rectangular section

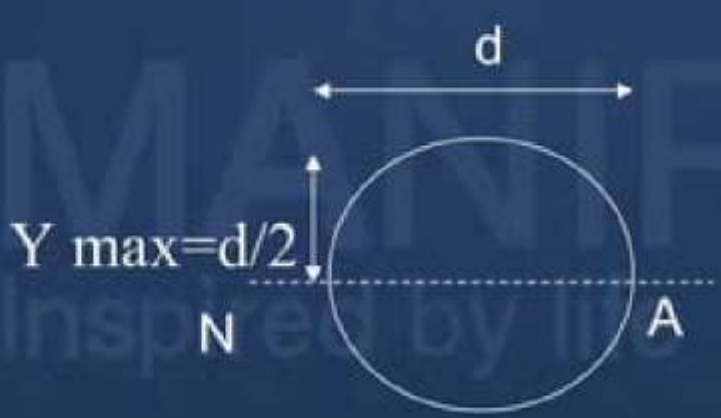


$$Z = I_{NA} / y_{max}$$

$$= 1/12(BD^3 - bd^3) / (D/2)$$

$$= (BD^3 - bd^3) / 6D$$

(3) Circular section

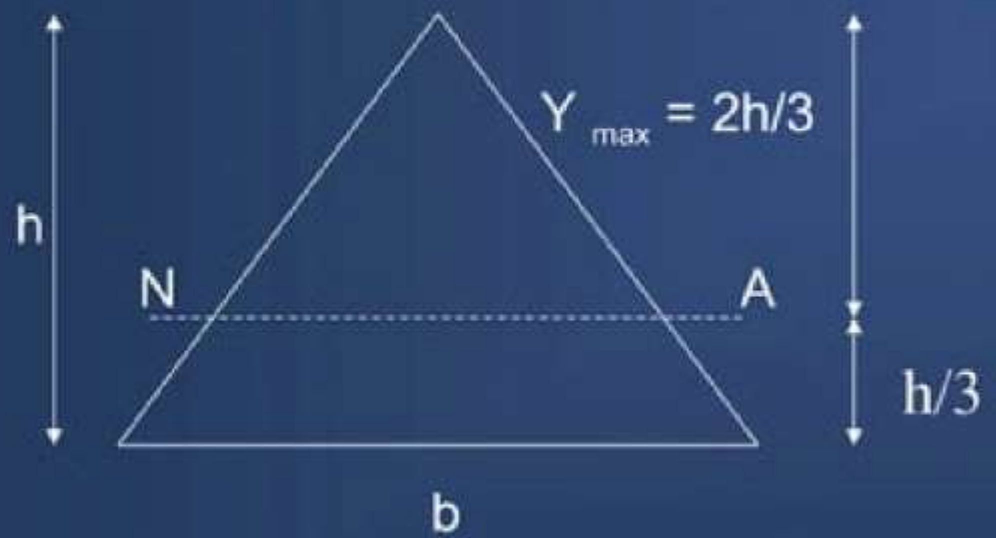


$$Z = I_{NA} / y_{max}$$

$$= (\pi d^4/64) / (d/2)$$

$$= \pi d^3/ 32$$

(4) Triangular section

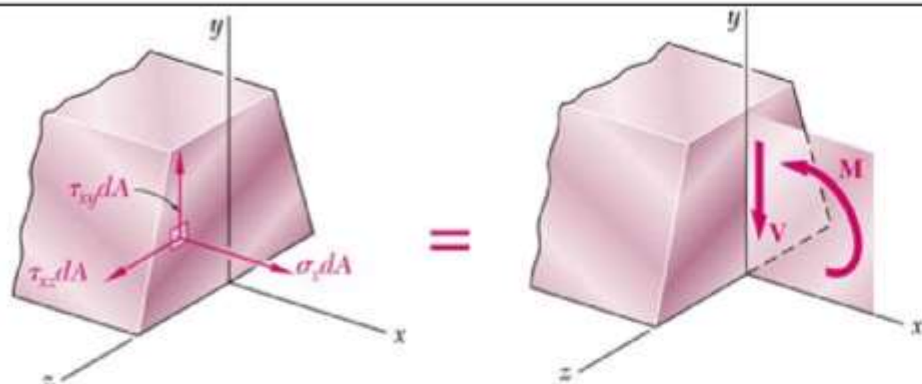


$$Z = I_{NA} / Y_{max}$$

$$= (bh^3 /36) / (2h/3)$$

$$= bh^2/24$$

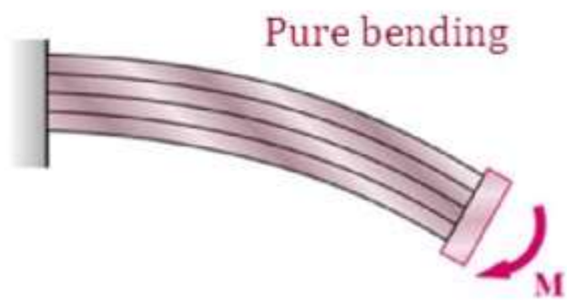
Shear Stress in Beams

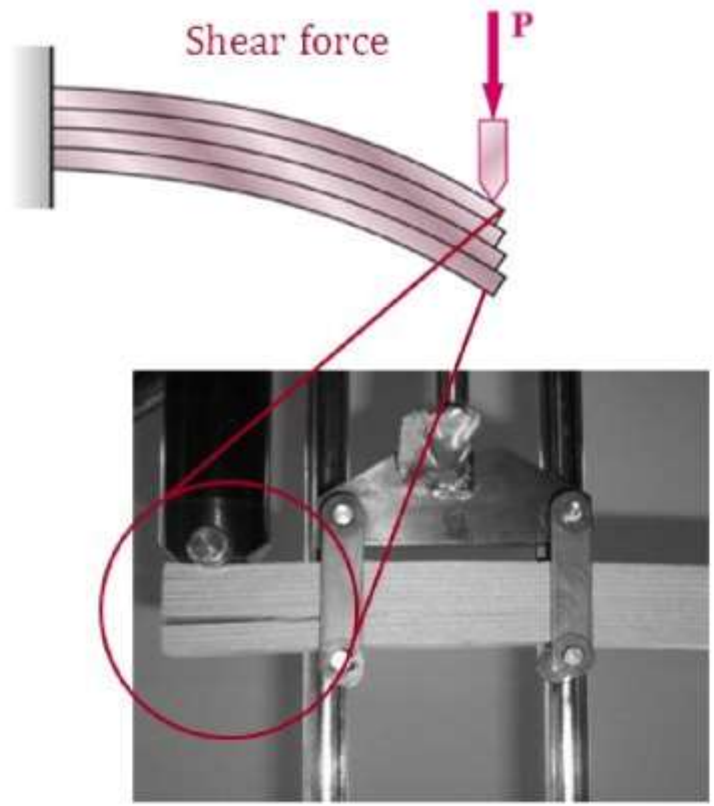


- Consider the effects of shear force (V).
- Already know how to find resulting axial force and moment due to stress σ_x from Chapter 4.
- We have two more equations for shear stress:
  - Total shear force in the y-direction: $\int \tau_{xy}\,dA = -V$
  - Total shear force in the z-direction: $\int \tau_{xz}\,dA = 0$

Shear Stress in Beams

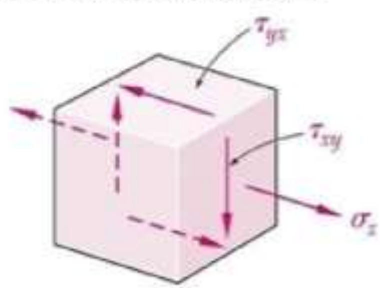
- Consider a cantilever beam composed of separate planks clamped at one end:





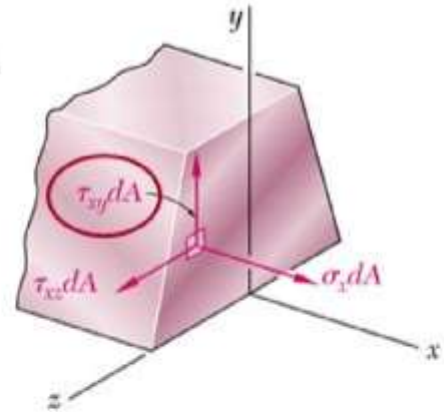
- Shear force causes tendency to "slide."

- Stresses are equal in horizontal and vertical directions.

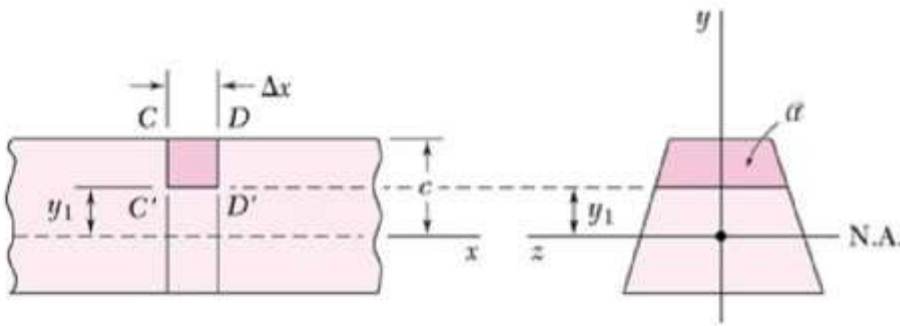


Shear Stress: Horizontal

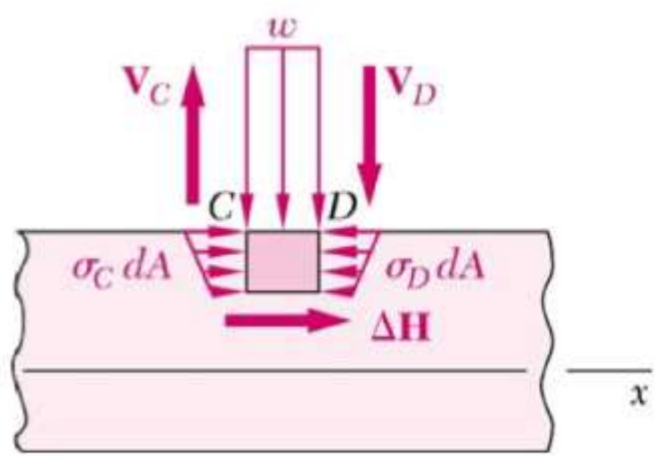
- Let us consider the horizontal component ($\tau_{yx} = \tau_{xy}$).



- Cut a section with cross-sectional area *a* at a distance y_1 above the centroid.



FBD →

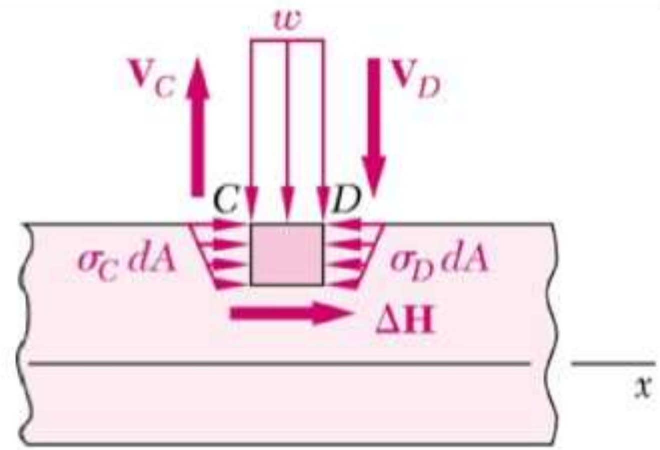


Shear Stress: Horizontal

- ΔH is the horizontal shearing force.
- Element width is Δx.
- Sum forces in x-direction:

$$\sum F_x = \Delta H + \int_a (\sigma_C - \sigma_D) dA = 0$$

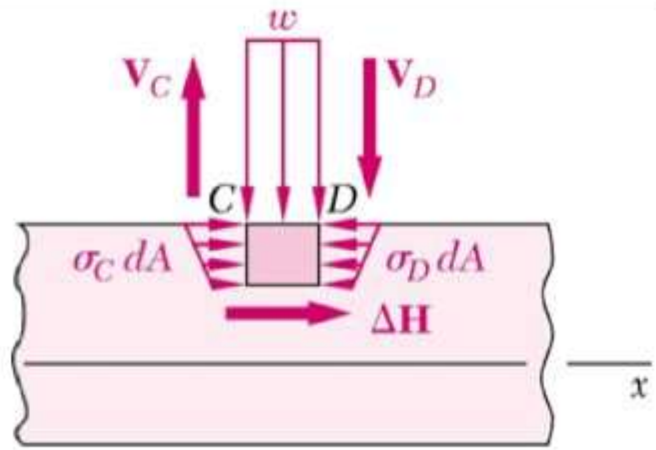
- Recall from chapter 4: $|\sigma| = \dfrac{My}{I}$



- Solve for ΔH and use equation for σ:

$$\Delta H = \int_a (\sigma_D - \sigma_C) dA = \int_a \left(\frac{M_D - M_C}{I} \right) y dA = \left(\frac{M_D - M_C}{I} \right) \int_a y dA$$

Shear Stress: Horizontal



- Recall first moment, Q, is defined as:

$$Q = \int_a y\,da$$

- The term M_D-M_C can be rewritten as:

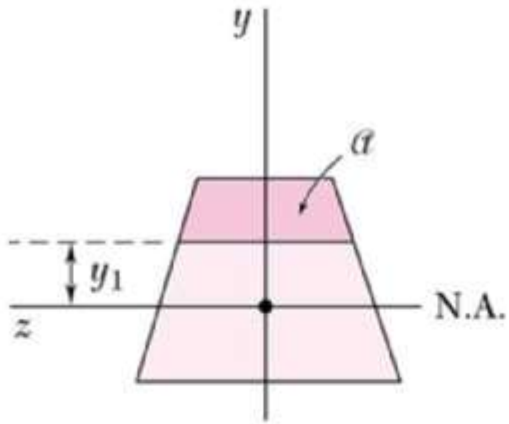
$$M_D - M_C = \Delta M = \frac{dM}{dx}\Delta x = V\Delta x$$

- Applying this to our equation for ΔH: $\boxed{\Delta H = \frac{VQ}{I}\Delta x}$

- We can rearrange this to define horizontal shear per unit length, q, called shear flow.

$$\boxed{q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I}}$$

Side Note on Q



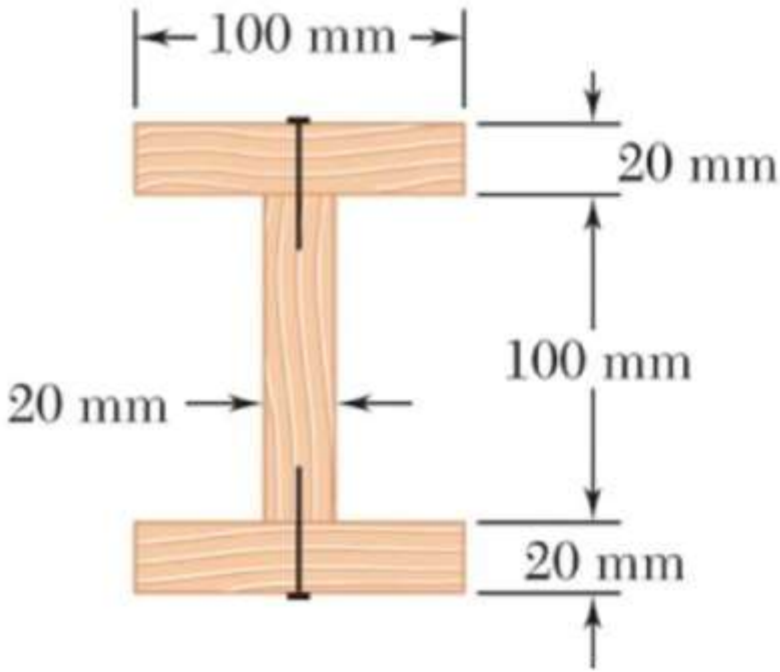
- Q is the definition of the first moment for the area above y_1 with respect to the x-axis (see Appendix A in textbook),

$$\boxed{Q = \int_a y\,dA = a\bar{y}}$$

where y bar is the distance between the centroid of the shaded section and the centroid of beam cross-section.

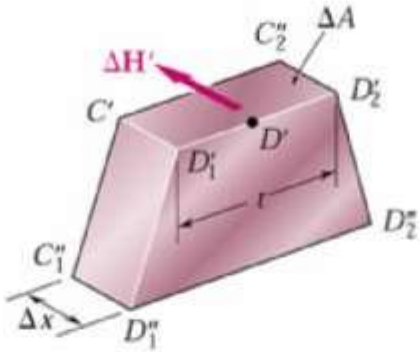
Example Problem

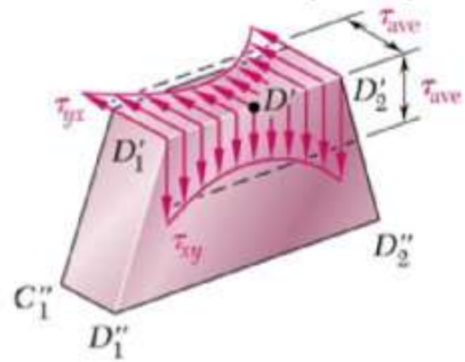
- A beam is made of three planks, 20 by 100 mm in cross-section, nailed together. Knowing that the spacing between nails is 25mm and that the vertical shear in the beam is V = 500 N, determine the shearing force in each nail.



Shear Stress: Vertical

- Now, let us consider the vertical component ($\tau_{xy} = \tau_{yx}$).





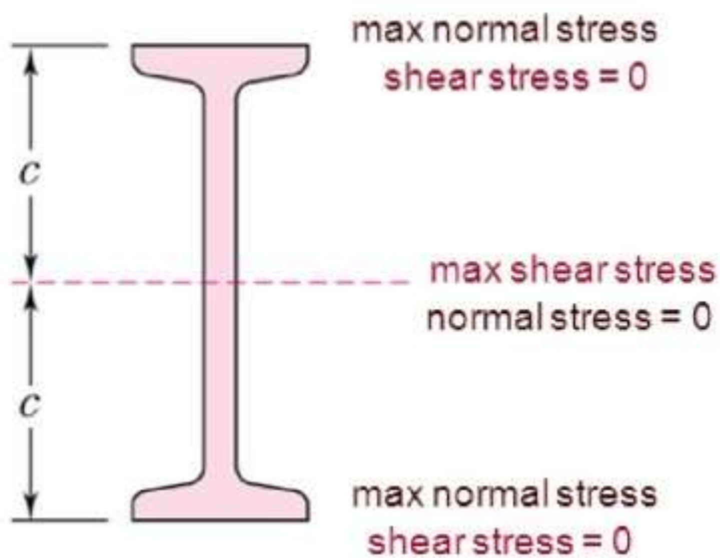
- We can calculate the average vertical shear stress on the cross-section.

$$\tau_{AVE} = \frac{\Delta H}{\Delta A} = \left(\frac{VQ}{I}\Delta x\right)\left(\frac{1}{t\Delta x}\right) = \frac{VQ}{It} = \tau_{AVE}$$

$$\boxed{\tau_{AVE} = \frac{VQ}{It}}$$

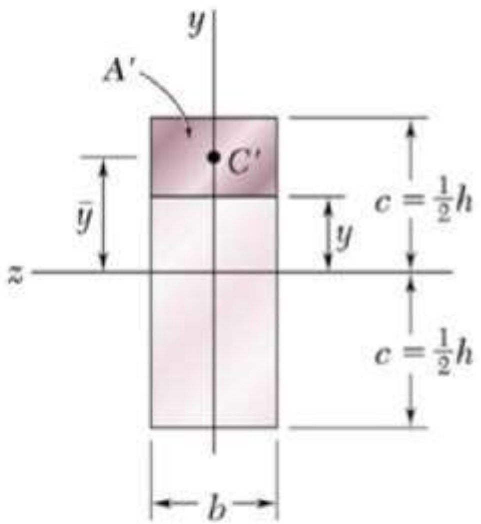
Shear Stress: Vertical

- So, where is τ_{AVE} maximum and minimum?
  - Use Q to find out.
  - $Q = 0$ at top and bottom surfaces
  - $Q =$ maximum somewhere in between



Shearing Stress in Common Shapes

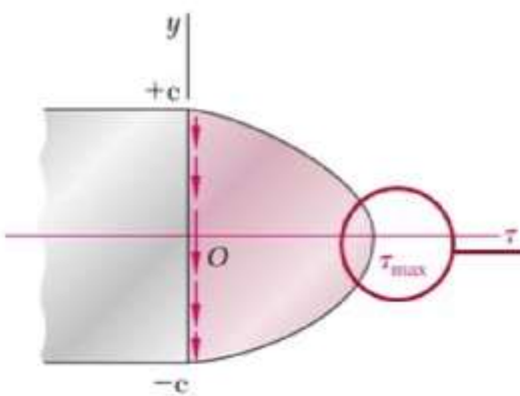
- Rectangular cross-section



$$Q = A\bar{y} = b(c-y)\frac{1}{2}(c+y) = \frac{1}{2}b\left(c^2 - y^2\right)$$

$$\tau_{xy} = \frac{VQ}{Ib} = \frac{V}{\left(b(2c)^3/12\right)}\frac{b\left(c^2-y^2\right)}{2b} = \frac{3V}{4bc^3}\left(c^2-y^2\right)$$

$$\boxed{\tau_{xy} = \frac{3V}{2A}\left(1-\frac{y^2}{c^2}\right)}$$

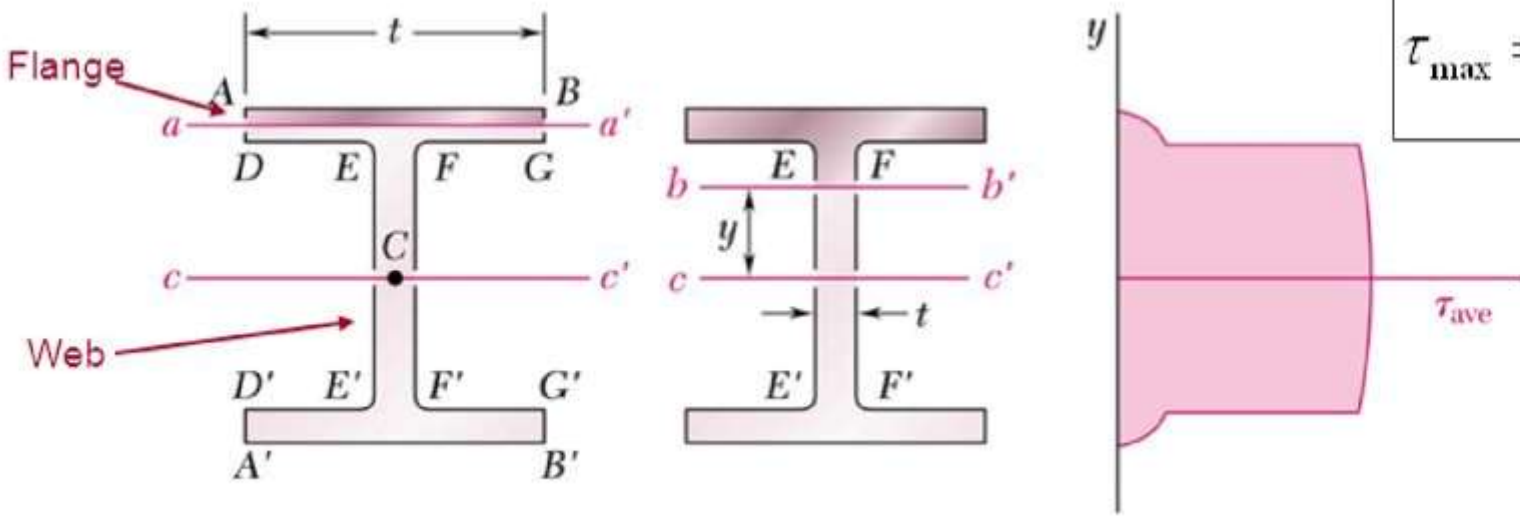


$$\boxed{\tau_{max} = \frac{3V}{2A}}$$

Shearing Stress in Common Shapes

- Beams with flanges
  - Vertical shear stresses are larger in the web than in the flange.
  - Usually only calculate the values in the web.
  - Ignore the effects of the small fillets at the corners.
  - Flanges have large horizontal shear stresses, which we will learn how to calculate later on.

$$\tau_{\max} = \frac{V}{A_{web}}$$



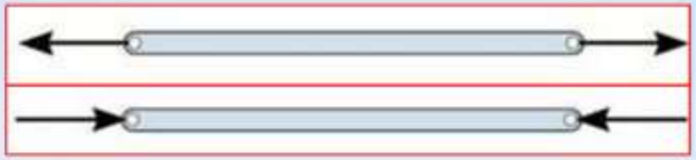
(a) (b) (c)

Trusses

Truss: is a structure composed of <u>slender members</u> (two-force members) joined together at their <u>end points</u> to support stationary or moving load.

❖ Each member of a truss is usually of uniform cross section along its length.

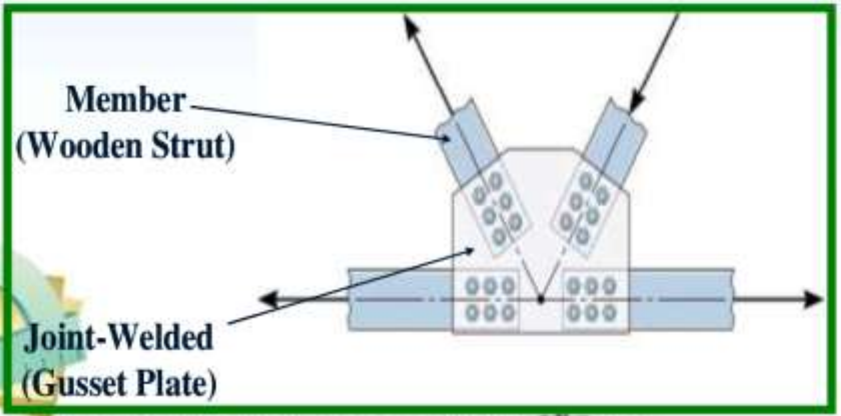
Calculation are usually based on following assumption:

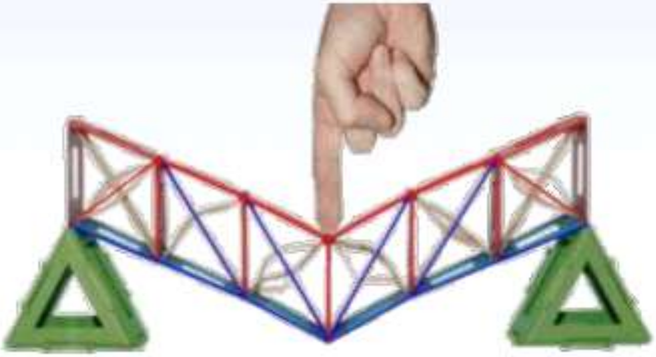


- The loads and reactions act only at the joint.
- Weight of the individual members can be neglected.
- Members are either under tension or compression.

<u>Joints:</u> are usually formed by <u>bolting</u> or <u>welding</u> the members <u>to</u> a common plate, called a gusset plate, or simply passing a large bolt through each member.

- ***Joints*** are modeled by <u>smooth pin connections</u>.





Analysis of Trusses

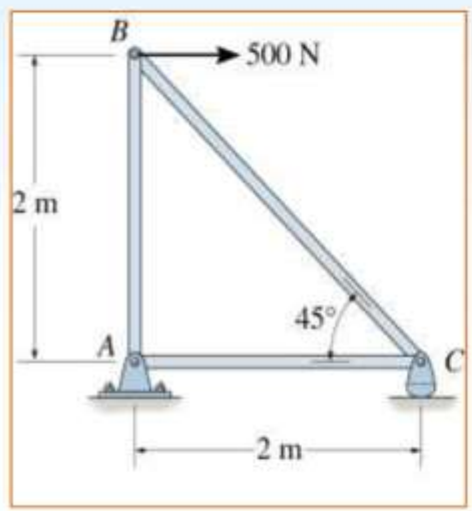
Truss Analysis

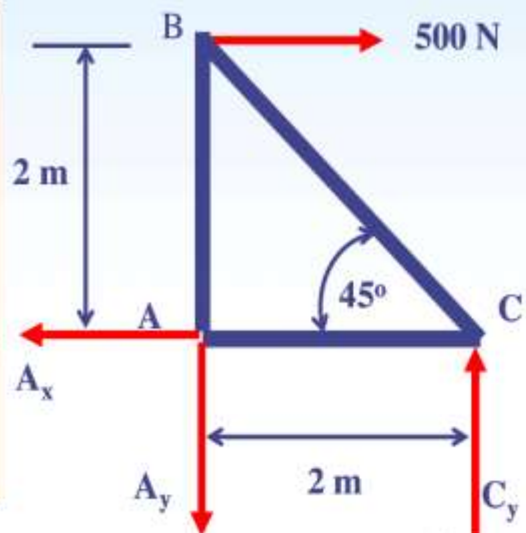
- **External equilibrium** → **To find the reaction forces**
- **Internal equilibrium** → **To find the force in each member**
  - **Method of joints**
  - **Method of sections**

External Equilibrium: to find the ***reaction forces***, follow the below steps:

1. Draw the ***FBD*** for the entire truss system.
2. Determine the ***reactions***. Using the equations of (2 D) which states:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_o = 0$$





Analysis of Trusses

Method of Joints: to find the *forces* in any *member*, choose a ***joint***, to which that member is connected, and follow the below steps:

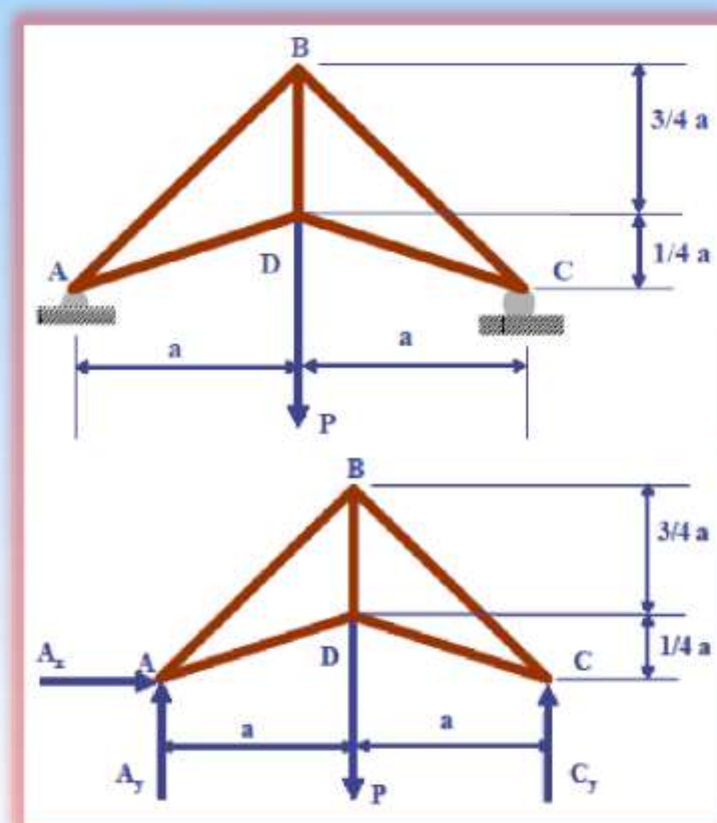
1. Draw the ***FBD*** for the entire truss system.
2. Determine the ***reactions***. Using the equations of (2 D) which states:

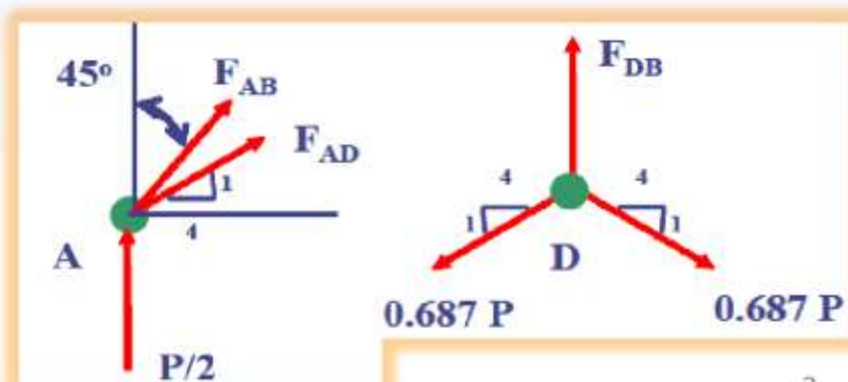
$$\boxed{\sum F_x = 0, \;\; \sum F_y = 0, \;\; \sum M_o = 0}$$

3. Choose the joint, and draw ***FBD*** of a ***joint*** with at least ***one known force*** and at most *two unknown forces*.
4. Using the equation of (**2 D**) which states:

$$\boxed{\sum F_x = 0, \;\; \sum F_y = 0}$$

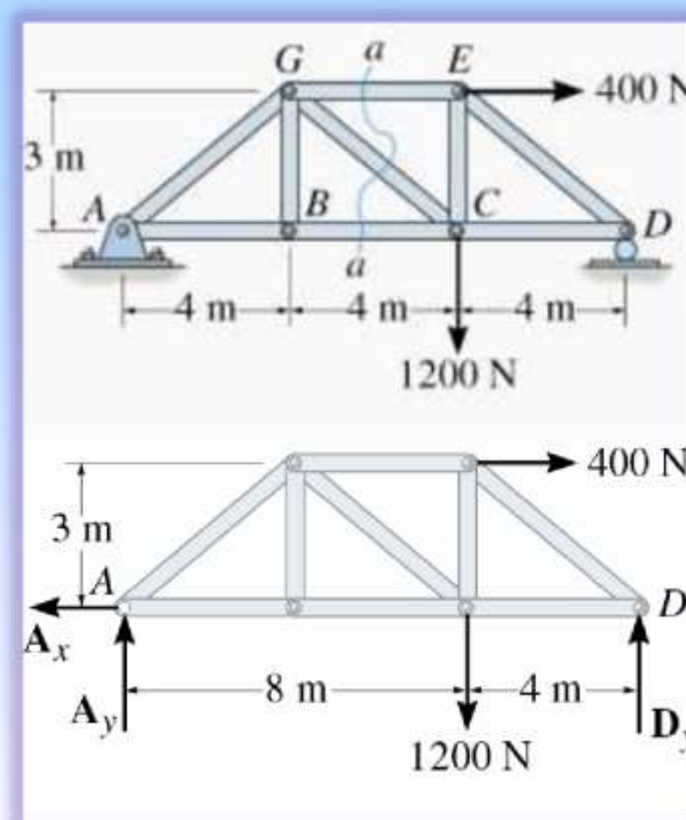
5. The *internal forces* are determined.
6. Choose another ***joint***.





Analysis of Trusses

Method of section (Internal equilibrium): to find the ***forces*** in any *member*, choose a ***section***, to which that *member* is appeared as an internal force, and follow the below steps:



1. Draw the ***FBD*** for the entire truss system.
2. Determine the ***reactions***. Using the equations of (**2 D**) which states:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_o = 0$$

3. Choose the ***section***, and draw ***FBD*** of that *section,* shows how the forces replace the sectioned members.
4. Using the equation of (**2 D**) which states:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_o = 0$$

5. The *internal forces* are determined.
6. Choose another *section* or *joint*.