Electrical Circuit Analysis (20A02301T)

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SYLLABUS

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY ANANTAPUR
(Established by Govt. of A.P., ACT No.30 of 2008)
ANANTHAPURAMU – 515 002 (A.P) INDIA

ELECTRICAL AND ELECTRONICS ENGINEERING

Course Code 20A02301T — ELECTRICAL CIRCUIT ANALYSIS — L T P C 3 0 0 3

UNIT - I Locus Diagrams & Resonance 8 Hrs

Series R-L, R-C, R-L-C and Parallel Combination with Variation of Various Parameters -Resonance-Series, Parallel Circuits, Frequency Response, Concept of Bandwidth and Q Factor.

UNIT - II Two Port Networks 9 Hrs

Two Port Network Parameters – Impedance – Admittance - Transmission and Hybrid Parameters and their Relations - Concept of Transformed Network - Two Port Network Parameters Using Transformed Variables.

UNIT - III Transient Analysis 12 Hrs

D.C Transient Analysis: Transient Response of R-L, R-C, R-L-C Series Circuits for D.C Excitation - Initial Conditions in network - Initial Conditions in elements - Solution Method Using Differential Equation and Laplace Transforms - Response of R-L & R-C Networks to Pulse Excitation. A.C Transient Analysis: Transient Response of R-L, R-C, R-L-C Series Circuits for Sinusoidal Excitations - Solution Method Using Differential Equations and Laplace Transforms.

SYLLABUS

UNIT - IV Fourier Transforms 10 Hrs

Fourier Theorem - Trigonometric Form and Exponential Form of Fourier series – Conditions of Symmetry - Line Spectra and Phase Angle Spectra - Analysis of Electrical Circuits to Non Sinusoidal Periodic Waveforms. Fourier Integrals and Fourier Transforms – Properties of Fourier Transforms and Application to Electrical Circuits.

UNIT - V Filters 9 Hrs

Filters – Low Pass – High Pass, Band Pass and Band Stop– RC, RL filters– derived filters and composite filters design – Attenuators – Principle of Equalizers – Series and Shunt Equalizers – L Type - T type and Bridged – T and Lattice Equalizers.

Textbooks:

1. William Hayt, Jack E. Kemmerly and Jamie Phillips, “Engineering Circuit Analysis”, McGraw Hill, 9th Edition, 2019.
2. A. Chakrabarti, “Circuit Theory: Analysis & Synthesis”, DhanpatRai& Sons, 2008. Reference Books:
1. M.E. Van Valkenberg, “Network Analysis”, 3rd Edition, Prentice Hall (India), 1980.
2. V. Del Toro, “Electrical Engineering Fundamentals”, Prentice Hall International, 2009.
3. Charles K. Alexander and Matthew. N. O. Sadiku, “Fundamentals of Electric Circuits” McGraw Hill, 5th Edition, 2013.
4. MahamoodNahvi and Joseph Edminister, “Electric Circuits” Schaum’s Series, 6th Edition, 2013.
5. John Bird, Routledge, “Electrical Circuit Theory and Technology”, Taylor & Francis, 5th Edition, 2014.

Unit-1:Locus Diagrams & Resonance

Def of Locus diagram:

"It is defined as the Locus of the current obtained for various values of the Variable element."

classification

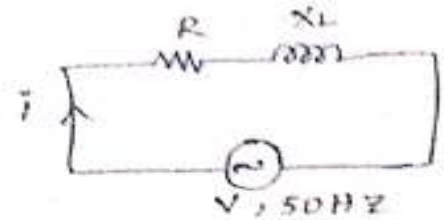
Locus diagrams are classified into two types

① series R-L, R-C and R-L-C circuits.

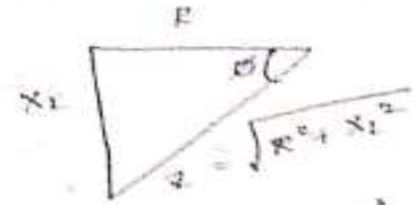
② Parallel combination of circuits.

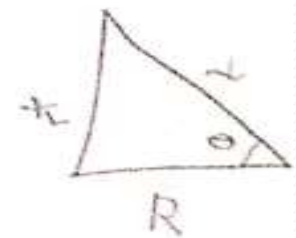
Unit-1:Locus Diagrams & Resonance contd...

Locus diagram of R-L circuit :-



Impedence diagram





According to KVL

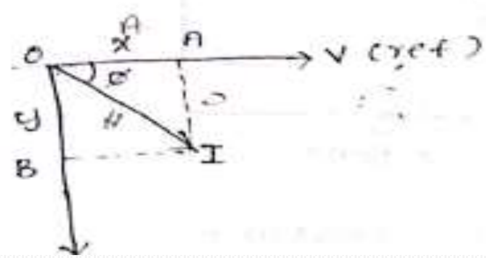
$V = I(R + jX_L)$

$V = IZ$, $Z = R + jX_L$

$I = \frac{V}{Z} \longrightarrow (1)$

Now , phasor diagram for R-L circuit is

* ($x = R$, $Y = $) *



Unit-1:Locus Diagrams & Resonance contd…

From the phasor diagram

$$\cos\phi = \frac{x}{I}$$

$$x = I\cos\phi \longrightarrow (2)$$

$$\sin\phi = \frac{OB}{I} = \frac{y}{-I}$$

$$y = -I\sin\phi \longrightarrow (3)$$

$$x^2 + y^2 = I^2\cos^2\phi + I^2\sin^2\phi$$

$$x^2 + y^2 = I^2(\cos^2\phi + \sin^2\phi)$$

Unit-1:Locus Diagrams & Resonance contd...

$$x^2+y^2 = I^2$$

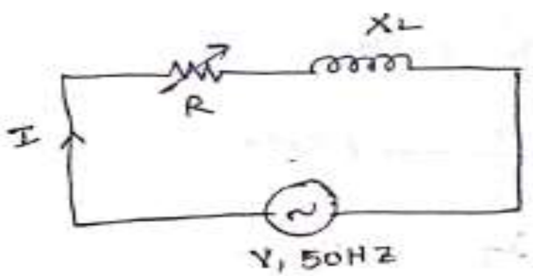
$$x^2+y^2 = \left(\frac{V}{Z}\right)^2$$

$$x^2+y^2 = \frac{V^2}{Z^2}$$

$$x^2+y^2 = \frac{V^2}{\left(\sqrt{R^2+X_L^2}\right)^{\cancel{2}}}$$

$$\boxed{\therefore x^2+y^2 = \frac{V^2}{R^2+X_L^2}} \longrightarrow (4)$$

case(i):- Variable R, constant X_L



Unit-1:Locus Diagrams & Resonance contd…

Here X_L is constant

$$y = -I \sin\phi$$

$$= -\frac{V}{Z} \times \frac{X_L}{Z}$$

$$= -\frac{V X_L}{R^2 + X_L^2}$$

$$y = -X_L \times \frac{V}{R^2 + X_L^2} \longrightarrow (5)$$

$$\frac{V}{R^2 + X_L^2} = \frac{-y}{X_L} \longrightarrow (6)$$

Substitute Eqn (6) in Eqn (4)

Unit-1:Locus Diagrams & Resonance contd…

$$x^2+y^2 = V \times \frac{V}{R^2+X_L^2}$$

$$x^2+y^2 = V \times \frac{-y}{X_L}$$

$$x^2+y^2+2y\cdot\frac{V}{2X_L}+\left(\frac{V}{2X_L}\right)^2-\left(\frac{V}{2X_L}\right)^2=0$$

$$x^2+\left(y+\frac{V}{2X_L}\right)^2=\left(\frac{V}{2X_L}\right)^2$$

$$\therefore\ x^2+\left(y+\frac{V}{2X_L}\right)^2=\left(\frac{V}{2X_L}\right)^2 \longrightarrow (7)$$

Now we know the circle equation

$$(x-x_1)^2+(y-y_1)^2=r^2 \longrightarrow (8)$$

Unit-1:Locus Diagrams & Resonance contd...

comparing eqn's (7) & (8), we get,

$x_1 = 0,$

$y_1 = \frac{-V}{2X_L}$

$r = \frac{V}{2X_L}$

$\therefore$ centre $= (x_1, y_1) = \left(0, \frac{-V}{2X_L}\right)$

radius $(r) = \frac{V}{2X_L}$.

Construction of locus diagram :-

$\cos\phi = \frac{R}{Z}$ & $I = \frac{V}{Z} \rightarrow I = \frac{V}{\sqrt{R^2 + X_L^2}}$

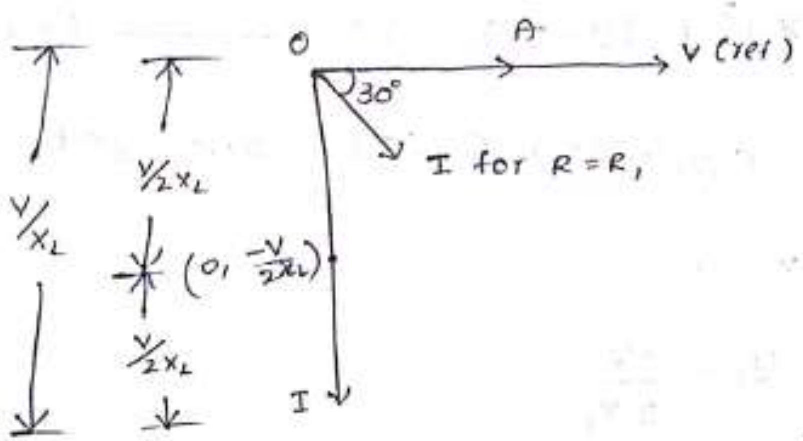
Suppose if $\phi = 30^\circ$ then $\cos 30^\circ = 0.866$

$\phi = 45^\circ$ then $\cos 45^\circ = 0.707$

$\phi = 60^\circ$ then $\cos\phi = 0.5$

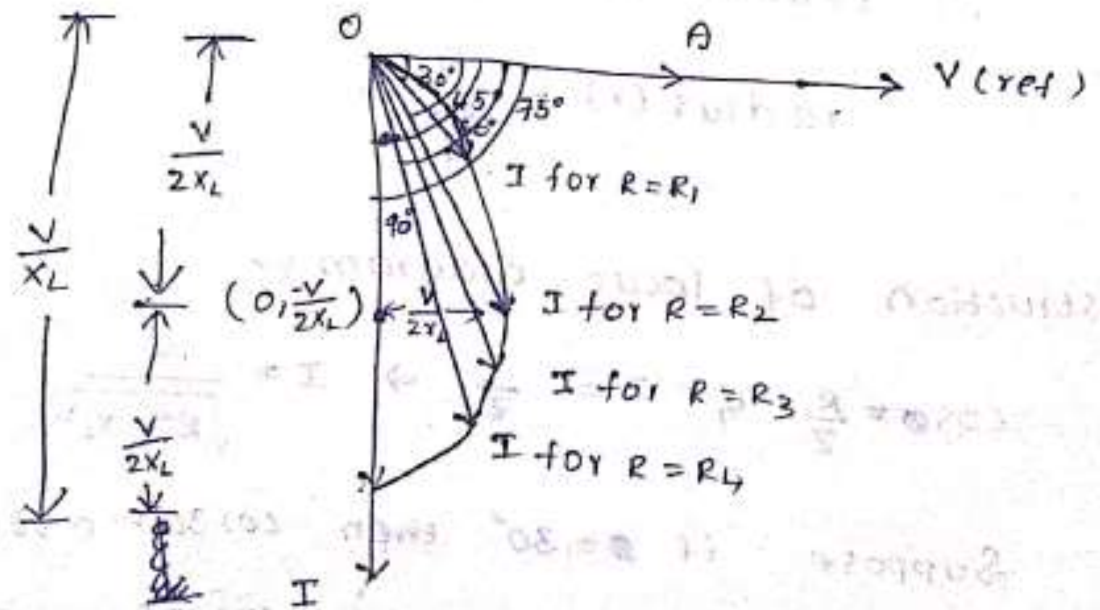
Unit-1:Locus Diagrams & Resonance contd...

If $\cos 30° = 0.866$ and compared due to other angle it is high value and we know that R is directly proportional to $\cos\phi$ and 'R' value is high then if we put high value of 'R' in current eqn then the magnitude of current is less. that means at 30° magnitude of current is less.



Unit-1:Locus Diagrams & Resonance

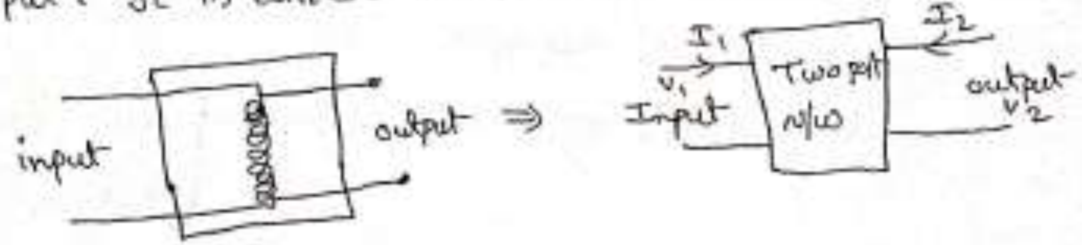
If angle increases 'R' value decreases then current values increases.



Unit-2: Two Port Networks

Two port n/w

It is the rectangular box represents a network, consists of two pairs of network [Four terminals (V_1, I_1, V_2, I_2)] where one pair of terminals can be designated as input, the other pair being output. It is called two port n/w.



Types of two port parameters

- In Two port n/w, Four variables are V_1, I_1, V_2, I_2
- out of Four variables, Two variables are dependent and other two variables are independent variables.
- These dependent & Independent variables are depends on the type of Parameter.

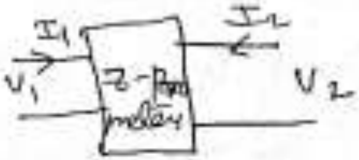
Unit-2: Two Port Networks contd…

① **Z-parameters (or) open ckt parameters:**

Z-parameters are represented as

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad -(1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad -(2)$$



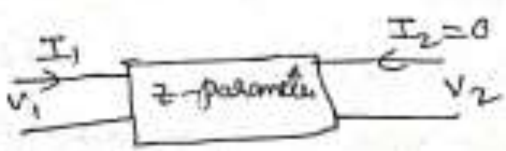
From the above equations

dependent variables : V_1 & V_2

Independent variables : I_1 & I_2

From eq (1) & (2) Z-parameters are $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

case(i): To find Z_{11} & Z_{21} by open ckting the output port i.e $I_2 = 0$.



Unit-2: Two Port Networks contd…

put $I_2 = 0$ in eq (1) & (2)

From eq(1) $V_1 = Z_{11} I_1$

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$ → Driving point Input Imped…

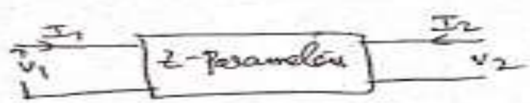
From eq (2)

$$V_2 = Z_{21} I_1$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$$ → Forward transfer Impedance

case(2): To find Z_{12} & Z_{22} by open circuiting port 1

i.e $I_1 = 0$



Put $I_1 = 0$ in eq (1) & (2)

From eq(1), $V_1 = Z_{12} I_2$

$$\boxed{Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}}$$ → Reverse transfer Impedance.

From eq (2)

$$V_2 = Z_{22} I_2$$

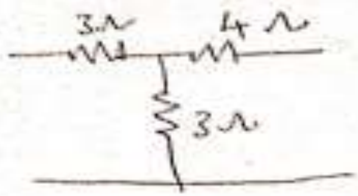
$$\boxed{Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}}$$ — Driving point output Impedance.

Unit-2: Two Port Networks contd...

Problems

Z-Parameters (Problem)

① Find Z-Parameters for the following ckt



Sol

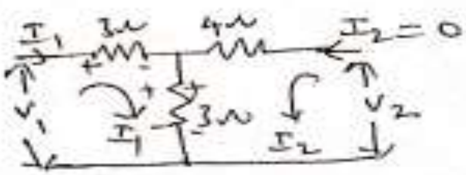
Z-Parameters

$V_1 = Z_{11} I_1 + Z_{12} I_2$ — ①

$V_2 = Z_{21} I_1 + Z_{22} I_2$ — ②

case (1) To find Z_{11} & Z_{21}, put $I_2 = 0$ $\left[Z_{11} = \frac{V_1}{I_1}, \ Z_{21} = \frac{V_2}{I_1} \right]$

Apply KVL for loop 1



$V_1 - 3I_1 - 3I_1 - 3I_2 = 0$

Put $I_2 = 0$

$V_1 - 6I_1 = 0$

$V_1 = 6I_1$

$\boxed{\frac{V_1}{I_1} = 6\,\Omega = Z_{11}}$

Unit-2: Two Port Networks contd…

Apply KVL for loop 2

$V_2 - 4I_2 - 3I_2 - 3I_1 = 0$

put $I_2 = 0$

$V_2 - 3I_1 = 0$

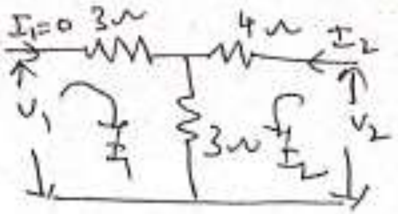
$V_2 = 3I_1$

$\boxed{\frac{V_2}{I_1} = Z_{21} = 3}$ Ω

Case (2): To obtain Z_{12} & Z_{22}, put $I_1 = 0$

$Z_{12} = \frac{V_1}{I_2}$, $Z_{22} = \frac{V_2}{I_2}$

apply KVL for loop 1



$V_1 - 3I_1 - 3I_1 - 3I_2 = 0$

put $I_1 = 0$

$V_1 - 3I_2 = 0$

$V_1 = 3I_2$

$\boxed{\frac{V_1}{I_2} = Z_{12} = 3\,\Omega}$

Unit-2: Two Port Networks contd…

apply KVL for loop 2

$$V_2 - 4I_2 - 3I_2 - 3I_1 = 0$$

put $I_1 = 0$

$$V_2 - 7I_2 = 0$$

$$V_2 = 7I_2$$

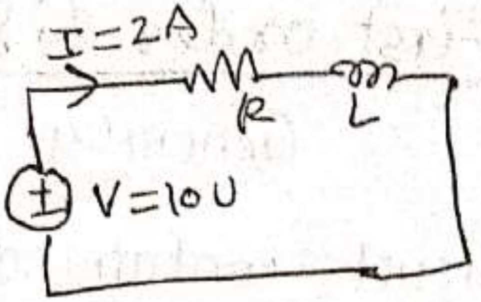
$$\boxed{\frac{V_2}{I_2} = z_{22} = 7\,\Omega}$$

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 3 & 7 \end{bmatrix}$$

Introduction

1. Steady state :

A circuit having voltage source or current source is said to be steady state if voltage and currents doesn't change with respect to time.



Ex : t=2 sec , V=10V , I=2A

t=5 sec , V=10V , I=2A

2. Transient state:

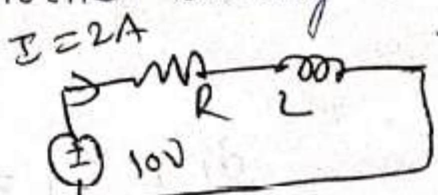
A circuit having voltage source, current source, circuit parameters like R, L & C. Now either change in voltage or current source or any one of the circuit parameters, then voltage and currents are changed from one state to another state is called transient state.

(OR)

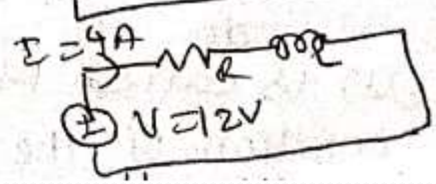
The behaviour of voltage and currents when it is changed from one steady state to another steady state is called transient state.

Ex: t = 2 sec, V = 10 V, I = 2A

t = 5 sec, V = 12 V, I = 4A



At t = 2Sec



At t = 5Sec

3. Transient time :

The time required for changing the voltage and currents from one state to another state is called Transient time.

4. Natural Response :

The circuit having circuit parameters like R, L, C with independent current and voltage sources, the response of output depends on the nature of the circuit, then the response is called natural response.

5 Transient Response:

The circuit having circuit parameters like R, L, C with independent voltage and current sources, then the energy storing elements (L&C) which delivers energy to the resistors, the response or o/p change with time and get saturated.

6 Forced Response:

The circuits having circuit parameters like R, L, C with dependent voltage and current sources, the response or output depends on the sources, then it is called forced response.

Initial Conditions

Initial conditions

S

Switch open switch close switch

(i) At $t=0$, switches are operative made

(ii) At $t=0^-$, it is the time just before closing the switch i.e. switch is about to close.

(iii) At $t=0^+$, It is the time just after ~~the~~ immediatly closing the switch

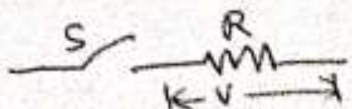
III. TRANSIENTS

Initial Conditions

(1) **Resistor (R) :**

→ The property of resistor is to opposes the flow of current

Case (1) : At $t = 0^-$:

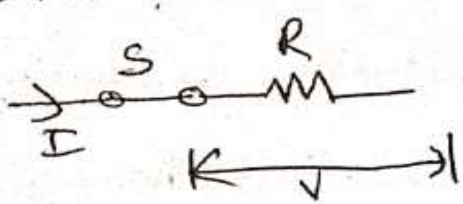


when switch is opens, The current doesn't flow.

Hence, $I = 0$

case (2) : At $t = 0^+$ [Transient State]

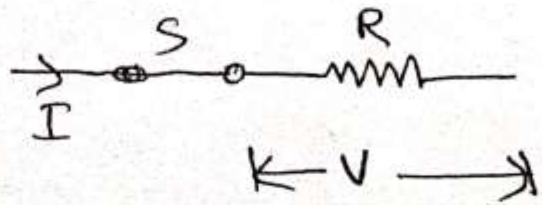
$t = 0^+$ means, The time just immediatly after closing switch.



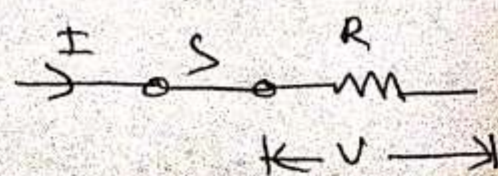
Hence $\boxed{V = IR}$

$\boxed{I = \frac{V}{R}}$

Case (3) : At $t = \alpha$ [steady state]



$t = \alpha$ means, The current flows continuously to resistor Then at infinity time

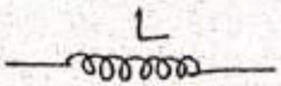


$$\boxed{V = IR}$$

$$\boxed{I = \frac{V}{R}.}$$

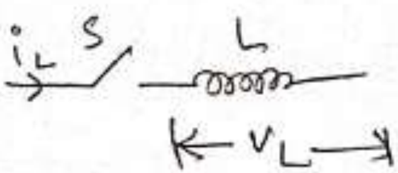
(2) Inductor (L)

→ Property of inductor is it opposes the sudden change in current

Symbol 

Case (1) : At $t = 0^-$

At $t = 0^-$ means switch is open

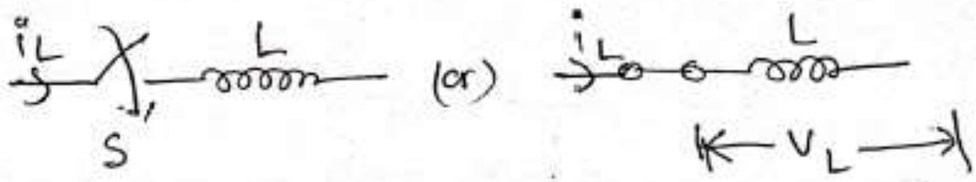


under this conditions, $i_L = 0$

Hence $\boxed{t = 0^-,\ i_L = 0}$

Case (2) : At $t = 0^+$ [Transient state]

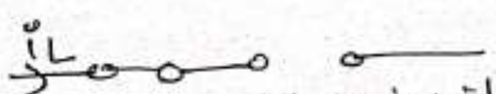
- At $t = 0^+$ means it is the time just immediatly after closing the switch.



- But due to inductor property, it doesn't allow sudden change in current.

Hence $\boxed{t = 0^+, \; i_L = 0}$, V_L = Full voltage

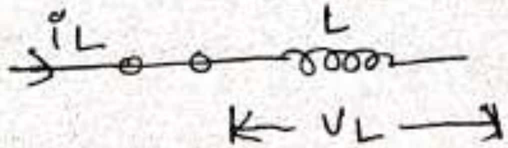
- Hence Inductor acts as open circuit



opencircuit

case (3) : At $t = \alpha$ (steady state]

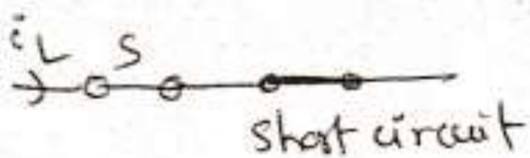
– when current flows continuously, then full current flows in the inductor



At $t = \alpha$, $i_L = i_{\text{Full current}}$

$V_L = 0$

→ Hence, Inductor acts as Short circuit

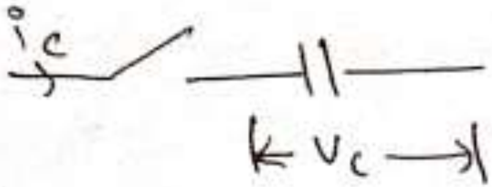


(3) Capacitor (C)

The property of capacitor is it doesn't allow sudden change in voltage.

case(1) : At $t = 0^-$

when switch is open

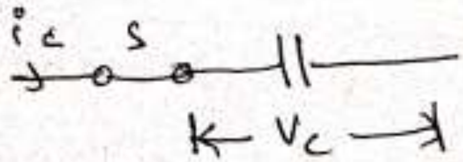


Hence,

$$\boxed{t = 0^-, \quad V_c = 0}$$

case(2): At $t=0^+$ [Transient state]

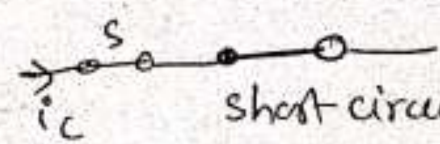
→ $t=0^+$, means after immediatly closing switch



→ But due to property of capacitor,

$$\boxed{\text{At } t=0^+, \quad V_c = 0}$$

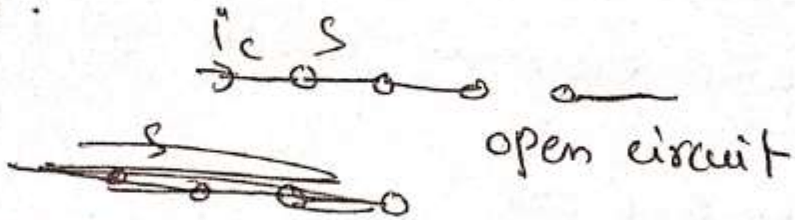
→ Hence, It acts as short circuit



short circuit

Case(3) : At $t=\infty$ (steady state)

→ under this conditions, Full voltage appears across the capacitor.



→ $\boxed{\text{At } t=\infty,\ v_c = V_{\text{full voltage}}}$ & $i_c = 0$

→ Hence capacitor acts as open circuit.

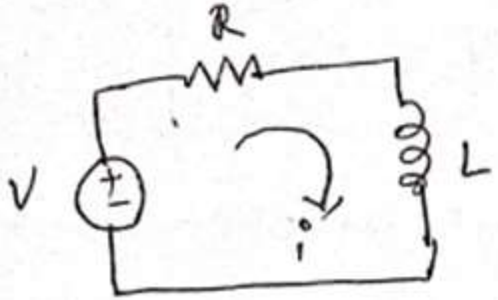
Summery

Parameter	At $t=0^-$	At $t=0^+$ (Transient state)	At $t=\infty$ (steady state)
Resistor (R)	$i=0$	$V=IR$ $I=V/R$	$V=IR$, $I=\frac{V}{R}$
Inductor (L)	$i_L=0$	$i_L=0$ Acts as open ckt	$i_L=I_{FL}$ acts as short ckt
capacitor (C)	$V_C=0$	$i_C=I_{FL}$ $V_C=0$ Acts as short ckt	$V_C=V_{FL}$ Acts as open ckt

First order differential equations

Generally KVL is applied to circuit containing R-L, R-C and R-L-C elements which gives differential equation

For eg: Take R-L ckt



KVL is applied to the ckt

$$V = IR + L\frac{di}{dt}$$

$$\therefore \frac{di}{dt} + \frac{R}{L}i = \frac{V}{L} \quad -(1)$$

eq (1) is first order differential equation

The solution of the differential equation is obtained by two ways.

1. Differential equations (D E) method
2. Laplace Transform method

1. D E method

In this method, the differential equations are of two types

1. Non-Homogeneous D.E
2. Homogeneous DE

1) Non-Homogeneous DE method

In this method, The General equation is

$$\frac{di}{dt} + Pi = Q \quad -(2)$$

From the example

$P = \frac{R}{L}, \quad Q = \frac{V}{L}$

The solution is

i(t) = complementary Function + Particular Integral

= Transient response + steady state

= Natural response + steady state

$$= i_{CF} + i_{PI}$$

$$= i_t + i_{ss}$$

$$= Ke^{-pt} + e^{-pt}\int Q \cdot e^{+pt}\,dt$$

$$= Ke^{-pt} + e^{-pt} Q \cdot \frac{e^{pt}}{P}$$

$$\boxed{i(t) = Ke^{-pt} + \frac{Q}{P}} \text{ Amps}$$

2) <u>Homogeneous DE method</u>

The General equation is

$$\frac{di}{dt} + Pi = 0 \quad -(3)$$

The solution is

$$i(t) = i_t + i_{ss}$$

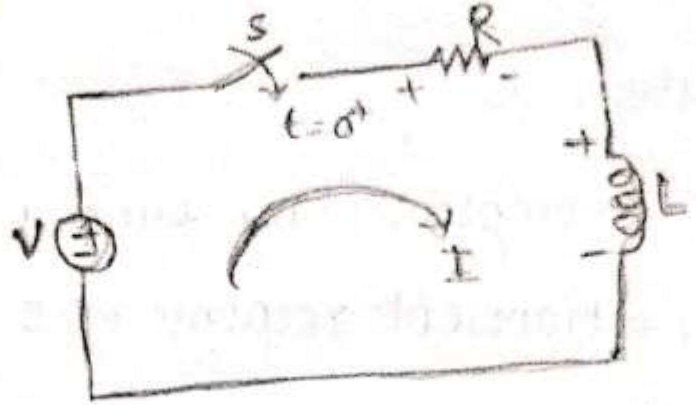
$$= Ke^{-Pt} + e^{-Pt}\int Q \cdot e^{Pt} dt$$

$$= Ke^{-Pt} + 0$$

$$\boxed{i(t) = Ke^{-Pt}} \quad \text{Amps}$$

* Transient Analysis of Series R-L circuit:

Series R-L circuit is excited by DC input which is connected through switch as shown in the figure.



Initial Conditions:

$$t = 0 \;, I = 0 \longrightarrow (1)$$

Now, apply KVL for the closed circuit

$$-V + IR + L\, di/dt = 0$$

$$V - IR - L\frac{di}{dt} = 0$$

$$V = IR + L\frac{di}{dt}$$

divide both sides with 'L'

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L} \longrightarrow (2)$$

It is a non-homogeneous equation

$$\frac{di}{dt} + P i = Q \quad , \text{ where } P = \frac{R}{L} , \ Q = \frac{V}{L}$$

The solution for eqn (2) is

$$i(t) = K e^{-Pt} + \frac{Q}{P}$$

$$i(t) = K e^{-\frac{R}{L} t} + \frac{V}{R} \longrightarrow (3)$$

Subst. eqn (1) in eqn (3)

$$0 = K e^{0} + \frac{V}{R}$$

$$\boxed{K = \frac{-V}{R}} \longrightarrow (4)$$

Subst. eqn (4) in eqn (3)

$$i(t) = -\frac{V}{R} e^{-\frac{R}{L}t} + \frac{V}{R}$$

$$\boxed{i(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)} \longrightarrow (5)$$

In case of series R-L circuit, the time constant

$$\tau = \frac{L}{R}$$

subst. τ in eqn (5)

$$i(t) = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right) \longrightarrow (6)$$

Now, $t = \tau$, then

$$i(t) = \frac{V}{R}(1 - e^{-1})$$

$$i(t) = \frac{V}{R}(1 - 0.367)$$

$$i(t) = 0.633\frac{V}{R}$$

at $t = 2\tau$, $i(t) = \frac{V}{R}(1 - e^{-2}) = 0.865\frac{V}{R}$

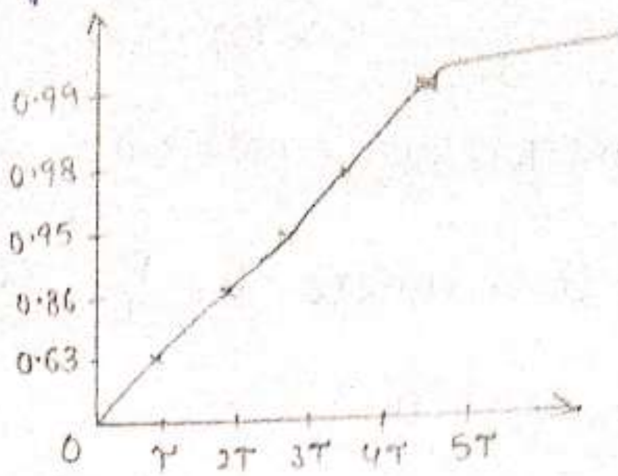
$t = 3\tau$, $i(t) = \frac{V}{R}(1 - e^{-3}) = 0.951\frac{V}{R}$

$t = 4T$, $i(t) = \frac{V}{R}(1 - e^{-4}) = 0.98\frac{V}{R}$

$t = 5T$, $i(t) = \frac{V}{R}(1 - e^{-5}) = 0.99\frac{V}{R}$

III. TRANSIENTS — R-L Circuit

Now, the graph b/w $i(t)$ vs t



Voltage across resistor:

$$V_R = iR$$

$$= \frac{V}{R}\left(1 - e^{\frac{-t}{\tau}}\right)R$$

$$\boxed{V_R = V\left(1 - e^{-t/\tau}\right)}$$

Voltage across inductor:

$$V = V_R + V_L$$

$$V_L = V - V_R$$

$$= V - V\left(1 - e^{\frac{-t}{\tau}}\right)$$

$$\boxed{V_L = Ve^{-t/\tau}}$$

Q: Find the current in series R-L series circuit having R = 2Ω and L = 10 H, while a DC voltage of 100 V is applied. What is the value of current after 5 sec of switch ON. what is the time taken by the current to reac half of its final value.

Sol: Given that

$R = 2\Omega$

$L = 10\ H$

$V = 100\ V$

$t = 5\ sec$

$$i(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$= \frac{100}{2}\left(1 - e^{-\frac{2}{10}t}\right)$$

$$= 50\left(1 - e^{-1/5\,t}\right)$$

at $t = 5$, $i = 50\left(1 - e^{-5/5}\right)$

$$\boxed{i = 31.606\ A}$$

(ii) Final value of current $i = \frac{V}{R} = \frac{100}{2} = 50$ amps

Half of its final value $i = \frac{50}{2} = 25$ amps

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$25 = 50\left(1 - e^{-\frac{2}{10}t}\right)$$

$$25 = 50\left(1 - e^{-\frac{t}{5}}\right)$$

$$25 = 50 - 50e^{-t/5}$$

$$-25 = -50e^{-t/5}$$

$$\frac{1}{2} = e^{-t/5}$$

$$\ln_e\left(\frac{1}{2}\right) = \ln_e e^{-t/5}$$

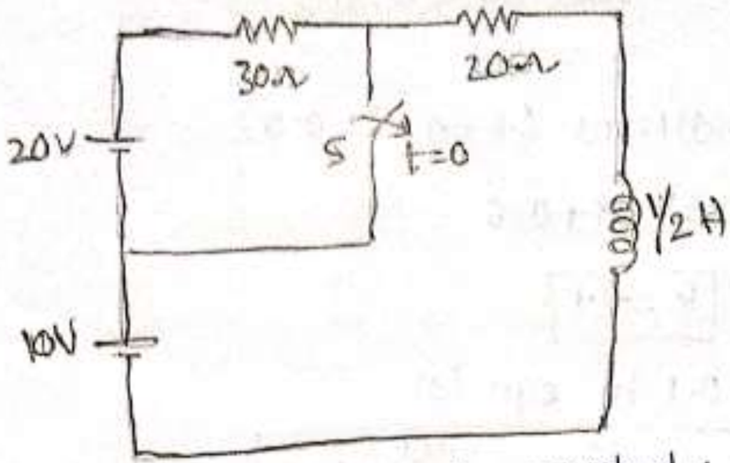
$$= -\frac{t}{5}$$

$$t = -5\ln\left(\frac{1}{2}\right)$$

$$t = -5 \times (-0.693)$$

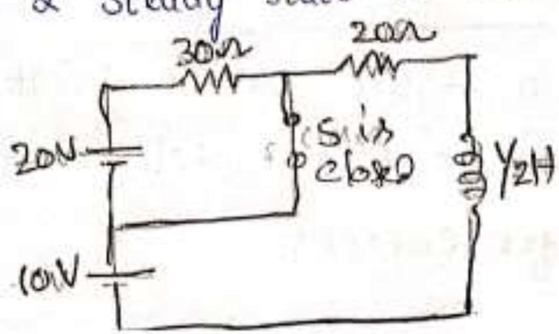
$$\boxed{t = 3.46 \text{ sec}}$$

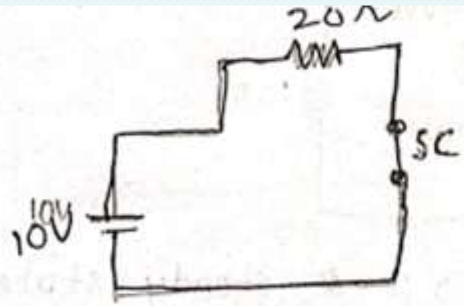
2. In the figure switch 'S' is closed and steady state is reached At t=0, the switch 'S' is opened, find $i_L(t)$ for $t>0$



Case 1 :

Ans: S is closed & steady state is reached :

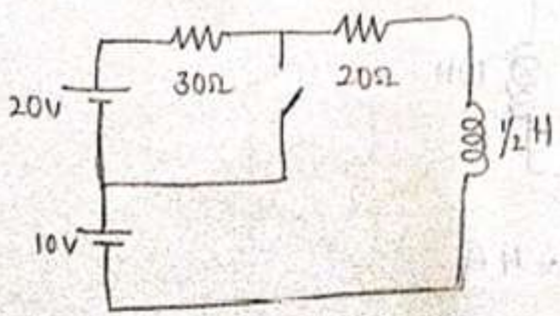


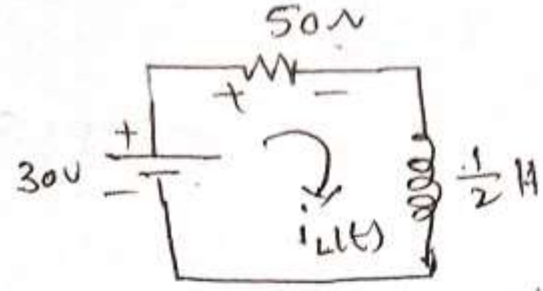


$$i = \frac{V}{R} = \frac{10}{20} = 0.5\,A$$

Case 2 :

At t=0 switch is opened.





Initial conditions

$t=0,\ i = 0.5A$ —(1)

Apply KVL to the ckt

$$-30 + 50\,i_L(t) + \frac{1}{2}\frac{di_L(t)}{dt} = 0$$

$$50\,i_L(t) + \frac{1}{2}\frac{di_L(t)}{dt} = 30$$

$$\frac{di_L(t)}{dt} + 100\,i_L(t) = 60$$

L(2)

It is like a non-homogeneous eqn, the solution is

$$i_L(t) = ke^{-Pt} + \frac{Q}{P} \quad [P=100, Q=60]$$

$$= ke^{-100t} + 0.6 \longrightarrow (3)$$

To find k:

Use initial conditions $(t=0, i=0.5)$

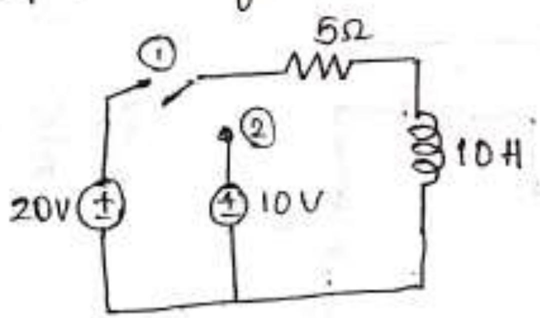
$$0.5 = K + 0.6$$

$$\boxed{K = -0.1}$$

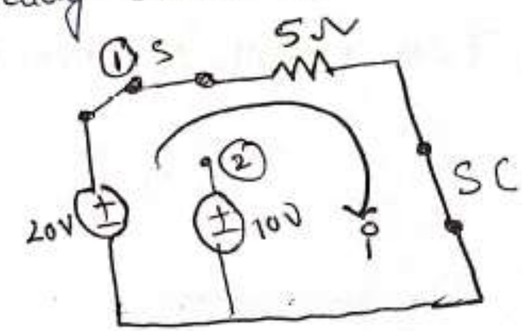
Substitute $k=-0.1$ in eqn (3)

$$\boxed{i_L(t) = -0.1\,e^{-100t} + 0.6}$$

3. In the ckt shown in figure switch is at position 1 and steady state is reached. At t = 0, switch is moved to position 2. Find the expression for current.

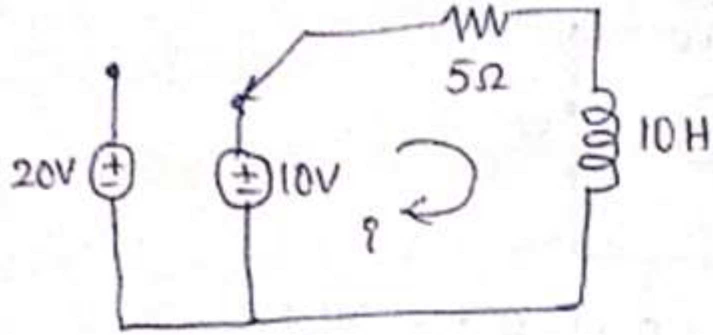


Ans: Switch is at position 1 & steady state is reached.



$$i = \frac{V}{R} = \frac{20}{5} = 4\,A$$

At $t=0$, switche is moved to position 2:



Initial conditions:

$$t=0 \quad , \quad i=4A$$

Now, apply KVL to the circuit

$$-10+5i+10\frac{di}{dt}=0$$

$$10\frac{di}{dt}+5i=10$$

$$\frac{di}{dt}+\frac{1}{2}i=1 \longrightarrow (1)$$

It is like a non-homogeneous eqn, the solution is

$$i(t) = Ke^{-Pt} + \frac{Q}{P} \quad [P = 1/2 , Q = 1]$$

$$= Ke^{-t/2} + 2 \longrightarrow (2)$$

To find k :

Use initial conditions, $(t=0, i=4\,A)$

$$4 = K + 2$$

$$\boxed{K = 2}$$

substitute $k=2$ in eqn (2)

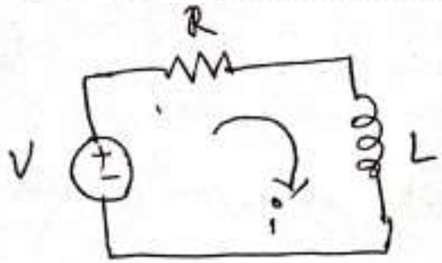
$$\boxed{i(t) = 2e^{-t/2} + 2.}$$

Problems on R-L Circuit

III UNIT NT\problem 2 RL.pdf

III UNIT NT\problem 5 RL.pdf

Take R-L ckt



KVL is applied to the ckt

$$V = IR + L\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L} = \frac{V}{L} \quad —(1)$$

eq (1) is first order differential equation.

The solution of the differential equation is obtained by two ways.

1. Differential equations (DE) method
2. Laplace Transform method

1) Non-Homogeneous DE method

In this method, The General equation is

$$\frac{di}{dt} + PI = Q \quad -(2)$$

The solution is

$$\boxed{i(t) = Ke^{-pt} + \frac{Q}{P}} \text{ Amps}$$

2) Homogeneous DE method

The General equation is

$$\frac{di}{dt} + Pi = 0 \quad -(3)$$

The solution is

$$\boxed{i(t) = Ke^{-pt}} \text{ Amps}$$

Formulas for Laplace Transforms

Laplace Transforms

Formulas

$f(t)$		$F(s)$
1	$\longrightarrow$	$\frac{1}{s}$
V	$\longrightarrow$	$\frac{V}{s}$
t	$\longrightarrow$	$\frac{1}{s^2}$
t^n	$\longrightarrow$	$\frac{n!}{s^{n+1}}$
e^{-at}	$\longrightarrow$	$\frac{1}{s+a}$
e^{at}	$\longrightarrow$	$\frac{1}{s-a}$

Inverse Laplace Transforms

$F(s)$		$f(t)$
$\frac{1}{s}$	$\longrightarrow$	1
$\frac{V}{s}$	$\longrightarrow$	V
$\frac{1}{s^2}$	$\longrightarrow$	t
$\frac{n!}{s^{n+1}}$	$\longrightarrow$	t^n
$\frac{1}{s+a}$	$\longrightarrow$	e^{-at}
$\frac{1}{s-a}$	$\longrightarrow$	e^{at}

Laplace Transform R-L Circuit

Laplace Transforms

$$t \cdot e^{-at} \longrightarrow \frac{1}{(s+a)^2}$$

$$t \cdot e^{at} \longrightarrow \frac{1}{(s-a)^2}$$

$$\sin\omega t \longrightarrow \frac{\omega}{s^2+\omega^2}$$

$$\cos\omega t \longrightarrow \frac{s}{s^2+\omega^2}$$

$$\frac{di}{dt} \longrightarrow sI(s) - i(0)$$

$$\int i\,dt \longrightarrow \frac{I(s)}{s}$$

$$e^{-at}\sin\omega t \longrightarrow \frac{\omega}{(s+a)^2+\omega^2}$$

$$e^{-at}\cos\omega t \rightarrow \frac{s}{(s+a)^2+\omega^2}$$

Inverse Laplace Transforms

$$\frac{1}{(s+a)^2} \longrightarrow t \cdot e^{-at}$$

$$\frac{1}{(s-a)^2} \Rightarrow t \cdot e^{at}$$

$$\frac{\omega}{s^2+\omega^2} \Rightarrow \sin\omega t$$

$$\frac{s}{s^2+\omega^2} \Rightarrow \cos\omega t$$

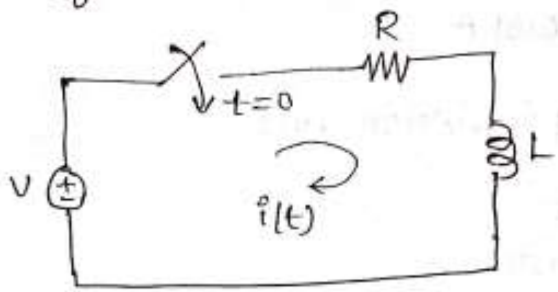
$$sI(s) \Rightarrow \frac{di}{dt}$$

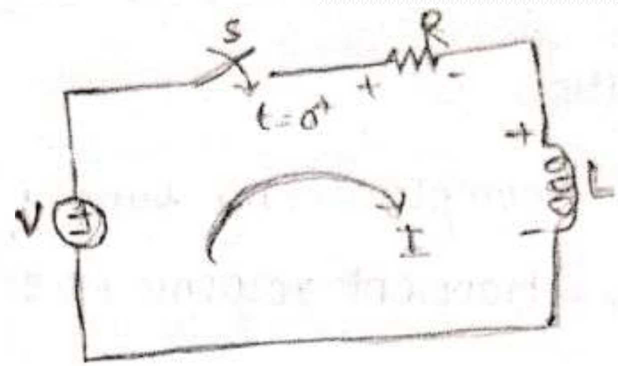
$$\frac{I(s)}{s} \Rightarrow \int i\,dt$$

$$\frac{\omega}{(s+a)^2+\omega^2} \Rightarrow e^{-at}\sin\omega t$$

Method 2 :

DC transient response analysis of series R-L circuit by Laplace transform method :





KVL equation, $-V + IR + L\,di/dt = 0$

$$V = Ri(t) + L\frac{di(t)}{dt} \longrightarrow (1)$$

Initial conditions:

$t=0,\ i=0$

In Laplace transform

$$L(V) = V/s$$

$$L[i(t)] = F(s)$$

$$L\left[\frac{di(t)}{dt}\right] = sI(s) - i(0) \longrightarrow (2)$$

↳ initial condition

Laplace Transform R-L Circuit

by apply laplace transform

$$\frac{V}{S} = R\,I(S) + L\left[S\,I(S)\right]$$

$$I(S)\left[R + SL\right] = \frac{V}{S}$$

$$I(S) = \frac{V/s}{R+SL}$$

$$I(S) = \frac{V/s}{L\left(S+\frac{R}{L}\right)}$$

$$I(S) = \frac{(V/L)}{S\left(S+\frac{R}{L}\right)} \longrightarrow (3)$$

Using partial fractions,

$$\frac{(V/L)}{S\left(S+R/L\right)} = \frac{A}{S} + \frac{B}{S+\frac{R}{L}}$$

$$\frac{A}{S} = \left.\frac{V/L}{S\left(S+\frac{R}{L}\right)}\right|_{S=0} = \frac{V/L}{R/L}$$

$$\boxed{A = \frac{V}{R}}$$

$$\frac{B}{s+R/L} = \left.\frac{V/L}{s(s+R/L)}\right|_{s=-\frac{R}{L}}$$

$$B = \frac{(V/L)}{-(R/L)}$$

$$\boxed{B = -\frac{V}{R}}$$

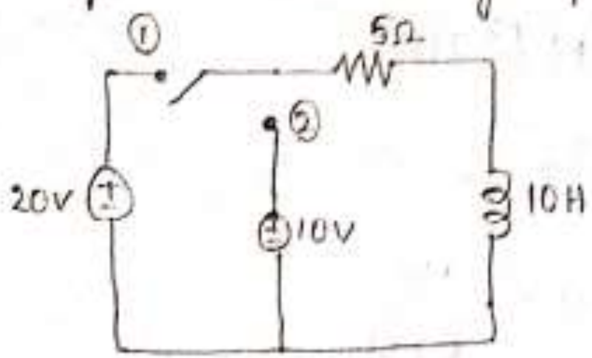
from eqn (3) $I(s) = \frac{V/R}{s} - \frac{V/R}{s+R/L}$

$$= \frac{V}{R}\left[\frac{1}{s} - \frac{1}{s+R/L}\right] \longrightarrow (4)$$

apply inverse Laplace

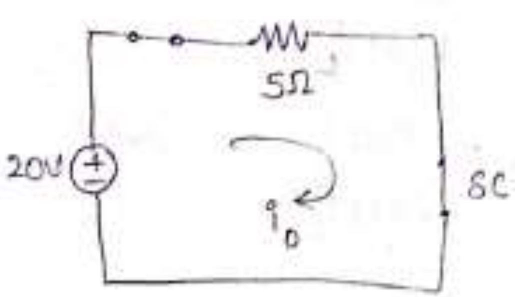
$$i(t) = \frac{V}{R}\left(1-e^{-\frac{R}{L}t}\right)$$

1. In the circuit shown in fig. switch is at position 1 & steady state is reached. At t=0 switch is moved to position 2. Find current through inductor using laplace transform method.



Case 1 :

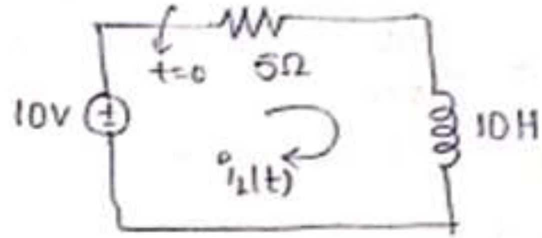
Ans: Switch is at position 1 & steady state is reached.



$$i_0 = \frac{20}{5} = 4\,A$$

Case 2 :

Switch is moved to position 2 at $t=0$



Initial condition,

$$t=0, i_0 = 4A \longrightarrow (1)$$

Apply KVL,

$$-10 + 5i_L(t) + 10\frac{di_L(t)}{dt} = 0$$

$$10\frac{di_L(t)}{dt} + 5i_L(t) = 10$$

$$\frac{di_L(t)}{dt} + \frac{1}{2} i_L(t) = 1 \longrightarrow (2)$$

apply laplace transform,

$$L\left[\frac{di_L(t)}{dt} + 0.5\, i_L(t)\right] = L[1]$$

$$[s I(s) - i(0)] + 0.5\, I(s) = \frac{1}{s}$$

$$I(s)(s+0.5) = \frac{1}{s} + 4$$

$$I(s) = \frac{1+4s}{s(s+0.5)} \longrightarrow (3)$$

$$\frac{1+4s}{s(s+0.5)} = \frac{A}{s} + \frac{B}{s+0.5}$$

$$\frac{A}{s} = \frac{1+4s}{s(s+0.5)}\Bigg|_{s=0}$$

$$A = \frac{1+4(0)}{0+0.5}$$

$$\boxed{A = 2}$$

$$\frac{B}{s+0.5} = \frac{1+4s}{s(s+0.5)}\Bigg|_{s=-0.5}$$

$$B = \frac{1+4(-0.5)}{-0.5}$$

$$= \frac{1-2}{-0.5}$$

$$\boxed{B = 2}$$

$$\therefore I(s) = \frac{2}{s} + \frac{2}{s+0.5}$$

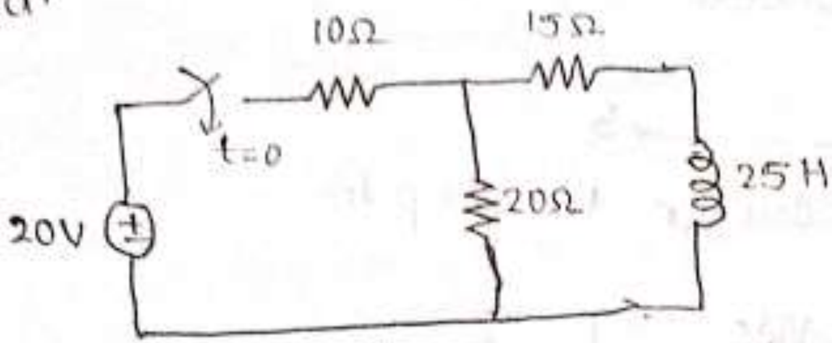
By using inverse Laplace

$$L^{-1} I(s) = L^{-1}\left\{\frac{2}{s}\right\} + L^{-1}\left\{\frac{2}{s+0.5}\right\}$$

$$i(t) = 2 + 2e^{-0.5t}$$

$$i(t) = 2(1 + e^{-0.5t}).$$

2 In the following figure, the switch is closed at $t=0$, find current through inductor using Laplace transform method.



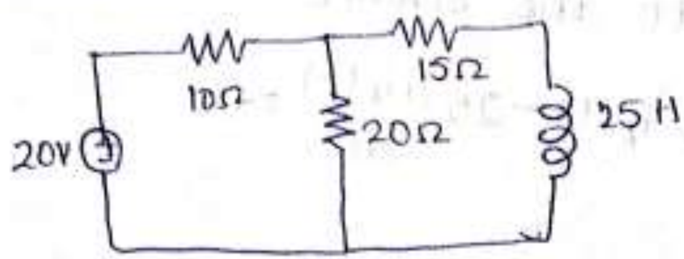
Case 1 :

Ans. Initial condition,

$$\text{at } t=0,\ i=0 \longrightarrow (1)$$

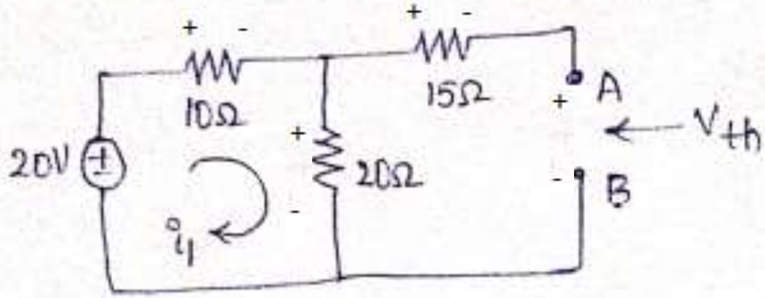
Case 2 :

Switch is closed at $t=0$



Apply Thevenin theorem to the above circuit.

Step 1: remove 25 H inductor from the circuit



Step 2: find V_{th}

apply KVL to the closed circuit $-20+10i_1+20i_1=0$

$$20-10i_1-20i_1=0$$

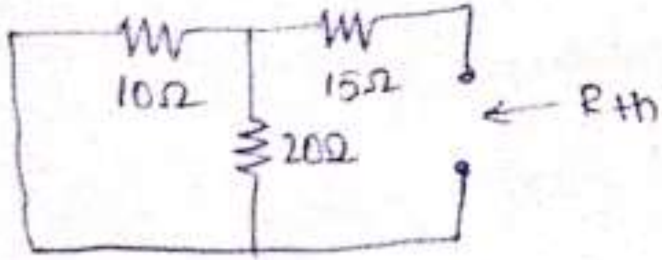
$$30i_1=20$$

$$i_1=0.66$$

$$-V_{th}+20\times0.66=0$$

$$\boxed{V_{th}=13.2\,V}$$

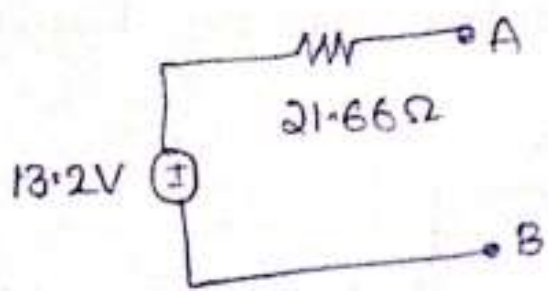
Step 3: To find R_{th}, short circuit the voltage source



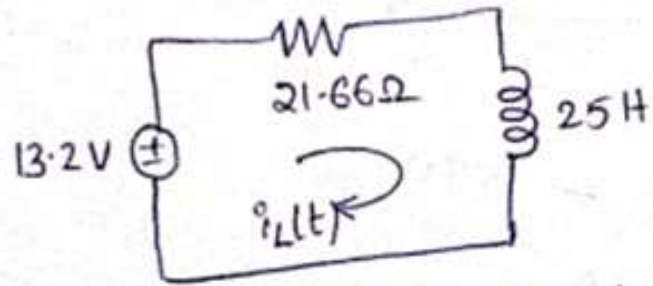
$$R_{th} = 15 + \frac{10 \times 20}{30}$$

$$\boxed{R_{th} = 21.66\Omega}$$

Step 4: Draw thevenin's equivalent circuit



Step 5: Add 25 H inductor to step 4.



Now apply KVL to the circuit

$$-13.2 + 21.66\, i_L(t) + 25 \frac{di_L(t)}{dt} = 0$$

$$13.2 - 21.66\, i_L(t) - 25 \frac{di_L(t)}{dt} = 0$$

$$21.66\, i_L(t) + 25 \frac{di_L(t)}{dt} = 13.2 \longrightarrow (2)$$

Apply laplace transform,

$$21.66\, i_L(s) + 25\left[s\, i_L(s) - I_L(0)\right] = \frac{13.2}{s}$$

$$21.66\, i_L(s) + 25\, s\, i_L(s) = \frac{13.2}{s}$$

$$i_L(s)\left[21.66 + 25s\right] = \frac{13.2}{s}$$

$$i_L(s) = \frac{13.2}{s(21.66+25s)} \quad \cancel{\longrightarrow (3)}$$

$$i_L(s) = \frac{\frac{13.2}{25}}{s\left(s+\frac{21.66}{5}\right)}$$

$$i_L(s) = \frac{13.2/25}{s(s+0.8664)}$$

$$i_L(s) = \frac{0.528}{s(s+0.8664)} \longrightarrow (3)$$

Let $\frac{0.528}{s(s+0.86)} = \frac{A}{s} + \frac{B}{s+0.86}$

$$0.528 = A(s+0.86)+Bs$$

$$0.528 = (A+B)s + 0.86A$$

$$A+B=0, \quad A = \frac{0.528}{0.86}$$

$$B=-0.613 \quad A = 0.613$$

$$I_L(s) = \frac{0.613}{s} - \frac{0.613}{s+0.86}$$

apply inverse laplace transform

$$i_L(t) = 0.613\left[1 - e^{-0.86}\right].$$

III UNIT NT\ADDITIONAL\R-C circuit both methods.pdf

III. TRANSIENTS

R-C Circuit

III UNIT NT\RC Laplace.pdf