VEMU INSTITUTE OF TECHNOLOGY

NEAR PAKALA, CHITTOOR-517112

(Approved by AICTE, New Delhi & Affiliated to JNTUA, Anantapuramu)

ELECTROMAGNETIC FIELDS

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ELECTROMAGNETIC FIELDS

UNIT I - Introduction

UNIT II - Static Electric Field

UNIT III - Static Magnetic Field

UNIT IV - Time Varying Fields And Maxwell's Equations

UNIT V - Electromagnetic Waves

COURSE OBJECTIVES

- ☐ To introduce the concept of co-ordinate systems and vector field
- ☐ To describe static electric fields, their behavior in different media, boundary conditions and electromagnetic potentials.
- ☐ To impart knowledge on the concept of static magnetic fields for simple configuration
- ☐ To Analyze the Maxwell's equations in differential and integral forms
- ☐ To understand the propagation of electromagnetic waves through different media.

COURSE OUTCOMES

- ☐ Differentiate different types of coordinate systems and use them for solving the problems of electromagnetic field theory.
- ☐ Interpret the concepts of static electric fields and apply boundary conditions on Electrostatic field.
- ☐ Develop concepts of static magnetic fields and apply boundary conditions.
- ☐ To use integral and point form of Maxwell`s equations for solving the problems of electromagnetic field theory .
- ☐ Describe the propagation of electromagnetic waves, Poynting vector and theorem.

ELECTROMAGNETIC FIELDS

UNIT I - INTRODUCTION

Sources and effects of electromagnetic fields - Vector fields - Different co-ordinate systems - Gradient, Divergence and Curl operation - Divergence theorem -Stoke's theorem - Coulomb's Law - Electric field intensity - Field due to point and continuous charges – Electric flux density - Gauss's law and application.

UNIT II - STATIC ELECTRIC FIELD

Electrical potential - Electric field and equipotential plots - Relationship between E and V - Electric field in free space, conductors, dielectric - Dielectric polarization, Electric field in multiple dielectrics - Boundary conditions, Poisson's and Laplace's equations - Capacitance energy density - Dielectric strength.

ELECTROMAGNETIC FIELDS

UNIT III - STATIC MAGNETIC FIELD

Lorentz Law of force, magnetic field intensity – Biot savart Law - Ampere's Law - Magnetic field due to straight conductors, circular loop, infinite sheet of current - Magnetic flux density in free space, conductor, magnetic materials - Boundary conditions - Scalar and vector potential - Magnetic force – Torque – Inductance – Energy density - Magnetic circuits.

UNIT IV- TIME VARYING FIELDS AND MAXWELL'S EQUATIONS

Faraday's laws, induced emf - Static and dynamic EMF, Maxwell's equations (differential and integral forms) - Displacement current - Relation between field theory and circuit theory.

UNIT V- ELECTROMAGNETIC WAVES

Electromagnetic wave generation equations - Uniform plane waves - Phase and group velocity, attenuation - Propagation in good conductors - Waves in free space, lossy and lossless dielectrics, conductors - Skin depth, Poynting theorem and vector.

TEXT BOOKS & REFERENCE BOOKS

TEXT BOOKS

Sl.No	Author(s)	Title of the Book	Publisher	Year of Publication
1.	Gangadhar K A, Ramanathan	Electromagnetic Field Theory	Khanna Publishers	2011
2.	William H. Hayt & Buck	Engineering Electromagnetics	Tata McGraw Hill	2012

REFERENCE BOOKS:

Sl.No	Author(s)	Title of the Book	Publisher	Year of Publication
1.	Meenakumari R & Subasri R	Electromagnetic Fields	New Age International Ltd Publishers	2010
2.	Mathew N. O. Sadiku	Principles of Electromagnetics	Oxford University Press	2010
3.	Kraus and Fleish	Electromagnetics with Applications	Tata McGraw Hill	2008
4.	Ashutosh Pramanik	Electromagnetism – Theory and Applications	PHI Learning Private Limited	2009
5.	Bhag Singh Guru and Hüseyin R	Electromagnetic field theory Fundamentals	Cambridge University Press	2009

SOURCES OF ELECTROMAGNETIC FIELDS

- ☐ Natural sources of electromagnetic fields
- ☐ Human-made sources of electromagnetic fields

EFFECTS OF ELECTROMAGNETIC FIELDS

- ☐ Low frequency and high frequency electromagnetic waves affect the human body in different ways.
- ☐ Human nervous system
- ☐ Birds and animals
- ☐ Human respiratory system
- ☐ Human memory loss
- ☐ Plants and Animals.
- ☐ Electrical components.

VECTOR FIELDS

Fields are classified as

Scalar field –Scalars are quantities characterized by magnitude only and algebraic sum.

Examples : atmospheric temperature and Pressure

Vector field - magnitude and direction

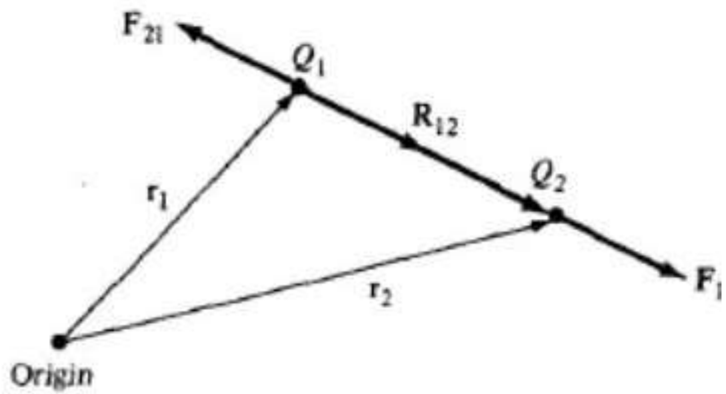
Examples : wind velocity and gravitational force in atmosphere.

Coulomb's Law

It states that the force F between two point charges Q_1 and Q_2 is

$$F = \frac{kQ_1Q_2}{R^2}$$

In Vector form



$$\mathbf{F}_{12} = \frac{Q_1Q_2}{4\pi\varepsilon_o R^3}\mathbf{R}_{12}$$

Or $$\mathbf{F}_{12} = \frac{Q_1Q_2\,(\mathbf{r}_2 - \mathbf{r}_1)}{4\pi\varepsilon_o|\mathbf{r}_2 - \mathbf{r}_1|^3}$$

Electric Field Intensity

Electric Field Intensity is the force per unit charge when placed in
electric field

$$E = \frac{F}{Q}$$

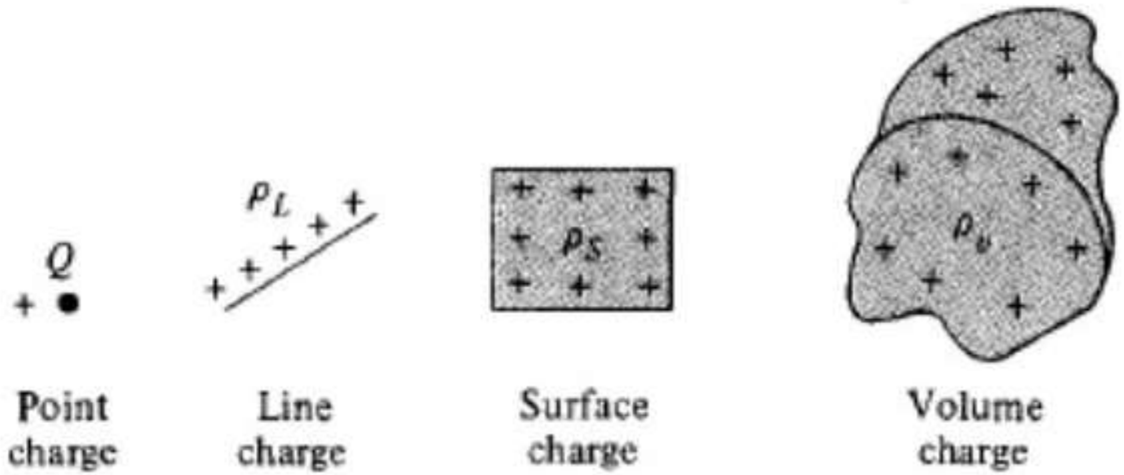
In Vector form

$$\mathrm{E} = \frac{Q(\mathbf{r} - \mathbf{r}')}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}'|^3}$$

If we have more than two point charges

Electric Field due to Continuous Charge Distribution

If there is a continuous charge distribution say along a line, on a surface, or in a volume



The charge element dQ and the total charge Q due to these cha distributions can be obtained by

$$dQ = \rho_v \, dv \rightarrow Q = \int_v \rho_v \, dv \qquad \text{(volume charge)}$$

The electric field intensity due to each charge distribution ρ_L, ρ_S a ρ_V may be given by the summation of the field contributed by t numerous point charges making up the charge distribution.

$$\mathbf{E} = \int \frac{\rho_L \, dl}{4\pi\varepsilon_o R^2} \mathbf{a}_R \qquad \text{(line charge)}$$

$$\mathbf{E} = \int \frac{\rho_S \, dS}{4\pi\varepsilon_o R^2} \mathbf{a}_R \qquad \text{(surface charge)}$$

Electric Flux Density

The electric field intensity depends on the medium in which t charges are placed.

Suppose a vector field D independent of the medium is defined by

$$D = \varepsilon_o E$$

The electric flux ψ in terms of D can be defined as

$$\Psi = \int \mathbf{D} \cdot d\mathbf{S}$$

The vector field D is called the electric flux density and is measure

Electric Flux Density

For an infinite sheet the electric flux density D is given by

$$\mathbf{D} = \frac{\rho_S}{2}\,\mathbf{a}_n$$

For a volume charge distribution the electric flux density D is gi
by

$$\mathbf{D} = \int \frac{\rho_v\,dv}{4\pi R^2}\,\mathbf{a}_R$$

Gauss Law

It states that the total electric flux ψ through any closed surface equal to the total charge enclosed by that surface.

$$\psi = Q_{enc}$$

$$\Psi = \oint d\Psi = \oint_S \mathbf{D} \cdot d\mathbf{S}$$

$$\text{Total charge enclosed } Q = \int \rho_v \, dv$$

Using Divergence Theorem

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{D} \, dv \qquad \text{(ii)}$$

Comparing the two volume integrals in (i) and (ii)

$$\rho_v = \nabla \cdot \mathbf{D}$$

This is the first Maxwell's equation.

It states that the volume charge density is the same as the diverge of the electric flux density.

Electric Potential

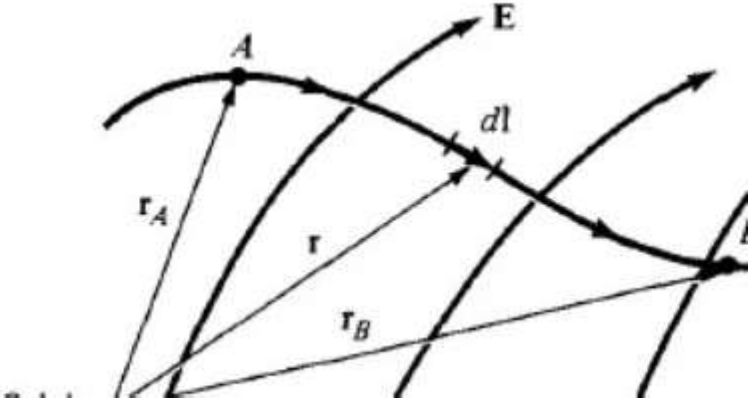
Electric Field intensity, E due to a charge distribution can be obtai
from Coulomb's Law.

or using Gauss Law when the charge distribution is symmetric.

We can obtain E without involving vectors by using the electric sca
potential V.

From Coulomb's Law the force on point charge Q is

$$\vec{F} = Q\vec{E}$$



The work done in displacing the charge

The total work done or the potential energy required in moving point charge Q from A to B is

$$W = -Q\int_A^B \vec{E}.dl$$

Dividing the above equation by Q gives the potential energy per u charge.

$$\frac{W}{Q} = -\int_A^B \vec{E}.dl \ = V_{AB}$$

V_{AB} is known as the potential difference between points A and B.

1. If V_{AB} is negative, there is loss in potential energy in moving from A to B (work is being done by the field), if V_{AB} is positive, the

The potential at any point due to a point charge Q located at the origi

$$V = \frac{Q}{4\pi\varepsilon_o r}$$

The potential at any point is the potential difference between t point and a chosen point at which the potential is zero.

Assuming zero potential at infinity, the potential at a distance r fr the point charge is the work done per unit charge by an external ag in transferring a test charge from infinity to that point.

$$V = -\int_{\infty}^{r} \vec{E}.d\vec{l}$$

For n point charges Q_1, Q_2, Q_3…..Q_n located at points with posit

vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3 ..... \vec{r}_n$ the potential at $\vec{r}$ is

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_o} \sum_{k=1}^{n} \frac{Q_k}{|\vec{r} - \vec{r}_k|}$$

If there is continuous charge distribution instead of point charges tl

the potential at $\vec{r}$ becomes

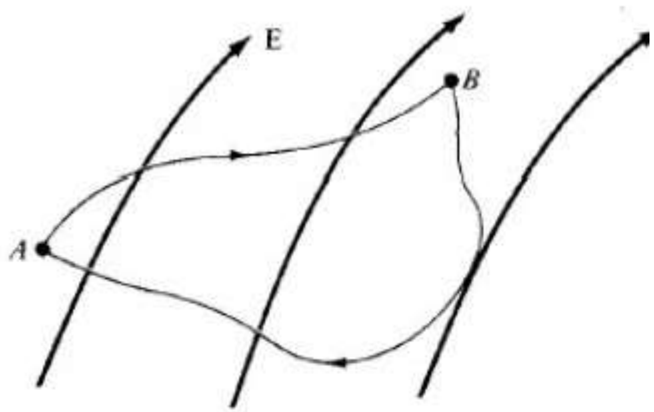
$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_o} \int_L \frac{\rho_L(\mathbf{r}')dl'}{|\mathbf{r} - \mathbf{r}'|} \qquad \text{(line charge)}$$

$$V(\mathbf{r}) = \frac{1}{} \int \frac{\rho_S(\mathbf{r}')dS'}{\phantom{|\mathbf{r} - \mathbf{r}'|}} \qquad \text{(surface charge)}$$

Relationship between E and V

The potential difference between points A and B is independent of t
path taken

$$V_{AB} = -V_{BA}$$



$$V_{AB} = -\int_{A}^{B} \vec{E}.d\vec{l} \quad \text{and} \quad V_{BA} = \int_{B}^{A} \vec{E}.d\vec{l}$$

$$V_{AB} + V_{BA} = \oint \vec{E}.d\vec{l} = 0$$

$$\oint \vec{E}.d\vec{l} = 0 \qquad \text{(i)}$$

Physically it means that no net work is done in moving a charge alo
a closed path in an electrostatic field.

Applying Stokes's theorem to equation (i)

$$\oint \vec{E}.d\vec{l} = \int (\vec{\nabla} \times \vec{E}).d\vec{S} = 0$$

$$\vec{\nabla} \times \vec{E} = 0 \qquad \text{(ii)}$$

Equation (i) and (ii) are known as Maxwell's equation for sta
electric fields.

Equation (i) is in integral form while equation (ii) is in different

Also

$$\vec{E} = -\nabla V$$

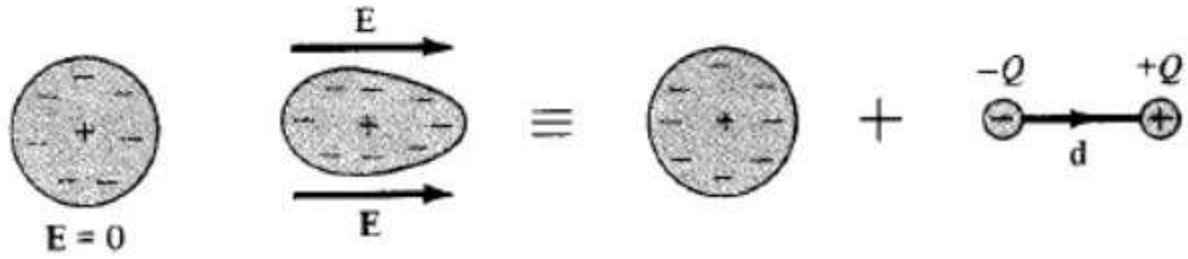
It means Electric Field Intensity is the gradient of V.

The negative sign shows that the direction of $\vec{E}$ is opposite to t direction in which V increases.

Polarization in Dielectrics

Consider an atom of the dielectric consisting of an electron cloud (- and a positive nucleus (+Q).

When an electric field $\vec{E}$ is applied, the positive charge is displac from its equilibrium position in the direction of $\vec{E}$ by $\vec{F_+} = Q\vec{E}$ wh the negative charge is displaced by $\vec{F_-} = Q\vec{E}$ in the oppos direction.



This distorted charge distribution is equivalent to the origi
distribution plus the dipole whose moment is

$$\vec{p} = Q\vec{d}$$

where $\vec{d}$ is the distance vector between -Q to +Q.

If there are N dipoles in a volume Δv of the dielectric, the total dip
moment due to the electric field

$$Q_1\mathbf{d}_1 + Q_2\mathbf{d}_2 + \cdots + Q_N\mathbf{d}_N = \sum_{k=1}^{N} Q_k\mathbf{d}_k$$

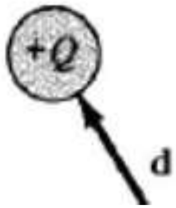
For the measurement of intensity of polarization, we defin
polarization $\vec{P}$ (coulomb per square meter) as dipole moment per u
volume

The major effect of the electric field on the dielectric is the creation dipole moments that align themselves in the direction of electric fiel

This type of dielectrics are said to be non-polar. eg: H_2, N_2, O_2

Other types of molecules that have in-built permanent dipole mome are called polar. eg: H_2O, HCl

When electric field is applied to a polar material then its perman dipole experiences a torque that tends to align its dipole moment in t direction of the electric field.

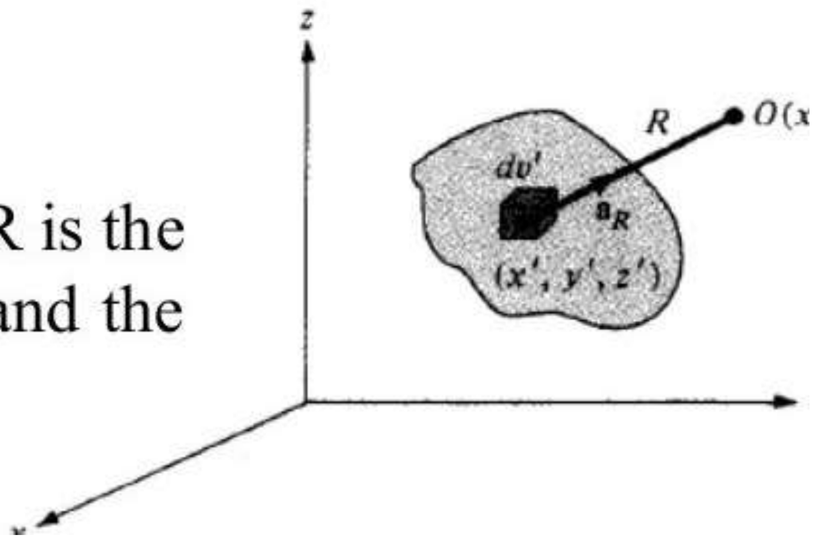


Field due to a Polarized Dielectric

Consider a dielectric material consisting of dipoles with Dip moment $\vec{P}$ per unit volume.

The potential dV at an external point O due to $\vec{P}dv'$

$$dV = \frac{\mathbf{P} \cdot \mathbf{a}_R \, dv'}{4\pi\varepsilon_0 R^2} \qquad \text{(i)}$$



where $R^2 = (x-x')^2+(y-y')^2+(z-z')^2$ and R is the distance between volume element dv' and the point O.

But $\mathbf{P} \cdot \mathbf{a}_R$ ... (1)

$$\frac{\mathbf{P} \cdot \mathbf{a}_R}{R^2} = \nabla' \cdot \frac{\mathbf{P}}{R} - \frac{\nabla' \cdot \mathbf{P}}{R}$$

Put this in (i) and integrate over the entire volume v' of the dielectri

$$V = \int_{v'} \frac{1}{4\pi\varepsilon_o}\left[\nabla' \cdot \frac{\mathbf{P}}{R} - \frac{1}{R}\nabla' \cdot \mathbf{P}\right] dv'$$

Applying Divergence Theorem to the first term

$$V = \int_{S'} \frac{\mathbf{P} \cdot \mathbf{a}'_n}{4\pi\varepsilon_o R} dS' + \int_{v'} \frac{-\nabla' \cdot \mathbf{P}}{4\pi\varepsilon_o R} dv' \qquad \text{(ii)}$$

where $\mathbf{a}_n$' is the outward unit normal to the surface dS' of the dielect where ρ_{ps} and ρ_{pv} are the bound surface and volume charge densities.

Bound charges are those which are not free to move in the dielectr material.

Equation (ii) says that where polarization occurs, an equivale volume charge density, ρ_{pv} is formed throughout the dielectric whi an equivalent surface charge density, ρ_{ps} is formed over the surface dielectric.

The total positive bound charge on surface S bounding the dielectric

$$Q_b = \oint \mathbf{P} \cdot d\mathbf{S} = \int \rho_{ps}\, dS$$

while the charge that remains inside S is

Total charge on dielectric remains zero.

$$\text{Total charge} = \oint_S \rho_{ps}\, dS + \int_v \rho_{pv}\, dv = Q_b - Q_b = 0$$

When dielectric contains free charge

If ρ_v is the free volume charge density then the total volume ch
density ρ_t

$$\rho_t = \rho_v + \rho_{pv} = \nabla \cdot \varepsilon_o \mathbf{E}$$

Hence

$$\rho_v = \nabla \cdot \varepsilon_o \mathbf{E} - \rho_{pv}$$

$$= \nabla \cdot (\varepsilon_o \mathbf{E} + \mathbf{P})$$

The effect of the dielectric on the electric field $\vec{E}$ is to increase $\vec{D}$ inside it by an amount $\vec{P}$.

The polarization would vary directly as the applied electric field.

$$\mathbf{P} = \chi_e \varepsilon_0 \mathbf{E}$$

Where χ_e is known as the electric susceptibility of the material

It is a measure of how susceptible a given dielectric is to electric fiel

Dielectric Constant and Strength

We know that

$$\mathbf{D} = \varepsilon_o \mathbf{E} + \mathbf{P} \quad \text{and} \quad \mathbf{P} = \chi_e \varepsilon_o \mathbf{E}$$

Thus

$$\mathbf{D} = \varepsilon_o (1 + \chi_e)\, \mathbf{E} = \varepsilon_o \varepsilon_r \mathbf{E}$$

or

$$\mathbf{D} = \varepsilon \mathbf{E}$$

where $\varepsilon = \varepsilon_o \varepsilon_r$

and

$$\varepsilon_r = 1 + \chi_e = \frac{\varepsilon}{\ }$$

No dielectric is ideal. When the electric field in a dielectric sufficiently high then it begins to pull electrons completely out of molecules, and the dielectric becomes conducting.

When a dielectric becomes conducting then it is called dielec breakdown. It depends on the type of material, humidity, temperat and the amount of time for which the field is applied.

The minimum value of the electric field at which the dielec breakdown occurs is called the dielectric strength of the dielec material.

or

The dielectric strength is the maximum value of the electric field th

Continuity Equation and Relaxation Time

According to principle of charge conservation, the time rate decrease of charge within a given volume must be equal to the outward current flow through the closed surface of the volume.

The current I_{out} coming out of the closed surface

$$I_{out} = \oint \mathbf{J} \cdot d\mathbf{S} = \frac{-dQ_{in}}{dt} \qquad \text{(i)}$$

where Q_{in} is the total charge enclosed by the closed surface.

Using divergence theorem

$$\oint \mathbf{J} \cdot d\mathbf{S} = \int \nabla \cdot \mathbf{J}\, dv$$

Equation (i) now becomes

$$\int_v \nabla \cdot \mathbf{J}\, dv = -\int_v \frac{\partial \rho_v}{\partial t}\, dv$$

or $$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t} \qquad \text{(ii)}$$

This is called the continuity of current equation.

Effect of introducing charge at some interior point of conductor/dielectric

According to Ohm's law

$$\mathbf{J} = \sigma \mathbf{E}$$

Equation (ii) now becomes

$$\nabla \cdot \sigma \mathbf{E} = \frac{\sigma \rho_v}{\varepsilon} = -\frac{\partial \rho_v}{\partial t}$$

or $$\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon}\rho_v = 0$$

This is homogeneous liner ordinary differential equation. By separa variables we get

$$\frac{\partial \rho_v}{\rho_v} = -\frac{\sigma}{\varepsilon}\partial t$$

Integrating both sides

$$\rho_v = \rho_{vo} e^{-t/T_r} \qquad \text{(iii)}$$

where

$$T_r = \frac{\varepsilon}{\sigma}$$

ρ_{vo} is the initial charge density (i.e., ρ_v at $t = 0$)

Equation (iii) shows that as a result of introducing charge at so interior point of the material there is a decay of the volume cha density ρ_v.

The time constant T_r is known as the relaxation time or the relaxat time.

Relaxation time is the time in which a charge placed in the interior

Boundary Conditions

If the field exists in a region consisting of two different media, t conditions that the field must satisfy at the interface separating t media are called boundary conditions

These conditions are helpful in determining the field on one side the boundary when the field on other side is known.

We will consider the boundary conditions at an interface separating

1. Dielectric (ϵ_{r1}) and Dielectric (ϵ_{r2})
2. Conductor and Dielectric
3. Conductor and free space

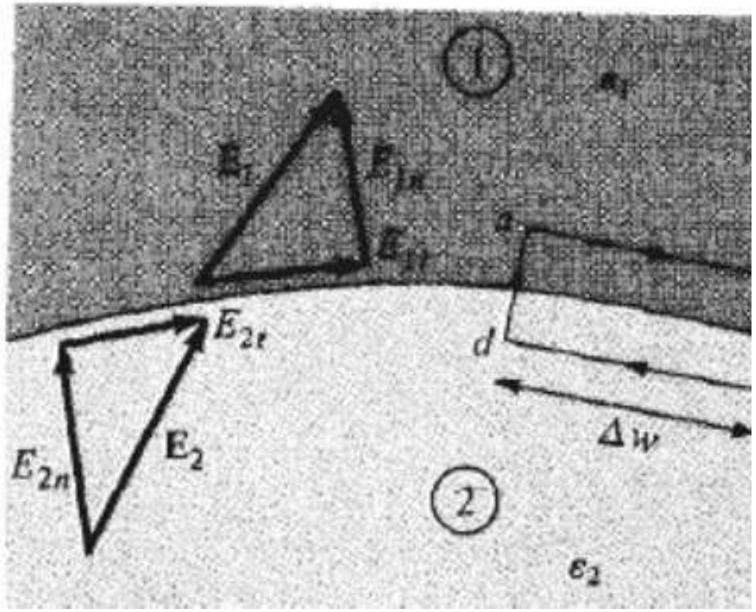
Boundary Conditions (Between two different dielectrics)

Consider the **E** field existing in a region consisting of two differe
dielectrics characterized by $\epsilon_1 = \epsilon_0 \epsilon_{r1}$ and $\epsilon_2 = \epsilon_0 \epsilon_{r2}$

$\mathbf{E}_1$ and $\mathbf{E}_2$ in the media 1 and 2 can be written as

$$\vec{E}_1 = \vec{E}_{1t} + \vec{E}_{1n} \quad \text{and} \quad \vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n}$$

But $$\oint \mathbf{E} \cdot d\mathbf{l} = 0$$



As $\Delta h \longrightarrow 0$

$$E_{1t} = E_{2t}$$

Thus the tangential components of **E** are the same on the two sides the boundary. **E** is continuous across the boundary.

But $$\mathbf{D} = \varepsilon\mathbf{E} = \mathbf{D}_t + \mathbf{D}_n$$

Thus

$$\frac{D_{1t}}{\varepsilon_1} = E_{1t} = E_{2t} = \frac{D_{2t}}{\varepsilon_2}$$

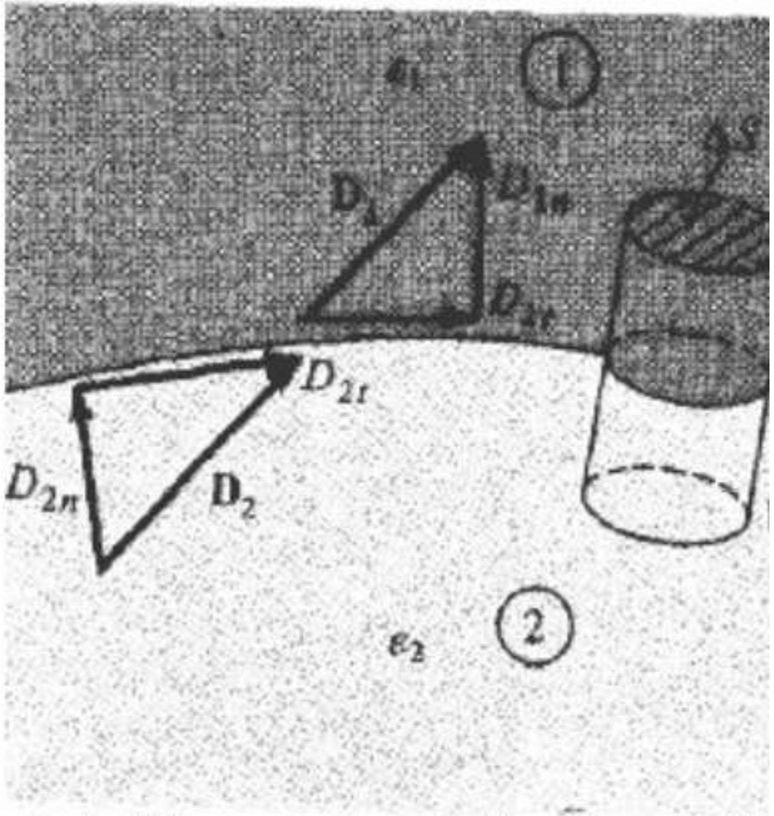
or D_{1t} D_{2t}

Applying $\oint \mathbf{D} \cdot d\mathbf{S} = Q_{enc}$

Putting $\Delta h \rightarrow 0$ gives

$$\Delta Q = \rho_S \Delta S = D_{1n} \Delta S - D_{2n} \Delta S$$

$$D_{1n} - D_{2n} = \rho_S$$



Where ρ_s is the free charge density placed deliberately at the bounda

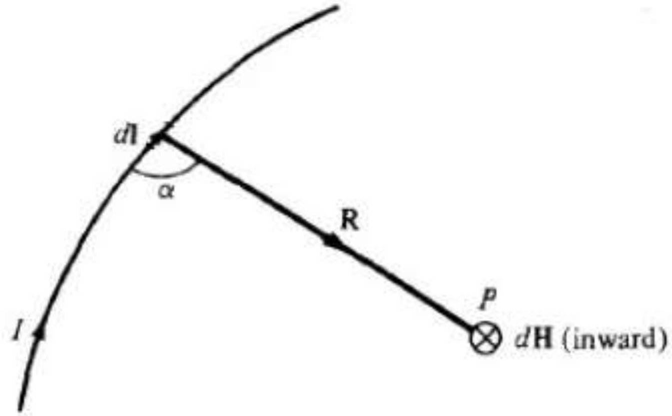
If there is no charge on the boundary i.e. $\rho_s = 0$ then

Biot-Savart's Law

It states that the magnetic field intensity d**H** produce at a point P the differential current element Idl is proportional to the product and the sine of angle α between the element and line joining P to t element and is inversely proportional to the square of distance between P and the element.

$$dH \propto \frac{I\,dl \sin\alpha}{R^2} \quad \text{or} \quad dH = \frac{I\,dl \sin\alpha}{4\pi R^2}$$

$$d\mathbf{H} = \frac{I\,d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2} = \frac{I\,d\mathbf{l} \times \mathbf{R}}{4\pi R^3}$$



Ampere's circuit Law

The line integral of the tangential component of **H** around a close p
is the same as the net current I_{inc} enclosed by the path.

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{\text{enc}}$$

Application of Ampere's law : Infinite Sheet Current

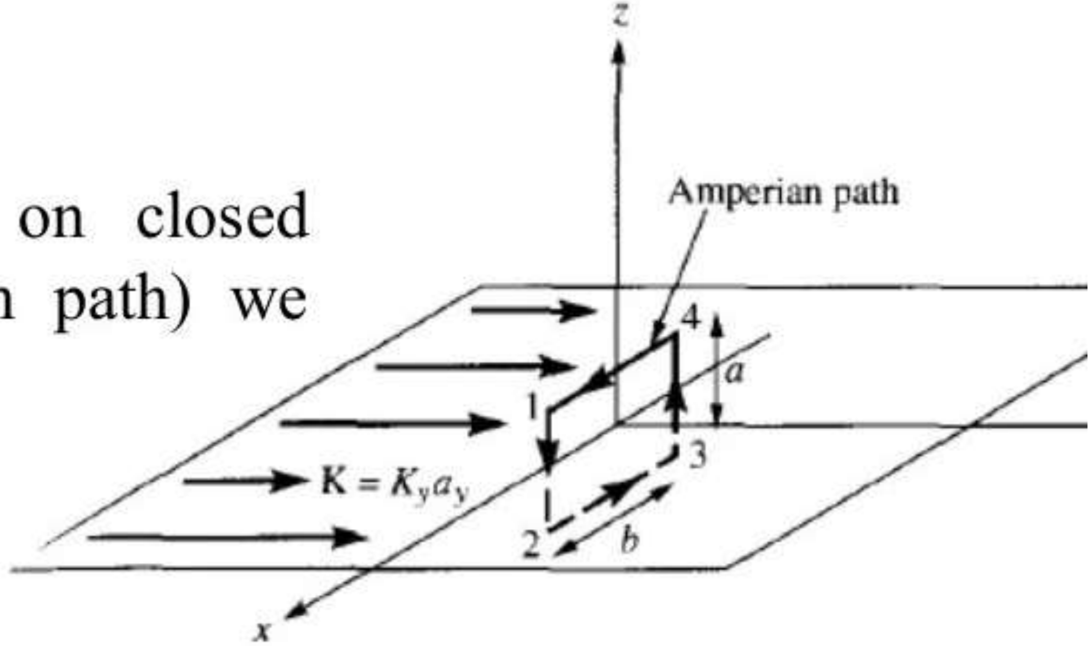
Consider an infinite current sheet in z = 0 plane.

If the sheet has a uniform current density then

$$\vec{K} = K_y \hat{a}_y$$

Applying Ampere's Law on closed rectangular path (Amperian path) we get

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc} = K_y b \quad \text{(i)}$$



The resultant $d\mathbf{H}$ has only an x-component.

Also **H** on one side of sheet is the negative of the other.

Due to infinite extent of the sheet, it can be regarded as consisting of such filamentary pairs so that the characteristic of **H** for a pair are the same for the infinite current sheets

$$\mathbf{H} = \begin{cases} H_o \mathbf{a}_x & z > 0 \end{cases} \qquad \text{(ii)}$$

Evaluating the line integral of **H** along the closed path

$$\oint \mathbf{H} \cdot d\mathbf{l} = \left(\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 \right) \mathbf{H} \cdot d\mathbf{l}$$

$$= 0(-a) + (-H_o)(-b) + 0(a) + H_o(b)$$

$$= 2H_o b \qquad \text{(iii)}$$

Comparing (i) and (iii), we get

$$H_o = \frac{1}{2} K_y \qquad \text{(iv)}$$

Using (iv) in (ii), we get

Generally, for an infinite sheet of current density **K**

$$\mathbf{H} = \frac{1}{2}\mathbf{K} \times \mathbf{a}_n$$

where a_n is a unit normal vector directed from the current sheet to t
point of interest.

Magnetic Flux Density

The magnetic flux density **B** is similar to the electric flux density **D** Therefore, the magnetic flux density **B** is related to the magnetic fi intensity **H**

$$\mathbf{B} = \mu_o \mathbf{H}$$

where μ_o is a constant and is known as the permeability of free spac

Its unit is Henry/meter (H/m) and has the value

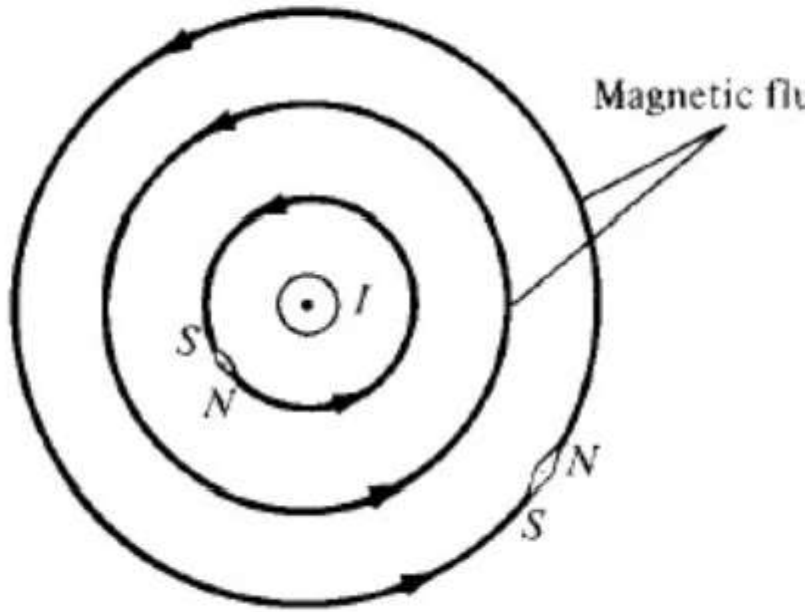
$$\mu_o = 4\pi \times 10^{-7} \text{ H/m}$$

The magnetic flux through a surface S is given by

$\int$

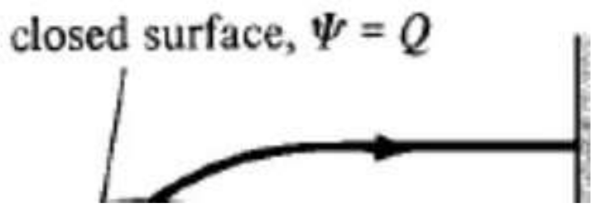
Magnetic flux lines due to a straight wire with current coming out of the page

Each magnetic flux line is closed with no beginning and no end and are also not crossing each other.



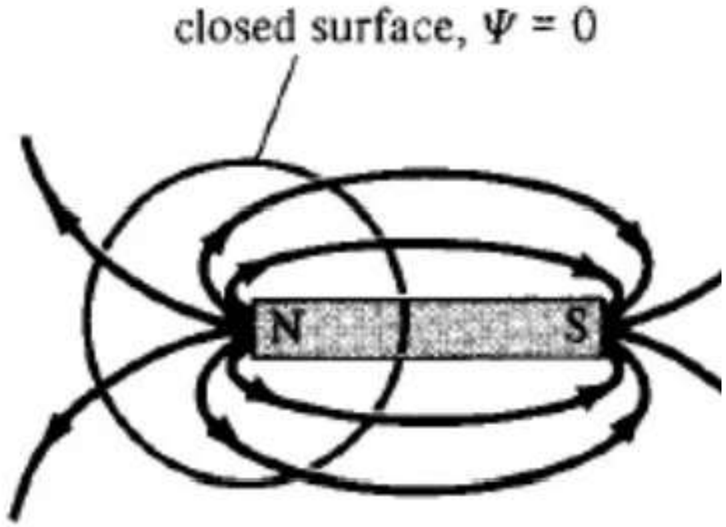
In an electrostatic field, the flux passing through a closed surface i the same as the charge enclosed.

$$\Psi = \oint \mathbf{D} \cdot d\mathbf{S} = Q$$



Magnetic flux lines are always close upon themselves,.

So it is not possible to have an isolated magnetic pole (or magnetic charges)



An isolated magnetic charge does not exist.

Thus the total flux through a closed surface in a magnetic field m be zero.

$$\oint \mathbf{B} \cdot d\mathbf{S} = 0$$

Applying Divergence theorem, we get

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{B}\, dv = 0$$

or $$\nabla \cdot \mathbf{B} = 0$$

This is Maxwell's fourth equation.

This equation suggests that magnetostatic fields have no source sinks.
Also magnetic flux lines are always continuous.

Faraday's law

According to Faraday a time varying magnetic field produces induced voltage (called electromotive force or emf) in a closed circu which causes a flow of current.

The induced emf (V_{emf}) in any closed circuit is equal to the time rate change of the magnetic flux linkage by the circuit. This is Faraday Law and can be expressed as

$$V_{emf} = -\frac{d\lambda}{dt} = -N\frac{d\Psi}{dt}$$

where N is the number of turns in the circuit and ψ is the flux throu each turn.

Transformer and Motional EMF

For a circuit with a single turn (N = 1)

$$V_{\text{emf}} = -\frac{d\Psi}{dt}$$

In terms of **E** and **B** this can be written as

$$V_{\text{emf}} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S} \qquad \text{(i)}$$

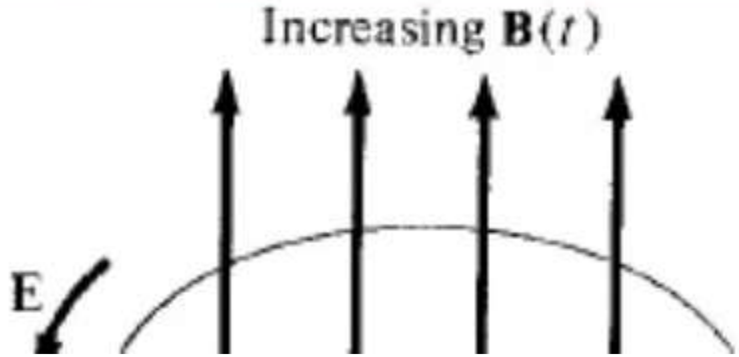
where ψ has been replaced by $\int_S \mathbf{B} \cdot d\mathbf{S}$ and S is the surface area the circuit bounded by a closed path L..

The variation of flux with time may be caused in three ways.

1. By having a stationary loop in a time-varying **B** field.
2. By having a time-varying loop area in a static **B** field.
3. By having a time-varying loop area in a time-varying **B** field.

Stationary loop in a time-varying B field (Transformer emf)

Consider a stationary conducting loop in a time-varying magnetic **B** field. The equation (i) becomes



This emf induced by the time-varying current in a stationary loop often referred to as transformer emf in power analysis since it is due the transformer action.

By applying Stokes's theorem to the middle term, we get

$$\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

Thus

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

This is one of the Maxwell's equations for time-varying fields.

2. Moving loop in static B field (Motional emf

When a conducting loop is moving in a static **B** field, an emf introduced in the loop.

The force on a charge moving with uniform velocity **u** in a magne field **B** is

$$\mathbf{F}_m = Q\mathbf{u} \times \mathbf{B}$$

The motional electric field $\mathbf{E}_m$ is defined as

$$\mathbf{E}_m = \frac{\mathbf{F}_m}{Q} = \mathbf{u} \times \mathbf{B}$$

Consider a conducting loop moving with uniform velocity **u,** the e induced in the loop is

By applying Stokes's theorem to equation (i), we get

$$\int_S (\nabla \times \mathbf{E}_m) \cdot d\mathbf{S} = \int_S \nabla \times (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{S}$$

$$\nabla \times \mathbf{E}_m = \nabla \times (\mathbf{u} \times \mathbf{B})$$

3. Moving loop in time-varying field

Consider a moving conducting loop in a time-varying magnetic fiel

Then both transformer emf and motional emf are present.

Thus the total emf will be the sum of transformer emf and motio
emf

$$V_{emf} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} + \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

also

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

Displacement Current

For static EM fields

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad \text{(i)}$$

But the divergence of the curl of a vector field is zero. So

$$\nabla \cdot (\nabla \times \mathbf{H}) = 0 = \nabla \cdot \mathbf{J} \qquad \text{(ii)}$$

But the continuity of current requires

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t} \neq 0 \qquad \text{(iii)}$$

Equation (ii) and (iii) are incompatible for time-varying conditions

Again the divergence of the curl of a vector field is zero. So

$$\nabla \cdot (\nabla \times \mathbf{H}) = 0 = \nabla \cdot \mathbf{J} + \nabla \cdot \mathbf{J}_d \qquad \text{(v)}$$

In order for equation (v) to agree with (iii)

$$\nabla \cdot \mathbf{J}_d = -\nabla \cdot \mathbf{J} = \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t}(\nabla \cdot \mathbf{D}) = \nabla \cdot \frac{\partial \mathbf{D}}{\partial t}$$

or

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{dt} \qquad \text{(vi)}$$

Putting (vi) in (iv), we get

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Maxwell's Equations in Final Form

Differential Form	Integral Form	Remarks
$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$	Gauss's law
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$	Nonexistence of isol magnetic charge*
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot d\mathbf{S}$	Faraday's law
$\partial \mathbf{D}$	$\int \quad \int \left(\quad \partial \mathbf{D} \right)$	

Introduction

- **Electromagnetic (EM) waves** were first postulated by James Clerk Maxwell and subsequently confirmed by Heinrich Hertz
- Maxwell derived a wave form of the electric and magnetic equations, revealing the wave-like nature of electric and magnetic fields, and their symmetry
- Because the speed of EM waves predicted by the wave equation coincided with the measured speed of light, Maxwell concluded that light itself is an EM wave
- According to Maxwell's equations, a spatially-varying electric field generates a time-varying magnetic field and *vice versa*
- Therefore, as an oscillating electric field generates an oscillating magnetic field, the magnetic field in turn generates an oscillating electric field, and so on
- These oscillating fields together form an electromagnetic wave

Speed of EM waves

- In the studies of electricity and magnetism, experimental physicists had determined two physical constants - the electric (ε_o) and magnetic (μ_o) constant in vacuum

- These two constants appeared in the EM wave equations, and Maxwell was able to calculate the velocity of the wave (i.e. the speed of light) in terms of the two constants:

$$c = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \approx 3.0 \times 10^8 \text{ m/s}$$

$\varepsilon_0 = 8.8542 \times 10^{-12}\ C^2\ s^2/kgm^3$ (permittivity of vacuum)

$\mu_0 = 4\pi \times 10^{-7}$ kgm/A^2s^2 (permeability of vacuum)

- Therefore the three experimental constants, ε_o, μ_o and c previously thought to be independent are now related in a fixed and determined way

Polarization of Electromagnetic Wave

The transverse EM wave is said to be polarized (more specifically, plane polarized) if the electric field vectors are parallel to a particular direction for all points in the wave

direction of the electric field vector **E** = direction of polarization

Example, consider an electric field propagating in the positive *z*-direction and polarized in the *x*-direction

$$\vec{E} = E_0 \sin(kz - \omega t)\hat{x}$$

$$c = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \quad \Rightarrow \quad \vec{B} = \left(\frac{1}{c}\right) E_0 \sin(kz - \omega t)\,\hat{y}$$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \quad \Rightarrow \quad \vec{S} = \varepsilon_0 c\, E_0^2 \sin(kz - \omega t)\hat{z}$$

Thank you