INTRODUCTION TO OPERATIONS RESEARCH

- The OR technique was initiated in England during II world war to solve strategic and tactical military problems.
- Then British and American military also started the same technique for effective utilization of military resources.
- Specialists in engineering, mathematics, economics, psychology, statistics and physical science were formed as group to do research on military problems.
- The same technique was spread to United states, France and Canada.
- After the succession of world war these teams were shifted towards industry and business sector to solve complex managerial problems.

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- In 1950 many colleges and universities introduced OR as a subject at degree level.
- In 1948 OR club was formed in England, later its name was changed as OR society of U.K.
- OR society of America was formed in 1950.
- In 1957 International federation of OR society was established with headquarters in London.
- In India OR unit was established in 1949 at Regional research laboratory Hyderabad for planning and organizing research.
- In 1953 an OR unit was established in the Indian statistical Institute, Calcutta for application of OR .methods in national planning and survey.

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- OR society of India (ORSI) was formed in 1957. It became the member of International federation of OR societies in 1959.
- In India Prof Mahalonobis made first application of OR to forecast trends of demand, availability of resources and for scheduling the complex schemes for developing the country’s economy.
- Today OR techniques are being used in various fields such as hospitals, libraries, city planning, transportation systems, crime investigation and industries.
- Some of Indian organizations using OR technique are: Indian Airlines, Railways, Defence, Tata Iron and Steel, Delhi Cloth mills, Fertilizer corporation of India.

Definitions:

- "O.R utilizes the planned approach and an interdisciplinary team in order to represent complex functional relationships as mathematical models for the purpose of providing a quantitative analysis". -- Thierauf and Klekamp.
- "O.R is a scientific approach to problem solving for executive management". –H.M.Wagner
- " O.R is a scientific knowledge through interdisciplinery team effort for the purpose of determining the best utilization of limited resources. – H.A.Taha
- OR is the art of winning wars without actually fighting them. – Auther Clark
- OR is the art of finding bad answers to problems to which otherwise worse answers are given. ---T.L.Saaty
- "O.R is essentially a collection of mathematical techniques and tools which in conjunction with system approach, are applied to solve practical decision problems of an economic or engineering nature". --- Daellenbach and George

Characteristics or significant features of O.R

- Decision making tool: Optimal solution to the problem.
- Scientific approach: Scientific methods, techniques and tools for analyzing and solving the problems.
- Interdisciplinary team approach: Different discipline members team analyze the cause and effect relationship between various parameters of the problem.
- System approach or wholistic approach: Indirect effect on all other parts is considered as a whole.
- O.R largely depends on digital computer.
- Objective: Locate the best or optimal solutions to the problem.

Models in O.R

- A model is a theoretical abstraction of a real life problem.

❖ Models are classified based on different criteria:

➢ Based on purpose:

1. Descriptive models: Describe some aspects based on observations, questionnaires etc. ex: Opinion poll
2. Predictive models: By answering what if type of questions can make predictions.
3. Prescriptive models(normative model): Predictive model repeatedly successful.

➢ Based on structure:

1. Iconic models: Represents as it is by scaling up or down.
2. Analogue models: One set of properties is used to represent another set of properties.
3. Symbolic models:

a) verbal models:

b) Mathematical models:

➢ Nature of environment:

i. Deterministic models:

ii. Probabilistic models:

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- Based on time response:

1. Static models:
2. Dynamic models:

- Based on method of solution:

1. Analytical models:
2. Heuristic models:
3. Simulation models:

General methods for solving O.R models

- Analytical Method: Mathematical tools like calculus, matrix, algebra, probability, graphs etc. are used for solving O.R problems.
- Numerical or Iterative method: when analytical method fail to solve complex problems numerical methods are used. Numerical methods involve trail and error procedure. An iterative process is one in which successive trails tend to approach an optimal solution.
- Monte-Carlo simulation : use of probability and random sampling concepts.

Phases of O.R (or) Methodology of O.R

- Formulate the problem.
- Construct the model.
- Acquire input data.
- Solve the model.
- Test the solution.
- Implement the solution.
- Modifying the model.
- Establishing control over the solution.

Scope of O.R

- O.R approach enables the executives to obtain best solution to any problem simple or complex, which will serve the interest of the organization as a whole.
- O.R compels the business managers to be quite explicit about their objectives, assumptions, and their visualization to constraints.
- While using O.R approach the manager has to consider all the variables which influence the decision and the manner these variables interact with each other.

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- The decision maker can conduct the experiments on the model by changing various conditions, to examine the effect of these changes and determine the best solution.
- The experiments can be conducted on the model causing any serious damage to the existing system.
- Computer can be used for obtaining solution, hence it is possible to solve complex problems.
- It quickly point out gaps in the data required to support workable solution to the problem.

Shortcomings of O.R

- Constructing complex O.R models is economical only for repetitive types of problems.
- Limitations on resources like money, time etc.
- Some times O.R models may not represent the realistic situations in which decisions have to be taken.
- Some times the basic data are subject to frequent changes which make the modification of O.R models quite expensive.

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- Many times the decision maker may not fully aware of the limitations of O.R model that he is using.
- One of the strongest limitations of O.R is the inability to transmit and communicate results to business executives who have to make decisions.

➢ Features of O.R solutions:

I. Economical:
II. Technically appropriate:
III. Reliable:
IV. Behaviorally appropriate:

Applications of O.R

1. Defence operations:

I. Tactical planning for requirements and use of weapon system.

II. Allocation of scarce resources to various defense operations.

III. To coordinate and integrate the activities of number of independent components (air force, army and navy) which gives maximum benefit to the military organization as a whole.

2. Industry and business:

I. Integrating the diversified activities of various sections so as to serve effectively the interest of the organization as a whole.

II. Forecasting the man power requirements ,assignment of jobs to men or machines, selection of suitable personal with due consideration of age and skills etc.

III. Determination of optimum number of persons for each service centre.

IV. Inventory control: Determination EOQ , production lot size, bidding policies and vendor analysis etc.

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V. Product mix, marketing and export planning
VI. Designing organization structures more effectively.
VII. Cash flow analysis, divided policies, investment decisions, long term capital requirements, auditing credit policies, claims and complaints procedures etc.
VIII. Scheduling and sequencing the production run by proper allocation of machines.
IX. Selection of location and sites for the production plant, location and size of warehouse.
X. Product selection, equipment selection.
XI. Maintenance policy and preventive maintenance.
XII. Sales forecasting.

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XIII. Distribution strategies, transportation planning.

XIV. Effectiveness of market research, selection of effective packing alternatives.

XV. Selection of advertising media with respect to cost and time.

XVI. To find optimum number of salesmen.

XVII. Replacement policies.

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3. **Government:** A government can use O.R for framing economic planning, planning of natural resources, social policies and energy planning.
4. **Agriculture:** O.R can be used to increase output by using the scarce resources in the best possible manner.
5. **Hospitals:**

I. To solve queuing problems in big hospitals

II. To solve administrative problems of hospital organization.

6. **Research and development:**

I. Reliability and economy of alternative designs.

II. Control of development projects.

III. Coordination of multiple research projects etc.

Techniques used in O.R

1. Linear programming.
2. Dynamic programming.
3. Queuing theory.
4. Inventory theory.
5. CPM and PERT techniques.
6. Game theory.
7. Replacement models.
8. Sequencing models.
9. Decision analysis models.
10. Simulation.

Linear programming

- Linear programming is one of the O.R technique.
- Linear programming is a mathematical technique for the purpose of allocating the limited resources in an optimum manner to achieve the objectives of the business, which may be maximum overall profit or minimum overall cost.
- It attempts to maximize or minimize a linear function of decision variables.
- The values of the decision variables are selected in such a way that they satisfy a set of constraints, which are in the form of linear inequality.

Terminology

1. Decision variables: The variables whose values are to be found so as to optimize the objective function.
2. Objective function: It states the determinants of the quantity either to be maximized or to be minimized.
3. Constraints: These are the restrictions imposed on decision variables. These may be in terms of availability of raw materials, machine hours, man hours etc.
4. Non negative condition: Each variable either zero or positive.
5. Linear relationship: It implies straight line or proportional relationship among the relevant variables.
6. Feasible solution : All such solutions which satisfy the constraints are called feasible solution, region comprising such solution is called feasible region.
7. Optimum solution: Optimum solution is the best of all feasible solution.
8. Formulation of LP model: Translating a problem in to a format of mathematical equation.

Assumptions in Linear programming

- There is a well defined objective function such as maximizing profit or minimizing cost.
- There are a number of restrictions
- The parameters are subject to variation in magnitude.
- The relationship expressed by constraints and the objective function are linear.
- The objective function is to be optimized w.r.t the decision variables involved in the phenomenon. The decision variables are non negative and represent real life situation.

Advantages of LP

- LP helps the management to make effective utilization of limited production resources.
- LP improves the quality of decision making by replacing rules of thumb.
- It provides feasibility in analyzing a variety of multi dimensional problems.
- It highlights the bottleneck occurs in a manufacturing shop, some machines may be overloaded while other remains idle. LP can strike optimal balance between the two situations.
- It helps in re-evaluation of the basic plan to meet changing conditions in the business.

Disadvantages of LP

- The assumptions that all relations are linear may not hold good in many real situations.
- In LP all coefficients and constraints are stated with certainty
- The solution many time is in fractions which may not remain optimal when rounded off.
- When the number of variables or constraints involved are quite large then it becomes necessary to use computers.
- It deals with only a single objective problems, where as in real life situations there may be more than one conflicting objectives.

Applications of LP

- Product mix
- Production planning
- Assembly line balancing
- Blending problem
- Media selection
- Travelling salesman problem
- Physical distribution
- Staffing problem
- Job evaluation and selection
- Agriculture
- Military
- Routing problems

Problem

A company is manufacturing two different types of products A and B. Each product has to be processed on two machines M1 and M2 . Product A requires 2 hours on machine M1 and 1 Hour on machine M2 . Product B requires 1 hour on Machine M1 and 2 hours on machine M2 . The available capacity of machine M1 is 104 hours and that of machine M2 is 76 hours. Profit per unit for the product A is Rs 6 and that for B is Rs 11.

Formulate the problem.

Solution:

	M1	M2	profit (Rs)
A	2	1	6
B	1	2	11
Availble hrs	104	76	

Objective function $\quad$ Max $Z = 6x+11y$

M1 constraint $\quad 2x+ y \leq 104$

M2 constraint $\quad x + 2y \leq 76$

$$x \geq 0, y \geq 0$$

Problem

The Xeon company owns a small paint factory that produces both interior and exterior house paints for wholesale distribution. Two basic raw materials, A and B, are used to manufacture the paints. The maximum availability of A is 6 tons a day that of B is 8 tons a day. The daily requirements of the raw materials per ton of interior and exterior paints are summarized in the following table.

Raw material	Tons of raw material per ton of paint	
	Exterior	Interior
Raw material A	1	2
Raw material B	2	1

A market survey has established that the daily demand for interior paint cannot exceed that of exterior paint by more than one ton. The survey also shows that the maximum demand for interior paint is limited to 2 tons daily. The wholesale price per ton 1s Rs.3000 for exterior paint and Rs.2000 for interior paint. Formulate the problem as LPP to determine number of units of interior and exterior paints should the company produce daily to maximize gross income.

Solution:

Raw material	Tons of raw material per ton of paint		
	Exterior	Interior	availability
Raw material A	1	2	6
Raw material B	2	1	8
Demand (tons)	1	2	
Price (Rs)	3000	2000	

Objective function Max $Z = 3000x+2000y$

A constraint $x+2y \le 6$

B constraint $2x + y \le 8$

Exterior constraint $x \le 1$,

Interior constraint $y \le 2$

$x \ge 0, \ y \ge 0$

Problem:

- A firm manufactures headache pills in two sizes A and B. Size A contains 2 grains of aspirin, 5 grains of bicarbonate and 1 grain of codeine. Size B contains 1 grain of aspirin, 8 grains of bicarbonate and 6 grains of codeine. It is found by users that it requires at least 12 grains of aspirin, 74grains of bicarbonate and 24 grains of codeine for providing immediate effect. It is required to determine the least number of pills a patient should take to get immediate relief. Formulate the problem as a standard LPP.

Problem:

- Three food products are available at costs of Rs 10,Rs 36 and Rs 24 per unit respectively. They contain 1000, 4000 and 2000 calories per unit respectively, and 200, 900,and 500 protein units per unit respectively. Required is the minimum cost diet containing at least 20000 calories and 3000 units of protein. Formulate the problem as a standard LPP.

Problem:

- The ABC company combines factors X and Y to form a product which must weigh 50 kgs. At least 20 kgs of X and no more than 40 kgs of Y can be used. The cost of X is Rs 25 per kg, and that of Y is Rs 10 per kg. To find the amounts of X and Y formulate the problem as LPP.

Problem:

- M/S ABCL company manufactures two types of cassettes, a video and an audio . Each video cassette takes twice as long to produce one audio cassette and the company would have time to make a maximum of 2000 per day if it is produced only audio cassettes. The supply of plastic is sufficient to produce 1500 per day of both audio and video cassettes combined. The video cassette requires a special testing and processing of which there are only 600 per day available. If the company makes a profit of Rs 3 and RS 5 per audio and video cassettes respectively, how many of each should be produced per day in order to maximize the profit.

Problem:

- A mining company is taking a certain kind of ore from two mines X and Y. The ore is divided in to three quality groups A,B and C. Every week the company has to supply 240 tons of A 160 tons of B and 44o tons of C. The cost per day for running the mine X is Rs 3000 while it is Rs 2000 for the mine Y. Each day X will produce 60 tons of A, 20 tons of B and 40 tons of C. The corresponding figures for Y are 20, 20 and 80. Develop the most economical production plan for finding the number of days for which the mines X and Y should work per week.

Problem:

- A person requires 10, 12 and 12 units of chemicals A, B and C respectively for his garden. A liquid product contains 5, 2 and 1 units of A, B and C respectively per jar. A dry product contains 1, 2 and 4 units of A , B and C per carton. If the liquid product sells for Rs 3 per jar and the dry product sells for Rs 2 per carton. How many of each should purchase to minimize the cost and meet the requirements? Formulate the above problem as LPP.

Problem:

- A company has two bottling plants. One located at Banglore and other at Mysore. Each plant produces 3 brands of drinks A,B and C. Banglore plant can produce (in one day) 1500 , 3000 and 2000 bottles of A, B and C respectively. The capacity of Mysore plant remains 1500,1000, and 5000 bottles per day of A, B and C respectively. A market survey indicates that during the month of April, there will be demand of 20000 bottles of A , 40000 bottles of B, and 44000 bottles of C. The operating cost per day for the plant at Banglore is Rs 600 . While the operating cost per day for the plant at Mysore is Rs 400 . For how many days each plant be run in April so as to minimize the production cost while still meeting the demand.

Problem:

Solve the following LPP by graphical method.

Max $Z = 2x_1 + 3x_2$

Subjected to constraints

$x_1 + x_2 \leq 1$

$3x_1 + x_2 \leq 4$

$x_1 \geq 0, x_2 \geq 0$

Problem:

Solve the following LPP by graphical method.

$\text{Max } Z = 5x_1 + 3x_2$

Subjected to constraints

$3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1 \geq 0, x_2 \geq 0$

Problem:

Solve the following LPP by graphical method.

Max $Z = 5x_1 + 3x_2$

Subjected to constraints

$x_1 + x_2 \leq 6$

$2x_1 + 3x_2 \geq 3$

$0 \leq x_1 \geq 3$,

$0 \leq x_2 \geq 3$

Problem:

Solve the following LPP by graphical method.

Min $Z = 1.5x_1 + 2.5x_2$

Subjected to constraints

$x_1 + 3x_2 \geq 3$

$x_1 + x_2 \geq 2$

$x_1 \geq 0$, $x_2 \geq 0$

Problem:

- Old hens brought at Rs2 each and young ones at Rs 5 each. The old hen lays 3 eggs per week and young ones lay 5 eggs per week, each egg being 30 paise worth. A hen (young or old) cost Re 1 per week to feed. I have only Rs 80 to spend for hens, how many of each kind I should buy to give a profit of more than Rs 6 per week. Assuming that I can not house more than 20 hens.
- Solution:

Max $Z= 0.3(3x_1+5x_2) - (x_1+x_2) = 0.5x_2 - 0.1x_1$

Subjected to constraints

$2x_1+5x_2 \leq 80$

$x_1+x_2 \leq 20$

$0.5x_2 - 0.1x_1 \geq 6$

$x_1 \geq 0 , x_2 \geq 0$

Problem:

Solve the following LPP by graphical method.

Max $Z= 8000x_1+7000x_2$

Subjected to constraints

$3x_1+x_2 \leq 6$

$x_1+x_2 \quad \leq 45$

$x_1 \leq 20$

$x_2 \leq 40$

$x_1 \geq 0$, $x_2 \geq 0$

Ans : x1=0, x2= 6 z=42000

Problem:

Solve the following LPP by graphical method.

Max $Z = 3x_1 - 2x_2$

Subjected to constraints

$x_1 + x_2 \leq 1$

$2x_1 + 2x_2 \geq 4$

$x_1 \geq 0$, $x_2 \geq 0$

Ans : Infeasible solution

Problem:

Solve the following LPP by graphical method.

Max $Z = x_1 + x_2$

Subjected to constraints

$x_1 + 2x_2 \leq 2000$

$x_1 + x_2 \leq 1500$

$x_2 \leq 600$

$x_1 \geq 0$, $x_2 \geq 0$

Ans : $x_1 = 1000$, $x_2 = 500$ $Z = 1500$

Problem:

Solve the following LPP by graphical method.

$\text{Max } Z = 5x_1 + 3x_2$

Subjected to constraints

$3x_1 + 5x_2 = 15$

$5x_1 + 2x_2 = 10$

$x_1 \geq 0 , x_2 \geq 0$

Ans : $x_1 = 20/19$ $x_2 = 45/19$

Problem:

Solve the following LPP by graphical method.

Max $Z = 3x_1 + 2x_2$

Subjected to constraints

$x_1 - x_2 \leq 1$

$x_1 + x_2 \geq 3$

$x_1 \geq 0 , x_2 \geq 0$

Ans : Unbounded solution

Simplex Method

- The Linear Programming with two variables can be solved graphically.
- If the linear programming problem has larger number of variables, the suitable method for solving is Simplex Method.
- The simplex method is an iterative process, through which it reaches ultimately to the minimum or maximum value of the objective function.

Terminology

- Slack variable: The variables are added to less than or equal to constraints to convert in to equality.
- Surplus variable: The variables are subtracted from greater than or equal to constraints to convert in to equality.
- Artificial variable: To avoid mathematical inconvenience (to avoid negative variable) some variable is added to constraints is known as artificial variable.

Simplex Method Procedure

- RHS of all constraints should be positive. If not convert in to positive.
- Convert all constraints to equality type by adding suitable variables.
- All Constraints write in the standard form. Standard form is nothing but all Constraints and objective function contains all the variables.
- Find initial basic feasible solution (IBFS) by putting (n-m) number of variables are zero in the order (x, S, A).
- Where n= no of variables

  m= no of constraints

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Draw the simplex table

		C_j					
C_B	Basic Variables	XB or Solution	x_1	x_2	s_1	s_2	Ratio
Z_j							
$C_j - Z_j$							

Contd…

- If all $(C_j – Z_j)$ values are less than or equal to zero , then the solution is optimal.
- If all $(C_j – Z_j)$ values are not less than or equal to zero , then the solution is not optimal.
- Then determine key column (In max case max value of $(C_j – Z_j)$ column taken as key column).
- Calculate ratio values.

$$\text{Ratio} = \frac{\text{solution}}{\text{corresponding key column element}}$$

Cond…

- The min ratio value taken as key row. (neglect negative value, zero and infinity)
- Key element is taken as intersection element of key row and key column.
- The variable corresponding to pivot column comes in to next simplex table and corresponding pivot row element goes out from simplex table.
- Draw the next simplex table with coefficients calculation as follows:

a) Divided by pivot row elements with pivot element will get elements in new table.

b) other than pivot row elements are calculated as

$$\text{New element} = \text{old element} - \frac{\text{CPRE X CPCE}}{\text{key element}}$$

Now calculate ($C_j - Z_j$) values for all columns, if all values are negative or zero then the solution is optimal.

Otherwise repeat the procedure until all ($C_j - Z_j$) values are negative or zero

Problem:

Solve the following LPP by using simplex method.

$$\text{Max } Z = 12x_1 + 15x_2$$

Subjected to constraints

$$4x_1 + 3x_2 \leq 12$$

$$2x_1 + 5x_2 \leq 10$$

$$x_1 \geq 0, \; x_2 \geq 0$$

Problem:

Solve the following LPP by using simplex method.

$\text{Max } Z = 2x_1 + 4x_2 + x_3 + x_4$

Subjected to constraints

$x_1 + 3x_2 + x_4 \leq 4$

$2x_1 + x_2 \leq 3$

$x_2 + 4x_3 + x_4 \leq 3$

$x_1 \geq 0 , x_2 \geq 0, x_3 \geq 0 , x_4 \geq 0$

Problem:

Solve the following LPP by using simplex method.

Min $Z = 2x_1 + 3x_2$

Subjected to constraints

$x_1 + x_2 \geq 5$

$x_1 + 2x_2 \geq 6$

$x_1 \geq 0$, $x_2 \geq 0$,

-M for max case and M for min case as coefficients of artificial variables in objective function.

Problem:

- An animal feed company must produce 200 kg of a mixture consisting of ingredient x_1 and x_2 daily. X_1 cost Rs 3 per kg and x_2 Rs 8 per kg. No more than 80 kgs of x_1 and at least 60 kgs of x_2 must be used. Find how much of each ingredient should be used if the company wants to minimize the cost.

Problem:

Solve the following LPP by using simplex method.

Max $Z = 4x_1 + 5x_2 - 3x_3$

Subjected to constraints

$x_1 + x_2 + x_3 = 10$

$x_1 - x_2 \geq 1$

$2x_1 + 3x_2 + x_3 \leq 30$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

Two Phase Method

- The problem solved in two phases.
- Phase-I: Write new objective function with only artificial variables. Ex: Max $W = -A_1 - A_2$

  constraints $a_{11} x_1 + a_{12} x_2 - s_1 + A_1 = b_1$

  $$a_{21} x_1 + a_{22} x_2 - s_2 + A_2 = b_2$$

  $$x_1, x_2, s_1, s_2, A_1, A_2 \geq 0$$

Solve by simplex method

- If $W^* = 0$ then go to phase –II.
- Else the problem is infeasible.
- Phase-II: Take the optimal table of phase-I and consider the original objective function. Test for optimality and obtain the optimal solution.

Problem

Solve the following LPP by using two phase method.

Min $Z = 5x_1 - 6x_2 - 7x_3$

Subjected to constraints

$x_1 + 5x_2 - 3x_3 \geq 15$

$5x_1 - 6x_2 + 10x_3 \leq 20$

$x_1 + x_2 + x_3 = 5$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

Ans: $x_2^* = 15/4$ $x_3^* = 5/4$ $Z^* = -125/4$

Problem.

Solve the following LPP by using two phase method.

Max $Z = -x_1 + 3x_2$

Subjected to constraints

$x_1 + 2x_2 \geq 2$

$3x_1 + x_2 \leq 3$

$x_1 \leq 4$

$x_1 \geq 0$, $x_2 \geq 0$

- Ans: $x_{1^*} = 0$, $x_{2^*} = 3$, $Z^* = 9$

Problem.

- Use simplex method to solve the following system of linear equations.

$x_1 - x_3 + 4x_4 = 3$

$2x_1 - x_2 = 3$

$3x_1 - 2x_2 - x_4 = 1$

$x_1, x_2, x_3, x_4 \geq 0$

Ans $x_1 = 5$, $x_2 = 7$, $x_3 = 2$, $x_4 = 0$

Problem.

- Use simplex method to solve the following system of linear equations.

  $x_1 + x_2 = 1$

  $2x_1 + x_2 = 3$

- Ans : $x_1 = 2$ $x_2 = -1$

Problem.

Solve the following LPP by using simplex method.

Min $Z = x_1 + 4x_2$

Subjected to constraints

$x_1 + x_2 \leq 3$

$-x_1 + x_2 \leq 1$

x_1 unrestricted, $x_2 \geq 0$,

- $X_1 = -1$, $x_2 = 0$ $Z^* = -1$

Special cases:

- Unbounded solution: In the key column all elements –ve or zero it is said to be unbounded.
- Multiple optimal : If any basic variable is not in basis and Cj- Zj value zero for that column then the solution having multiple optimal, next alternative solution can be determined by taking that column as key column and continue the procedure.
- Infeasible solution: In two phase method the optimum value of auxilary objective function is non zero, then it is having infeasible solution. In big M method in the optimal table basis contains artificial variable, it is having infeasible solution.

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- Degeneracy solution: In the ratio column two values are minimum and equal, then it is said to be degeneracy. If it is in optimal table it is said to be permanent degeneracy. If it is in middle table it is said to be temporary degeneracy.

Problem.

Solve the following LPP by using simplex method.

Max $Z = 4x_1 + 10x_2$

Subjected to constraints

$2x_1 + x_2 \leq 50$

$2x_1 + 5x_2 \leq 100$

$2x_1 + 3x_2 \leq 90$

$x_1 \geq 0$, $x_2 \geq 0$

- Ans : $x_1 = 0$, $x_2 = 20$ or $x_1 = 75/4$, $x_2 = 25/2$

  $Z = 200$

Problem.

Solve the following LPP by using simplex method.

Min $Z = x_1 - x_2$

Subjected to constraints

$x_1 + x_2 \leq 1$

$2x_1 + x_2 \geq 6$

$x_1 \geq 0 , x_2 \geq 0$

- Ans : Infeasible solution

Problem.

Solve the following LPP by using simplex method.

Max $Z= 5x_1+8x_2$

Subjected to constraints

$4x_1 + 6x_2 \leq 24$

$2x_1+ x_2 \leq 18$

$3x_1+ 9x_2 \leq 36$

$x_1 \geq 0 , x_2 \geq 0$

- Ans :x1=0, x2 = 4 z = 32

Problem.

Solve the following LPP by using simplex method.

Max $Z = 2x_1 + 2x_2$

Subjected to constraints

$x_1 - x_2 \geq -1$

$-0.5x_1 + x_2 \leq 2$

$x_1 \geq 0 , x_2 \geq 0$

- Ans : Unbounded solution.

Duality Introduction

- For every linear programming problem whether maximization or minimization has associated with it another mirror image problem based on same data.
- The original problem is called primal problem while the mirror image problem is called dual problem.
- If the original problem is a maximization problem then the dual problem is minimization problem.
- If the original problem is a minimization problem then the dual problem is maximization problem.
- In either case the final table of the dual problem will contain both the solution to the dual problem and the solution to the original problem.
- The solution of the dual problem is readily obtained from the original problem solution if the simplex method is used.
- The variables of dual problem are called shadow variables.

Dual Problem Formulation

- The number of constraints in the original problem is equal to the number of dual variables.
- The number of constraints in the dual problem is equal to the number of variables in the original problem.
- The original problem profit coefficients appear on the right hand side of the dual problem constraints.
- If the original problem is a maximization problem then the dual problem is a minimization problem. Similarly, if the original problem is a minimization problem then the dual problem is a maximization problem.
- The original problem has less than or equal to (≤) type of constraints while the dual problem has greater than or equal to (≥) type constraints.
- The coefficients of the constraints of the original problem which appear from left to right are placed from top to bottom in the constraints of the dual problem and vice versa.

Contd…

Primal

Max $Z = C_1x_1 + C_2x_2$ + ----- + C_nx_n

Subjected to constraints

$a_{11}x_1 + a_{12}x_2$ +---- + $a_{1n}x_n \leq b_1$

$a_{21}x_1 + a_{22}x_2$ + ------ + $a_{2n}\, x_{2n} \leq b_2$

|

|

|

$a_{m1}x_1 + a_{m2}x_2$ + ------ + $a_{mn}\, x_n \leq b_n$

x_1 , x_2 , ---- x_n all ≥ 0

$$\text{Min } W = b_1y_1 + b_2y_2 + \text{-----} + b_my_m$$

Subjected to constraints

$$a_{11}y_1 + a_{21}y_2 + \text{----} + a_{m1}y_m \geq C_1$$

$$a_{12}y_1 + a_{22}y_2 + \text{------} + a_{m2}y_m \geq C_2$$

|

|

|

$$a_{1n}y_1 + a_{2n}y_2 + \text{------} + a_{mn}y_m \geq C_n$$

$$y_1, y_2, \text{----} y_m \text{ all} \geq 0$$

Characteristics of the dual problem

- Dual of the dual is primal.
- If either the primal or dual problem has a solution, then the other also has a solution and their optimum values are equal.
- If any of the two problems has only an infeasible solution, then the value of objective function of the other is unbounded.
- The value of the objective function for any feasible solution of the primal is less than the value of the objective function for any feasible solution of the dual.
- If either the primal or dual problem has an unbounded solution, then the solution to the other problem is infeasible.
- If the primal has a feasible solution but the dual does not have, then the primal will not have a finite optimum solution and viceversa.

Advantages of duality

- It yields number of powerful theorems.
- Computational procedure can be considerably reduced by converting it in to dual, if the primal problem contains a large number of constraints and a small number of variables.
- Solution of the dual checks the accuracy of the primal solution for computational errors.
- It gives additional information as to how the optimum solution changes as a result of the changes in the coefficients and the formulation of the problem. (This is termed as post optimality or sensitivity analysis.)
- It indicates fairly close relationship exists between linear programming and duality.
- Economic interpretation of the dual helps the management in making future decisions.

Problem

Obtain the dual for the following problem

Max $Z = 5x_1 + 3x_2$

Subjected to constraints

$x_1 + x_2 \leq 2$

$5x_1 + 2x_2 \leq 10$

$3x_1 + 8x_2 \leq 12$

$x_1 \geq 0$, $x_2 \geq 0$

Problem

Obtain the dual for the following problem

$\text{Max } Z = x_1 + x_2 + x_3$

Subjected to constraints

$2x_1 + x_2 + 2x_3 \leq 2$

$4x_1 + 2x_2 + x_3 \leq 2$

$x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$,

Problem

Obtain the dual for the following problem

Max $Z = 3x_1 + 4x_2$

Subjected to constraints

$2x_1 + 3x_2 \leq 16$

$5x_1 + 2x_2 \geq 20$

$x_1 \geq 0$, $x_2 \geq 0$

Problem

Obtain the dual for the following problem

$\text{Min } Z = 5x_1 - 6x_2 + 4x_3$

Subjected to constraints

$3x_1 + 4x_2 + 6x_3 \geq 9$

$x_1 + 3x_2 + 2x_3 \geq 5$

$7x_1 - 2x_2 - x_3 \leq 10$

$x_1 - 2x_2 + 4x_3 \geq 4$

$2x_1 + 5x_2 - 3x_3 \geq 3$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0,$

Problem

Obtain the dual for the following problem

Max $Z = 3x_1 + 4x_2$

Subjected to constraints

$x_1 + x_2 \geq 4$

$-x_1 + 3x_2 \geq -4$

$x_1 \geq 0, x_2 \geq 0$

Problem

Obtain the dual for the following problem

Max $Z = 5x_1 + 10x_2$

Subjected to constraints

$2x_1 - 3x_2 \leq 7$

$x_1 + 2x_2 = 4$

$x_1 \geq 0$, $x_2 \geq 0$

Problem

Obtain the dual for the following problem

$\text{Max } Z = 3x_1 + 5x_2 + 7x_3$

Subjected to constraints

$x_1 + x_2 + 3x_3 \leq 10$

$4x_1 - x_2 + 2x_3 \geq 15$

$x_1 \geq 0$, $x_2 \geq 0$, x_3 unrestricted in sign,

Problem

Obtain the dual for the following problem

$$\text{Min } Z = 2x_1 + 3x_2 + 4x_3$$

Subjected to constraints

$$2x_1 + 3x_2 + 5x_3 \geq 2$$

$$3x_1 + x_2 + 7x_3 = 3$$

$$x_1 + 4x_2 + 6x_3 \leq 5$$

$x_1 \geq 0$, $x_2 \leq 0$, x_3 unrestricted in sign,

Duality Theorems

I. Weak duality theorem: If x and y be two basic feasible solutions to the primal and dual problem respectively, then min W ≥ max Z.

II. Fundamental theorem of duality: If x^* and y^* be the optimal solutions of primal and dual problem respectively, then $W^* = Z^*$

III. Dual of the dual is primal.

Complementary slackness condition(CSC): Let x and y be the feasible solutions to the primal and dual problems. Let u and v be the slack and surplus variables to the respective primal and dual problems. Then x an y is optimal if and only if xv=0, and yu=0

Problem

- A pension fund manager is considering investing in two shares A and B. It is estimated that (1) share A will earn a dividend of 12% per annum and share B 4% per annum. (2) Growth in the market value in one year of share A will be 10 paise per Re 1 invested and in B 40 paise per Re 1 invested. He requires to invest the minimum total sum which will give dividend income of at least Rs 600 per annum and growth in one year of at least Rs 1000 on the initial investment. You are required to (1) state the mathematical formulation of the problem. (2) compute the minimum sum to be invested to meet the manager's objective by using dual simplex method.

Problem

- Using CSC solve the following problem.
- Min $Z = 2x_1 + 12x_2 + 3x_3$

$x_1 + 2x_2 + x_3 \geq 8$

$x_1 + 6x_2 + 2x_3 \geq 2$

$x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$,

TRANSPORTATION PROBLEMS

INTRODUCTION:-

- It is special kind of LPP in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the sources and destinations respectively, such that the total cost of transportation is minimized.
- The transportation problem is to transport various amounts of a single homogeneous commodity, that are initially stored at various origins, to different destinations in such a way that the total transportation cost is a minimum.

Mathematical Formulation:

- Let there be 'm' origins , ith origin possessing a_i units of a certain product, where as there are n destinations (n may or may not be equal to m) with destinations b_j units. Costs of shipping of an item from each of m origins (sources) to each of destinations are known either directly or indirectly in terms of mileage, shipping hours, etc. Let c_{ij} be the cost of shipping one unit product from ith origin (source) to the jth destination and x_{ij} be the amount to be shipped from ith origin to the jth destination.
- It is also assumed that total availabilities $\sum a_i$ satisfy the total requirements $\sum b_j$, i.e
- $\sum a_i = \sum b_j$ $(i=1,2\ldots m \; ; \; j=1,2\ldots n)$

Contd…

- The problem now is to determine non-negative (≥ 0) values of xij satisfying both, the availability constraints:
- $\sum x_{ij} = a_i$ for i=1,2…..m
- As well as the requirement constraints:
- $\sum x_{ij} = b_j$ for j=1,2….n
- And minimizing the total cost of transportation (shipping)
- $Z = \sum \sum (x_{ij})(c_{ij})$ objective function
- It may be observed that the constraint equations and the objective function are all linear in xij, so it may be looked like a Linear Programming problem.
- This special type of LPP will be called a Transportation Problem (T.P)

Feasible solution, Basic Feasible solution and Optimum solution:

- Feasible solution : A set of non-negative individual allocations ($x_{ij} \geq 0$) which simultaneously removes deficiencies is called a feasible solution.
- Basic feasible solution: A feasible solution to a m-origin, n-destination problem is said to be basic if the number of positive allocations are m+n-1 ,i.e one less than sum of rows and columns
- If the number of allocations in a basic feasible solution are less than **m+n-1**, it is called degenerate BFS otherwise, non-degenerate BFS.
- Optimum solution: A feasible solution (not necessary basic) is said to be optimal if it minimizes the total transportation cost

Types of Transportation problem:-

- There are two different types of transportation problem.

  Balanced Transportation Problem:- A transportation problem is said to be balanced if sum of the availability is equal to sum of the requirements. **i.e. $\sum a_i = \sum b_j$**

- **Unbalanced Transportation problem:** A transportation problem is said to be unbalanced if sum of the availability is not equal to sum of the requirements. **i.e. $\sum a_i \neq \sum b_j$**
- **First solution to balanced Transportation problem**
- Methods of finding an optimal solution of the transportation problem will consist of two main steps

  (i). Finding initial basic feasible solution.

  (ii) Optimality Test.

Finding initial basic feasible solution.

(i)Northwest corner rule.

(ii) Least cost entry method.

(iii) Vogel's Approximation Method

- To Optimality Test, The IBFS must satisfy two conditions
- Number of allocations (N) = m+n-1.where m=no. of rows ,n=no. of columns
- These allocations should be in the independent position.

North - west Corner Rule:

Step 1 : The first assignment is made in the cell occupying the upper left hand (north-west) corner of the transportation table .the maximum possible amount is allocated there .that is ,X_{11} =min (a_1,b_1).this value of X_{11} is than entered in the cell (1,1) of the transportation table.

Step 2:

- If $b_1>a_1$, move vertically downwards to the second row and make the second allocation of amount x_{21} =min (a_2, b_1-x_{11}) in the cell (2, 1).
- If $b_1<a_1$, move horizontally right-side to the second column and make second allocation of amount x_{12}=min (a_1-x_{11}, b_2) in cell (1, 2).
- If $b_1=a_1$, there is a tie for the second allocation. one can make the second allocation of magnitude x_{12}=min(a_1-a_1,b_2)=0 in the cell (1,2) or x_{21}=min(a_2,b_1-b_1)=0 in the cell of (2,1) .
- Step 3: Start from the new north-west corner of the transportation table and repeat steps 1 and 2 until all the requirements are satisfied.

LOWESTCOST ENTRY METHOD (LCEM) OR MATRIX MINIMUM METHOD

- **Step1.** Determine the smallest cost in the cost matrix of the transportation table. Let It be (cij). allocate xij = min (ai.bj) in the cell (i,j).
- **Step2.** (i) If xij=ai, cross-out the ith row of the transportation table and decrease bj by ai. Go to step 3. (ii) If xij= bj, cross out the jth column of the transportation table and decrease ai by bj.go to step 3.

  (iii)If xij =ai=bj, cross –out either the ith or jth column but not both.
- **Step3.** Repeat steps 1and2 for the resulting reduced transportation table until all the requirements are satisfied .whenever the minimum cost is not unique, make an arbitrary choice among the minima.

VOGEL'S APPROXIMATION METHOD (VAM)

- **Step1.** For each row of the transportation table identify the smallest and next-to-smallest cost. Determine the differences between them for each row .these are called "penalties". Put them alongside the transportation table by enclosing them in the parentheses against the respective rows .similarly; compute these penalties for each column.
- **Step2.** Identify the row or column with the largest penalty among all the rows and columns. If a tie occurs ,use any arbitrary tie breaking choice .let the largest penalty correspond to ith row and let cij be the smallest cost in the ith row. Allocate the largest possible xij =min (ai,bj) in the cell (i,j)and cross-out the ith row or the jth row column in the usual manner.
- **Step3.** Again compute the column and row penalties for the reduced transportation table and then go to step 2.repaet the procedure until the requirement satisfied.

Problem.

- A company has three warehouses A,B and C and four stores w, x, y and z. The warehouses have altogether surplus of 150 units of a given commodity as follows

  A – 50, B – 60, C – 40

The four stores need the following amounts.

w – 20, x – 70, y – 50, z – 10

Cost (in rupees) of shipping one unit of commodity from various warehouses to different stores it as follows.

		Stores			
		w	x	y	z
	A	50	150	70	60
	B	80	70	90	10
Warehouse	C	15	87	79	81

1) Formulate the problem as LPP model.
2) Write out the transportation schedule by using North- West corner method, least cost method, and vogels method.
3) Find the associated transportation cost (only IBFS)

Problem.

- Find the initial feasible solution for the following problem by three methods.

To From	W1	W2	W3	W4	capacity
F1	21	16	25	13	11
F2	17	18	14	23	13
F3	32	27	18	41	19
Requirement	6	10	12	15	

Ans: N-W method total cost = 1095

Least cost method total cost = 922

Vogels method total cost = 796

Optimal Solution

- Optimal Solution can be obtained by two methods:

Stepping stone method :

Modified distribution method (MODI)

To get optimal Solution by this method

1) Write the table of allocation found by the initial solution method satisfying the condition that there are m+n-1 number of allocations.
2) Consider the allocated cells. Find the simplex multipliers U_i, (i= 1,2,3 ....m) , V_j (j= 1, 2 3....n)

   such that for each occupied cell (i, j)

$C_{ij} = U_i + V_j$ where C_{ij} is individual cost of an allocation from source i to destination j.

To start with any one of the U_i or V_j value is assumed to be zero, then applying the equation, the other values of U_i and V_j are determined for occupied cells.

3) Consider unoccupied cells : Find the cell evaluation value for each unoccupied cell by using the equation.

$$C_{ij\ bar} = C_{ij} - (U_i + V_j)$$

If all the cell evaluation values are positive the solution is optimal. If all the cell evaluation values are not positive the solution is not optimal and hence follow the procedure for shifting.

Procedure of Shifting

- Select the cell corresponding to the most negative c_{ij} bar value.
- To find the path for this cell : Path is nothing but the route be travel from this cell back to itself by a series of alternating horizontal and vertical jumps from one occupied cell to another without a direct reversal of route.
- Now mark alternatively + and – signs for the cells at the corners of the route assuming that starting point to be + ve.
- Now list the allocations in the negative corner of this path and select minimum negative corner allocation.

  Shift this minimum negative corner allocation.

  That is add and subtract this minimum - ve corner allocation to cells at the +ve and -ve corners of the path respectively to get the altered allocations along the path.

  Form a new table with these altered allocations, check whether the number of allocations equal to m+n-1 in the new table. If the number of allocations are not equal to m+n-1 follow the procedure of resolving degeneracy and proceed. Repeat the above procedure until getting optimal.

Problem.

- Solve the following transportation problem.

							Availability
	5	3	7	3	8	5	3
	5	6	12	5	7	11	4
	2	1	3	4	8	2	2
	9	6	10	5	10	9	8
Req	3	3	6	2	1	2	

Problem

- Solve the following transportation problem.

	I	II	III	IV	V
A	12	4	9	5	9
B	8	1	6	6	7
C	1	12	4	7	7
D	10	15	6	9	1

The availability at sources I, II, III, IV and V are 40,20,50,30 and 40 respectively. The requirement at destinations A,B, C and D are 55, 45, 30 and 50 respectively. Find the optimal solution using MODI method. Find whether solution is unique or not? If not find alternative solution

Problem.

- There are three factories located at places P,Q and R. These factories supply products to whole sale agents at places at S, T and W. The weekly capacities of factories P Q and R 76, 82 and 77 units respectively. The weekly requirement of agents S,T and W are 72, 102 and 41 units respectively. The unit transportation cost in rupees from P to S , T and W are 5, 8 and 8 respectively, from Q to S , T and W are 16, 25 and 15 respectively, from R to S , T and W are 9, 16 and 25 respectively. Determine the following.

a) optimal transportation schedule

b) Minimum cost of transportation

c) Is the solution unique? Justify your answer.

Use the solution obtained by least cost method to check for optimality.

Problem.

- A Priyanshu enterprises has 3 factories at location A, B and C and which supplies 3 warehouses located at D, E and F. Monthly factories capacities are 10,80 and 50 units respectively. Monthly warehouse requirements are 75,20 and 50 units respectively. Unit shipping cost are given as

		Warehouse		
		D	E	F
	A	5	1	7
Factory	B	6	4	6
	C	3	2	5

The penalty cost for not satisfying demand at the warehouses D, E, F are Rs 5, 3, and Rs2 per unit respectively. Determine the optimal distribution for priyanshu using transportation technique.

Problem.

- A company has factories at four different places which supply warehouses A,B,C,D and E. Monthly factory capacities are 200, 175,150 and 325 respectively. Month ly warehouse requirements are 110, 90,120 230 and 160 respectively. Unit shipping costs are given in table. The costs are in RS.

	To	A	B	C	D	E
	1	13	-	31	8	20
From	2	14	9	17	6	10
	3	25	11	12	17	15
	4	10	21	13	-	17

Shipment 1 to B and from 4 to D are not possible. Determine the optimum distribution to minimize shipping costs.

Problem

- Industrial shop wants to purchase the following quantities of cutting tools.

Tool type	T1	T2	T3	T4	T5
Quantity	150	100	75	250	200

Combining all types of tools the following four manufacturers are ready to supply not more than quantities stated below.

Manufacturer	M1	M2	M3	M4
Total quantity	300	250	150	200

The shop that estimates that is profit per day will vary with manufacturer as shown in the matrix below. How should orders be placed. What is the maximum profit.

	T1	T2	T3	T4	T5
M1	11	14	17	9	6
M2	12	13	18	7	4
M3	10	14	19	8	5
M4	13	11	16	10	7

ASSIGNMENT PROBLEMS

- Allocation of men to machines.
- Allocation of managers to offices.
- Allocation of jobs to persons
- Assumptions:
- Each resource should be allocated to one job and each job requires only one resource.
- The assignment method is also called Hungarian method or flood technique.
- Def: If there are n facilities and n jobs with details related to the effect of each facility for each job known, the problem is to assign one facility to one and only one job so that the given measure of effectiveness is optimized.
- Condition for assignment m = n

Comparison between assignment and transportation problems

Assignment	Transportation
1. m= n	1. m is not equal to n
2. Sum of demand = Sum of supply	2. Sum of demand is not equal to Sum of supply
3. Only one allocation is possible for each row and each column	3. More than one allocation is possible.

Formulation of assignment model

- Min Z = sigma i=1 to n sigma j=1 to n (Cij xij)
- Xij = 1 for occupied cell
- Xij = 0 for unoccupied cell

sigma i=1 to n xij =1   i = 1,2,----n

sigma j=1 to n xij =1   j = 1,2,----n

Procedure:

- **Step 1.** Subtract the minimum element of each row in the cost matrix [c_{ij}] from every element of the corresponding row.
- **Step 2.** Subtract the minimum element of each column in the reduced matrix obtained in the step 1 from every element of the corresponding column.
- **Step 3.** (a) Starting with row 1 of the matrix obtained in step 2, examine rows successively until a row with exactly one zero element is found. Mark ( ), at this zero , as an assignment will be made there. Mark (X) at all other zeros in the column (in which we mark) to show that they cannot be used to make other assignments. Proceed in this way until the last row is examined.
- (b) After examining all the rows completely, proceed similarly examining the columns. Examine columns starting with column 1 until a column containing exactly one unmarked zero is found. Mark (,) at this zero and cross (mark X) at all zeros of the row in which is marked. Proceed in this way until the last column is examined.
- (c) Continue these operations (a) and (b) successively until we reach to any of the two situations.

  (i) all the zeros are marked or crossed.

  Or (ii) the remaining unmarked zeros lies at least two in each row and column.

  In case (i), we have a maximal assignment (assignment as much as we can) and in case (ii) still we have some zeros to be treated for which we use the trial and error method to avoid the use of highly complicated algorithm. Now there are two possibilities: If it has an assignment in every row and every column (i.e. total number of marked zeros is exactly n), then the complete optimal assignment is obtained. If every row has no assignment then apply the lines procedure

Contd…

- **Step 4**: Lines procedure: draw the minimum number of lines to cover the zeros at least once. For that rule to draw minimum number of lines Tick the rows that do not assignments In that marked (ticked) row(s) for the cross off zeros put a tick mark in the columns having In that marked column, for the assignments put tick mark in the row Continue until the chain procedure ends
- **Step 5**: Draw the lines for un marked rows and marked columns and find the minimum element in uncovered line elements then modify the matrix with the three conditions Add the minimum element where the two lines are intersecting Subtract the minimum element where the line is not passing through elements Leave the elements as it is if the line is passing through the elements
- **Step 6:** continue the procedure until all the rows have assignments i.e N=n

Problem.

- A department head has four subordinates and has four tasks have to be performed. Subordinates differ in efficiency and tasks differ in their intrinsic difficulty. Time each man would take to perform each task is given in the effectiveness matrix. How the tasks should be allocated to each person so as to minimize the total man hours.

Subordinates

		I	II	III	IV
	A	8	26	17	11
	B	13	28	4	26
Tasks	C	38	19	18	15
	D	19	26	24	10

Problem.

- A car hire company has one car at each five depots a,b,c,d,and e. A customer requires a car in each town namely A,B,C,D and E. Distance (in kms) between depots and towns are given in the following distance matrix.

	a	b	c	d	e
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

How should car to be assigned to customers so as minimize distance travelled

Problem.

- A company is forced when the problem of assigning six different machines to five different jobs. The costs are estimated as follows (in hundreds of rupees)

	1	2	3	4	5
1	2.5	5	1	6	1
2	2	5	1.5	7	3
3	3	6.5	2	8	3
4	3.5	7	2	9	4.5
5	4	7	3	9	6
6	6	9	5	10	6

Problem.

- A company has five jobs to be done. The following matrix shows the return in rupees on assigning i th machine to j th job. Assign the five jobs to five machines so as to maximize the total expected profit.

		jobs				
		A	B	C	D	E
	1	5	11	10	12	4
	2	2	4	9	3	5
Machines	3	3	12	5	14	6
	4	0	14	4	11	7
	5	7	9	8	12	5

Problem.

- A job shop has purchased five new machines of different type. There are five available locations in the shop where a machine could be installed. Some of these locations more desirable than others for particular machines because of their proximity to work centres which would have a heavy work flow to and from these machines. Therefore the objective is to assign the new machines to the available locations in order to minimize the total cost of material handling. The estimated cost per unit time of material handling involving each of the machines is given below for the respective locations. Locations 1,2,3,4, and 5 are not considered suitable for machines A,B,C,D and E respectively. Find the optimal solution.

  - Locations

		1	2	3	4	5
	A	X	10	25	25	10
	B	1	X	10	15	2
M/CS	C	8	9	X	20	10
	D	14	10	24	X	15
	E	10	8	25	27	x

Problem.

- A company has 4 plants w,x,y, and z each of which can produce any one of the four products P,Q,R, and S. The production cost and sales revenue differ from one plant to another. Determine the optimal combination of plants product which will maximize the profit of the company. Details of production cost and sales revenue have been shown.
- Sales revenue in thousands of rupees

Product

	P	Q	R	S
W	70	60	66	75
X	73	63	68	72
Y	55	58	60	62
Z	63	68	71	76

- Production cost in thousands of rupees

Product

	P	Q	R	S
W	62	58	60	71
X	71	58	62	71
Y	50	51	53	59
Z	61	66	65	70

Problem.

- An air line operates seven days a week has time table shown below. Crews must have a minimum layover time of 5 hours between flights. Obtain the pair of flights that minimizes layover time away from home for any given pair the crew will be based at the city that results in smaller layover.
- Delhi- Jaipur
- Flight no Departure arrive
- 1 7am 8am
- 2 8am 9am
- 3 1:30pm 2:30pm
- 4 6:30pm 7:30pm
- Jaipur- Delhi
- Flight no Departure arrive
- 101 8am 9:15am
- 102 8:30am 9:45am
- 103 12 noon 1:15pm
- 104 5:30pm 6:45pm
- For each pair mention the town where the crew should be based.

Travelling salesman problem

- **Travelling salesman (Routing) problem**: suppose a salesman wants to visit a certain number of cities allotted to him. He knows the distance(or cost or time) of journey between every pair of cities , usually denoted by cij i.e from city i to j .His problem is to select such a route that starts from his home city, passes through each city once and only once, and returns to his home town in the shortest possible distance (or at the least cost or in least time)
- Symmetrical: The problem is said to be symmetrical if the distance (or cost or time) between every pair of cities is independent of the direction of his journey.
- Asymmetrical: The problem is said to be asymmetrical, if for one or more pair of cities, the distance (cost or time) changes with the direction. For example, flying from East to West usually takes longer time than from West to East on account of prevailing winds.
- Formulation of Travelling salesman problem as Assignment:
- Suppose cij is the distance from city I to j and xij=1 if the sales man goes directly from city I to j and xij=0 otherwise. Then minimize $\sum\sum$ (xij)(cij) with the additional restriction that xij must be so chosen that no city visited twice before the tour of all cities is completed. In particular he can't go directly from city i to I itself. This possibility may be avoide in the minimization process by adopting the convention cij=∞ which ensures that xij an never be unity.

- The distance (or cost or time) matrix for this problem is given below
- From A1 A2 An
- A1 ∞ C12 C1n
- A2 C21 ∞ C2n
- . . . .
- An Cn1 Cn2 ∞
- To solve the travelling salesman problem, first express the problem as an assignment problem and solve in the usual manner. If the assignment schedule satisfy the extra condition (no city visit twice) that is the solution of travelling salesman problem otherwise the next best solution is next minimum element to zero.

Problem.

- A salesman has to visit five cities A,B,C,D and E. The distances (in hundreds of miles) between the five cities are as follows.

	A	B	C	D	E
•A	-	7	6	8	4
•B	7	-	8	5	6
•C	6	8	-	9	7
•D	8	5	9	-	8
•E	4	6	7	8	-

If the salesman starts from city A and has to come back to city A. Which route be selected so that the total distance travelled is minimum.

Problem.

- Solve the travelling salesman problem in the matrix shown below.

To

	1	2	3	4	5
1	α	6	12	6	4
2	6	α	10	5	4
3	8	7	α	11	3
4	5	4	11	α	5
5	5	2	7	8	α

GAME THEORY

- **Introduction:**
- **Game theory is a decision theory applicable to competitive situations.**
- **It is usually used when two or more individuals or organizations with conflicting objectives try to make decisions.**
- **Decision made by one decision maker affects the decision made by one or more of the decision makers.**
- **Game theory is based on “minimax principle” which states that each competitor will act so as to minimize his maximum loss or maximize his minimum gain.**

Game theory applicable situations:

- Two players struggling to win at chess.
- Candidates fighting an election.
- Firms struggling to maintain their market shares.
- Two enemies planning war tactics etc.

Contd…

- **Assumptions of a game:**
- There are finite number of competitors, called players.
- A list of finite or infinite numbers of possible courses of action (alternatives) is available to each player. The list need not be the same for each player. Such a game is said to be in normal form.
- A play is played when each player chooses one of his courses of action. The choices are assumed to be made simultaneously, so that no player knows his opponent's choice until he has decided his own course of action.
- Every play is associated with an outcome, known as payoff (generally money or some other quantitative measures for the satisfaction) which determine a set of gains, one to each player. Here a loss is considered a negative gain. Thus after each play of the game, one player pays to others an amount determined by the courses of actions chosen.
- All players act rationally and intelligently.
- Each player attempts to maximize his gain or minimize loss.
- Each player makes individual decision without direct communication.
- Complete relevant information is known to each player. i.e., players are conservative.

Contd…

- **Zero-sum game:** Let there be n competitors and N_i (i = 1, 2, …, n) number of courses of action available to each competitor. Then, if the algebraic sum of payments to all the competitors after a play of the game is played is equal to zero, such a game is termed as a zero-sum game.
- It is very clear in the above example. If the outcome for the player A for play is +1, the corresponding outcome for B is -1, hence the algebraic sum of payments or outcomes to both the players is equal to zero.
- **Two-person Zero-sum game:** A game involving two persons is called two-person game and the algebraic sum of gains and losses of both the players is equal to zero, such a game is called Two-person Zero-sum game. i.e., in Two-person Zero-sum game the gains of one player are equal to the losses of the other player. Two-person games are also called rectangular games.

- **n – person game:** A game involving n players, is called a n – person game.

  Pay-off matrix: It is a table which shows how payments should be made at the end of a play or the game.

  Strategy of a player: This refers to the predetermined rule by which a player decides as to how to use his/her courses of action in the different plays of a game, from his own list of available courses of action during the game. Generally, we employ two types of strategies, namely, (i) pure strategy (ii) mixed strategy.
- **Pure strategy:** A pure strategy is that in which one knows, in advance of all plays that he will always choose only one particular course of action. Thus pure strategy is a decision rule always to select the same course of action. Every course of action is a pure strategy.
- **Mixed strategy:** A mixed strategy is that in which a player decides, in advance to choose one of his courses of action in accordance with some fixed probability distribution, Thus in case of mixed strategy we associate probability to each course of action (each pure strategy). The pure strategies which are used in a mixed strategy with non-zero probabilities are termed as supporting strategies.
- **Optimal strategy:** It is also called as maximin or minimax criterion of optimality. If a player has to choose out of worst possible outcomes of all his potential strategies, he will choose the strategy that corresponds to the best of these worst outcomes. Such a strategy is called optimum strategy.

Contd…

- **Maximin and minimax criterion (statement):** "A player lists his worst possible outcomes and then he chooses that strategy which corresponds to the best of these worst outcome."
- Value of the game and fair game: The maximum guaranteed gain to player A (maximizing player) if both the players use their best strategies is called the value of the game and is denoted by 'v' which is unique.
- **Fair game:** A game whose value is zero is called a fair game.
- **Characteristics of game theory:** The following are the chief characteristics for the classification of various games.
- Number of persons or groups playing the game.
- Number of activities which may be finite or infinite.
- Type of strategy.
- Number of alternatives (choices) available to each player during a particular play, which may be finite or infinite.
- How much information about the past activities of other players is available to the players. It may be complete or partly or not at all.
- Pay-off may be such that the gains of some players may and may not be direct losses of other players.

- **Various methods** (as per syllabus) **to solve the rectangular two-person zero-sum games:**
- The maximin-minimax principle
- Dominance property
- Graphical method (in case of games without saddle point/mixed strategies)

- **Saddle point:** If max min value of game coincides with min max value of that game then the game is said to have saddle point or equilibrium point. This implies that a game with saddle point is that in which the players use pure strategies i.e. they choose the same course of action throughout the game. The point of intersection of their pure strategies used, is known as a saddle point. The gain at the saddle point gains the value of the game.
- **Strictly determined game:** A game with optimal pure strategies is sometimes called strictly determined.
- **Note:** If a matrix involves more than one saddle point then there exists more than one optimum solutions of the game.
- **Rule for detecting a saddle point:**
- Select the minimum of each row and encircle them. Put the column maximums in small squares. A point, if appears in the matrix, which is within the circle and the square both, is a saddle point.
- **Dominance property:** Generally the dominance property is used to reduce the size of the pay-off matrix. The concept of dominance can be applied to any two-person zero-game with any number if strategies for each player. For a payoff matrix of large size, the rule of dominance can be used to reduce its size by carefully eliminating certain rows and/or columns prior to final analysis to determine the optimum strategy selection for each player.

Rule to reduce the size of payoff matrix:

- If all the elements in a row (say i^{th} row) of a payoff matrix are less than or equal to the corresponding elements of the other row (say j^{th} row) then the player A will never choose the i^{th} strategy or in other words the i^{th} strategy is dominated by the j^{th} strategy.
- If all the elements in a column (say r^{th} column) of a payoff matrix are greater than or equal to the corresponding elements of the other column (say s^{th} column) then the player B will never choose the r^{th} strategy or in other words the r^{th} strategy is dominated by the s^{th} strategy.
- A pure strategy may be dominated if it is inferior to average of two more other pure strategies.
- **Graphical method:** If a problem has no saddle point, then we will reduce the given pay-off matrix to (2 x n) or (m x 2) size, if possible, by applying any/some/all of the above rules of dominance. It is obvious that if one player has only two strategies the other will also use two strategies. Graphical method is helpful in finding out which of the two strategies can be used. The advantage of this method is relatively fast.
- The graphical method or geometrical method consists of two graphs.
- The payoff (gains) available to player A versus his strategies options,
- The payoff (losses) faced by player B versus his strategies options.
- Consider the following 2 x n payoff matrix of a game without saddle point.

Contd…

- Player A has two strategies A_1 and A_2 with probability of their selection and respectively,
- To plot the expected payoff, we draw two parallel lines one unit apart and mark a scale on each of them. These two lines represent the two strategies available to player A. Then we draw lines to represent each of B's strategies. To represent B's 1st strategy (B_1), we join on scale 1 to on scale 2. This line will represent the expected payoff of the line with as x-axis and as y-axis. Similarly, other payoff lines can be drawn.
- The lower boundary of these lines will give the minimum expected payoff and the highest point on this lower boundary will then give the maximum expected payoff of player A and hence the optimum value of . The two optimum strategies for B are then given by the two lines which pass through this maximum point. Thus the 2 x n game is reduced to 2 x 2 game which can be easily solved by any of the methods discussed earlier.
- The m x 2 games are also treated in the same manner except that minimax point is the lowest point on the upper boundary of the straight lines (instead of highest point on the lower boundary).

Contd…

		Player B	
		I	II
Player A	I	V11	V12
	II	V21	V22

$$x1 = \frac{V22 - V21}{(V11+V22) - (V12+V21)} \quad , \quad x2 = 1 - x1$$

$$y1 = \frac{V22 - V12}{(V11+V22) - (V12+V21)} \quad , \quad y2 = 1 - y1$$

$$V = \frac{V22V11 - V12V21}{(V11+V22) - (V12+V21)}$$

Problem.

- Solve the game whose pay off matrix is follows.
- PlayerB

		B1	B2	B3	B4
PlayerA	A1	-5	2	0	7
	A2	5	6	4	8
	A3	4	0	2	-3

Problem.

- Solve the game whose pay off matrix is follows.

		Player B		
		1	2	3
Player A	1	-2	16	-2
	2	-5	-8	-4
	3	-5	20	-9

Problem.

- Solve the game whose pay off matrix is follows.
- PlayerB

		B1	B2	B3	B4
PlayerA	A1	1	7	3	4
	A2	5	5	4	5
	A3	7	2	1	3
	A4	6	6	3	4

Problem.

- Solve the game whose pay off matrix is follows.
- PlayerQ

PlayerP		1	2	3	4	5
	1	10	5	2	9	1
	2	8	6	5	7	8
	3	3	5	4	6	9
	4	6	7	3	3	2

Problem.

- Solve the game whose pay off matrix is follows using dominance concept.
- PlayerB

		1	2	3	4	5
PlayerA	A1	2	4	3	8	4
	A2	5	6	3	7	8
	A3	6	7	9	8	7
	A4	4	2	8	4	3

Problem.

- Solve the game whose pay off matrix is follows using dominance concept.
- PlayerB

		1	2	3	4
PlayerA	A1	3	2	4	0
	A2	3	4	2	4
	A3	4	2	4	0
	A4	0	4	0	8

Problem.

- using the concept of dominance solve the following game.
- PlayerB

		I	II	III	IV	V
PlayerA	I	3	5	4	9	6
	II	5	6	3	7	8
	III	8	7	9	8	7
	IV	4	2	8	5	3

Problem.

- Solve the game whose pay off matrix is follows.
- PlayerA
- PlayerB

		A1	A2	A3
PlayerB	B1	0	-2	7
	B2	2	5	6
	B3	3	-3	8

Problem.

- Solve the game whose pay off matrix is follows.
- PlayerB

		B1	B2	B3	B4
• PlayerA	A1	3	2	4	0
	A2	2	4	2	4
	A3	4	2	4	0
	A4	0	4	0	8

- Solve the game whose pay off matrix is follows.
- PlayerA

		A1	A2	A3
PlayerB	B1	9	1	4
	B2	0	6	3
	B3	5	4	8

Problem.

- Solve the following game graphically.

		B					
		1	2	3	4	5	6
A	1	2	-2	3	4	3	7
	2	-4	6	5	1	4	6

- Solve the following game graphically.

		B			
		1	2	3	4
A	1	4	4	5	1
	2	6	5	4	8

Problem.

- Solve the following game graphically.

		B	
		1	2
	1	3	-5
	2	1	-1
A	3	2	-3
	4	-1	3
	5	0	1

Limitations of game theory

- The assumptions that the player have the knowledge about their own pay offs and pay offs of others is rather unrealistic. He can only make a guess of his own and his rivals stratagies.
- As the number of players increase in the game, the analysis of gaming strategies becomes increasingly complex and difficult. In practice there are many firms in an oligopoly situations and game theory can not be very helpful in such situations.
- The assumptions of maxmini and minimax show that the players are risk averse and have complete knowledge of the strategies. These do not seem practical.
- Rather than each player in an oligopoly situation working under uncertain conditions, the players will allow each other to share the secrets of business in order to work out a collusion. Thus the mixed strategies are also not very useful.
- No base for calculating pay off: There is no correct formula and scientific base for calculating pay offs of game.
- Non dynamic or static nature: In markets generally the changes are frequent and fast. The game theory gives least scope to work out the strategy immediately unless some ground work to known pay off is made.

QUEUING THEORY

- Ex: Airport operations, emergency medical services, ration shop, banking sector, tool room operations, car service garages, computer job process, check posts, petrol bunk, library issue counter etc.
- The other name for a waiting line is a “Queue” and a system involving a waiting line is called as a queuing system.
- Waiting line will be formed as in two situations:
- 1. Single service facilities
- 2. Multi service facilities

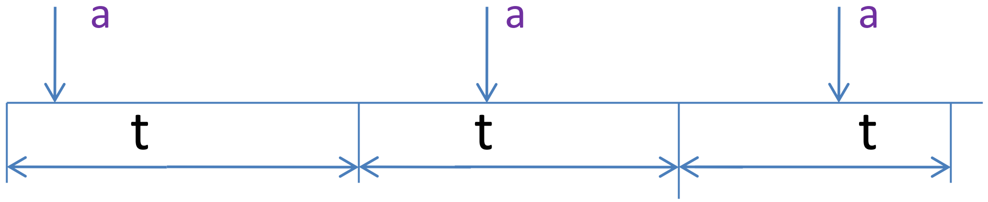
Contd…

- Structure of a single service queuing system:
- Arrivals → service facility → departure
- Structure of a multi service queuing system:
- service facility
- Arrivals → service facility → Departure
  service facility

Elements of a queuing system:

1. Source of customers.
2. Queue size
3. Queuing discipline (method of selection)
4. service facility or service channel
5. Service distribution.

1. Source of customers

- Characteristics of source of customers:
- I. Size of the source
- II. The pattern of arrival to the system
- III. Attitude of the customer
- I. Size of the source:-
- a. The size of population of customers has considerable effect on the ,models of queues.
- b. The calling source may be finite or infinite.
- II. The pattern of arrival to the system:-
- a. Scheduled arrivals
- b. Unscheduled arrivals
- a. Scheduled arrivals:
- This process describes measures are made to describe the pattern of arrivals namely arrival rate and interval time.
- 

Contd…

- b. Unscheduled arrivals (Random arrivals):
- Probability has to determine.
- Random arrivals are described by Poisson distribution.
- Probability of n arrivals in time t is given by

$$P(n) = \frac{e^{-\lambda} - (\lambda t)^{n}}{n!}$$

λ = mean arrival rate = reciprocal to mean interval time.

Contd…

- III. Attitude of the customer:
- The customers being patient or impatient.
- Impatient customers may refuse to enter the queue or leave the queue before receiving service.
- A patient customer is assumed.
- 2. Queue size
- I t may be finite or infinite.

3.Queuing discipline (method of selection)

- a. First come first serve (FCFS)
- b. Last come first serve (LCFS)
- c. Service in random order.

Contd…

- 4. service facility or service channel:
- I. single service channel
- II . Multi service channel
- III. Series of service channel.
- I. single service channel:
- In this only one server. Arrivals form one line and get service.
- II . Multi service channel:

  There are number of parallel channels, each with one server. Several customers can be served simultaneously.

III. Series of service channel:

  Customer pass successively through a number of channels before service is completed.

Contd….

- 5. Service distribution (Departure):
- If service times are random, it has been found that they are best described by exponential probability distribution.
- The number of customers served per unit of time is called service rate.
- The probability of n complete services in time t

$$= \frac{(\mu t)^{n} e^{-\mu t}}{n!}$$

Queue models:

- Model-I: (M/M/1),(FCFS/α/ α)

(Poisson arrival, exponential service, single channel, first come first served, maximum allowable customers being infinity in the system and infinite population model.)

- Model-II: (M/M/1),(FCFS/n/N)

(Poisson arrival, exponential service, single channel, first come first served, maximum allowable customers being infinity in the system and finite population model.)

- Model-III: (M/M/1),(FCFS/N/ α)

(Poisson arrival, exponential service, single channel, first come first served, maximum allowable customers being restricted in the system and infinite population model.)

- Model-IV: (M/M/C),(FCFS/α/ α)

(Poisson arrival, exponential service, multi channel, first come first served, maximum allowable customers being infinity in the system and infinite population model.)

Model-I: (M/M/1),(FCFS/α/ α)

(Poisson arrival, exponential service, single channel, first come first served, maximum allowable customers being infinity in the system and infinite population model.)

- Formulae:

1. Traffic intensity or probability that the server is busy = $p = \lambda/\mu$
2. The probability that the system is empty =

$P_0 = 1-P = 1- \lambda/\mu$

3. The steady state probability of n customers in the system $P_n = P_0 P^n$

$= (1- \lambda/\mu)(\lambda/\mu)^n$

4. Average number of customers in the system= (waiting line + service) $= Ls = \lambda/(\mu - \lambda)$
5. . Average number of customers in queue=$Lq=\lambda^2/ \mu(\mu - \lambda)$
6. Average time a customer spends in a system $Ws=1/(\mu-\lambda)$
7. Average waiting time of a customer in a queue $Wq=\lambda/ \mu(\mu - \lambda)$
8. Average length of non empty queue (length of queue which is formed time to time) $Ln = \mu / (\mu - \lambda)$
9. Average waiting time in a non emty queue $Wn = Ws = 1 / (\mu - \lambda)$

Problem.

- In a medium scale industry , a tool and cutter grinder operator finds that the time spent on each tool has an exponential distribution with mean 25minutes. If he grinds tools in the order in which they come in arrival tools for grinding is approximately Poisson with an average rate of 11 per 8 hours day. What is the operators expected idle time each day. How many jobs are ahead of the average tool just brought in?

Problem.

- In a big CNC m/c shop, there is only one CNC programmer to write programs using G and M codes. Since the programmer work varies in length, the programming rate is randomly distributed approximating a Poisson distribution with mean service rate of 7 programs per hour. The jobs for programming arrive at a rate of 4 per hour during the entire 8 hours work day. If the programmer is valued at Rs 35 per hour, determine the following.

1. Programmer utilization.
2. The time (in percentage) that an arriving job for programming has to wait.
3. Average system time.
4. Average cost due to waiting on the part of the programmer.

Problem.

- Arrivals at a public telephone booth are considered to be Poisson with an average time of 8 min between one arrival and next. The length of the telephone calls is assumed to be exponentially distributed with a mean value of 2min.
- 1. What will be the probability that a person arriving at the booth will have to wait.
- 2. Determine the average queue length that is formed from time to time.
- 3.The telephone department is interested to install a second booth if convinced that an arrival would expect to have to wait at least 5min for the phone. Determine the increase in flow of arrivals which will justify a second booth.

Problem.

- A self service store employs one cashier at its counter. Nine customers arrive on an average every 5 minutes, while the cashier can serve 10 customers in 5 minutes. Assuming Poisson distribution for arrival rate and exponential distribution for service rate, find
- 1. Average number of customers in the system.
- 2. Average number of customers in the queue. (or) average queue length.
- 3. Average time a customer spends in the system.
- 4. Average time a customer waits before being serrved.

Problem.

- A person repairing radios find that the time spent on the radio sets has been exponential distribution with mean 20 min. If the radios are repaired in the order in which they come in and their arrival is approximately Poisson with an average rate of 15 per 8 hour day. What is the repair man's expected idle time each day? How many jobs are ahead of the average set just brought in?

Problem.

- A branch of Punjab national bank has only one typist. Since the typing work varies in length (number of pages to be typed), the typing rate is randomly distributed approximating a Poisson distribution with mean service rate of 8 letters per hour. The letters arrive at a rate of 5 per hour during the entire 8 hour work day. If the type writer is valued at Rs1.50 per hour, determine
- 1) equipment utilization.
- 2) The percent time that an arriving letter has to wait.
- 3)Average system time
- 4) Average cost due to waiting on type writer.

Problem.

- On average 96 patients per 24 hour day requires the service of an emergency clinic. Also on average, a patient requires 10 minutes of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it costs the clinic Rs 100 per patient treated to obtain an average servicing time of 10 minutes and that of each minutes of decrease in this average time would cost Rs 10 per patient treated. How much would have to be budgeted by the clinic to decrease the average size of queue from1⅓ patients to ½ patient?

Model-II: (M/M/1),(FCFS/n/N)

(Poisson arrival, exponential service, single channel, first come first served, maximum allowable customers being infinity in the system and finite population model.)

- Formulae:
- Probability of an empty system = P_0

$$= \frac{1}{\sum_{n=0}^{N} \frac{N!\ (\lambda/\mu)^n}{(N-n)!}}$$

- Probability of n customers in the system = P_n =

$$\frac{\frac{N!\ (\lambda/\mu)^n}{(N-n)!}}{\sum_{n=0}^{N} \frac{N!\ (\lambda/\mu)^n}{(N-n)!}}$$

Expected number of customers in the system = Ls = $\sum_{n=0}^{N} nP_n = N - (\mu/\lambda)(1-P_0)$

Expected number of customers in the queue = Lq = $N - \{(\lambda + \mu)/\lambda\}(1-P_0)$

Problem.

- A mechanic repairs 4 machines, the mean time between service requirements is 5 hours for each machine and forms an exponential distribution. The mean repair time is 1 hour and also follows the same distribution pattern. Machine down time costs Rs 25 per hour and the mechanic costs Rs 55 per day.
- a) Find the expected number of operating machines.
- b) Determine the expected down time cost per day.
- c) Would it be economical to engage two mechanics, each repairing only two machine?

Model-III: (M/M/1),(FCFS/N/ α)

(Poisson arrival, exponential service, single channel, first come first served, maximum allowable customers being restricted in the system and infinite population model.)

- Formulae: $P_0 = \dfrac{1-P}{1-P^{N+1}}$
- $Pn = \dfrac{1-P}{1-P^{N+1}} P^n$
- $L_S = \dfrac{P\,[1-(1-N)P^N + NP^{N+1}]}{(1-P)\,(1-P^{N+1})} = L_q + 1 - P_0$
- $L_q = \dfrac{[(1-NP^{N-1} + (N-1)P^N]}{(1-P)\,(1-P^{N+1})} P^2$
- $\lambda' = \mu(1-P_0)$
- $Ws = Ls/\lambda'$,
- $Wq = Lq / \lambda'$

Problem.

- A one person barber shop has 6 chairs to accommodate people waiting for haircut. Assume customers who arrive when all 6 chairs are full, leave without entering the barber shop. Customers arrive at the average rate of 3 per hour and spend an average of 15 minutes in the barber shop. Then find
- a) The probability a customer can get directly in to the barber chair up on arrival.
- b) Expected number of customers waiting for a hair cut.
- c) Effective arrival rate.
- d) The time a customer can expect to spend in the barber shop.

Problem.

- At a railway station only one train is handled at a time. The railway yard is sufficient only for two trains to wait while the other is given signal to leave the station. Trains arrive to the station at an average rate of 6 per hour and the railway station can handle them on an average of 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady state probabilities for the various number of trains in the system. Also find the average waiting time of a train coming in to the yard.

Problem.

- In a railway marshaling yard goods trains arrive at a rate of 30 trains per day. Assuming that the inter arrival time follows an exponential distribution and service time distribution is also exponential with an average of 36 minutes. Calculating the following.
- a) Average number of customers in the system.
- b) The probability that the queue size exceeds 10.
- c) If the input of trains increases to an average of 33 per day, what will be the change in a) and b).

Problem.

- Customers arriving at an industrial consultant office is according to Poisson distribution at the rate of 28 per hour. The waiting room can accommodate not more than 14 customers. Consultation time per customer is exponential with a mean rate of 20 per hour. Find the effective arrival rate at the consultant office. What is the expected service and sent from the consultant office.

Model-IV: (M/M/C),(FCFS/α/ α)
(Poisson arrival, exponential service, multi channel, first come first served, maximum allowable customers being infinity in the system and infinite population model.)

- Formulae:
- Probability of an empty system = P_0
- $$= \frac{1}{\sum_{n=0}^{C-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^C}{C!} \frac{C\mu}{(C\mu - \lambda)}}$$
- Average number of customers in the system= Ls = $\frac{\lambda \mu(\lambda/\mu)^C}{(C-1)!\,(C\mu - \lambda)^2} P_0 + \lambda/\mu$
- Average number of customers waiting in the queue= Lq = Ls - λ/ μ
- $= \frac{\lambda \mu(\lambda/\mu)^C}{(C-1)!\,(C\mu - \lambda)^2} P_0$
- Average time a customer spends in the system = Ws = Ls /λ =
- $\frac{\mu(\lambda/\mu)^C}{(C-1)!\,(C\mu - \lambda)^2} P_0 + 1/\mu$
- Average waiting time of a customer spends in the queue= Wq = Lq /λ =
- $\frac{\mu(\lambda/\mu)^C}{(C-1)!\,(C\mu - \lambda)^2} P_0$

Contd…

- Probability that a customer has to wait =
- $$P(n \geq C) = \frac{\mu(\lambda/\mu)^C}{(C-1)!\,(C\mu-\lambda)} P_0$$
- Probability that a customer enters the service without waiting= $1 - P(n \geq C) = 1 - \frac{\mu(\lambda/\mu)^C}{(C-1)!\,(C\mu-\lambda)} P_0$

Average number of idle servers =

C – avg no of customers served

Utilization rate = $P = \lambda/C\mu$

Efficiency of M/M/C model =

$$\frac{\text{avg no of customers served}}{\text{Total no of customers}}$$

Problem.

- A tax consulting firm has three counters in its office to receive people who have problems concerning their income, wealth and sale taxes on the average 48 persons arrive in an8 hours day. Each tax advisor spends 15 minutes on an average on an arrival. If the arrivals are Poisonally distributed and service times are according to exponential distribution, find
- a) The average number of customers in the system.
- b) Average number of customers waiting to be served.
- c) Average time a customer spends in the system.
- d) Average waiting time for a customer.
- e) The number of hours each week a tax advisor spends performing his job.
- f) The probability that a customer has to wait before he gets service.
- g) The expected number of idle tax advisors at any specified time.

Problem.

- Ships arrive at a port at the rate of one in every 4 hours with exponential distribution of inter arrival times. The time a ship occupies a berth for unloading has exponential distribution with an average of 10 hours. If the average delay of ship waiting for berth is to be kept below 14 hours. How many berths should be provided at the port?

Problem.

- A library wants to improve its service facilities in terms of the waiting time of its borrowers. The library has 2 counters at present and borrowers arrive according to Poisson distribution with arrival rate 1 every 6 minutes and service time follows exponential distribution with a mean of 10 minutes. The library has relaxed its membership rules and a substantial increase in the number of borrowers is expected. Find the number of additional counters to be provided. If the arrival rate is expected to be twice the present value and average waiting time of the borrowers must be limited to half the present value.

Problem.

- A bank has two tellers working on saving accounts. The first teller handles with drawls only while the second teller handles deposits only. It has been found that the service time distribution for the deposits and with drawls both is exponential with mean service time 3 minutes per customer. Depositors are found to arrive in a Poisson fashion throughout the day with mean arrival rate 16 per hour. With drawers also arrive in a Poisson fashion with mean arrival rate 14 per hour.
- a) What would be the effect on the average waiting time for depositors and withdrawers if each teller could handle both with drawls and deposits?
- b) What would be the effect if this could only be accomplished by increasing the service time to 3.5 minutes?

Problem.

- At present a servicing department provides answers through one channel, which an average Can deal 24 enquiries per hour at a cost of Rs 3 per enquiry. Increasingly the customer are complaining that they have to wait for a long time and the department is considering alternative arrangements. There is either a two channel system costing Rs 100 per hour and service rate 15 per hour in each, or a three channel system costing Rs 125 per hour and service rate of 10 per hour in each. Customers arrive at the rate of 20 per hour. Average time a customer is in this system. $\frac{(P_K)^K}{k!\,(1-p)^2\,K\mu} P0 + \frac{1}{\mu}$

- $P0 = \frac{k!(1-P)}{(P_K)^K + k!\,(1-P)\,\{\Sigma^{K-1}_{n=0} \frac{(P_K)^n}{n!}\}}$

You are required to calculate

- a) the average time a customer is in the system under the present arrangement.
- b) the extra charges per enquiry that would need to be made to recover the extra cost of each of the two arrangements proposed.

Problem.

- A branch of Punjab national bank has only one typist. Since the typing work varies in length (number of pages to be typed), the typing rate is randomly distributed approximating a Poisson distribution with mean service rate of 8 letters per hour. The letters arrive at a rate of 5 per hour during the entire 8 hour work day. If the waiting time cost of the letters is Rs Rs1.50 per hour, determine average system time and the total lost time cost.

  There is a possibility of either installing an additional type writer of the same type or replacing the present one by a better and faster type writer. The data are given below

	service rate	daily rental cost(Rs)
Present type writer	8	2.5
Proposed type writer	12	4.5

Suggest the better alternative.

Terminology:

- Balking: Customers in most of the cases, do not have long period patience to join and wait for the turn; may be having lack of time. On seeing customer's waiting in a queue for the service, they may not like to join the queue. This is called balking.
- Reneging: There are some customers who have already joined the queue, but on account of slow service rate, or some other reasons, break the queue and leave the system without getting service. This is called reneging.
- Jockeying: Some service units have more channels to serve the customers. This is multi channel facilities. All the channels are busy serving the customers but either some servers operate speedily or customers in the queue demand the type of services which consume less time and as a result the potential queue decreases very fast. In case of such events , customers in the other queue break their queue and join the shorter queue . This is called jockeying.

SEQUENCING

- Definition: Suppose there are n jobs (1,2,3…n) each of which has to be processed one at a time at each of m machines A,B,C… The order of processing each job through machines is given. The time that each job must require on each machine is known. The problem is to find a sequence among $(n!)^m$ number of all possible sequences for processing the jobs so that the total elapsed time for all the jobs will be minimum.
- Mathematically:
- Ai = time for i job on machine A
- Bi = time for i job on machine B etc.
- Ti = time from start of first job to completion of last job
- Then the problem is to determine for a machine a sequence of jobs i_1, i_2,i_3, ------i_n Permutations of the integers which will minimize T.

Terminology

- Number of machines: It means the service facilities through which a job must pass before it is completed.
- Ex: A book to be published has to be processed through composing, printing, binding etc.
- Processing order: It refers to the order in which various machines required for completing the job.
- Processing time: It means the time required by each job on each machine. The notations T_i will denote the processing time required i th machine.

Contd…

- Idle time on a machine: This is the time for which a machine remains idle during the total elapsed time. The notations x_{ij} is used to denote idle time of machine j between the end of (i-1) th job and start of I th job.
- Total elapsed time : This is the time between starting the first job and completing the last job. This also includes idle time and is denoted by T.
- No passing rule: This rule means that passing is not allowed. i.e the same order of jobs is maintained over each machine. If each of the n jobs is to be processed through two machines A and B in the order AB, then this rule means that each job will go to machine A and then to B.

Assumptions:

- No machine can process more than one operation at a time.
- Each operation once started, must be performed until completion.
- A job is an entity. i.e even though the job represents a lot individual parts, no lot may be processed by more than one machine at a time.
- Each operation must be completed before any other operation, which must precede can begin.
- Time intervals for processing are independent of the order in which operations are performed.
- There is only one of each type of machine.
- A job is processed as soon as possible subject to ordering requirements.
- All jobs are known and are ready to start processing before the period under consideration begins.
- The time required to transfer jobs between machines is negligible.

Cases:

1. N jobs and two machines A and B all jobs processed in the order AB.
2. N jobs and three machines A, Band C all jobs processed in the order ABC.
3. Two jobs and m machines. Each job is to be processed through the machines in a prescribed order.
4. Problems with n jobs and m machines.

Johnson's method (n jobs on two machines)

- 1. Examine A_i's and, B_i's for i=1,2,3----nand find min [A_i, B_i]
- 2. a) If this minimum be A_k for i= k do the kth job first of all.
- b) If this minimum be B_r for i= r do the rth job last of all.
- 3. a) If there is a tie for minima $A_k = B_r$ do the kth job first of all and the rth job last of all.
- b) If the tie for minimum occurs among the A_i 's select the job corresponding to minimum of B_i's and do it first of all.
- c) If the tie for minimum occurs among the B_i 's select the job corresponding to minimum of A_i's and do it in last of all . Repeat same.
- 4. Cross out already assigned and repeat above steps until all jobs have been assigned.

Problem.

- There are five jobs , each of which must go through the two machines A and B in the order AB. Processing times are given below.

Job	1	2	3	4	5
Time for m/c A	5	1	9	3	10
Time for m/c B	2	6	7	8	4

- Determine a sequence for five jobs that will minimize the elapsed time T. Calculate the total idle time for the machines in this period.

Problem.

- Find a sequence that will minimize the total elapsed time required to complete following tasks. Calculate the total idle time for the machines in this period.

Tasks	A	B	C	D	E	F	G	H	I
Time m/c I	2	5	4	9	6	8	7	5	4
Time m/cII	6	8	7	4	3	9	3	8	11

Problem.

- Seven jobs are to be operated on m/c M1 and M2 in that order. The time duration in hours is given below.

Jobs	A	B	C	D	E	F	G
Time m/c I	15	19	9	10	12	6	8
Time m/CII	20	13	12	10	16	8	5

Problem.

- A book binder has one printing press, one binding m/c and manuscripts of seven books. The times required for performing printing and binding operations for different books are shown below.

Book	1	2	3	4	5	6	7
Printing days	20	90	80	20	120	15	65
Binding days	25	60	75	30	90	35	50

Decide the optimum sequence of processing of books in order to minimize the total time required to turn out all the books.

Problem.

- Determine an optimum sequence to process the various types of fan blades each day from the following information, so as to minimize the total elapsed time.
- Type

Type	1	2	3	4	5	6
Number to be Processed per day	4	6	5	2	4	3
Time on m/cA(min)	4	12	14	20	8	18
Time on m/cB(min)	8	6	16	22	10	2

Also work out the total elapsed time for an optimum sequence. What is the total idle time on m/cA? on m/c B?

Processing n jobs through 3 machines

- Only three machines A,B and C are involved.
- Each job is processed in the order ABC.
- No passing of job is permitted.
- The actual processing times are as follows

Job	1	2	3	4---	n
m/c A	A_1	A_2	A_3	A_{4----}	A_n
m/c B	B_1	B_2	B_3	B_{4----}	B_n
m/c C	C_1	C_2	C_3	C_{4----}	C_n

The following any one or both must be satisfied, then only the problem will be solved.

1) The smallest processing time on m/c A is greater than or equal to the largest processing time on m/c B.
2) The smallest processing time on m/c C is greater than or equal to the largest processing time on m/c B.

Then the problem can be converted in to n jobs two m/c s by taking

$G_i = A_i + B_i$

$H_i = B_i + C_i$ in the order GH.

Problem.

- We have five jobs , each of which must go through three machines A,B and C, in the order ABC. Processing time (in hrs) are as given below.

Job	1	2	3	4	5
Time on m/c A	16	20	12	14	22
Time on m/c B	10	12	4	6	8
Time on m/c C	8	18	16	12	10

- Determine a sequence for the five jobs that will minimize the total elapsed time. Find also the idle time of machines A,B and C.

Problem.

- A publisher proposes to publish five different books, the manuscripts which have already been submitted to him for publication. He wants to bring all the books out in the market in as short a period as can be allowed. Each book has to be processed through the following (in the order). 1. composing 2. Printing 3. Binding before these can be brought out in the market. The time taken by each of the above process is known to the publisher and it as follows.

Book	1	2	3	4	5
Composing	40	90	80	60	50
Printing	50	60	20	30	40
Binding	80	100	60	70	110

Determine the optimum sequence of giving the manuscript to the press and also find total completion time and idle time on printing machine and binding machine.

Problem.

- A certain furniture manufacturer has five jobs which must go through stages X,Y,Z in the order XYZ. The processing time at each stage it as follows.

Job	1	2	3	4	5
Time for X	14	20	12	14	16
Time for Y	4	6	8	6	10
Time for Z	10	16	8	4	4

- What should be the sequence of the jobs ? Work out the total elapsed time relating to optimum sequence.

Problem.

- A certain manufacturer has to process 6 items through two stages of production viz assembling and polishing. The time taken for each of these items at the different stages is given below in a appropriate units.
- Items 1 2 3 4 5 6
- Time for

Items	1	2	3	4	5	6
Time for assembly:	8	10	6	7	9	14
Polishing:	5	9	10	8	12	8

Find an optimum sequence in which these items are to be processed so as to minimize the total processing time.

Suppose a third stage of production viz packing is added with processing time for the said items as follows.

Items	1	2	3	4	5	6
Time for packing:	13	15	14	16	15	12

Find sequence in which these six items are to be processed so as to minimize the time taken to process all the items through all the three stages of production.

Problem.

- Six jobs are to be processed at three machines is in the order BAC. The time taken by each job on the three machines is given below. Each machine can process one job at a time.

Jobs	1	2	3	4	5	6
Time on m/cA	30	40	20	10	50	35
Time on m/cB	50	80	90	70	60	75
Time on m/cC	40	80	70	60	20	45

- Determine the optimum sequence for the jobs and total elapsed time.

PROJECT MANAGEMENT

- Project is a vision and any research is an accident. Once there is a vision and it is supplemented by hand work and then it becomes a reality.
- a) Planning and organization
- b) controlling the time and cost parameters and coordinating with higher authorities.
- For planning and extension of activities, in-charge of project operation takes care while for evaluation and reporting part the project control incharge takes care.
- Critical path method(CPM)
- Program evaluation and review technique.
- Crashing project duration.

Contd….

- CPM: Duport company developed. Controlling and maintenance of chemical plant. 1959
- PERT: Same period it was developed by US Navy department.
- Project planning: site selection, logistic planning man power planning, procurement planning, financial planning, operation planning, contract and marketing planning, project evaluation and sensitivity analysis.
- Project control: It maintains human resource, plans and manages structure, designs overall quality policy.

a) General evaluation of each department

b) Resolving grievances and updating the higher authorities

c) Work break down structure.

Activity on arrow (AOA)

Activity on node (AON)

CPM example: bridge construction, workshop organization, preparing for university exams

- Activity: Part of total project plan.
- 1. Parallel activity 2. concurrent activity
- 3. dummy activity
- Network:
- Beginning and completion of an activity is a node called event.
- Preceding activity, successor activity,
- The diagram showing interrelationship of occurrence of all activities in a situation is called a network diagram.

Contd…

- Early start time,
- Latest finish time
- LST-LFT= Float
- Total float = T= Ls – Es = Lf – Ef
- Total float is the maximum float by which the activity can be delayed.
- Free float is a part of total float. It is found by subtracting slack of head event from total float.
- Free float =T – slack of head event
- Total duration with free floats enjoyed will not extended the total duration of the project.
- Independent float = free float – slack of tail event
- Interfering float: It is a part of total float. It is the slack in the head event and is the difference of the finish time of all previous activities and the earliest start time of successor activities.
- Critical path: It is a path on which all activities have zero float.

Problem.

- From the following activity table, draw a network diagram and find the critical path.

Activity:	1-2	1-3	2-4	2-5	3-5	4-5	5-6
Duration:	6	8	3	5	9	6	8

Problem.

- From the following activity table, draw a network diagram and find the critical path.

Activity:	1-2	1-3	1-4	2-3	2-5	3-4	3-5
Duration:	6	8	10	7	7	10	6
4-5	4-7	5-6	6-8	7-8			
dummy	6	4	7	2			

Problem.

- From the following activity table, draw a network diagram and find the critical path.

Activity:	1-2	1-3	1-4	2-3	2-5	3-4	3-5
Duration:	8	14	8	10	12	dummy	9

4-6	5-6	6-7
3	5	7

Problem.

- Draw a network diagram and make forward and backward passes and also find the critical path. Also calculate total float, free float, independent float and interfering float.

Activity:	1-2	2-3	2-4	3-5	3-6	4-5
Duration:	3	9	11	7	4	4

4-7	5-8	6-8	7-8
8	6	3	9

Problem.

- Estimated times for the jobs of a project are given below.
-

Job	A	B	C	D	E	F	G
Time	13	5	8	10	9	7	7

H	I	J	K	L
12	8	9	4	17

The constraints governing the job are as follows.

A and B are start jobs , A controls C, D and E. B controls F and J. G depends up on C, H depends on D, E and F controls I and L, K follows J,L is also controlled by K. G,H,I and L are the last jobs. Draw the network, determine float for each activity project duration and critical path.

Problem.

- Tasks A, B, C….. H, I constitute a project. The precedence relationships are A < D, A < E, B < F, D < F, C < G, C < H, F < I, G < I.
- Draw a network to represent the project and find the minimum completion time of the project when time in days of each task is as follows.

• Task	A	B	C	D	E	F	G
• Time	8	10	8	10	16	17	18
	H	I					
	14	9					

PERT

- Optimistic time : It is an optimistic approach about completion time of an activity. If all the circumstances, natural, physical, remain in favour, then the estimated completion time is dented as t_0.
- Normal or moderate regular time: It is a regular time period or usual time slot of complete execution of an activity.
- Pessimistic time: It is the time given for completion of an activity in which all the adverse cases are considered. It is a pessimistic approach for activity timings.
- Average time of an activity = $\frac{t_0 + 4t_n + t_p}{6}$

Standard deviation $S = \frac{t_p - t_0}{6}$ = square root of the sum of variances of critical activities

s^2 = Variance

- Mean = $\bar{x}$

$z = \frac{x - \bar{x}}{s}$

Problem.

- From the given activity table and three types of estimates, find 1) draw the net work diagram. 2) the critical path, 3) standard deviation of the critical path. 4) find the probability of completion the project in due time. 5) what is the probability of completing the project earlier by 10% of the time. 6) What is the probability of completing project by allowing 15% more time.

Activity:	1-2	1-3	2-3	2-4	3-4	3-5	4-5
t_0 :	2	2	7	6	6	6	2
t_n :	6	3	11	14	7	7	6
t_p :	10	4	15	16	14	14	10

Problem.

- From the given activity table and three types of estimates, find 1) draw the net work diagram. 2) the critical path, 3) standard deviation of the critical path.

Activity:	1-2	1-3	1-4	2-5	3-4	3-6	4-6	5-6
• t_0 :	2	2	7	6	0	6	2	7
t_n :	6	3	11	14	0	7	6	8
t_p :	10	4	15	16	0	14	10	15

Problem.

- The time estimates (in weeks) for the activities of a PERT network are given below.

Activity:		1-2	1-3	1-4	2-5	3-5	4-6	5-6
• t_0	:	1	1	2	1	2	2	3
t_n	:	1	4	2	1	5	5	6
t_p	:	7	7	8	1	14	8	15

a) Draw the project network and identify all the paths through it. b) Determine the expected project length. C) calculate the standard deviation and variance of the project length. d) what is the probability that the project will be completed. i) at least 4 weeks earlier than expected time. ii) no more than 4 weeks later than expected time. E)If the project due date is 19 weeks, what is the probability of not meeting the due date. F) the probability that project will be completed on schedule, if the scheduled time is 20 weeks. G) What should be the scheduled completion time for the probability completion to be 90%?

CRASHING

- In real life there is a cut throat competitions. The management possessing longer scale production units always face competitions of introducing and pushing their products in the market.
- Each new product that is introduced first in the market will capture the market and other that follows will have hard time to do. The management that introduces the product late will have to bear additional expenses.
- 1.Continuous advertisements 2. High profit margin to the retailers or stockiest.
- There are some activities whose regular duration for completion can be shortened by applying additional resources. One has to pay for such extra resources.
- There are two impacts on compressing an activity.
- 1) total project duration decreases. 2) cost of project increases.
- Normal time (tn), normal cost (cn)
- By employing extra resources, the activity duration can be shortened. This process of shortening has an upper limit, i.e. there is always a time beyond which one can not shortened the time.
- The minimum time to do an activity is called crash time and is denoted as Tc.
- The cost of one incurs for reducing the time slot from tn to tc is called crash cost. It is denoted as Cc.
- Compression period = tn – tc
- Compression cost = Cc- Cn
- The ratio $\frac{Cc - Cn}{tn - tc}$ cost time ratio or cost slope.

Problem.

- Draw the network from the following table. Find the critical path from the table. Crash the time for maximum possible period with out affecting the critical path.

Activity:	1-2	2-3	2-4	3-5	4-5
t_n :	5	7	6	5	8
t_c :	4	6	5	4	5
C_n :	1000	100	200	100	300
C_c :	1600	170	400	500	600
Cost/slope:	600	70	200	400	100

Problem.

- Draw the network and critical path from the following. Crash the time to a maximum possible period with out affecting the critical path.

Activity:	1-2	2-3	2-4	3-5	4-5
t_n :	5	7	6	5	8
t_c :	4	6	5	4	5
C_n :	1000	100	200	200	300
C_c :	1600	200	400	600	570
Cost/slope:	600	100	200	400	90

DYNAMIC PROGRAMMING

- Richard Bellman developed this technique in early 1950's. This is also called recursive optimization.
- Dynamic means decisions are taken in different stages. Programming means selection of optimum one.
- Discrete, continuous, deterministic as well as probabilistic models can be solved by this technique.
- Large problems spitted in to small problem.

Bellman's principle of optimality

- An optimal policy (set of decisions) has the property that what ever the initial state and decisions are the remaining decisions must constitute an optimal policy with regard to state resulting from the first decision.
- Fn(x)=max(r(dn) + fn-1(T(x,dn))
- The problem which does not satisfy the principle of optimality can not be solved by dynamic programming.
- Capital budgeting, LPP, shortest path, allocation (cargo loading)

Problem.

- A manufacturing company has three sections producing different cosmetic products A,B and C respectively. The management has allocated Rs 50000 for expanding the production facilities. The production can be increased either by adding new machines or replacing some old machines by automatic machines. The cost of adding and replacing the machines along with the expected returns in different sections is given in the table below. Select a set of expansion plans which may yield the maximum returns.
- Alternatives Product A Product B Product C

Alternatives	Product A		Product B		Product C	
	Cost(Rs)	Return(Rs)	Cost(Rs)	Return(Rs)	Cost(Rs)	Return(Rs)
1. No expansion	0	0	0	0	0	0
2. Add new machine	10000	20000	20000	30000	5000	12000
3. Replace old machine	15000	25000	30000	45000	10000	15000

Problem.

- An oil company has 8 units of money available for exploration of three sites. If oil is present at a site, the probability of finding it depends up on the amount allocated for exploiting the site as given below.

Site	units of money								
	0	1	2	3	4	5	6	7	8
1	0.0	0.0	0.1	0.2	0.3	0.5	0.7	0.9	1.0
2	0.0	0.1	0.2	0.3	0.4	0.6	0.7	0.8	1.0
3	0.0	0.1	0.1	0.2	0.3	0.5	0.8	0.9	1.0

- The probability that oil exists at sites,1, 2 and 3 is 0.4, 0.3 and 0.2 respectively. Find the optimal allocation of money.

Problem.

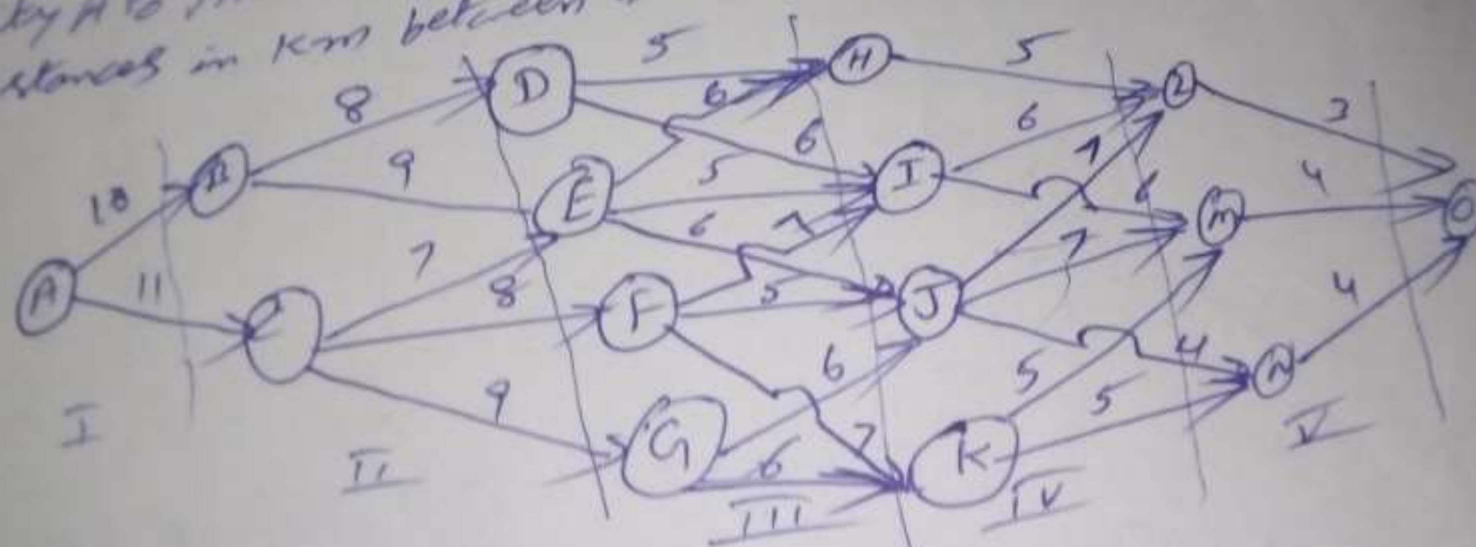
- A cosmetics manufacturing company is interested in selecting the advertising media for its product and frequency of advertising in each media. The data collected over the past two years regarding frequency of advertising in three media of news paper, radio and TV and related sales of the product give following results.

Frequency	Expected sales in thousands of rupees		
	TV	Radio	News paper
1	220	150	100
2	275	250	175
3	325	300	225
4	350	320	250

The cost of advertising in news paper Rs 500 per appearance while in radio and TV, it is Rs 1000 and Rs 2000 respectively. The budget provides Rs 4500 per week for advertisement. The problem is of determining the optimal combination of advertising media and frequency.

Problem.

(5) The possible routes joining the different cities are given. The expected
outcome is to locate the shortest path from any of the cities to any
other city on this diagram, we have shown a directed graph between
15 cities. Here we are required to find shortest path from the
city A to the last city K. The figures on the arrows show the
distances in km between the cities.



stage I

A to B = 10*

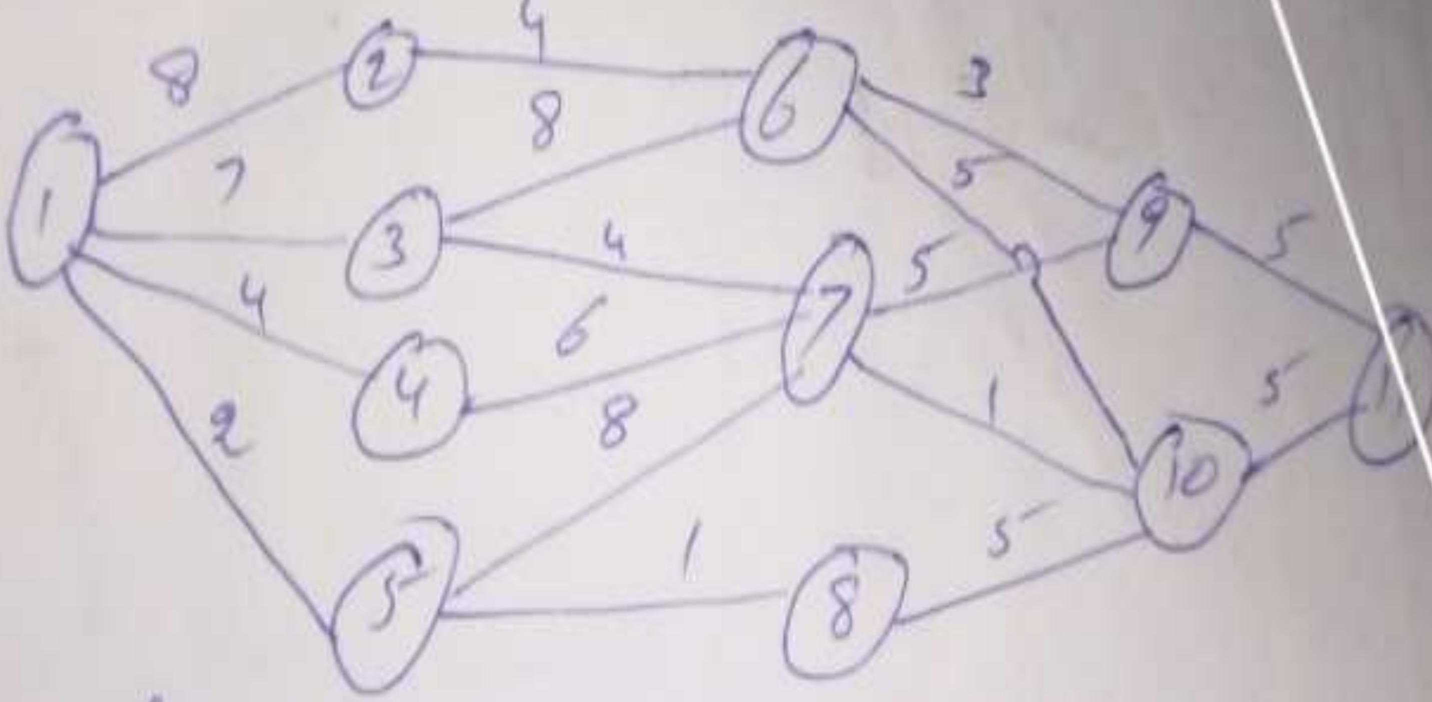
B 10

C 11

start

Problem.

(6) A distance network consists of eleven nodes which are ... shown in fig. find the shortest path from node 1 to node 11 and corresponding distances.



Problem.

- Solve the following model of the optimal subdividing of a cable of length 10 units in to three parts such that their product of their lengths is maximized, using dynamic programming technique.
- (Max $Z= Z1Z2Z3$
- $Z1+Z2+Z3=10$
- $Z1, Z2,Z3\geq 0$)

Problem.

- Solve the following LPP using dynamic programming technique.
- Max Z = $10x_1 + 30x_2$
- $3x_1 + 6x_2 \leq 168$
- $12x_2 \leq 240$
- $x_1, x_2 \geq 0$

REPLACEMENT MODELS

- Replacement theory is concerned with the problem of replacement of machines, electricity bulbs, men etc. due to their deteriorating efficiency, failure, or break down. Replacement is carried out under the following situations.
- 1. When existing items have outlived their effective lives and it may not be economical with them anymore.
- 2. Items which might have been destroyed either by accident or otherwise.

Contd….

- The above two situations can be categorized in to following four categories.
- 1. Replacement of items that deteriorates with time. Eg: machine tools, vehicles, equipment, building etc.
- 2. Replacement of items which do not deteriorates with time but fail completely after certain time of use. Eg: electric bulbs, T.V parts etc.
- 3. Replacement of an equipment that becomes out of date due to new development. Eg: mechanized accounting system by computer system. Ordinary weaving looms by automatic looms etc.
- 4. The existing working staff in an organization gradually diminishes due to death, retirement and other reasons. These replacements thus needed.

Contd…

- The steps involved for replacement problem: -
- 1. Identify the items to be replaced and also their failure mechanism. There are two types of failures. a) gradual b) sudden.
- In gradual failures the equipment has resale or salvage value.
- In sudden failure value of equipment is high. To avoid the cost of sudden failure the concern should try to predict when such failures are likely to occur and try to replace the item before it actually fails.
- 2. Collect the data relating to depreciation cost and the maintenance cost over a time period from the available sources for the items which follow gradual failure mechanism. In the case of items following sudden failure mechanism collect the data for failure rates cost of replacement for failed items and cost off preventive replacement.
- 3. On using the above data, suitable model in OR may be evolved for determining the exact time of replacing the involved items.

Replacement of items that deteriorates with time.(without change in money value):

- To find optimum replacement period of items to this. We will consider basically two costs.
- 1. Maintenance cost and operating cost which increases as the equipment ages.
- 2. depreciation cost which diminishes with age of the equipment.
- The optimum replacement of the items is according to following rules.
- 1. If the scrap value of the equipment is zero. i.e depreciation cost is not given, then replace the equipment when maintenance cost becomes greater than the current average cost.
- 2. If we are given the resale value or the depreciation cost, the maintenance cost and cost of equipment, then the optimum replacement period is determined by the minimum value of the average cost to date.

Contd….

- Annual cost of machine at any time t = capital cost – scrap value + maintenance cost at time t = C-S+f(t)
- maintenance cost during n years is = $\int_0^n f(t)\,dt$
- Total cost $T = C-S(t) + \int_0^n f(t)\,dt$
- The average annual total cost incurred on the item per year during n years =
- $T_a = \dfrac{C-S(t) + \int_0^n f(t)\,dt}{n}$
- To determine the optimum period for replacing the machine, the function is differentiated with respect to n and equal to zero.
- $\dfrac{dT_a}{dn} = -\dfrac{1}{n^2}[C-S(t)] - \dfrac{1}{n^2}\int_0^n f(t)\,dt - \dfrac{1}{n} f(n) = 0$

- $f(n) = \dfrac{C-S(t)}{n} + \dfrac{1}{n}\int_0^n f(t)\,dt$
- i.e. $f(n) = T_a$
- $f(n) = T_a$ is minimum for T provided that f(t) is non decreasing and f(0) = 0.
- Hence an item should be replaced when the average cost to date becomes equal to the current maintenance cost.

Rs
Average cost
Average operating cost
Average depreciation
optimum
TIME

Problem.

- The following table gives the running costs per year and resale values after a certain equipment whose purchase price is Rs 6500. At what year is the replacement due optimality.

Year	1	2	3	4	5	6	7	8
Running cost (Rs)	1400	1500	1700	2000	2400	2800	3000	3900
Resale value (Rs)	4000	3000	2200	1700	1300	1000	1000	1000

Problem.

- A machine shop has a press Which is to be replaced as it wears out. A new press is installed now and an optimum replacement plan is to be for next 7 years after which the press is no longer required. Following data are available.

Year	1	2	3	4	5	6	7
cost of new m/c (Rs)	500	525	550	600	650	725	800
Resale value (Rs)	250	125	75	50	40	25	0
Operating cost (Rs)	150	200	250	300	375	450	575

Find an optimum replacement policy.

Problem.

- Machine A costs of Rs 80000, annual operating costs are Rs2000 for the first year and they increase by Rs 15000 every year. (for example in the fourth year the operating cost is Rs47000). Determine the least age at which to replace the machine. If the optimum replacement is followed, what will be the average yearly cost of operating and owning the machine?(assume that resale value of the machine is zero when replaced and that future costs are not discounted.)
- Another machine B costs Rs100000. Annual operating costs for first year is Rs 4000 and they increase by Rs7000 every year. The firm has a machine of type A which is one year old . Should the firm replaced it with B and if so when?

Problem.

- A transport company owns three mini buses each which was purchased for Rs80000. The costs of running a bus together with resale value are as follows

Year	1	2	3	4	5	6	7	8
Running cost (Rs)	3000	3600	4800	5000	8000	11200	15000	20000
Resale value (Rs)	70000	61000	55000	49000	32000	20000	10000	5000

- Two of these buses are two years old while the third one is one year old. The company contemplates replaces the buses by two full size buses, each such bus containing 50% more seating capacity than a mini bus. Estimate of running cost and resale value of each of new buses are given below. While each such bus would cost Rs 120000.

Year	1	2	3	4	5	6	7	8
Running cost (Rs)	3400	3900	4700	5800	7200	9000	12000	16000
Resale value (Rs)	100000	92000	86000	81000	76000	66000	54000	40000

Should the mini buses be replaced with new full sized buses? If not why? If yes when?

Replacement of items that deteriorates with time.(with change in money value $V^{n-1} = \{1/(r+1)\}^{n-1}$):

- The cost of a new machine is Rs 5000. The maintenance cost during the nth year is given by $R_n = 500 \times (n-1)$, n= 1,2,3 ….suppose that the discount rate per year is 0.05. After how many years it will be economical to replace the machine by a new one.

Problem.

- A manufacturer is offered two machines A and B. A is priced at Rs 5000 and running costs are estimated at Rs 800 for each of the first five years, increasing by Rs 200 per year in the 6 th and subsequent years. Machine B which has the same capacity as A, costs Rs 2500 but will have running costs of Rs 1200 per year for six years increasing by Rs 200 per year there after. If money is worth 10% per year, which machine should be purchased.(Assume that machines will eventually be sold for scrap at negligible price.)

Problem.

- Let the value of money be assumed be 10% per year and suppose that machine A is replaced after every three years, where as machine B is replaced after every six years. The yearly costs of both the machines are given as under:

Year	1	2	3	4	5	6
m/c A	1000	200	400	1000	200	400
m/c B	1700	100	200	300	400	500

Determine which machine should be purchased.

Replacement of items that fail completely

- In real life, the failure of a certain item occurs all of a sudden instead of gradual deterioration. Eg: electric bulb, TV parts etc which results completely breakdown of a system. The breakdown implies loss in production, idle inventory, immediate replacement of item may not be available, idle labour and many other losses, so that the failure of the item puts the organization to a heavy loss. Using the probability distribution of failure time of the item, following two types of replacement policies have been adopted.

Contd….

- Individual replacement policy: under this policy, an item is immediately replaced after its failure.
- Group replacement policy: under this policy, decision is taken as to when all the item must be replaced irrespective of fact that items have failed or have not failed, with the provision that if any item fails before the optimum time, it may be replaced individually. Such policy generally requires two fold consideration namely.

a) The rate of individual replacement during the period

b) The total cost incurred for individual and group replacement policy during selected interval of time.

Contd….

- The period for which total cost incurred is minimum will be the optimum period for replacement. Thus for the formation of group replacement one should know the probability of failure, loss incurred due to these failures, cost of individual replacement and cost of group replacements. The rules for calculating time of group replacement and total cost involved is given below.
- One should replace the group of items at the end of ith period, if the cost of individual replacement for ith period is greater than the average cost per period through the end of ith period.
- One should not replace the group of items at the end of ith period, if the cost of individual replacement at the end of (i-1)th period is less than the average cost per period through the end of ith period.

Problem.

- The management of a large hotel is considering the periodic replacement of light bulbs fitted in its rooms. There are 500 rooms in the hotel and each room has six bulbs. The management is now following the policy of replacing the bulbs as they fail at a total cost of Rs3 per bulb. The management feels that this cost can be reduced to Re1 by adopting the periodic replacement method. On the basis of information given below evaluate the alternative and make a recommendation to the management.

Months of use:	1	2	3	4	5
%of bulbs failing by that month	10	25	50	80	100

Problem.