

# ELECTRICAL DISTRIBUTION SYSTEM & AUTOMATION

**(20A02702a)**

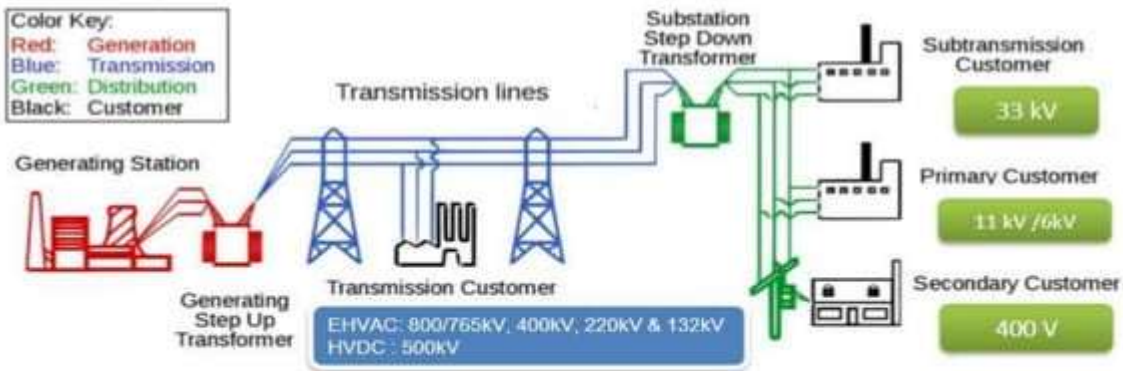
**IV BTECH ISEM**



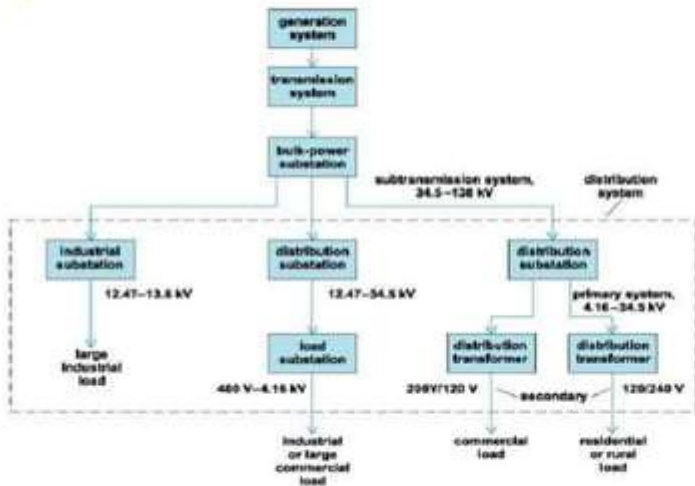
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# Introduction

- In general, the definition of an electric power system includes a generating, a transmission, and a distribution system.

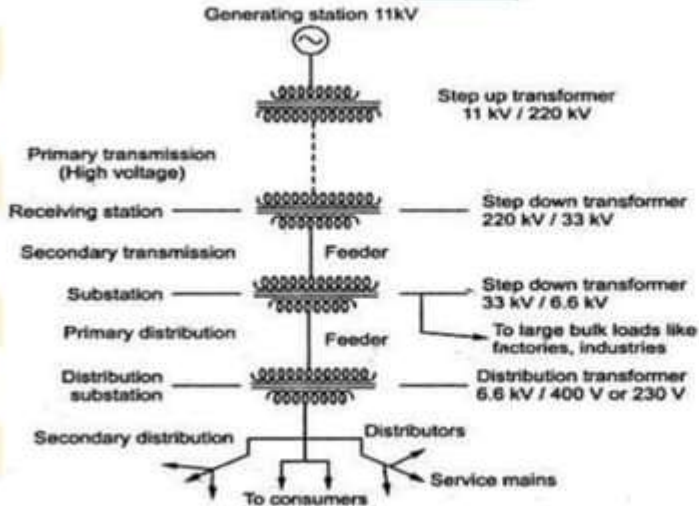


# Distribution System Layout



- Sub-transmission line carry voltages from major TLS.
- Mainly from 34.5 kv to 69 kv, this power is sent to regional distribution Substation, sometimes voltage is Tapped along the way for use in Industrial or large commercial operations

# SLD of GTD



# Distribution System

"The part of power system which distributes electrical power for local use is known as DISTRIBUTION SYSTEM."

- This system is the electrical system between the substation fed by the transmission system and consumer meter.
- Distribution line generally consist of
  1. Feeders
  2. Distributers
  3. Service mains

## Feeders

- A Feeder is conductor which connects the substation to the area where power is to be distributed .
- Feeder are used to feed the electrical power from the generating station to the substation
- No tapings are taken from the feeder
- So the current in it remains the same throughout
- Main consideration in the design of feeder is the Current carrying capacity.

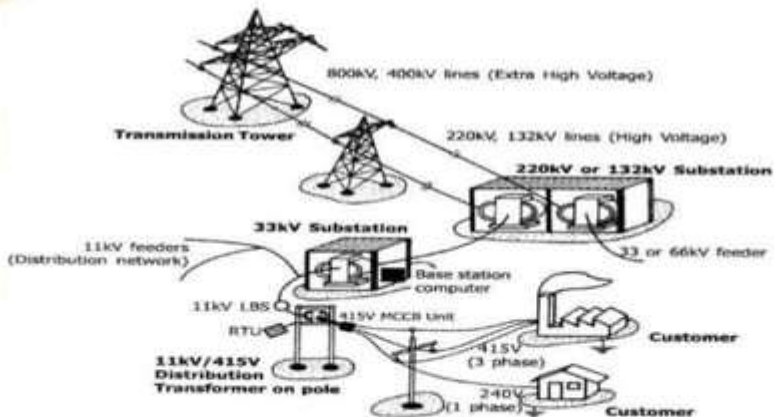
## Distributer

- A distributer is a conductor from which tapings are taken from pole mounted transformer to the consumer
- The current through a distributer is not constant because tapings are taken at various places along its length
- Voltage drop is main consideration
- Limit of variation is 6% of rated at consumer

# Service mains

- A service mains is a generally a small cable which connects the distributor to the consumer 's meter.

The connecting links between the distributor and the consumer terminals.



# Distribution system

Distribution system is a part of power system, existing between distribution substations and consumers.

It is further classified on the basis of voltage

Primary distribution system- 11 KV or 6.6 KV or 3.3 KV

Secondary distribution system- 415 V or 230 V

Classification Of Distribution System: It can be classified under different considerations as; 1. Type Of Current: a) AC Distribution System b) DC Distribution System

2. Type Of Construction: a) Overhead System b) Underground System

3. Type Of Service: a) General Lighting & Power b) Industrial Power c) Railway d) Streetlight etc

4. Number Of Wires: a) Two Wire b) Three Wire c) Four Wire

5. Scheme Of Connection: a) Radial Distribution System b) Ring or Loop Distribution System c) Interconnected Distribution System



# AC distribution

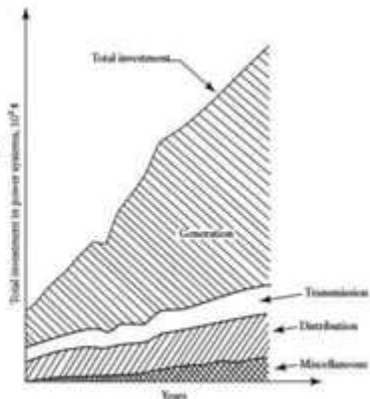
- AC distribution system is the electrical system between the step-down substation fed by the transmission system and the consumers' meters.
- The AC distribution system is classified into
  - Primary distribution system
  - Secondary distribution system.
- **Why Study of Electrical Distribution System is Necessary?**
- In recent years the investment trends show that the percentage of electrical plants in service by the production, i.e, GTD major investment as been in generation followed closely in second by distribution.
- The major expenses for O & M has been in production sector followed by distribution sector
- The economic importance of the distribution system is very high, and the amount of investment involved needs careful planning, design, construction and operation

# Distribution System Planning

*Distribution System planning* is essential to ensure that the growing demand for electricity can be satisfied by addition of DS which are technically adequate and also economical.

The *objective* of distribution system planning is to assure that the growing demand for electricity, in terms of increasing growth rates and high load demand, can be satisfied in an optimum way by additional distribution systems

First *Distribution system planners* must determine the load magnitude and its geographic location. Then the DS must be placed and sized in such a way as to serve the load at maximum efficiency, being cost effectiveness by minimizing feeder losses and construction costs, while considering the constraints of service reliability.



# Factors Effecting DSP

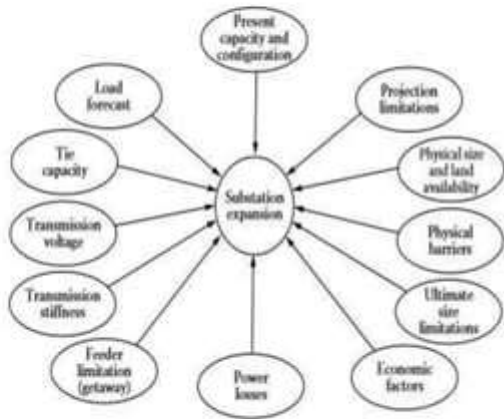
Demand of ever increasing power capacity, Higher distribution voltages, automation and special control are some of the parameter which effect the DSP.

In general the factors effecting may include scarcity in availability of land in urban areas, ecological considerations and limitation on fuel choices, investment etc.

- **Load Forecasting**

- The load demand growth of geographic area is the most important factor which influences the expansion Of DS.
- Thus the forecasting of load increase and the system reaction to these increases is the most important in planning process.
- Load forecasting is done in two ways based on time, long range forecasting for 15-20 years and short range for 5 years.
- These forecasting would predict the future loads in details and the final loads are predicted in kVA

## Factors affecting DSP



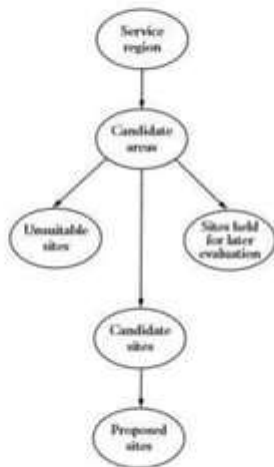
# Substation Expansion

- The planner makes the decision based on tangible and intangible information.
- For example the forecasted load, load demand and load growth may require a substation expansion or new substation.

## Substation site selection

- The distance from the load centers and from the existing sub-transmission lines as well as other limitations, such as availability of land, its cost, and land use regulations, is important.
- The substation siting process can be described as a screening procedure through which all possible locations for a site are passed

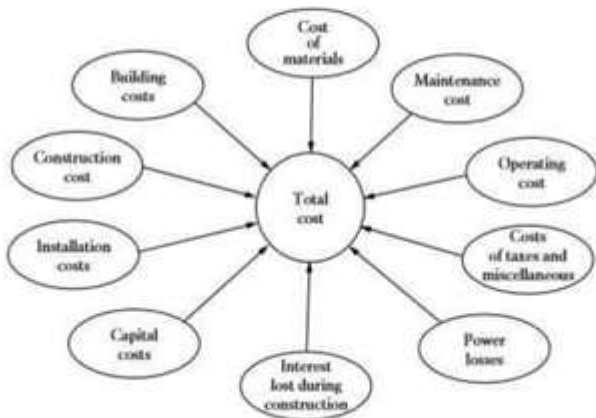
# Substation Site Selection



- Service region is the area Under Evaluation
- Candidate areas are divided Into three groups:
  1. Sites which are unsuitable
  2. Sites which are initially selected but rejected after panning process.
  3. Sites which needs to be studied in detail

## Other Factors

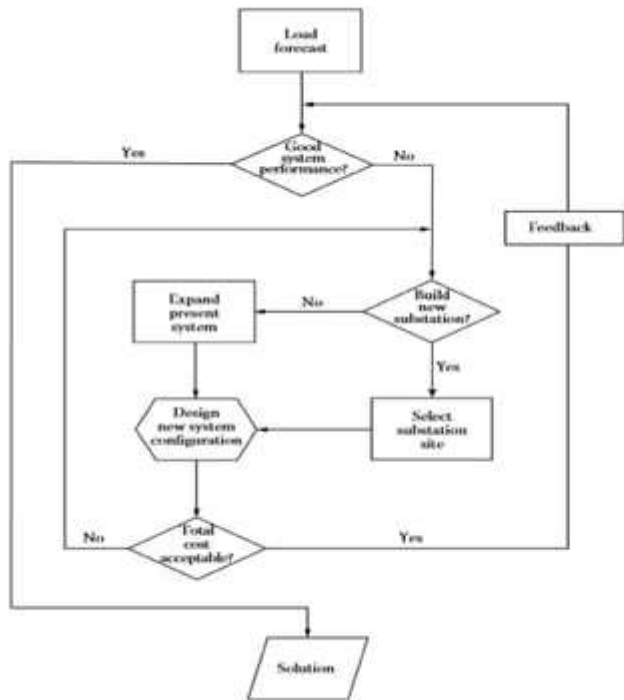
- Once the loads are predicted and assigned, then other factors are considered like, primary voltage selection, feeder route selection, number of feeders, conductor size selection and total cost are taken into account for final planning process.



# Present Planning techniques

- Computer based : many electric distribution system planners in the industry utilize computer programs
- Such as load flow programs, radial or loop load flow programs, short-circuit and fault-current calculation programs, voltage drop calculation programs, and total system impedance calculation programs, as well as other tools such as load forecasting, voltage regulation, regulator setting, capacitor planning, reliability, and optimal siting and sizing algorithms.
- The computers do perform calculations more expeditiously than other methods and free the distribution engineer from detailed work. The engineer can then spend time reviewing results of the calculations, rather than actually making them.





Typical Distribution System Planning Process

# Distribution System Planning Models

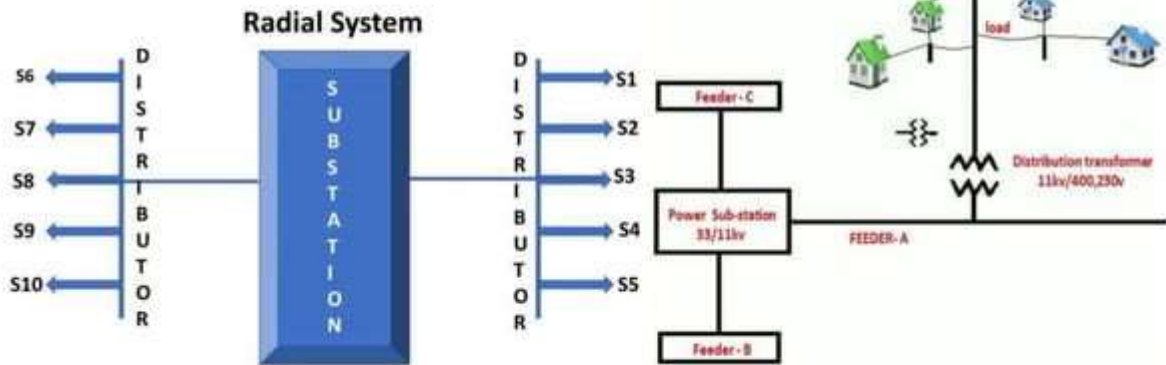
- In general, distribution system planning has a complex procedure due to a large number of variables involved and mathematical requirements.
- Therefore, mathematical models are developed to represent the system and can be employed by distribution system planners to investigate and determine optimum expansion patterns or alternatives by selecting the following models:
  1. Optimum substation locations
  2. Optimum substation expansions
  3. Optimum substation transformer sizes
  4. Optimum load transfers between substations and demand centers
  5. Optimum feeder routes and sizes to supply the given loads subject to numerous constraints to minimize the present worth of the total costs involved

## Sub-transmission System

- The sub-transmission system is defined as the circuits which deliver energy from the transmission system to the primary distribution system.
- But usually the sub-transmission system are supplied from the transmission sub-stations and still referred to as sub-transmission.
- Earlier many of the sub-transmission system were the transmission lines, but due the increase in the load growth and demand for more power led to the reduction of voltage from 220kV to 33kV which are found in the sub-transmission lines.
- Mainly the distribution system are considered to be consisting four main components : Its sub-transmission system, the substation itself, the feeder system and the consumers.
- The distribution substation reduces the voltage to the lower primary voltage in the range of 22 to 11kV for local distribution.
- The local transformer reduces the voltages to 430/415V or 230/240V with a variation of +/- 6 percent at the consumer end.

# Sub-transmission network configuration

- Radial distribution system



# Sub-transmission network configuration

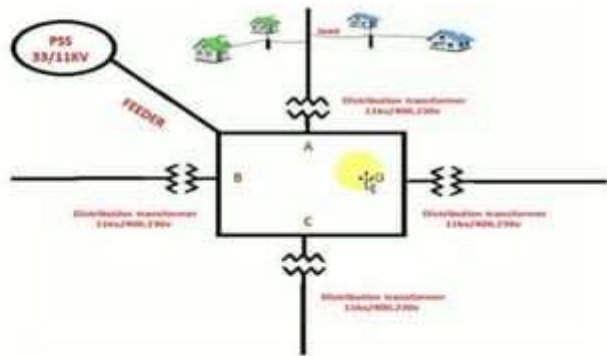
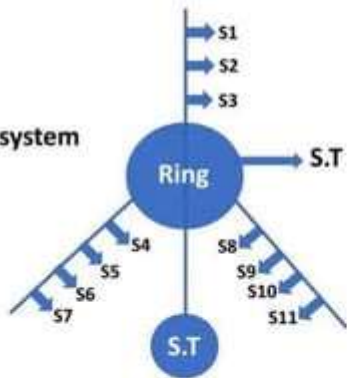
- **Radial distribution system**

- This system is used only when substation or generating station is located at the center of the consumers.
- In this system, different feeders radiate from a substation or a generating station and feed the distributors at one end.
- Thus, the main **characteristic of a radial distribution system** is that the power flow is in only one direction.
- Although this system is simplest and least expensive, it is not highly reliable.
- A major **drawback of a radial distribution system** is, a fault in the feeder will result in supply failure to associated consumers as there won't be any alternative feeder to feed distributors.

# Sub-transmission network configuration

- Ring main distribution system

Ring main system



# Sub-transmission network configuration

- **Ring main distribution system**

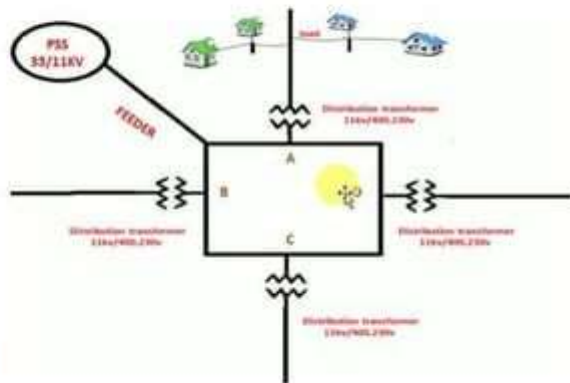
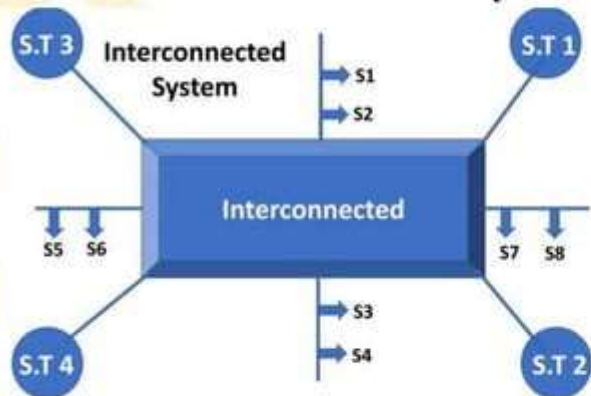
- Here, each distribution transformer is fed with two feeders but in different paths.
- The feeders in this system form a loop which starts from the substation bus-bars, runs through the load area feeding distribution transformers and returns to the substation bus-bars.
- Ring main distribution system is the most preferred due to its following advantages.

## **Advantages of ring main distribution system**

- There are fewer voltage fluctuations at consumer's terminal.
- The system is very reliable as each distribution transformer is fed with two feeders. That means, in the event of a fault in any section of the feeder, the continuity of the supply is ensured from the alternative path.

# Sub-transmission network configuration

- **Interconnected distribution system**





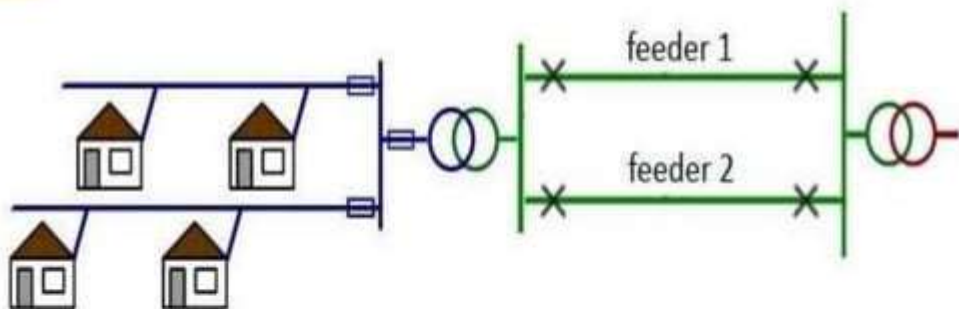
## Sub-transmission network configuration

- **Interconnected distribution system**

- When a ring main feeder is energized by two or more substations or generating stations, it is called as an interconnected distribution system.
- This system ensures reliability in an event of transmission failure.
- Also, any area fed from one generating stations during peak load hours can be fed from the other generating station or substation for meeting power requirements from increased load.

## Sub-transmission network configuration

- **Parallel feeders distribution system**



## Sub-transmission network configuration

- **Parallel feeders distribution system**

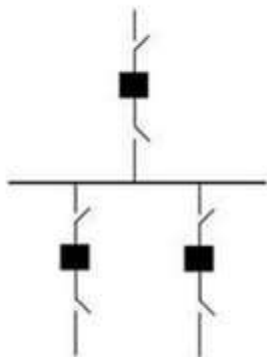
- The disadvantage of a radial system can be minimized by introducing parallel feeders.
- The initial cost of this system is much more as the number of feeders is doubled.
- Such system may be used where reliability of the supply is important or for load sharing where the load is higher

# Substation Bus Schemes

- The electrical substation is a junction point where two or more transmission lines terminate.
- In reality, most EHV and HV substations can be the point where more than half a dozen of lines terminate.
- In many large transmission substations, the total numbers of lines terminating exceeds one or two dozen.
- A substation bus scheme is the arrangement of overhead bus bar and associated switching equipment (circuit breakers and isolators) in a substation.
- The operational flexibility and reliability of the substation greatly depends upon the bus scheme.
- **Types of bus scheme:**
  - Single Bus
  - Main Bus and Transfer Bus
  - Double Bus and Double Breaker
  - Ring or Mesh Bus
  - Break and Half

# Substation Bus Schemes

## Single Bus Configuration



Single Bus Configuration

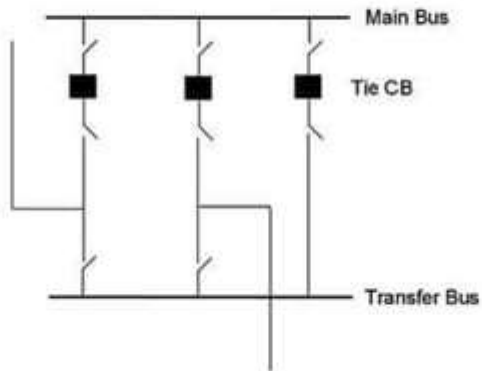
# Substation Bus Schemes

- As the name implies, the single bus substation configuration consists of all circuits connected to a main bus.
- A fault on the bus or between the bus and circuit breaker will result in an outage of the entire bus or substation.
- Failure of a single circuit breaker will also result in an outage of the entire bus.
- Maintenance of any circuit breaker requires shutdown of the corresponding circuit/line and maintenance of the bus requires a complete shutdown of the bus.
- A bypass switch across the breaker should be used for maintenance of the corresponding breaker. Circuit protection is disabled in this case.



# Substation Bus Schemes

## Main and Transfer Bus Configuration



Main and Transfer Bus Configuration

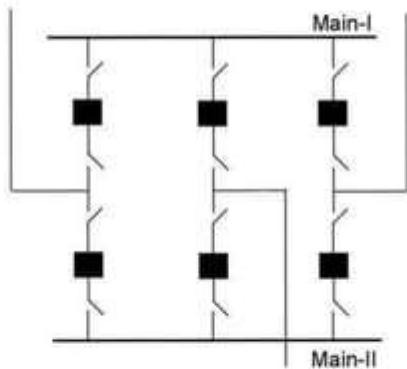
# Substation Bus Schemes

- In this arrangement one or more busses is added to the single bus substation scheme.
- One or more circuit breakers may be used in this arrangement to make connections between the main and transfer bus.
- For maintenance of a circuit breaker, the transfer bus is energized by closing the isolator switches to the transfer bus, then the circuit breaker to be maintained is opened and isolated on both sides.



# Substation Bus Schemes

## Double Bus Double Breaker



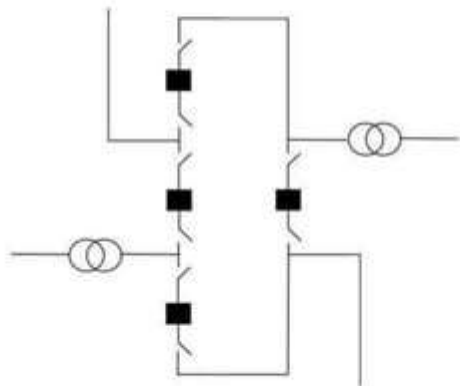
Double Bus Double Breaker Configuration

# Substation Bus Schemes

- This configuration utilizes two buses and two breakers per circuit.
- Both buses are normally energized and any circuit can be removed for maintenance without an outage on the corresponding circuit.
- Failure of one of the two buses will not interrupt a circuit because all of the circuits can be fed from the remaining bus and isolating the failed bus.
- Substations with the double bus double breaker arrangement require twice the equipment as the single bus scheme but are highly reliable.
- Load balancing between buses can be achieved by shifting circuits from one bus to the other.
- This scheme is typically found in EHV transmission substations or generating stations.

# Substation Bus Schemes

## Ring Bus Configuration



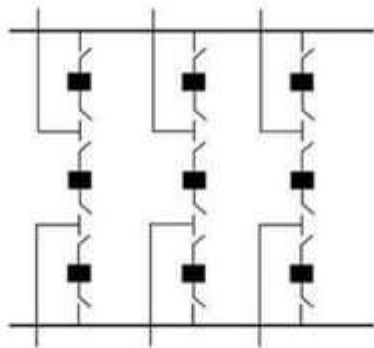
Ring Bus Configuration

# Substation Bus Schemes

- In the ring bus configuration, as the name implies, the circuit breakers are connected to form a ring, with isolators on both sides of each breaker.
- Each circuit is fed from both sides.
- This scheme has good operational flexibility and high reliability, any of the circuit breakers can be opened and isolated for maintenance without interruption of service.

# Substation Bus Schemes

## Breaker and Half Configuration



Breaker and Half Configuration

# Substation Bus Schemes

- When expansion of the substation is required to accommodate more circuits, can easily be expanded to the One and Half breaker configuration.
- This configuration uses two main buses, both of which are normally energized with three breakers connected between the buses.
- In this bus configuration, three breakers are required for every two circuits - hence the "one and half" name.
- Think of it as, to control one circuit requires one full and a half breaker.
- The middle breaker is shared by both circuits, similar to a ring bus scheme where each circuit is fed from both sides.
- Any circuit breaker can be isolated and removed for maintenance purposes without interrupting supply to any of the other circuits.
- Additionally, one of the two main busses can be removed for maintenance without interruption of service to any of the other circuits.

## Substation Bus Schemes - Comparison

Type of Bus Scheme	Advantages	Disadvantages
Single bus	<ol style="list-style-type: none"><li>1. Lowest cost</li></ol>	<ol style="list-style-type: none"><li>1. Failure of bus or any circuit breaker results in shutdown of entire substation</li><li>2. Difficult to do any maintenance</li><li>3. Bus cannot be extended without completely de-energizing substation</li><li>4. Can be used only where loads can be interrupted or have other supply arrangements</li></ol>

## Substation Bus Schemes - Comparison

Type of Bus Scheme	Advantages	Disadvantages
Main-and-transfer	<ol style="list-style-type: none"><li>1. Low initial cost.</li><li>2. Any breaker can be taken out of service for maintenance.</li></ol>	<ol style="list-style-type: none"><li>1. Requires one extra breaker for the bus tie.</li><li>2. Switching is somewhat complicated when maintaining a breaker.</li></ol>



## Substation Bus Schemes - Comparison

Type of Bus Scheme	Advantages	Disadvantages
Double bus–double breaker	<ol style="list-style-type: none"><li>1. Each circuit has two dedicated breakers</li><li>2. Has flexibility in permitting feeder circuits to be connected to either bus</li><li>3. Any breaker can be taken out of service for maintenance</li><li>4. High reliability</li></ol>	<ol style="list-style-type: none"><li>1. Most expensive</li><li>2. Would lose half the circuits for breaker failure if circuits are not connected to both buses</li></ol>

## Substation Bus Schemes - Comparison

Type of Bus Scheme	Advantages	Disadvantages
Ring bus	<ol style="list-style-type: none"><li>1. Low initial cost</li><li>2. Flexible operation for breaker maintenance</li><li>3. Any breaker can be removed for maintenance without interrupting load</li><li>4. Requires only one breaker per circuit</li><li>5. Does not use main bus</li><li>6. Each circuit is fed by two breakers</li></ol>	<ol style="list-style-type: none"><li>1. If a fault occurs during a breaker maintenance period, the ring can be separated into two sections</li><li>2. Breaker failure during a fault on one of the circuits causes loss of one additional circuit</li></ol>

## Substation Bus Schemes - Comparison

Type of Bus Scheme	Advantages	Disadvantages
Breaker-and-a-half	<ol style="list-style-type: none"><li>1. Most flexible operation</li><li>2. High reliability</li><li>3. Breaker failure of bus side breakers removes only one circuit from service</li><li>4. Either main bus can be taken out of service at any time for maintenance</li><li>5. Bus failure does not remove any feeder circuits from service</li></ol>	<ol style="list-style-type: none"><li>1. 1½ breakers per circuit</li><li>2. High Cost</li></ol>

## Rating of Distribution Substation

- Substations receive Electric energy from Generating stations or other Substations via long Transmission lines with 110/230/400 kV system voltage.
- Substations are equipped with Transformers to step down the voltage to 11/33/66 kV to feed HT distribution lines.
- Which is expressed in kilovolt-amperes (kVA) or megavolts- amperes (MVA), and indicates **the amount of power that can be transferred through the transformer**. Distribution substation transformers are typically in the range of 3 kVA to 25 MVA.
- The MVA capacity of these Transformers is called as the rating of the Substation.

# Bus-bars or Buses

- **Bus-bars (also called buses)** can be found throughout the entire power system, from generation to industrial plants to electrical distribution boards. Bus-bars are used to carry large current and to distribute current to multiple circuits within switchgear or equipment.
- Plug-in devices with circuit breakers or fusible switches may be installed and wired.
- Originally, bus-bars consisted of **uncovered copper conductors supported on insulators**, such as porcelain, mounted within a non-ventilated steel housing. This type of construction was adequate for current ratings of 225–600 A.



## Bus-bars or Buses

- The bus-bars were also covered with insulation for safety and to permit closer spacing of bars of opposite polarity in order to achieve lower reactance and voltage drop.



## Central Role of Computer in DSP – System Approach

- Distribution system planners use **computer based tools** to perform the **tedious calculations** necessary for **system analysis**.
- It has only been in the **past few years** that technology has provided the means for planners to truly take a **system approach to the total design and analysis**.
- The system approach using **computer tools** starts by **examining** the types of **information required and its sources**.
- The methodology used is that the **information generated** using the tool which are actually final **decisions and information** are **passed from one stage** of the design process to another.
- It is thus noted that the **human engineer must evaluate** the information generated and add his or her input. Finally, the results must be displayed for use and stored for later reference.
- The system approach seeks to automate as much of the process as possible, ensuring in the process that the various transformations of information are made as efficiently as possible

## Central Role of Computer in DSP – System Approach

- A database management system (DBMS) that stores, retrieves, and modifies various data on distribution systems



- In addition to the database management program and the network analysis programs.
- One such new tool is network editor.
- The features of the network editor may include network objects, for example, feeder line sections, secondary line sections, distribution transformers, or variable or fixed capacitors, control mechanisms, and command functions.

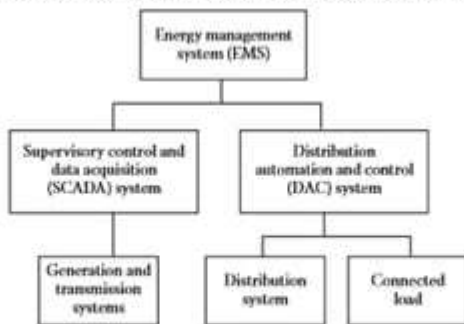


## Distribution System Automation

- The main **purpose** of an electric power system is to **efficiently generate, transmit, and distribute electric energy**.
- The operations involved aims at geographically and functionally complex **monitoring and control** systems.
- The **supervisory control and data acquisition (SCADA)** system involves generation and transmission systems.
- The **distribution automation and control (DAC)** system oversees the distribution system, including connected load.
- Automatic monitoring and control features have long been a part of the SCADA system.
- The term distribution automation has a very broad meaning, and additional applications are added every day.
- To some people, it may mean a communication system at the distribution level that can control customer load and can reduce peak-load generation through load management.

## Distribution System Automation

- To others, the distribution automation may mean an **unattended distribution substation** that could be considered attended through the use of an on-site microprocessor.
- The microprocessor, located at a distribution substation, can continuously monitor the system, make operating decisions, issue commands, and report any change in status to the distribution dispatch center, store it on-site for later use, or forget it, depending on the need of the utility



Thank You

## Site Selection Of Sub-station:

- The site should be near the load center keeping in view the future load growth.
- Access road to the site for smooth movement of construction machines, equipments and transformers.
- Good Roadways to construction site and shorter distance to rail head are desired.
- The site should be chosen to avoid soil filling, earth removal etc. The requirement of soil filling and earth removal takes time and increases total cost of substation.

- Historical data of worst flood is taken into account to avoid water logging of the substation in case of possibility of flood. Flood plains and wetlands are avoided.
- Atmospheric conditions like salt and suspended chemical contaminants influence selection of equipments and maintenance requirements.
- Interference with communication signals. The construction company have to take permission from the appropriate authority.

➤ Electric and magnetic field strength are of particular concern especially for Ultra High Voltage (UHV) systems at 765 kV, 1200 kV or above. Research organisations has shown the impact of strong Electric/magnetic fields due to UHV substations and lines on human health. Such new concerns are also required to be addressed properly.

➤ Forest land, sanctuaries and national parks are avoided. Almost all governments has laid stringent rules to comply for approval of forest land and wild life sanctuary. The usual process takes time to get approval from the concerned authorities. This process delays the construction activities.

- Approval is also required from aviation authority. Substation should be away from airport and defence establishments.
- Water supply and sewage system are the two most important facilities to be given due consideration.

## **Factors affecting the selection of primary feeder rating:**

- The nature of the load connected
- The load density of the area served
- The growth rate of the load
- The need for providing spare capacity for emergency operations
- The type and cost of circuit construction employed
- The Design and capacity of the substation involved
- The type of regulating equipment used
- The quality of service required
- The continuity of service required



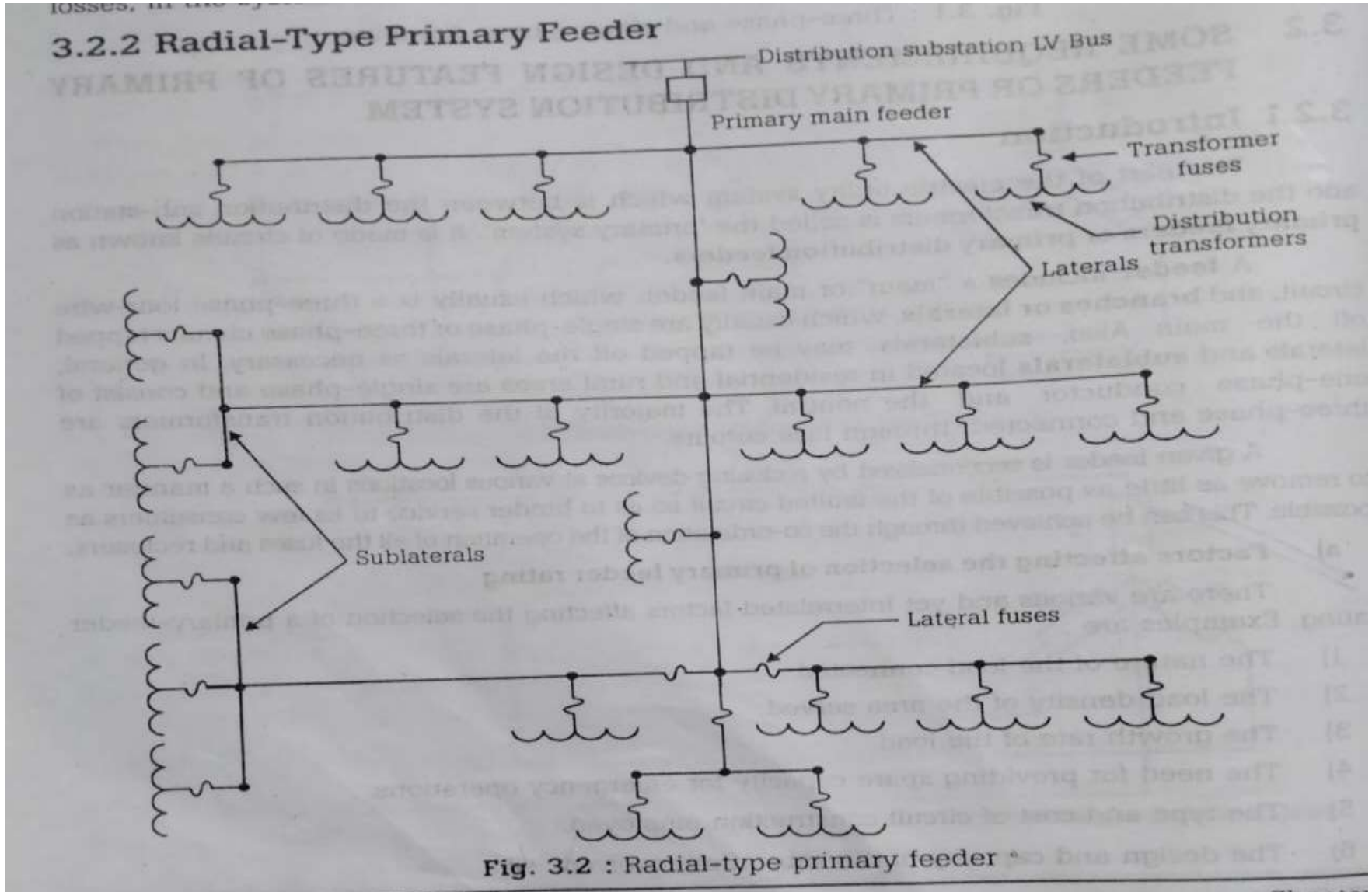
## **primary feeder :**

The part of the electric utility system which is between the distribution sub station and distribution transformers is called the primary system. It is also known as **primary feeders or primary distribution feeders.**

There are two types of primary feeders

- 1.Radial-type primary feeder
- 2.loop-type primary feeder

# 1. Radial-type primary feeder:



- The simplest and the lowest cost and therefore the most common form of primary feeder is the radial feeder. As shown in above.
- The main primary feeder branches into various primary laterals or branches, which in turn separates into several sub laterals to serve all the distribution transformers.
- In general, the main feeder and sub feeders are three phase three wire or four wire circuits and the laterals are three or single phase.

- In this system, the current magnitude is the greatest in the circuit conductors that leave the substation, the current magnitude continuously lesser out towards the end of the feeder as lateral and sub laterals are tapped off the feeder.
- The reliability of service continuity of the radial primary feeder is low. A fault occurrence at any location on the radial primary feeder causes a power outage for every consumer on the feeder unless the fault can be isolated from the source by a disconnecting device such as a fuse, sectionalizer, disconnecting switch or reclosure.

# Modified radial-type primary feeder with tie and sectionalizing switches:

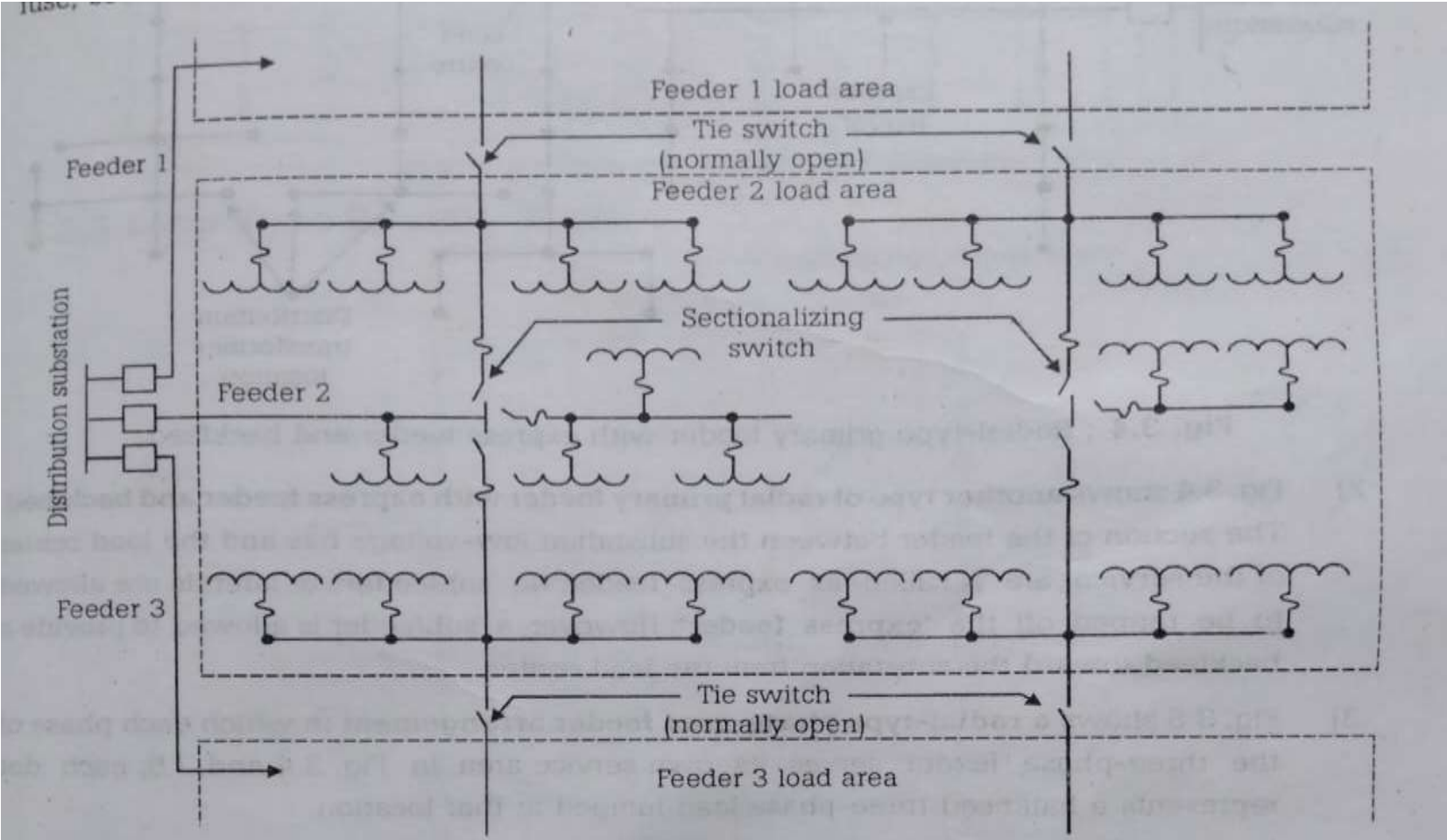


Fig. 3.3 : Radial-type primary feeder with tie and sectionalizing switches

- In this system provides fast restoration of service to customers by switching unfaulted sections of the feeder to an adjacent primary feeder or feeders.
- The fault can be isolated by opening the associated disconnecting devices on each side of the faulted section.

# Radial feeder with express feeder and backfeed:

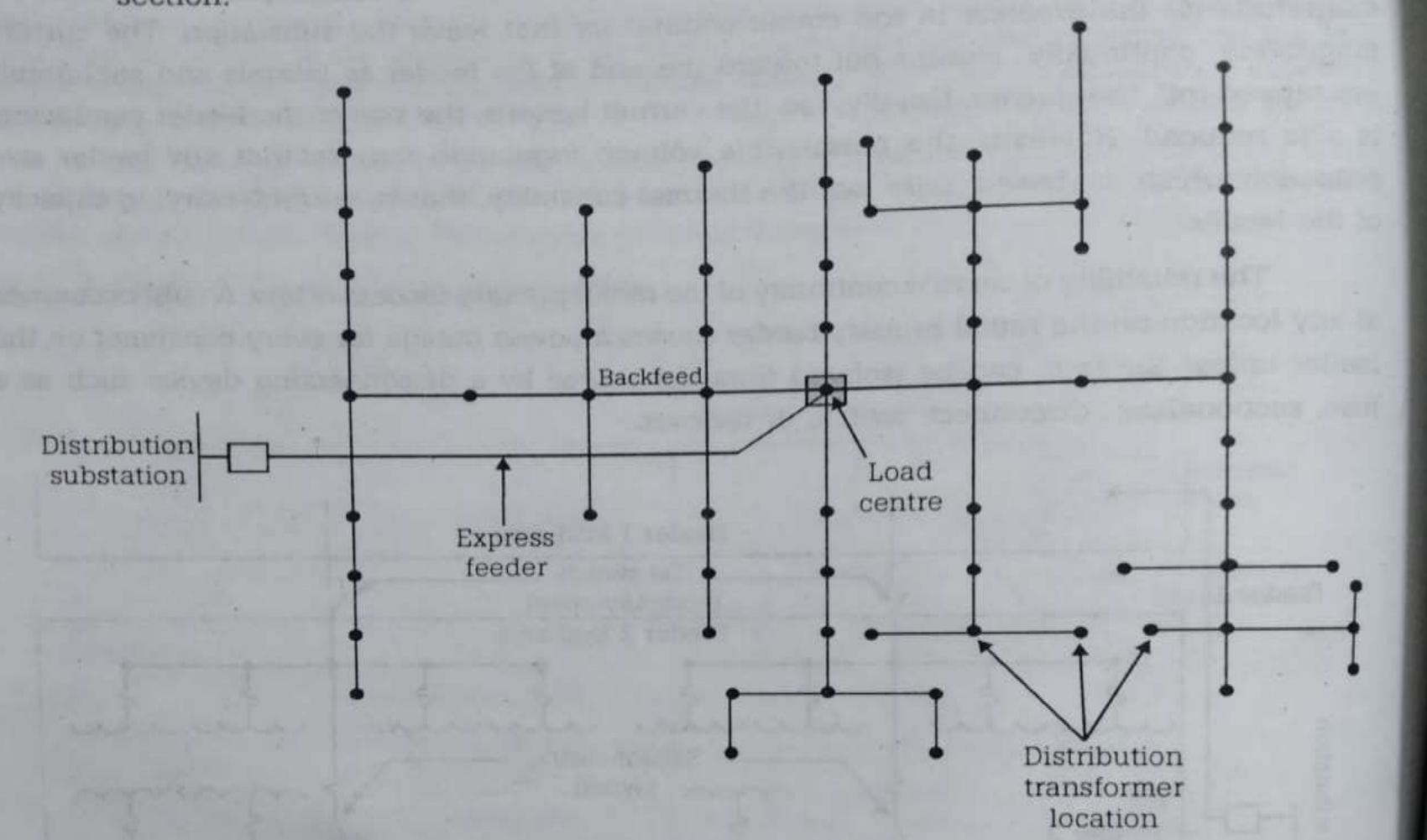
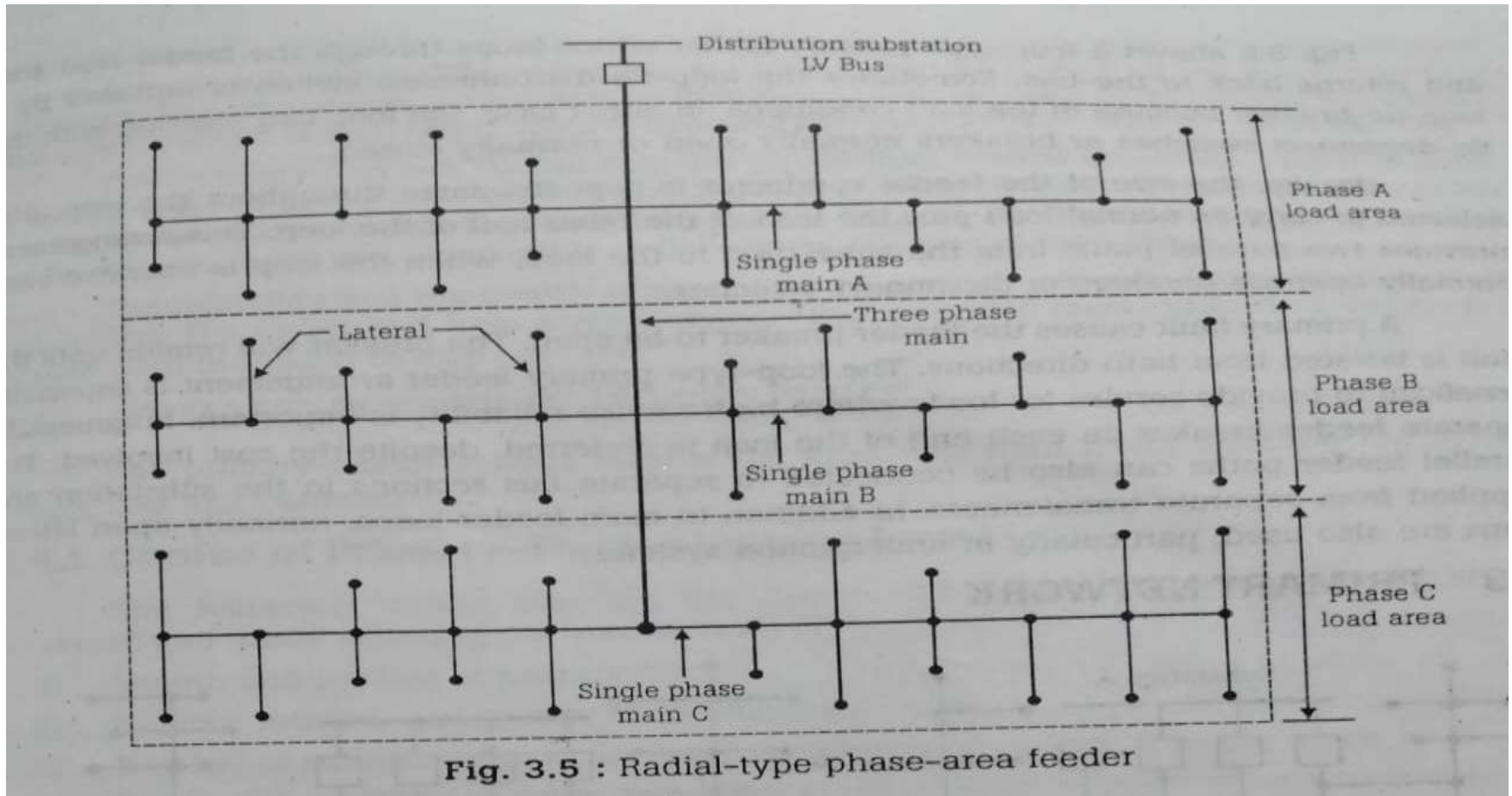


Fig. 3.4 : Radial-type primary feeder with express feeder and backfeed

- The section of the feeder between the substation low-voltage bus and the load center of the service area is known as express feeder. no sub feeders or laterals are allowed to be tapped off the express feeder.
- However, a sub feeder is allowed to provide a backfeed towards the substation from the load center.



# Radial type phase area feeder arrangement:



In this system each phase of the three-phase feeder serves its own service area. Each dot represents a balanced three phase load lumped at that location.

# Loop type primary feeder:

## 3.2.3 Loop-Type Primary Feeder

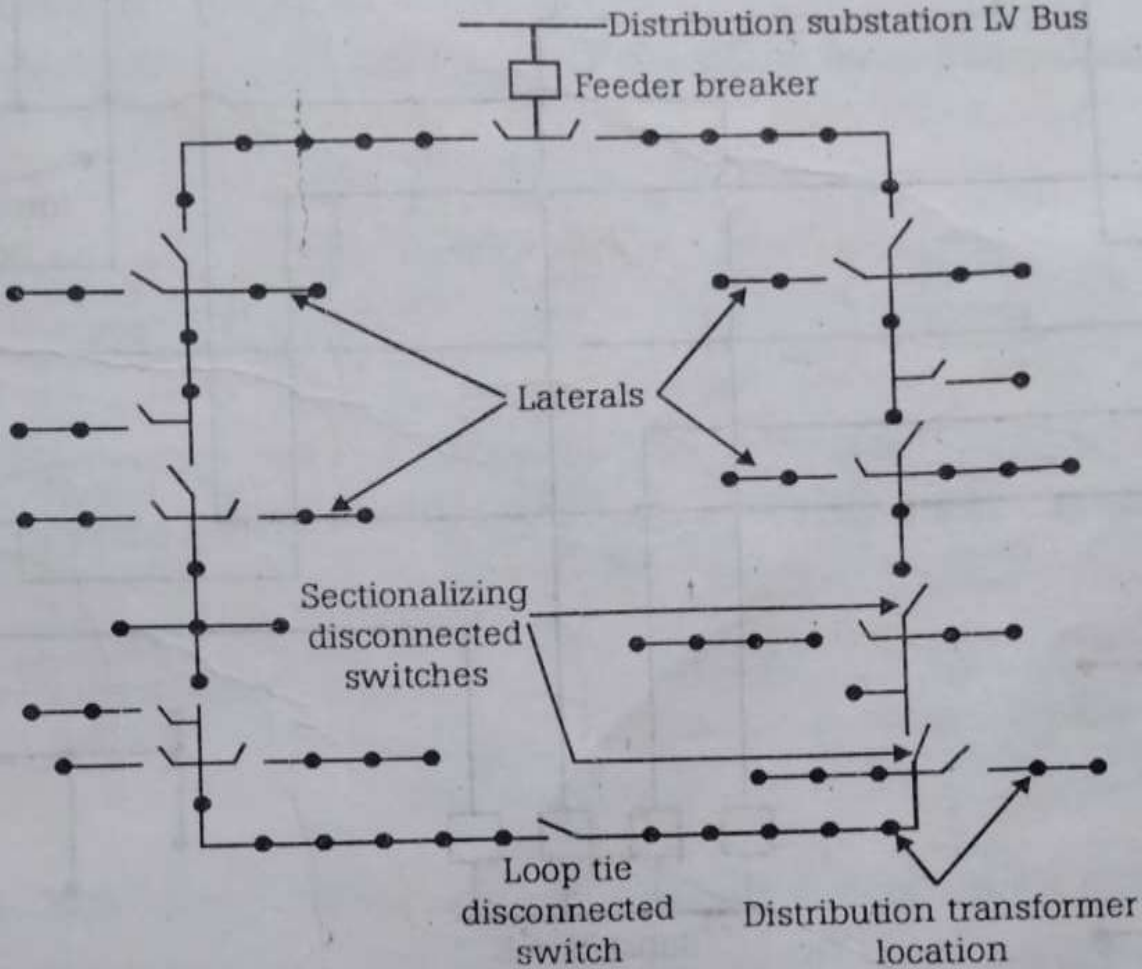


Fig. 3.6 : Loop-type primary feeder

- In this loop type primary feeder which loops through the feeder load area and returns back to the bus.
- Usually the size of feeder conductor is kept the same throughout the loop. It is selected to carry its normal load plus the load of the other half of the loop.
- This arrangement provides two parallel paths from the substation to the load, when the loop is operated with normally open tie breakers or disconnect switches.

- The loop type primary feeder arrangement is especially beneficial to provide service for loads where high service reliability is required.
- The main drawback of this system is separate feeder breaker on each end of the loop is preferred, so it leads to high cost.

## **Primary Feeder Voltage levels:**

- 1.Primary feeder length.
- 2.Primary feeder loading
- 3.Number of distribution substations.
- 4.Rating of distribution substations.
- 5.Numuber of sub-transmission voltage.
- 6.Number of customers affected by a specific outage.
- 7.Voltage drops
- 8.Power loss.
- 9.Load Projections.
- 10.Equipment availability costs.
- 11 Company policies

## **Primary Feeder Loading:**

Primary feeder loading is defined as the **loading of a feeder during peak load conditions** as measured at the substation.

**The following are the factors affecting the design of primary feeder loading,**

1. Nature and Density of the feeder loads connected.
2. Growth rate and reserve capacity requirements for emergency.
3. Continuity, reliability and quality of service.
4. Primary feeder voltage levels and regulation requirements.

5. Location and Capacity of the distribution substation.

6. Type and cost of construction and operating cost factors.

7. Alternate supply provision made.

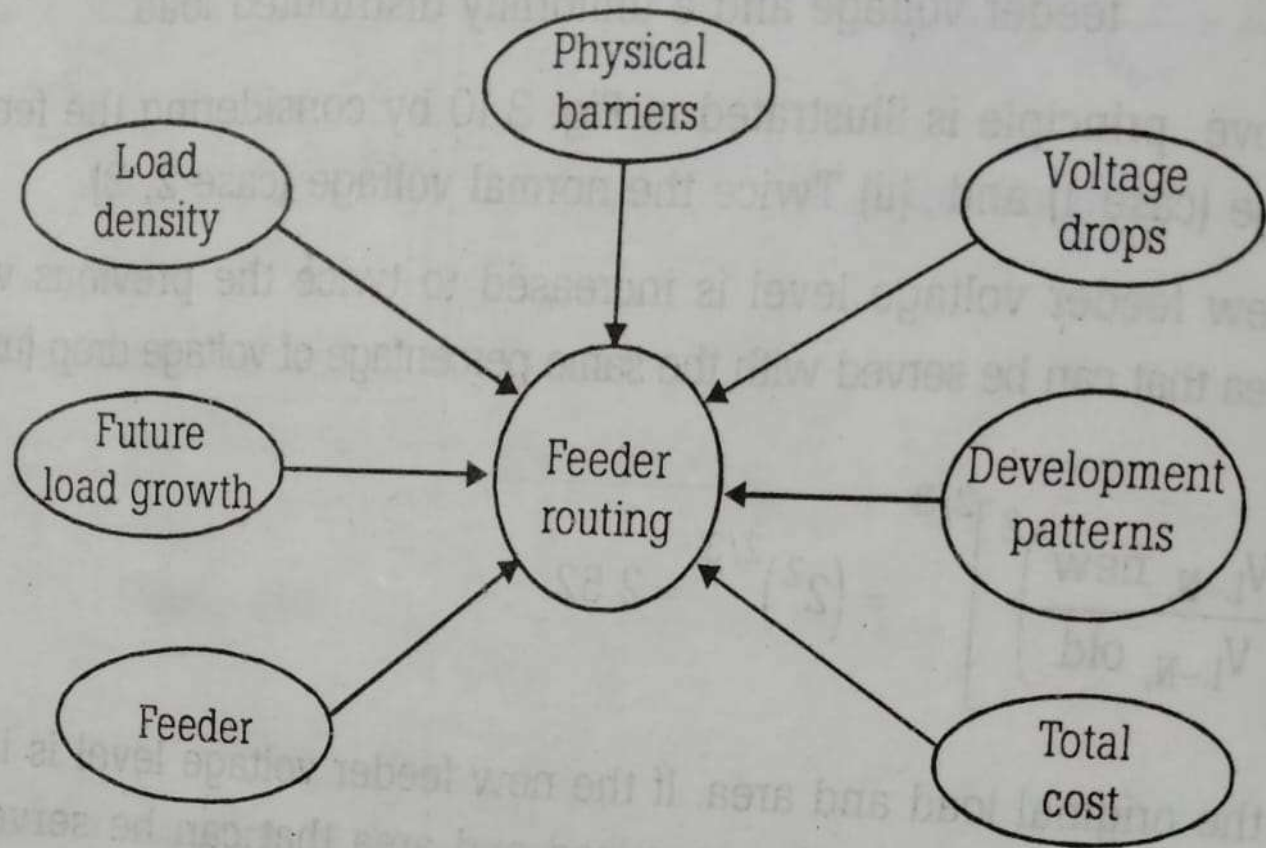
Some other additional factors are,

8. Feeder routing.

9. Number of feeders.

10. Selection of conductor size.

### 3.5.1 Factors Affecting Feeder Routing Decisions



**Fig. 3.11** : Factors affecting feeder routing decisions



### 3.5.2 Factors Affecting Number of Feeders

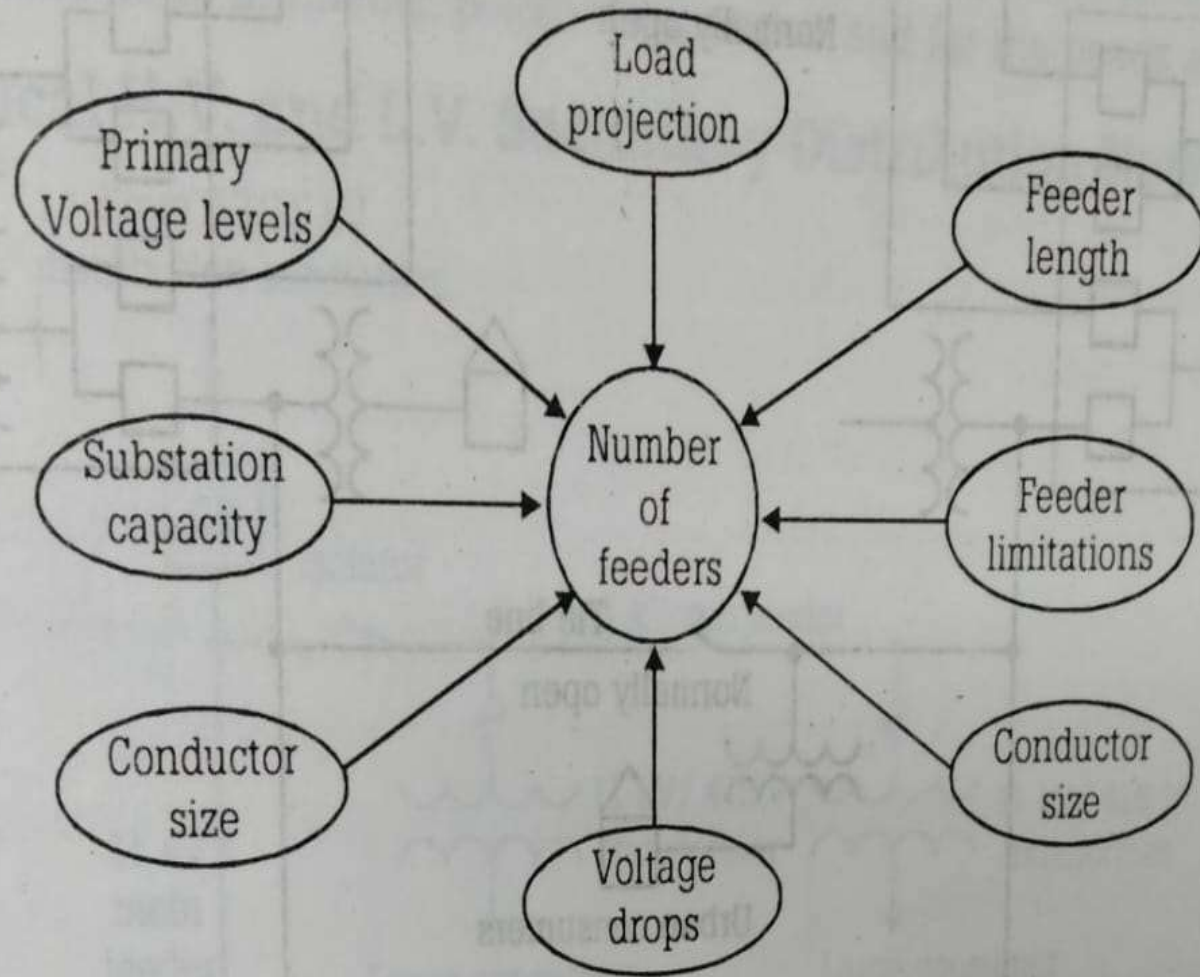


Fig. 3.19 Factors affecting number of feeders

### 5.3 Factors Affecting Conductor Size Selection

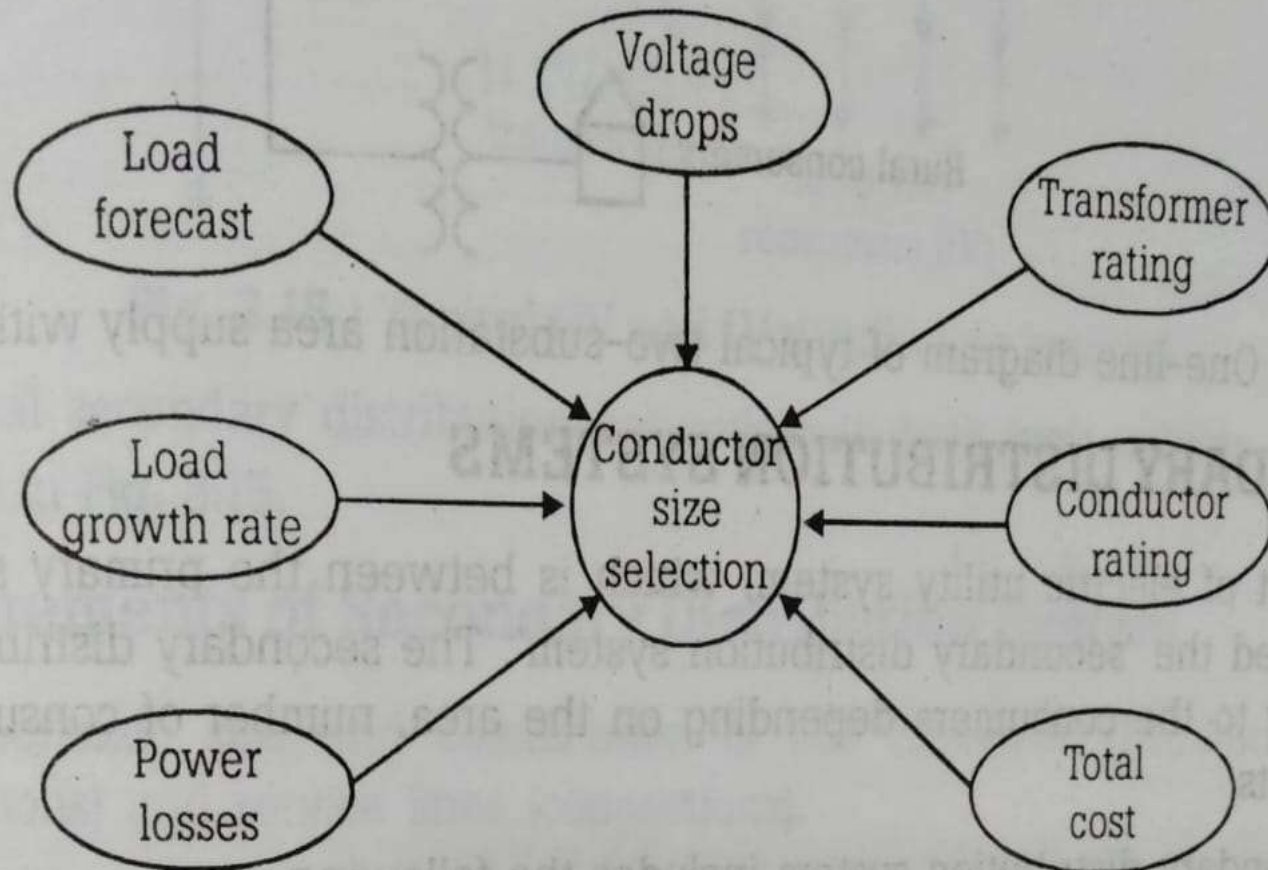



Fig. 3.13 : Factors affecting conductor size selection



# Load Flow Analysis of Radial Distribution Networks

**M Murali**

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# Overview

- Introduction
- Per unit conversion
- Data preparation
- Backward/ forward sweep load flow
- Simulation studies

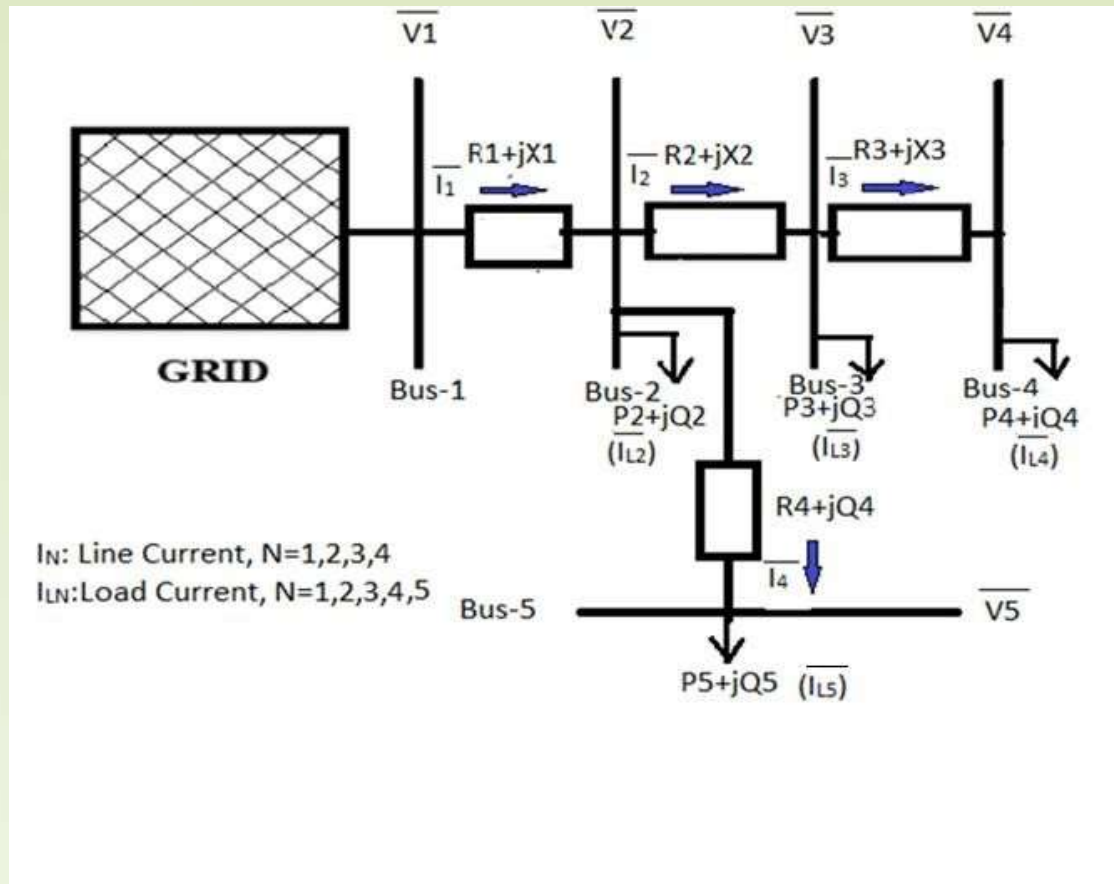
# Introduction

- ❖ Load flow studies are performed to know the steady state operating condition of a power system.
- ❖ In power systems, powers are known quantities rather than currents. Thus, the equations used in load flow studies are in terms of power.
- ❖ These power flow equations are nonlinear and solved by iterative techniques.
- ❖ Load flow studies are used for planning, operation, economic scheduling and exchange of power between utilities.
- ❖ Transient, contingency and stability studies are required load flow analysis.

## Why backward forward sweep load flow algorithm is used for distribution networks?

- ❖ Due to the following special features Newton Raphson and other transmission network algorithms are failed with distribution network:
  - ✓ Radial and weakly meshed networks
  - ✓ High R/X ratio
  - ✓ Distributed generation

# Typical radial distribution network topology: Single line diagram



In load flow approach, all quantities are converted to per unit values.

# Need of per unit conversion

- ❖ An interconnected power system has different voltage levels. So, transformation of all the impedances to a single voltage level is required.
- ❖ It gives a clear idea of relative magnitudes of various quantities such as voltage, current, power and impedance.
- ❖ The per unit values vary within narrow range and this makes the computational analysis easier for complex power systems.
- ❖ The per unit values of voltage, current and impedance are same for both primary and secondary side of transformer.



# Per unit calculations

- ❖ The per unit value of any quantity is the ratio of actual value to its base value. It is a unit less quantity.
- ❖ In power systems computation, usually three phase base MVA and line to line base voltage in kV are selected to specify the base.
- ❖ The base current ( $I_B$ ) is calculated as,

$$I_B = \frac{S_B}{\sqrt{3}V_B}$$

- ❖ The base impedance ( $Z_B$ ) is calculated as,

$$Z_B = \frac{V_B/\sqrt{3}}{I_B} = \frac{V_B^2}{S_B} = \frac{(kV_B)^2}{MVA_B}$$

where,  $S_B$  is base volt-ampere (VA) and  $V_B$  is base voltage (V)

# Changing of base of per unit quantities

- ❖ In power system, the per unit impedance of generators and transformers are based on their own ratings and the per unit impedance of lines are based on their own ohmic values.
- ❖ For power system analysis, all the per unit impedance values must be calculated at same base.
- ❖ Let,  $S_B^{old}$ ,  $V_B^{old}$  and  $Z$  ( $\Omega$ ) are old base Volt-ampere, old base voltage and actual impedance, respectively.
- ❖ The per unit value with old and new base values are:

$$Z_{pu}^{old} = \frac{Z}{Z_B^{old}} = Z \frac{S_B^{old}}{(V_B^{old})^2}, \quad Z_{pu}^{new} = \frac{Z}{Z_B^{new}} = Z \frac{S_B^{new}}{(V_B^{new})^2}$$

- ❖ The new per unit value in terms of old is:

$$Z_p^{new} = Z_p^{old} \frac{S_B^{new}}{S_B^{old}} \left( \frac{V_B^{old}}{V_B^{new}} \right)^2$$

# Data preparation

- ❖ To perform load flow analysis the following data must be known:
  - ✓ Base MVA of the network
  - ✓ Base kVA of the network
  - ✓ Voltage mismatch accuracy
  - ✓ Maximum number of iterations
  - ✓ Resistance and reactance of each branch of network
  - ✓ Real and reactive power demand at each and every nodes of the network

# Bus/Node data file preparation

- ❖ Starting from substation bus assign a number to each buses sequentially
- ❖ For a bus, all the data must be in single row
- ❖ Column 1: enter the buses number sequentially
- ❖ Column 2: enter the buses code (0: load bus, 1: slack bus and 2: voltage controlled bus)
- ❖ Column 3 and 4: enter the initial voltage magnitude and angle to each buses as '1' per unit and '0' (zero) degree respectively
- ❖ Column 5 and 6: enter the load demand of each buses in MW and MVAR respectively

# Line data file preparation

- ❖ Connectivity of buses must be known
- ❖ For a branch of network, all the data must be in single row
- ❖ Column 1 and 2: enter the branch buses numbers in sequence
- ❖ Column 3 : enter the branch resistance
- ❖ Column 4: enter the branch reactance

# Equations involved in load flow analysis

- ❖ Load current ( $\bar{I}_i$ ) at bus ' $i$ ' is calculated,

$$\bar{I}_i = \frac{P_i - jQ_i}{\bar{V}_i^*}$$

- ❖ Branch current ( $\overline{I_i^b}$ ) for branch ' $im$ ' is calculated by using,

$$\overline{I_i^b} = \bar{I}_m + \sum_{i \in \Gamma} I_i$$

- ❖ The voltage at receiving end ( $\bar{V}_m$ ) is calculated as,

$$\bar{V}_m = \bar{V}_i - \overline{I_i^b} (R_{im} + jX_{im})$$

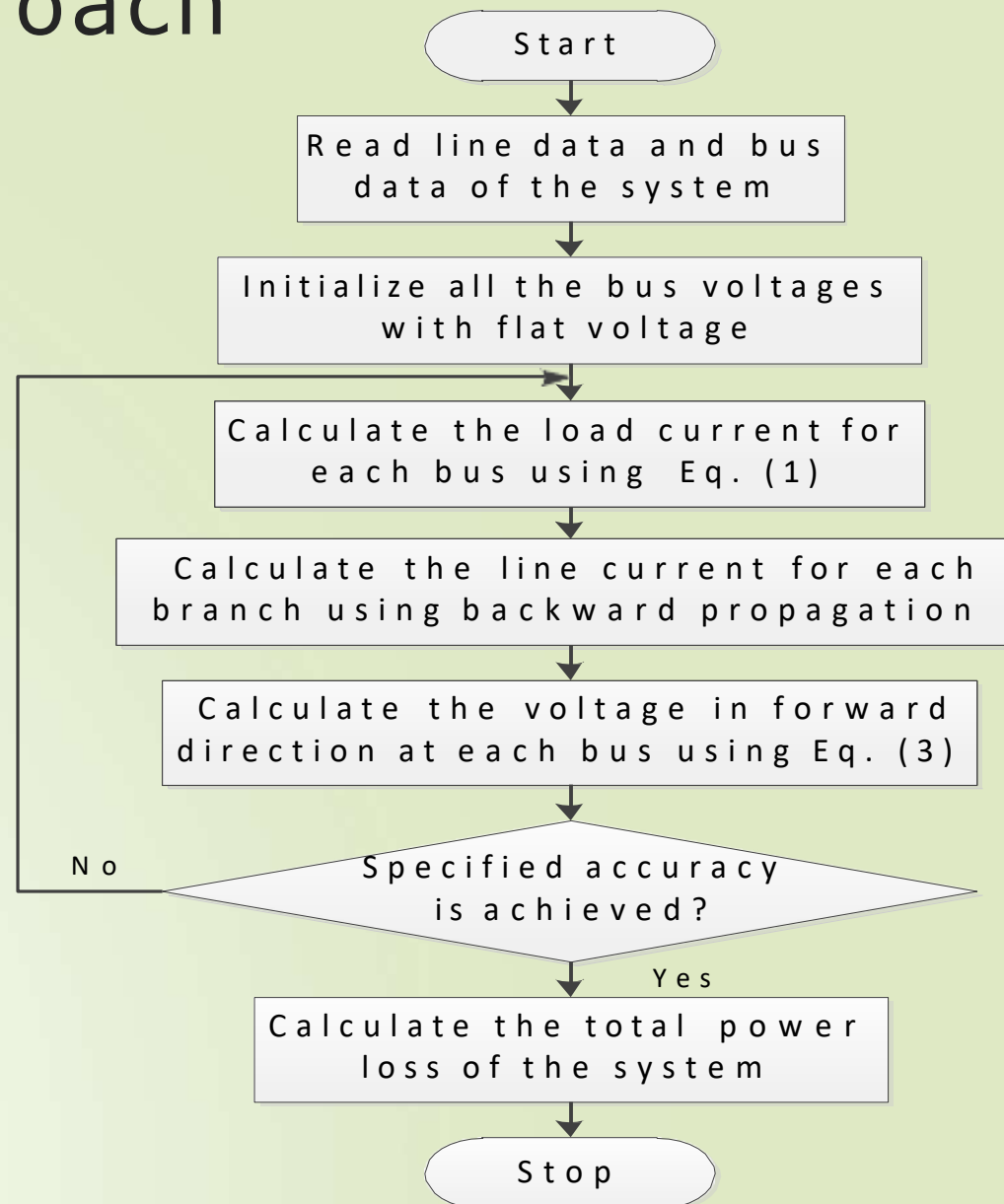
where,  $i \in [1, \dots, N]$ ,  $im \in [1, \dots, N - 1]$

$N$  is total number of bus,  $\bar{V}_i$  is sending end voltage

$\Gamma$  is a set of all the nodes which are located beyond node ' $m$ '

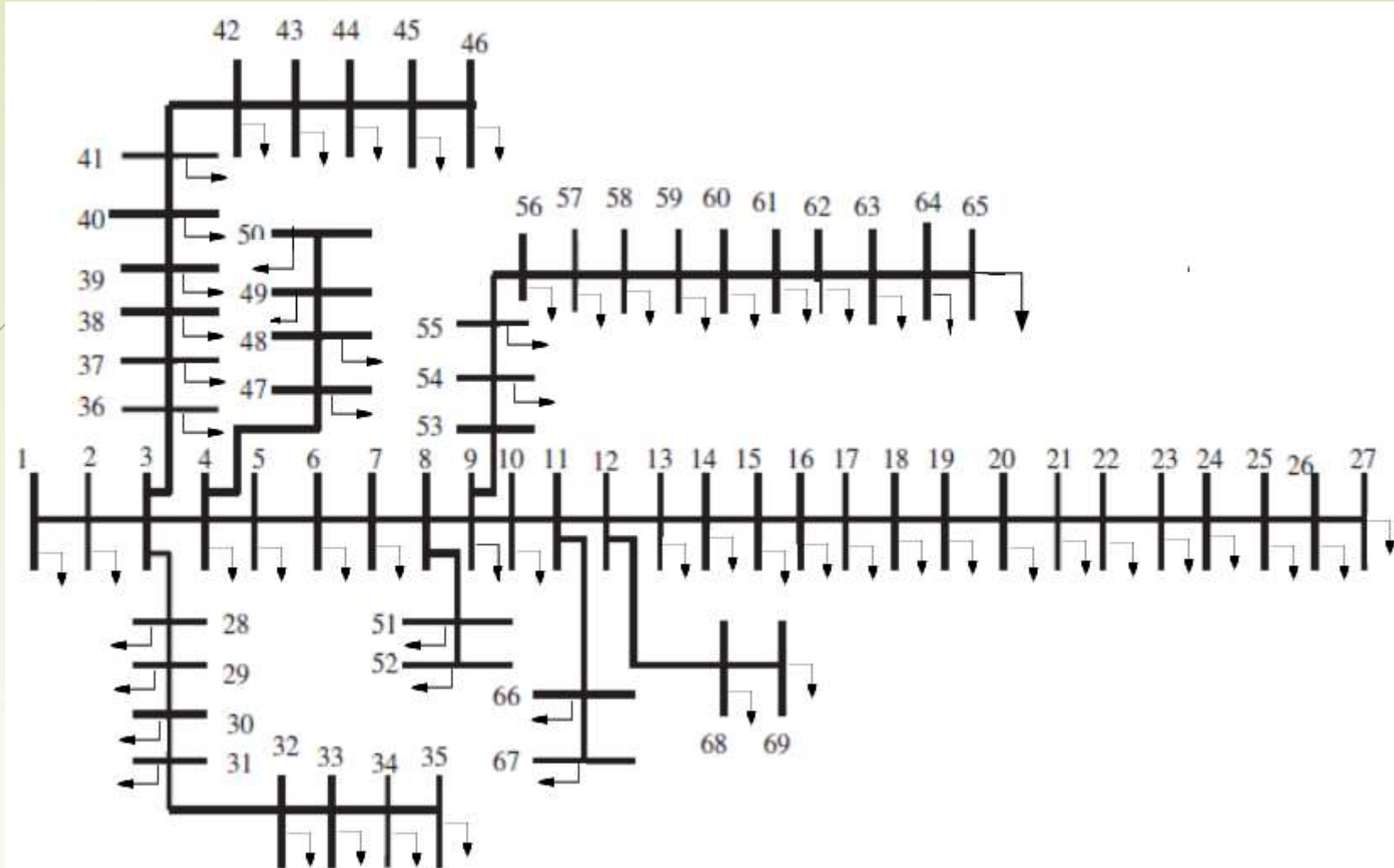
$R_{im}$  and  $X_{im}$  are the resistance and reactance of branch ' $im$ '

# Flowchart of backward forward sweep approach



# 69-node radial distribution network

14





# Node data of 69-node radial distribution network

```

basemva =10 ; accuracy = 0.0001;
maxiter=100; basekv=12.66;
%Distribution System data (69 bus system)
%
% Bus      Bus      Voltage  Angle   Load   Load
% no.      code     mag.     (degree) (kW)    (kVAR)
%
busdata=[

```

%	Bus no.	Bus code	Voltage mag. (pu)	Angle (degree)	Load (kW)	Load (kVAR)
	1	1	1	0	0	0
	2	0	1	0	0	0
	3	0	1	0	0	0
	4	0	1	0	0	0
	5	0	1	0	0	0
	6	0	1	0	2.6	2.2
	7	0	1	0	40.4	30
	8	0	1	0	75	54
	9	0	1	0	30	22
	10	0	1	0	28	19
	11	0	1	0	145	104
	12	0	1	0	145	104
	13	0	1	0	8	5.5
	14	0	1	0	8	5.5
	15	0	1	0	0	0
	16	0	1	0	45.5	30
	17	0	1	0	60	35
	18	0	1	0	60	35
	19	0	1	0	0	0
	20	0	1	0	1	0.6
	21	0	1	0	114	81
	22	0	1	0	5.3	3.5
	23	0	1	0	0	0
	24	0	1	0	28	20
	25	0	1	0	0	0
	26	0	1	0	14	10
	27	0	1	0	14	10
	28	0	1	0	26	18.6
	29	0	1	0	26	18.6
	30	0	1	0	0	0

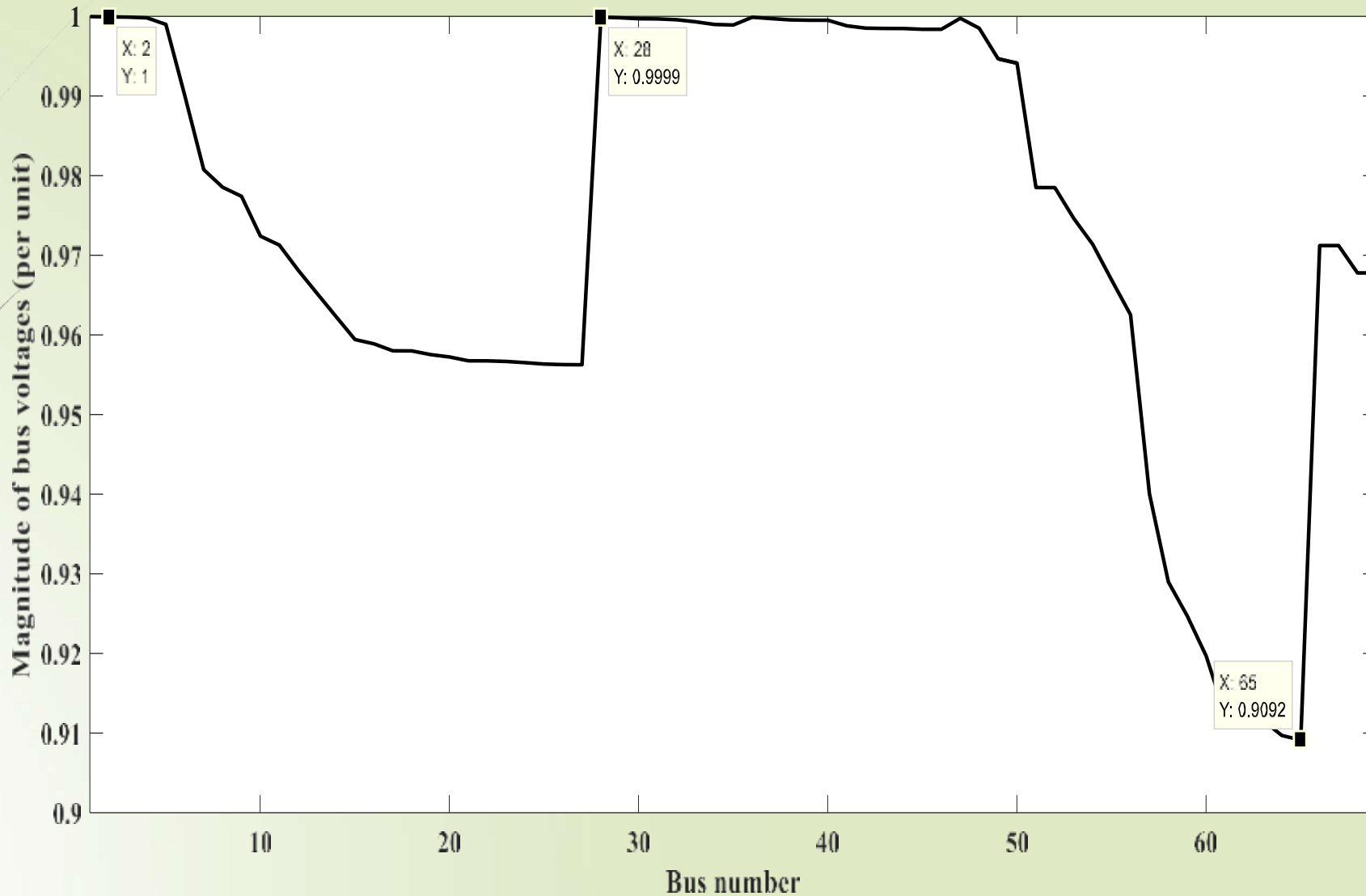
31	0	1	0	0	0
32	0	1	0	0	0
33	0	1	0	14	10
34	0	1	0	19.5	14
35	0	1	0	6	4
36	0	1	0	26	18.55
37	0	1	0	26	18.55
38	0	1	0	0	0
39	0	1	0	24	17
40	0	1	0	24	17
41	0	1	0	1.2	1
42	0	1	0	0	0
43	0	1	0	6	4.3
44	0	1	0	0	0
45	0	1	0	39.22	26.3
46	0	1	0	39.22	26.3
47	0	1	0	0	0
48	0	1	0	79	56.4
49	0	1	0	384.7	274.5
50	0	1	0	384.7	274.5
51	0	1	0	40.5	28.3
52	0	1	0	3.6	2.7
53	0	1	0	4.35	3.5
54	0	1	0	26.4	19
55	0	1	0	24	17.2
56	0	1	0	0	0
57	0	1	0	0	0
58	0	1	0	0	0
59	0	1	0	100	72
60	0	1	0	0	0
61	0	1	0	1244	888
62	0	1	0	32	23
63	0	1	0	0	0
64	0	1	0	227	162
65	0	1	0	59	42
66	0	1	0	18	13
67	0	1	0	18	13
68	0	1	0	28	20
69	0	1	0	28	20

# Line data of 69-node radial distribution network

%	ns	nr	R	X
linedata				
=	[1	2	0.0005	0.0012
	2	3	0.0005	0.0012
	3	4	0.0015	0.0036
	4	5	0.0251	0.0294
	5	6	0.366	0.1864
	6	7	0.3811	0.1941
	7	8	0.0922	0.047
	8	9	0.0493	0.0251
	9	10	0.819	0.2707
	10	11	0.1872	0.0691
	11	12	0.7114	0.2351
	12	13	1.03	0.34
	13	14	1.044	0.345
	14	15	1.058	0.3496
	15	16	0.1966	0.065
	16	17	0.3744	0.1238
	17	18	0.0047	0.0016
	18	19	0.3276	0.1083
	19	20	0.2106	0.0696
	20	21	0.3416	0.1129
	21	22	0.014	0.0046
	22	23	0.1591	0.0526
	23	24	0.3463	0.1145
	24	25	0.7488	0.2745
	25	26	0.3089	0.1021
	26	27	0.1732	0.0572
	3	28	0.0044	0.0108
	28	29	0.064	0.1565
	29	30	0.3978	0.1315
	30	31	0.0702	0.0232

31	32	0.351	0.116
32	33	0.839	0.2816
33	34	1.708	0.5646
34	35	1.474	0.4873
3	36	0.0044	0.0108
36	37	0.064	0.1565
37	38	0.1053	0.123
38	39	0.0304	0.0355
39	40	0.0018	0.0021
40	41	0.7283	0.8509
41	42	0.31	0.3623
42	43	0.041	0.0478
43	44	0.0092	0.0116
44	45	0.1089	0.1373
45	46	0.0009	0.0012
4	47	0.0034	0.0084
47	48	0.0851	0.2083
48	49	0.2898	0.7091
49	50	0.0822	0.2011
8	51	0.0928	0.0473
51	52	0.3319	0.1114
9	53	0.174	0.0886
53	54	0.203	0.1034
54	55	0.2842	0.1447
55	56	0.2813	0.1433
56	57	1.59	0.5337
57	58	0.7837	0.263
58	59	0.3042	0.1006
59	60	0.3861	0.1172
60	61	0.5075	0.2585
61	62	0.0974	0.0496
62	63	0.145	0.0738
63	64	0.7105	0.3619
64	65	1.041	0.5302
11	66	0.2012	0.0611
66	67	0.0047	0.0014
12	68	0.7394	0.2444
68	69	0.0047	0.0016];

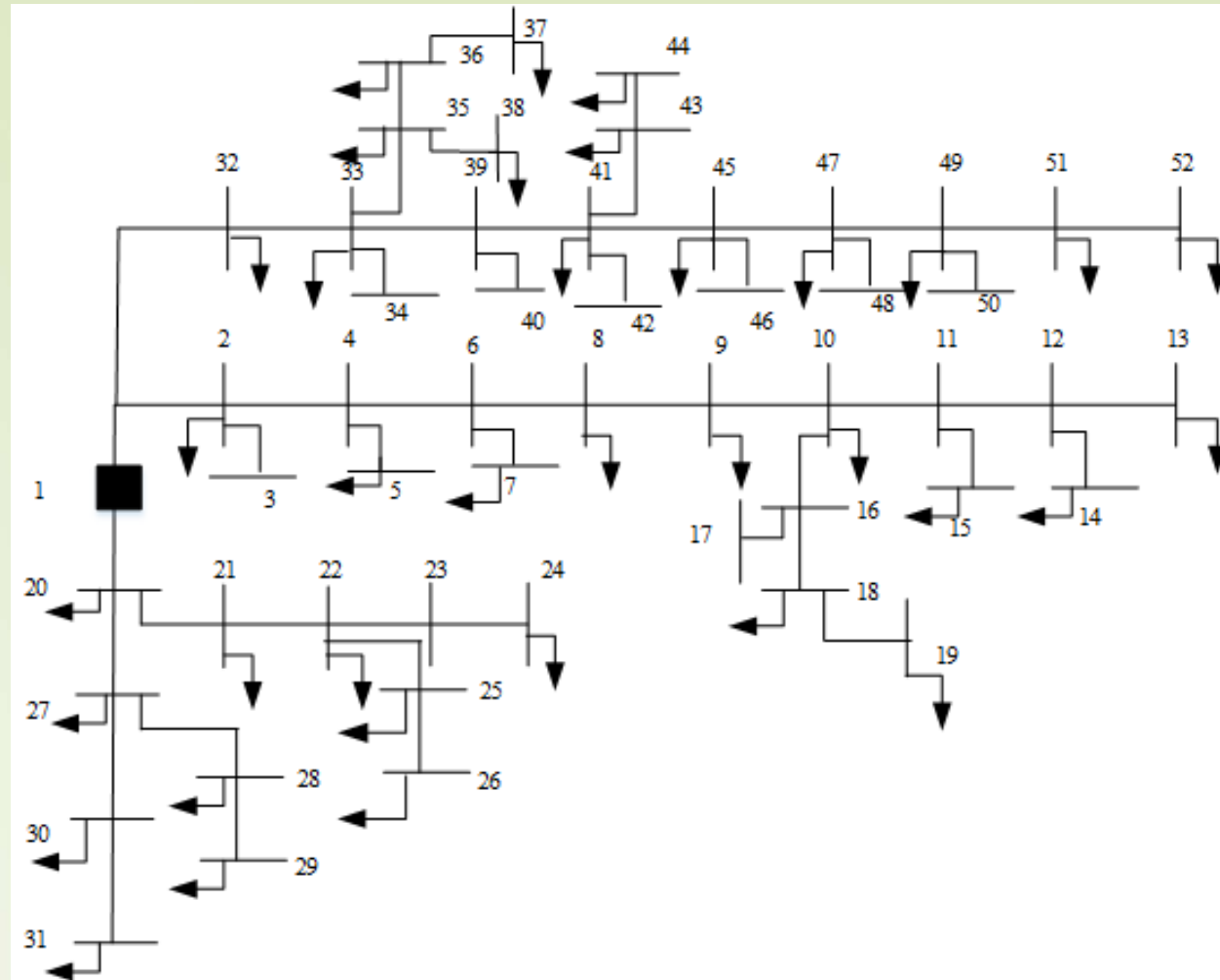
# Voltage magnitude plot of 69-node radial distribution network



# Bus voltages of 69-node radial distribution network

v=[	1.0000 + 0.0000i	0.9989 + 0.0002i
	1.0000 -	0.9999 - 0.0001i
	0.0000i	0.9997 - 0.0002i
	0.9999 -	0.9996 - 0.0002i
	0.0000i	0.9995 - 0.0002i
	0.9998 -	0.9995 - 0.0002i
	0.0001i	0.9988 - 0.0004i
	0.9990 -	0.9986 - 0.0005i
	0.0003i	0.9985 - 0.0005i
	0.9901 +	0.9985 - 0.0005i
	0.0009i	0.9984 - 0.0005i
	0.9808 +	0.9984 - 0.0005i
	0.0021i	0.9984 - 0.0005i
	0.9786 +	0.9998 - 0.0001i
	0.0024i	0.9985 - 0.0009i
	0.9774 +	0.9947 - 0.0033i
	0.0025i	0.9941 - 0.0037i
	0.9724 +	0.9785 + 0.0024i
	0.0039i	0.9785 + 0.0024i
	0.9713 +	0.9747 + 0.0029i
	0.0042i	0.9714 + 0.0033i
	0.9681 +	0.9669 + 0.0039i
	0.0051i	0.9626 + 0.0045i
	0.9652 +	0.9400 + 0.0109i
	0.0059i	0.9289 + 0.0140i
	0.9623 +	0.9246 + 0.0153i
	0.0066i	0.9196 + 0.0169i
	0.9594 +	0.9122 + 0.0178i
	0.0074i	0.9119 + 0.0179i
	0.9589 +	0.9115 + 0.0179i
	0.0075i	0.9096 + 0.0181i
	0.9580 +	0.9090 + 0.0182i
	0.0077i	0.9713 + 0.0042i
	0.9580 +	0.9713 + 0.0042i
	0.0077i	0.9678 + 0.0052i
	0.9575 +	0.9678 + 0.0052i];
	0.0079i	
	0.9572 +	
	0.0080i	
	0.9568 +	

# 52-node radial distribution network



# Node data of 52-node radial distribution network

```

nbus=52;
basemva = 1 ; accuracy = 0.0001;
maxiter=100; basekv=11;
% Distribution System data (52 bus system)
% Bus Bus Voltage Angle Load Load
% No. code mag. (Degr e e) (kW) (kVAR)
busdat
a=[

```

Bus No.	Bus code	Voltage mag. (pu)	Angle (Degr e e)	Load (kW)	Load (kVAR)
1	1	1	0	0	0
2	0	1	0	81	39
3	0	1	0	135	65
4	0	1	0	108	52
5	0	1	0	108	52
6	0	1	0	27	13
7	0	1	0	54	26
8	0	1	0	135	65
9	0	1	0	81	39
10	0	1	0	67	32
11	0	1	0	27	13
12	0	1	0	27	13
13	0	1	0	108	52
14	0	1	0	54	26
15	0	1	0	94	45
16	0	1	0	67	33
17	0	1	0	67	33
18	0	1	0	108	52
19	0	1	0	81	39
20	0	1	0	108	52
21	0	1	0	94	46
22	0	1	0	81	39
23	0	1	0	108	52
24	0	1	0	108	52
25	0	1	0	102	50

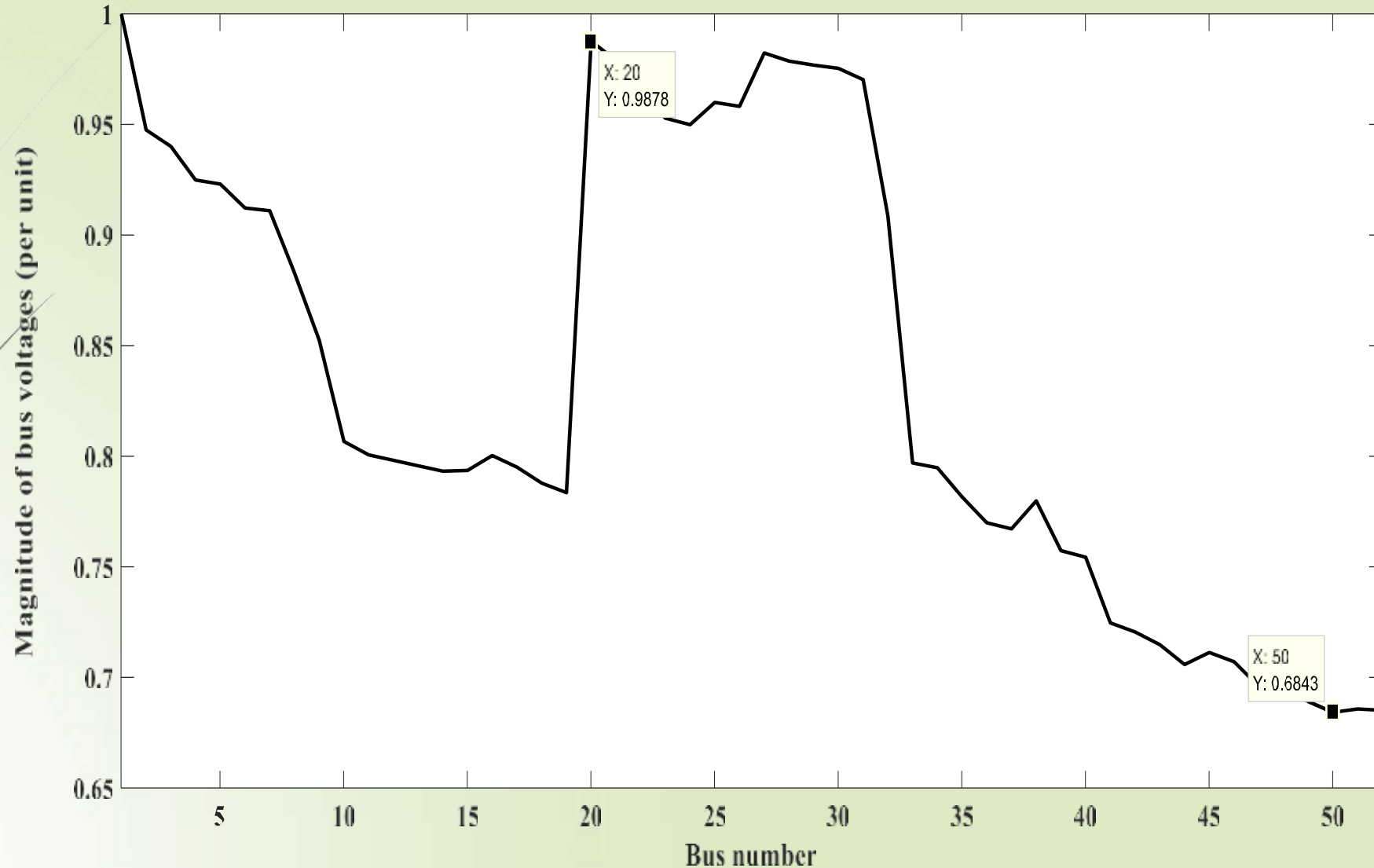
26	0	1	0	41	20
27	0	1	0	108	52
28	0	1	0	162	79
29	0	1	0	68	33
30	0	1	0	68	33
31	0	1	0	95	46
32	0	1	0	41	20
33	0	1	0	121	59
34	0	1	0	41	20
35	0	1	0	41	20
36	0	1	0	135	66
37	0	1	0	81	40
38	0	1	0	68	33
39	0	1	0	95	46
40	0	1	0	108	52
41	0	1	0	41	20
42	0	1	0	95	46
43	0	1	0	27	13
44	0	1	0	122	59
45	0	1	0	108	52
46	0	1	0	81	39
47	0	1	0	68	33
48	0	1	0	41	20
49	0	1	0	68	33
50	0	1	0	81	39
51	0	1	0	108	52
52	0	1	0	41	20];

# Line data of 52-node radial distribution network

%	ns	nr	R (pu)	X (pu)
linedata=	[ 1	2	0.0258	0.0111
	2	3	0.043	0.0185
	2	4	0.0129	0.0056
	4	5	0.0129	0.0056
	4	6	0.0086	0.0037
	6	7	0.0172	0.0074
	6	8	0.0215	0.0093
	8	9	0.0258	0.0111
	9	10	0.043	0.0185
	10	11	0.0129	0.0056
	11	12	0.0086	0.0037
	11	15	0.043	0.0185
	12	13	0.0301	0.013
	12	14	0.0344	0.0148
	10	16	0.0129	0.0056
	16	17	0.0516	0.0222
	16	18	0.043	0.0185
	18	19	0.0344	0.0148
	1	20	0.0086	0.0037
	20	21	0.0129	0.0056
	21	22	0.0258	0.0111
	22	23	0.043	0.0185
	23	24	0.0215	0.0093
	22	25	0.0258	0.0111
	25	26	0.0344	0.0148

20	27	0.0086	0.0037
27	28	0.0129	0.0056
28	29	0.0215	0.0093
27	30	0.0344	0.0148
30	31	0.043	0.0185
1	32	0.0344	0.0148
32	33	0.043	0.0185
33	34	0.0344	0.0148
33	35	0.0301	0.013
35	36	0.0344	0.0148
36	37	0.0215	0.0093
35	38	0.0172	0.0074
33	39	0.0215	0.0093
39	40	0.0172	0.0074
39	41	0.0215	0.0093
41	42	0.0258	0.0111
41	43	0.0387	0.0167
43	44	0.043	0.0185
41	45	0.0129	0.0056
45	46	0.0301	0.013
45	47	0.0215	0.0093
47	48	0.0129	0.0056
47	49	0.0129	0.0056
49	50	0.0344	0.0148
49	51	0.0129	0.0056
51	52	0.0086	0.0037];

# Voltage magnitude plot of 52-node radial distribution network

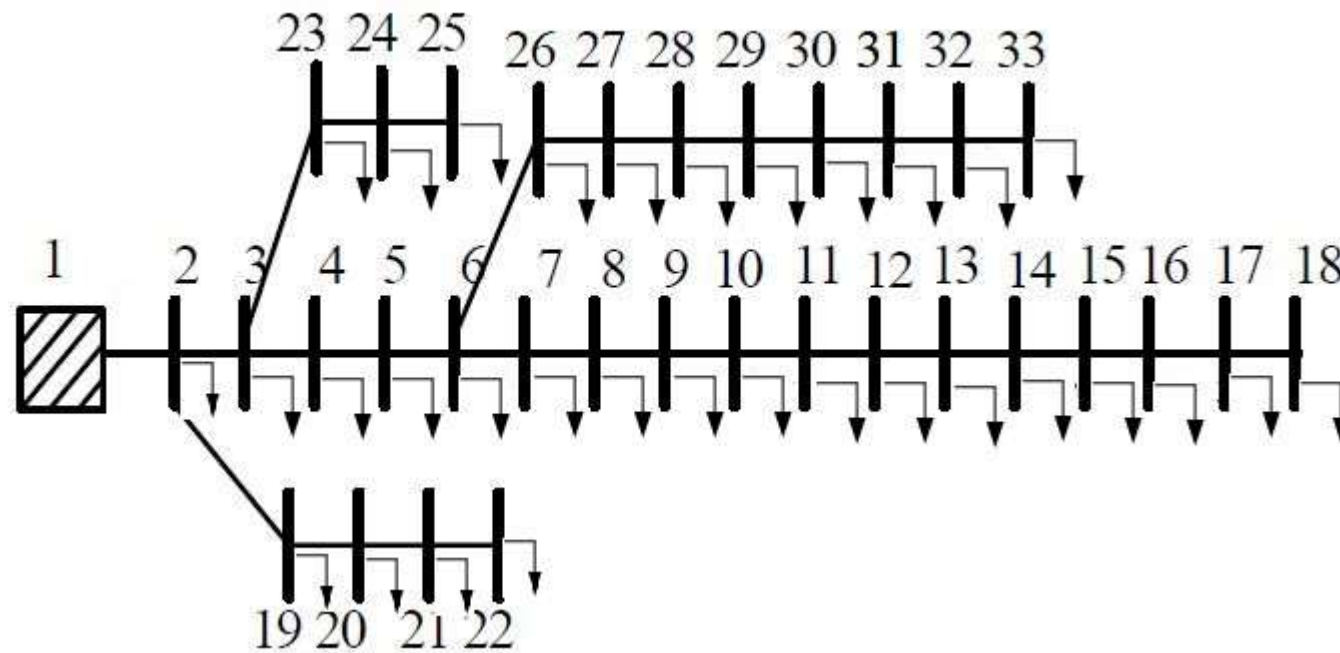




# Node voltages of 52-node radial distribution network

v=[	1.0000 + 0.0000i	0.9824 + 0.0008i
	0.9476 + 0.0019i	0.9788 + 0.0009i
	0.9402 + 0.0022i	0.9769 + 0.0010i
	0.9250 + 0.0027i	0.9755 + 0.0011i
	0.9231 + 0.0027i	0.9704 + 0.0013i
	0.9123 + 0.0031i	0.9087 + 0.0031i
	0.9111 + 0.0032i	0.7970 + 0.0068i
	0.8830 + 0.0041i	0.7949 + 0.0069i
	0.8525 + 0.0051i	0.7817 + 0.0073i
	0.8068 + 0.0067i	0.7700 + 0.0078i
	0.8007 + 0.0069i	0.7672 + 0.0079i
	0.7982 + 0.0070i	0.7799 + 0.0074i
	0.7957 + 0.0071i	0.7574 + 0.0079i
	0.7933 + 0.0071i	0.7544 + 0.0080i
	0.7936 + 0.0071i	0.7247 + 0.0089i
	0.8004 + 0.0069i	0.7206 + 0.0090i
	0.7951 + 0.0072i	0.7148 + 0.0091i
	0.7879 + 0.0074i	0.7058 + 0.0094i
	0.7836 + 0.0075i	0.7113 + 0.0092i
	0.9878 + 0.0005i	0.7071 + 0.0093i
	0.9791 + 0.0009i	0.6959 + 0.0096i
	0.9647 + 0.0015i	0.6950 + 0.0097i
	0.9529 + 0.0020i	0.6891 + 0.0098i
	0.9500 + 0.0021i	0.6842 + 0.0100i
	0.9601 + 0.0017i	0.6857 + 0.0099i
	0.9583 + 0.0018i	0.6851 + 0.0099i];

# 33-node radial distribution network



# Node data and line data of 33-node radial distribution network

```

basemva =100 ; accuracy = 0.0001;
maxiter=100; basekv=12.66;
%Distribution System data (33 bus
system)
%
% Bus Bus Voltage Angle Load Load
% no. code mag. (degree (kW) (kVAR)
busdata
=[
1 1 1 0 0 0
2 0 1 0 100 60
3 0 1 0 90 40
4 0 1 0 120 80
5 0 1 0 60 30
6 0 1 0 60 20
7 0 1 0 200 100
8 0 1 0 200 100
9 0 1 0 60 20
10 0 1 0 60 20
11 0 1 0 45 30
12 0 1 0 60 35
13 0 1 0 60 35
14 0 1 0 120 80
15 0 1 0 60 10
16 0 1 0 60 20
17 0 1 0 60 20
18 0 1 0 90 40
19 0 1 0 90 40
20 0 1 0 90 40
21 0 1 0 90 40
22 0 1 0 90 40
23 0 1 0 90 50
24 0 1 0 420 200
25 0 1 0 420 200
26 0 1 0 60 25
27 0 1 0 60 25
28 0 1 0 60 20
29 0 1 0 120 70
30 0 1 0 200 600
31 0 1 0 150 70
32 0 1 0 210 100
33 0 1 0 60 100

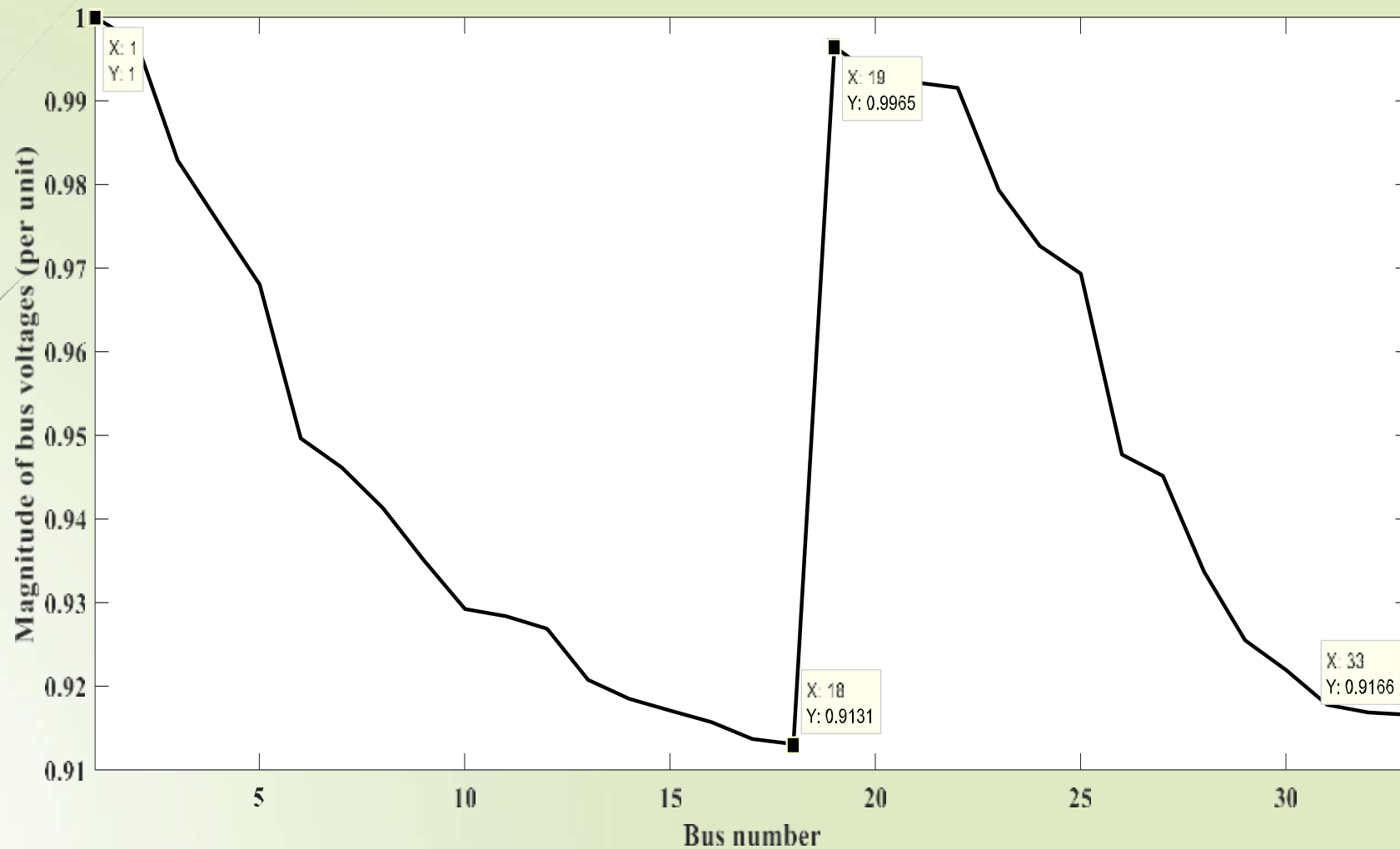
```

```

%
% ns nr R X
linedata=
[1 2 0.0922 0.047
2 3 0.493 0.2511
3 4 0.366 0.1864
4 5 0.3811 0.1941
5 6 0.819 0.707
6 7 0.1872 0.6188
7 8 0.7114 0.2351
8 9 1.03 0.74
9 10 1.044 0.74
10 11 0.1966 0.065
11 12 0.3744 0.1238
12 13 1.468 1.155
13 14 0.5416 0.7129
14 15 0.591 0.526
15 16 0.7463 0.545
16 17 1.289 1.721
17 18 0.732 0.574
2 19 0.164 0.1565
19 20 1.5042 1.3554
20 21 0.4095 0.4784
21 22 0.7089 0.9373
3 23 0.4512 0.3083
23 24 0.898 0.7091
24 25 0.896 0.7011
6 26 0.203 0.1034
26 27 0.2842 0.1447
27 28 1.059 0.9337
28 29 0.8042 0.7006
29 30 0.5075 0.2585
30 31 0.9744 0.963
31 32 0.3105 0.3619
32 33 0.341 0.5302];

```

# Voltage magnitude plot of 33-node radial distribution network



# Node voltages of 33-node radial distribution network

v=	[	1.0000 + 0.0000i
		0.9970 + 0.0003i
		0.9829 + 0.0016i
		0.9755 + 0.0028i
		0.9681 + 0.0039i
		0.9497 + 0.0022i
		0.9462 - 0.0016i
		0.9413 - 0.0010i
		0.9351 - 0.0022i
		0.9292 - 0.0032i
		0.9284 - 0.0031i
		0.9269 - 0.0029i
		0.9208 - 0.0043i
		0.9185 - 0.0056i
		0.9171 - 0.0062i
		0.9157 - 0.0065i
		0.9137 - 0.0077i
		0.9131 - 0.0079i
		0.9965 + 0.0001i
		0.9929 - 0.0011i
		0.9922 - 0.0014i
		0.9916 - 0.0018i
		0.9794 + 0.0011i
		0.9727 - 0.0004i
		0.9694 - 0.0011i
		0.9477 + 0.0029i
		0.9452 + 0.0038i
		0.9337 + 0.0051i
		0.9255 + 0.0063i
		0.9219 + 0.0080i
		0.9178 + 0.0066i
		0.9169 + 0.0062i
		0.9166 + 0.0061i];

# Summary of the results

28

<b>Parameters</b>	<b>69-node system</b>	<b>52-node system</b>	<b>33-node system</b>
Minimum voltage of system	0.9092	0.6843	0.9131
Minimum voltage node number	65	50	18
Maximum voltage node number	2	20	2
Power loss of the network (kW)	225.0021	887.3782	202.6762

# Reference...

- ▮ S. Ghosh and D. Das, "Method for load-flow solution of radial distribution networks," *Proc. IEE, Generation, Transmission, Distribution*, vol. 146, no. 6, pp. 641–648, 1999.
- ▮ M. E. Baran and F. F. Wu, "Network reconfiguration in distribution systems for loss reduction and load balancing," *IEEE Trans. Power Delivery*, vol. 4, no. 2, pp. 1401–1407, Apr. 1989.
- ▮ J. S. Savier and D. Das, "Impact of network reconfiguration on loss allocation of radial distribution systems," *IEEE Trans. Power Delivery*, vol. 22, no. 4, pp. 2473–2480, Oct. 2007.
- ▮ D. Thukaram, H.P. Khincha, H.P. Vijaynarasimha, "Artificial neural network and support vector machine approach for locating faults in radial distribution systems," *IEEE Trans Power Deliv.* vol. 20, no. 2 I, pp. 710–721, 2005.

Thank You...



# Reactive Power Compensation of Distribution Systems with Shunt Capacitors

**M MURALI**

**Associate Professor, Department of EEE,**

**VEMU IT**



# What is reactive power compensation?

- ▮ Reactive power compensation is an approach to locally supply the reactive power demand to the consumer. This results in:
  - ▮ Reduction of power loss
  - ▮ Improvement of bus voltage profile
  - ▮ power factor correction

This is also called volt/var control...

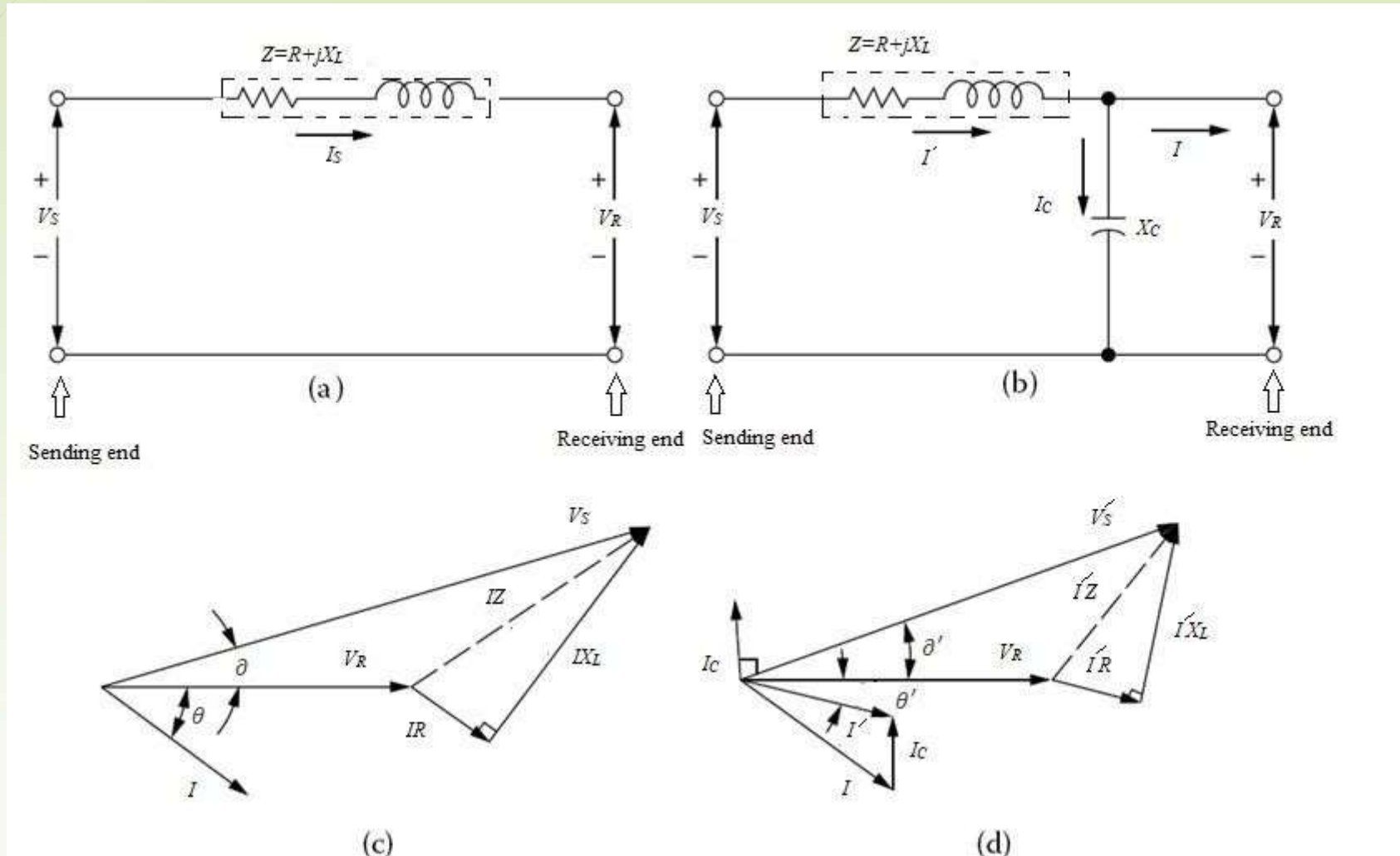
# Different reactive power compensation techniques for distribution networks

- ▣ Capacitor
- ▣ Synchronous condenser
- ▣ Custom power devices
- ▣ Distributed generation units

# Shunt Capacitor placement

- ▮ *Shunt capacitors, that is, capacitors connected in parallel with lines, are used extensively in distribution systems.*
- ▮ Shunt capacitors modify the characteristic of an inductive load by drawing a leading current.
- ▮ With shunt capacitor placement to a feeder, the magnitude of the source current can be reduced, the power factor can be improved, and consequently the voltage drop between the sending end and the load is also reduced.

# Phasor diagram for a feeder section with and without capacitor placement



# Voltage drop computation with capacitor placement

- ▮ Voltage drop in feeders, or in short transmission lines, with lagging power factor can be approximated as:

$$VD = I_R R + I_X X_L$$

where,

$R$  is the total resistance of the feeder circuit,  $\Omega$

$X_L$  is the total inductive reactance of the feeder circuit,  $\Omega$

$I_R$  is the real power (or in-phase) component of the current, A

$I_X$  is the reactive power (or out-of-phase) component of the current lagging the voltage by  $90^\circ$ , A

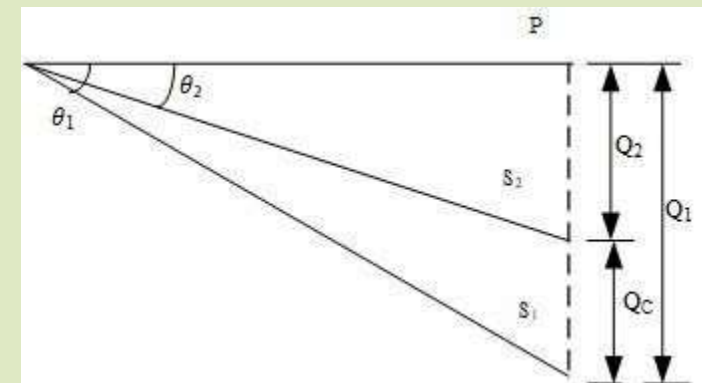
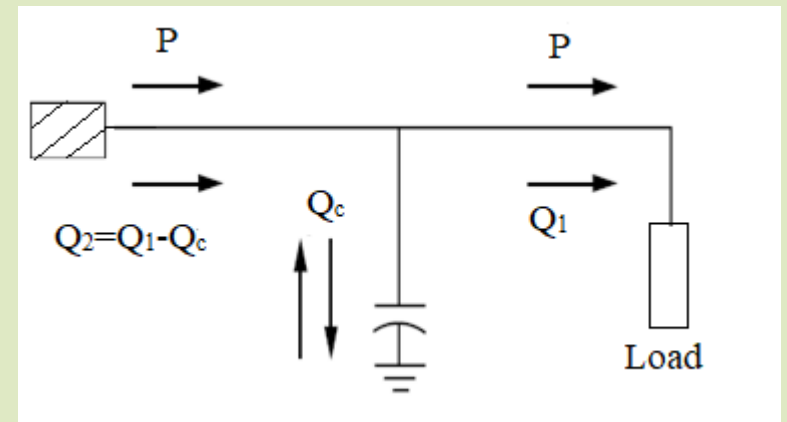




# Capacitor placement for the power factor correction: Power Triangle

- When a shunt capacitor of  $Q_c$  kVA is installed at the load, the power factor can be improved from  $\cos \theta_1$  to  $\cos \theta_2$ , where,

$$\cos \theta_2 = \frac{P}{S_2} = \frac{P}{(P^2 + Q_2^2)^{1/2}} = \frac{P}{[P^2 + (Q_1 - Q_c)^2]^{1/2}}$$



# Power factor correction

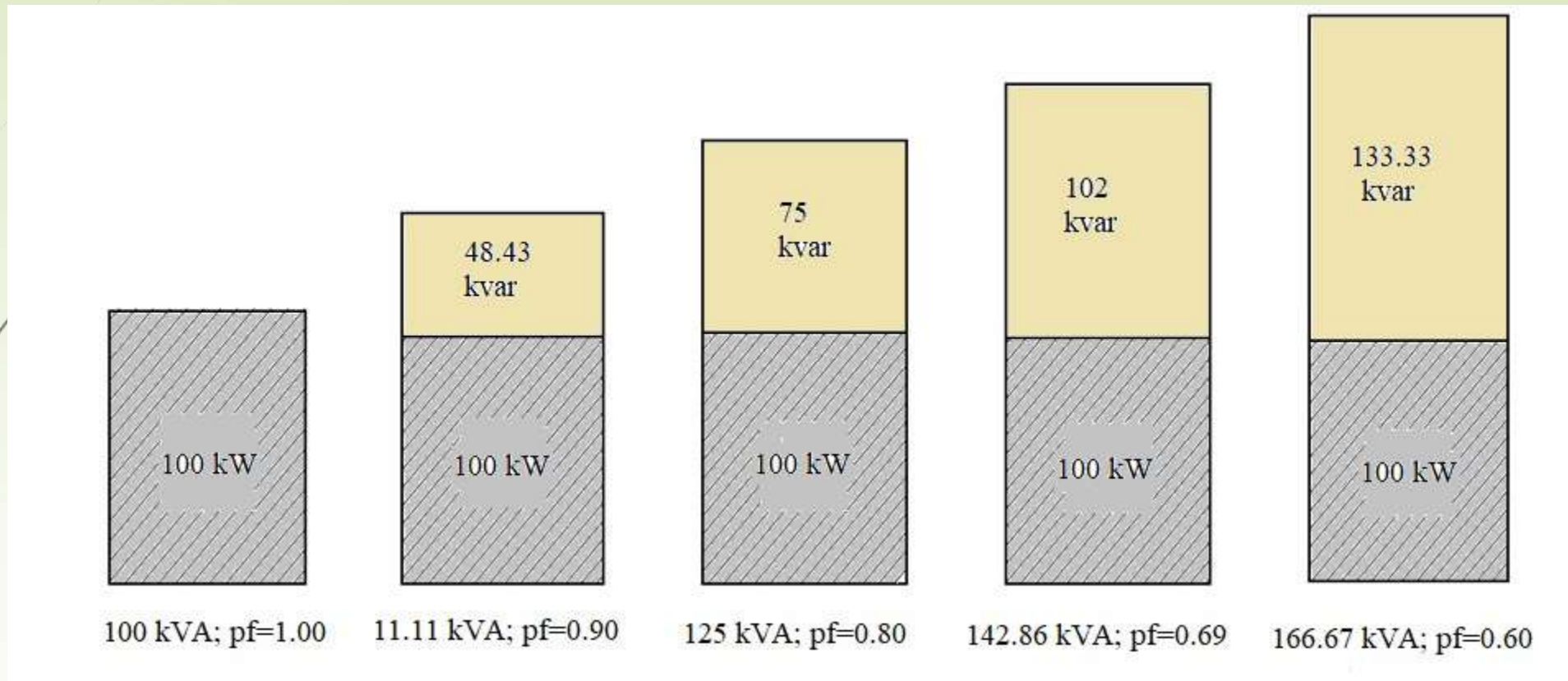
It gives a multiplier to determine the kVAr requirement. It is based on the following formula:

$$Q = P(\tan\theta_{orig} - \tan\theta_{new}) = P \left\{ \sqrt{\frac{1}{PF_{ori}^2} - 1} - \sqrt{\frac{1}{PF_{ne}^2} - 1} \right\}$$

where,

- Q is the required compensation in kVAr
- P is the real power kW
- $PF_{orig}$  is the original power factor
- $PF_{new}$  is the desired power factor

# Illustrative example of the effect of reactive power on power factor



Ref. T. Gonen. *Electric Power Distribution System Engineering*

# Concepts of leading and lagging power factor

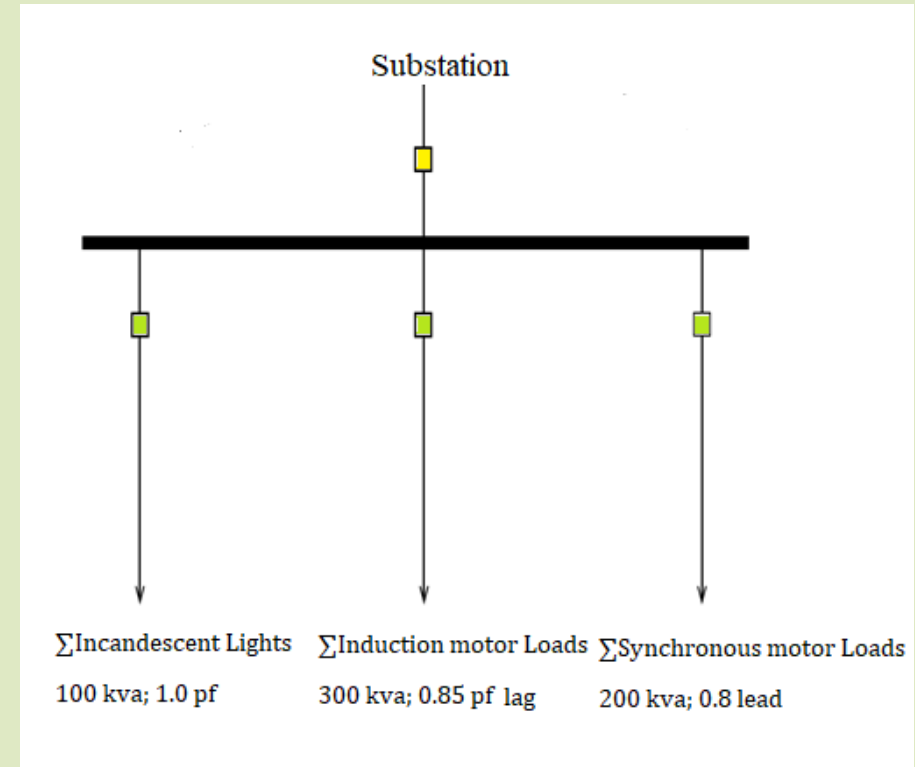
- ▮ The concept of “lagging” and “leading” power factors depends on the directions of the flows of real and reactive powers.
- ▮ For a given load, the power factor is *lagging* if the load consumes reactive power. It is *leading* if the load supplies reactive power.
- ▮ An induction motor has a lagging power factor, since it consumes reactive power from the source to meet its magnetizing requirements. But a capacitor (or an overexcited synchronous motor) supplies reactive power and thus has a leading power factor,

# Economy in power factor correction: Concept of economic power factor

- ▮ Since the capacitor can compensate the reactive current drawn by the load, it results in a reduced total current, which in turn causes less power losses.
- ▮ Thus, the power factor correction produces economic savings in capital expenditures and fuel expenses through a release of kilovolt-amp capacity.
- ▮ The economic power factor is the point at which the economic benefits of adding shunt capacitors just equal the cost of the capacitors.

# Example...

- ▮ Assume that a substation supplies three different kinds of loads, i.e., incandescent lights, induction motors, and synchronous motors, as shown in Figure. Based on the given data in Figure, determine the following:
  - ▮ a. The apparent, real, and kVARs of each load
  - ▮ b. The total apparent, real, and reactive powers of the power that should be supplied by the substation
  - ▮ c. The total power factor of the substation



## Solution...

□ Since incandescent lights are basically a unity power factor load, hence,  $S_1 = P_1 = 100$  kW and reactive power  $Q_1 = 0$ .

□ For induction motor loads operating power factor of 0.85 lagging,  $S_2 = 300$  kVA

Real power consumption  $P_2 = 300 \times 0.85 = 255$  kW

□ For synchronous motor load, the apparent power  $S_3 = 200$  kVA. Thus, the real power consumption,  $P_3 = (200 \text{ kVA}) \cos \theta = 200 \times 0.8 = 160$  kW

## Solution...

- ❖ Reactive power consumption ( $Q_2$ ) of the induction motor

load is,  $Q_2 = \sqrt{(300)^2 - (255)^2} = \sqrt{90,000 - 65,025} \cong 158 \text{ kVAr}$

- ❖ Reactive power consumption ( $Q_3$ ) of the

synchronous motors is  $Q_3 = - \sqrt{(200 \text{ kVA})^2 - (160$

$\text{kW})^2} \sqrt{\hspace{10em}}$

$$= -\sqrt{40,000 - 25,600}$$

$$= - 14,400$$

$$= -120 \text{ kVAr}$$



## Solution...

At the substation, the total real power consumption is

$$\begin{aligned}P_{total} &= P_1 + P_2 + P_3 \\ &= 100 + 255 + 160 \\ &= 515 \text{ kW}\end{aligned}$$

Total Reactive power consumption is:

$$\begin{aligned}Q_{total} &= Q_1 + Q_2 + Q_3 \\ &= 0 + 158 - 120 \\ &= 38 \text{ kVAr}\end{aligned}$$

Reactive power

- Induction motor load required = 158 kVAr
- Synchronous motor supplied = 120 kVAr
- Substation must supply = 38 kVAr

## Solution...

The kVA of the substation is  $S_{total}$

$$= \frac{\sqrt{P_{tot}^2 + Q^2}}{to}$$
$$= \sqrt{515^2 + 38^2}$$
$$= \sqrt{266,669}$$
$$= 516.4 \text{ kVA}$$

The power factor of the substation is,

$$= \frac{515 \text{ kW}}{516.4 \text{ kVA}}$$
$$= 0.997 \text{ lagging}$$

## Example...

- ▮ A synchronous generator is rated 1250 kVA. It is operating at a rated load of 1250 kVA at 0.85 power factor lagging. An additional load of 150 kW at 0.85 power factor is to be added. Determine the value of capacitors needed in order not to overload the generator.

### Solution:

- ❖ The generator provides real power  $P = 1250 \times 0.85 = 1062.5 \text{ kW}$
- ❖ The generator provides reactive power  $Q = \sqrt{1250^2 - 1062.5^2} = 658.48 \text{ kVAr}$

## Solution...

- ❖ With an additional load of 150 kW, the generator needs to supply  $P_{new} = 1062.5 + 150 = 1212.5$  kW
- ❖ Since the generator is rated 1250 kVA, with the additional load it can provide  $Q_{new} = \sqrt{1250^2 - 1212.5^2} = 303.88$  kVAr
- ❖ The additional load demands the reactive power of  $Q_{add} = \sqrt{\left(\frac{150}{0.85}\right)^2 - 150^2} = 92.96$  kVAr
- ❖ The total reactive power demand is  $Q_d = Q + Q_{add} = 658.48 + 92.96 = 751.44$  kVAr
- ❖ However, the generator can only provide 303.88 kVAr
- ❖ Therefore, the capacitor rating should be  $Q_c = 751.44 - 303.88 = 447.56$  kVAr

## Exercise 1...

- ▮ Assume that a load consumes 80 kW and 60 kVAr lagging. It is required that its power factor is to be improved to 0.9 using capacitor bank so that the real power consumption of the load remains same. Determine the size of the capacitor bank.

# Computerized method to determine the economic power factor

- ▮ Firstly, a load flow digital computer program can be employed to determine the kVA, kV, and kVAR at annual peak level for the whole system.
- ▮ Shunt capacitors are applied to a particular bus for correcting to an initial power factor, for example, 0.8.
- ▮ Then, a load flow run is performed to determine the total system losses.
- ▮ Additional capacitors are applied to the bus to increase the power factor by 0.01, and another load flow run is made.
- ▮ This process of iteration is repeated until the power factor becomes unity.

# Computerized method to determine the economic power factor

- After determining the economic power factor, the additional capacitor size required can be calculated as:

$$\Delta Q_c = P_{PK}(\tan \phi - \tan \theta)$$

Where

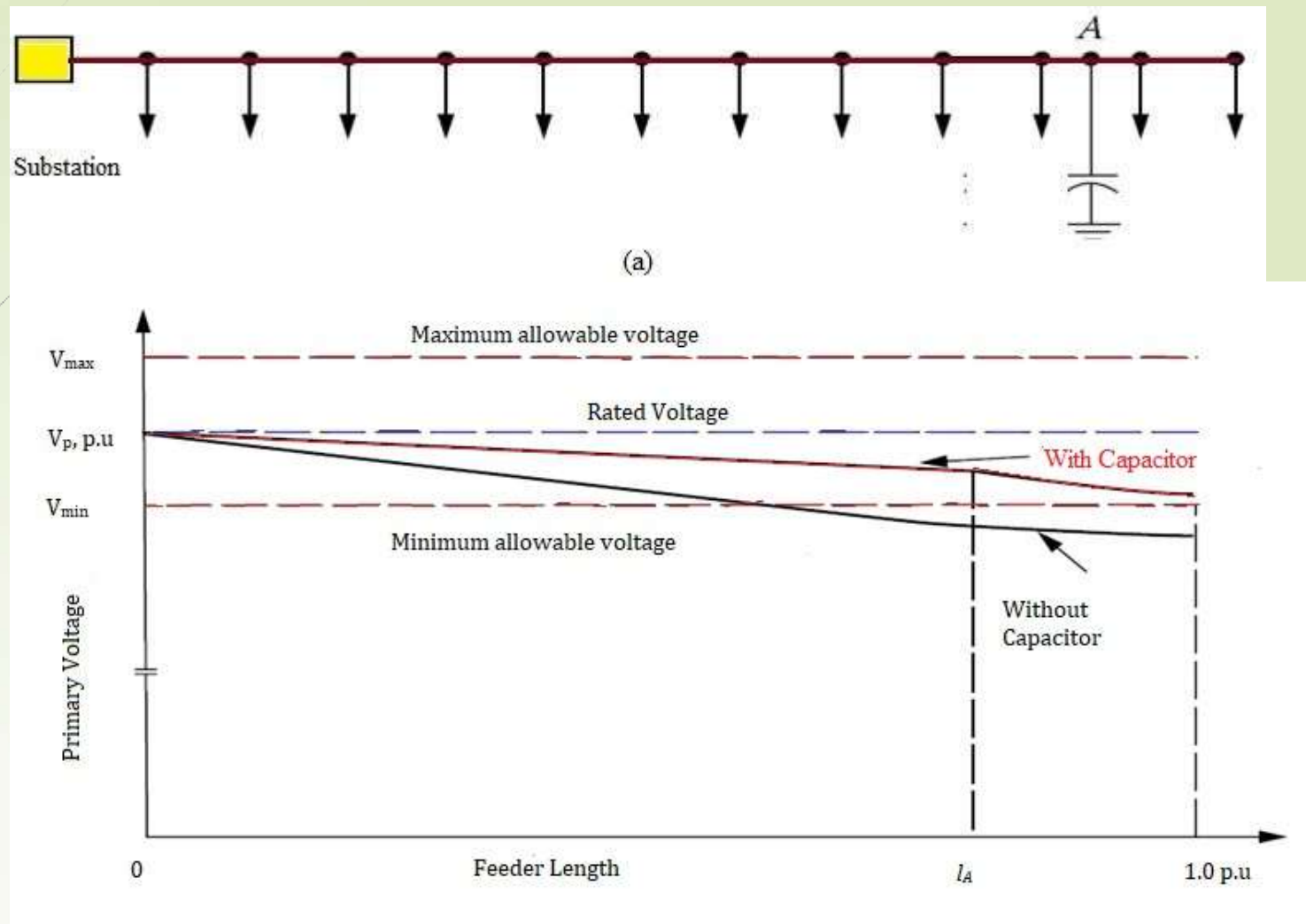
$\Delta Q_c$  is the required capacitor size, kVAr

$P_{PK}$  is the system demand at annual peak, kW

$\tan \phi$  is the tangent of original power factor angle

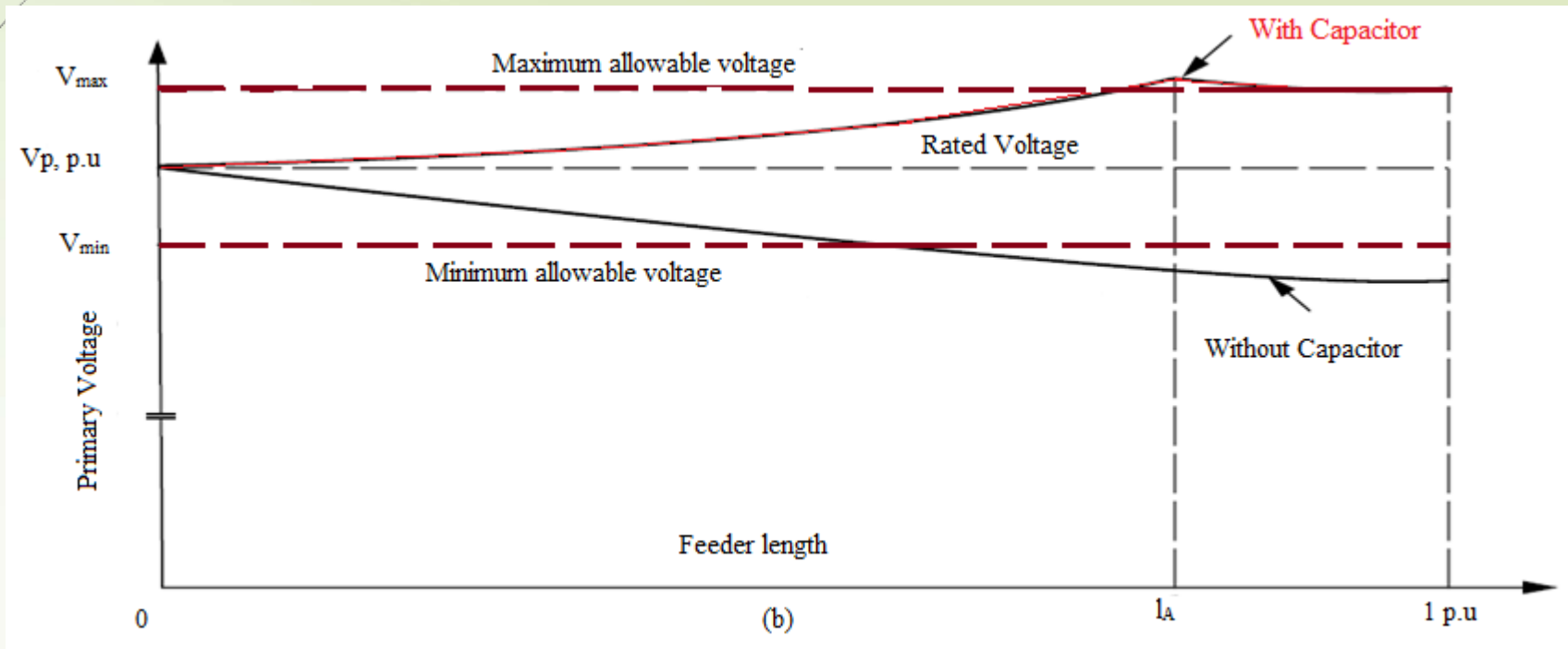
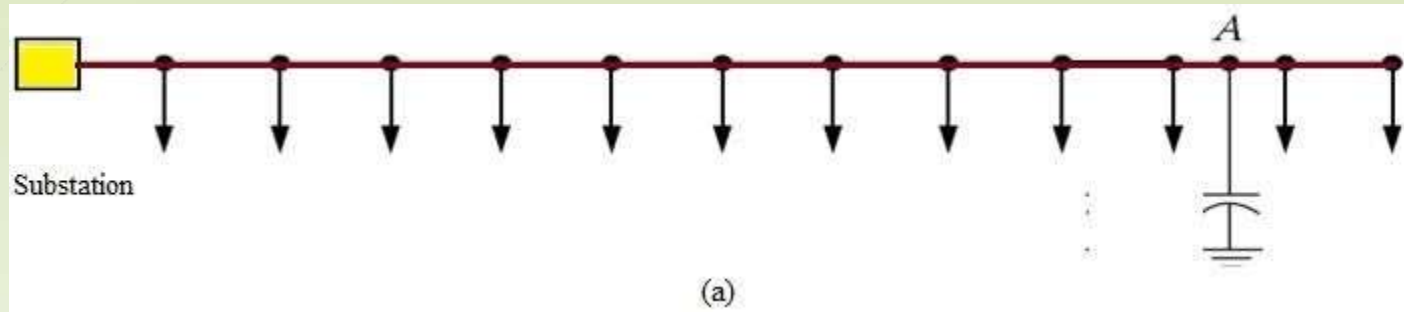
$\tan \theta$  is the tangent of economic power factor angle

# Effect of Capacitor placement on the voltage profile of distribution feeder line at rated load





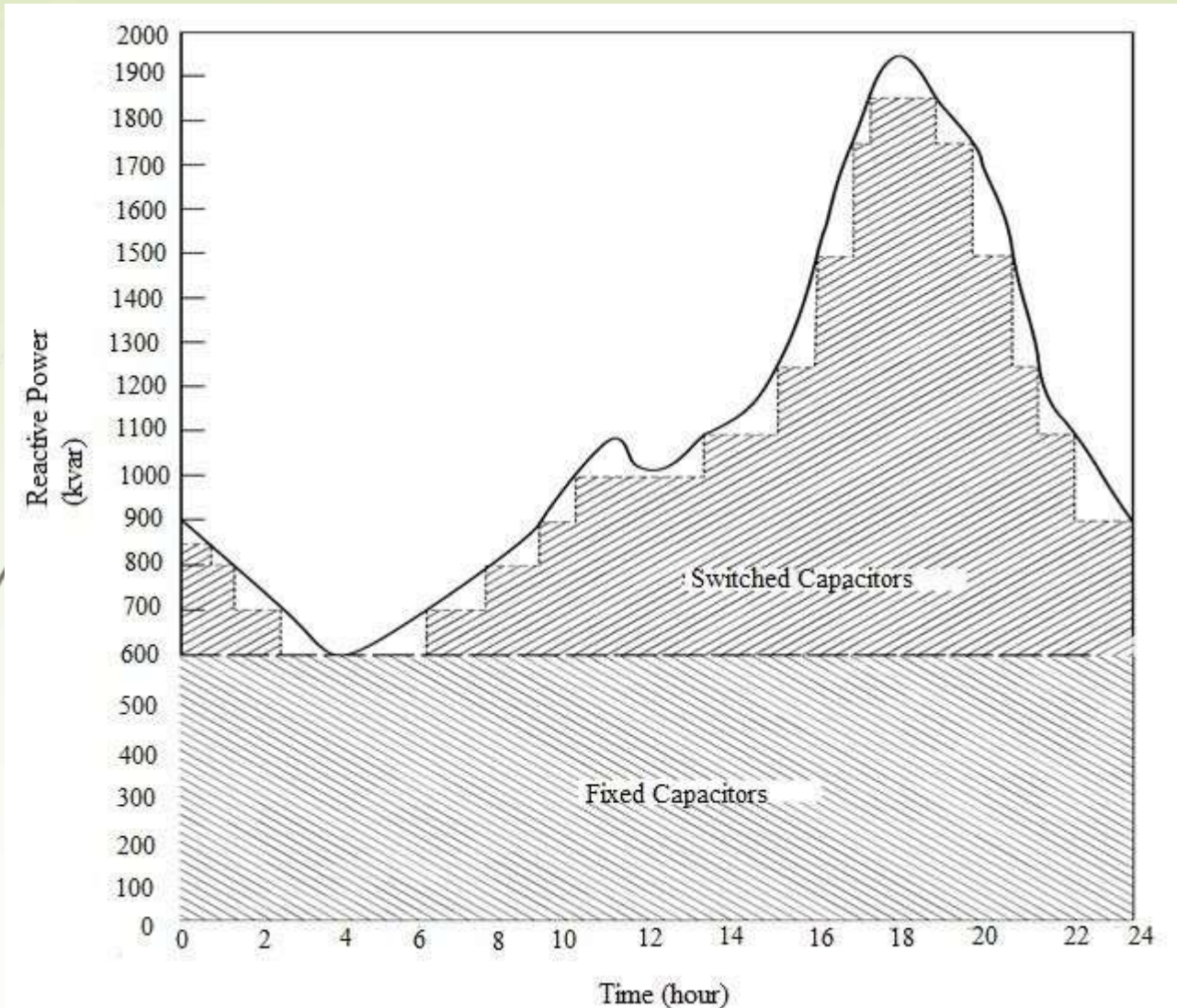
# Effect of Capacitor placement on the voltage profile of distribution feeder line at light load



## Effect of Capacitor placement on the voltage profile of distribution feeder line

- ▮ If only fixed-type capacitors are installed, there will be voltage rise problem during light load conditions.
- ▮ Therefore, some of the capacitors are installed as *switched capacitor banks* so they can be switched off during light-load conditions.
- ▮ Thus, the *fixed capacitors* are sized so that their presence would not cause voltage rise at light loading.

# Switched capacitor bank



- However, in practice, the number of steps or blocks is selected to be much less than the ones shown in the figure due to the additional expenses involved in the installation of the required switchgear and control equipment.

# Capacitor sizing

- Many utilities apply the following rule of thumb to determine the size of the switched capacitors:

$$\frac{\text{kVAr from (switched + fixed) capacitors}}{\text{kVAr of peak reactive feeder load}} \geq 0.70$$

Some utilities use the following rule of thumb: *The total amount of fixed and switched capacitors for a feeder is the amount necessary to raise the receiving-end feeder voltage to maximum at 50% of the peak feeder load.*

# Economic benefits of capacitor placement for distribution networks

- ▣ Released distribution substation capacity
- ▣ Reduced energy losses
- ▣ Reduced voltage drop and consequently improved voltage regulation
- ▣ Released capacity of feeder and associated apparatus
- ▣ Postponement or elimination of capital expenditure due to system improvements and/or expansions
- ▣ Revenue increase due to voltage improvements

# Total financial benefits of capacitor placement

- Therefore, the total benefits due to the installation of capacitor banks can be summarized as:

$$\begin{aligned}\sum \Delta\$ &= (\textit{Demand reduction}) + (\textit{Energy reduction}) + (\textit{Revenue increase}) \\ &= (\Delta\$_G + \Delta\$_T + \Delta\$_S + \Delta\$_F) + \Delta\$_{ACE} + \Delta\$_{BEC}\end{aligned}$$

$\Delta\$_G$  is the released generation capacity beyond maximum generation capacity at original power factor, kVA

$\Delta\$_T$  is the released transmission capacity beyond maximum transmission capacity at original power factor, kVA

$\Delta\$_S$  is the released distribution substation capacity beyond maximum substation capacity at original power factor, kVA

$\Delta\$_F$  is the annual benefits due to released feeder capacity, \$/year

Thank You...

# **ECONOMIC JUSTIFICATION OF CAPACITORS**



# Overview

2

- ❖ Benefits due to Released Generation Capacity
- ❖ Benefits due to Released Transmission Capacity
- ❖ Benefits due to Released Distribution Substation Capacity
- ❖ Benefits due to Reduced Energy Losses
- ❖ Benefits due to Reduced Voltage Drops
- ❖ Benefits due to Released Feeder Capacity
- ❖ Financial Benefits due to Voltage Improvement
- ❖ Total Financial Benefits due to Capacitor Installations

# Economic Justification of capacitors- Intro

3

- Loads on electric utility systems include two components
  - active power - generated at power plants
  - reactive power - provided by either power plants or capacitors
- shunt power capacitors - the most economical source to meet the reactive power requirements of inductive loads and transmission lines operating at a lagging power factor
- When reactive power is provided only by power plants, size of the system component increases

- Capacitors reduces
  - reactive power demand
  - Line currents
  - losses and loadings are reduced in distribution lines, substation transformers, and transmission lines
- Depending upon the uncorrected power factor of the system, the installation of capacitors can
  - **increase generator and substation capability** for additional load at least 30%
  - increase individual circuit capability,
  - the voltage regulation also increases , approximately 30%–100%.

- the economic benefits force capacitor banks to be **installed on the primary distribution system** rather than on the secondary.
- the economic benefits derived from **capacitor installation** can be summarized as
  - Released generation capacity
  - Released transmission capacity
  - Released distribution substation capacity
  - Additional advantages in distribution system
    - a. Reduced energy (copper) losses
    - b. Reduced voltage drop and consequently improved voltage regulation
    - c. Released capacity of feeder and associated apparatus
    - d. Elimination of capital expenditure due to system improvements and/or expansions
    - e. Revenue increase due to voltage improvements

# Benefits due to Released Generation Capacity

6

- The released generation capacity due to the installation of capacitors can be calculated approximately from.

$$\Delta S_G = \begin{cases} \left[ \left( 1 - \frac{Q_c \times \cos^2 \theta}{S_G} \right)^{1/2} + \frac{Q_c \times \sin \theta}{S_G} - 1 \right] S_G & \text{when } Q_c > 0.10 S_G \\ Q_c \times \sin \theta & \text{when } Q_c \leq 0.10 S_G \end{cases}$$

- $\Delta S_G$  is the released generation capacity beyond maximum generation capacity at original power factor, kVA
- $S_G$  is the generation capacity, kVA
- $Q_c$  is the reactive power due to corrective capacitors applied, kvar
- $\cos \theta$  is the original (or uncorrected or old) power factor before application of capacitors

- Therefore, the annual benefits due to the released generation capacity can be expressed as

$$\Delta S_G = \Delta S_G \times C_G \times i_G$$

- $\Delta S_G$  is the annual benefits due to released generation capacity, \$/year
- $\Delta S_G$  is the released generation capacity beyond maximum generation capacity at original power factor, kVA
- $C_G$  is the cost of (peaking) generation, \$/kW
- $i_G$  is the annual fixed charge rate applicable to generation

# Benefits due to Released Transmission Capacity

8

- The released transmission capacity due to the installation of capacitors can be calculated approximately as

$$\Delta S_T = \begin{cases} \left[ \left( 1 - \frac{Q_c \times \cos^2 \theta}{S_T} \right)^{1/2} + \frac{Q_c \times \sin \theta}{S_T} - 1 \right] S_T & \text{when } Q_c > 0.10 S_T \\ Q_c \times \sin \theta & \text{when } Q_c \leq 0.10 S_T \end{cases}$$

- $\Delta S_T$  is the released transmission capacity beyond maximum transmission capacity at original power factor, kVA
- $S_T$  is the transmission capacity, kVA

The annual benefits due to the released transmission capacity can be found as

$\Delta S_T$  is the annual benefits due to released transmission capacity, \$/year

$$\Delta S_T = \Delta S_T \times C_T \times i_T$$

$\Delta S_T$  is the released transmission capacity beyond maximum transmission capacity at original power factor, kVA

$C_T$  is the cost of transmission line and associated apparatus, \$/kVA

$i_T$  is the annual fixed charge rate applicable to transmission

# Benefits due to Released Distribution Substation Capacity

9

- The released distribution substation capacity due to the installation of capacitors can be found approximately from

$$\Delta S_s = \begin{cases} \left[ \left( 1 - \frac{Q_c^2 \times \cos^2 \theta}{S_s^2} \right)^{1/2} + \frac{Q_c \times \sin \theta}{S_s} - 1 \right] S_s & \text{when } Q_c > 0.10 S_s \\ Q_c \times \sin \theta & \text{when } Q_c \leq 0.10 S_s \end{cases}$$

$\Delta S_s$  is the released distribution substation capacity beyond maximum substation capacity at original power factor, kVA

$S_s$  is the distribution substation capacity, kVA

The annual benefits due to the released substation capacity can be calculated as

$\Delta S_s$  is the annual benefits due to the released substation capacity, \$/year

$\Delta S_s$  is the released substation capacity, kVA

$C_s$  is the cost of substation and associated apparatus, \$/kVA

$i_s$  is the annual fixed charge rate applicable to substation

$$\Delta S_s = \Delta S_s \times C_s \times i_s$$



# Benefits due to Reduced Energy Losses

10

- The annual energy losses are reduced as a result of decreasing copper losses due to the installation of capacitors. The conserved energy can be expressed as

$$\Delta ACE = \frac{Q_{c,3\phi} R (2 S_{L,3\phi} \sin \theta - Q_{c,3\phi}) 8760}{1000 \times V_{L-L}^2}$$

$\Delta ACE$  is the annual conserved energy, kWh/year

$Q_{c,3\phi}$  is the three-phase reactive power due to corrective capacitors applied, kvar

$R$  is the total line resistance to load center,  $\Omega$

$Q_{L,3\phi}$  is the original, that is, uncorrected, three-phase load, kVA

$\sin \theta$  is the sine of original (uncorrected) power factor angle

$V_{L-L}$  is the line-to-line voltage, kV

The annual benefits due to the conserved energy can be calculated as

$\Delta ACE$  is the annual benefits due to conserved energy, \$/year  $\Delta \$_{ACE} = \Delta ACE \times EC$

$EC$  is the cost of energy, \$/kWh

# Benefits due to Reduced Voltage Drops

The following advantages can be obtained by the installation of capacitors into a circuit

1. The effective line current is reduced, and consequently, both IR and  $IX_L$  voltage drops are decreased, which results in improved voltage regulation.
2. The power factor improvement further decreases the effect of reactive line voltage drop.

The percent voltage drop that occurs in a circuit can be expressed as

%VD is the percent voltage drop

$S_{L,3\phi}$  is the three-phase load, kVA

r is the line resistance, Ohms/m

x is the line reactance, Ohms/ m

l is the length of conductors, m

$V_{L-L}$  is the line-to-line voltage, kV

$$\%VD = \frac{S_{L,3\phi}(r \cos \theta + x \sin \theta)l}{10 \times V_{L-L}^2}$$

- After the application of the capacitors, the system voltage rise due to the improved power factor and the reduced effective line current
- the voltage drops due to IR and IXL are minimized.
- The approximate value of the percent voltage rise along the line can be calculated as

$$\%VR = \frac{Q_{c,3\phi} \times X \times l}{10 \times V_{L-L}^2}$$

Voltage-rise phenomenon through every transformer from the generating source to the capacitors occurs due to the application of capacitors

It is independent of load and power factor of the line and can be expressed as

$\%VR_T$  is the percent voltage rise through the transformer  $\%VR_T = \left( \frac{Q_{c,3\phi}}{S_{T,3\phi}} \right) \cdot x_T$

$S_{T,3\phi}$  is the total three-phase transformer rating, kVA

$x_T$  is the percent transformer reactance

# Benefits due to Released Feeder Capacity

13

- The feeder capacity is restricted by allowable voltage drop rather than by thermal limitations.
- The installation of capacitors decreases the voltage drop and consequently increases the feeder capacity
- The feeder capacity can be calculated as 
$$\Delta S_F = \frac{(Q_{cap})x}{x \sin \theta + r \cos \theta} \text{ kVA}$$

The annual benefits due to the released feeder capacity can be calculated as

$\Delta S_F$  is the annual benefits due to released feeder capacity, \$/year 
$$\Delta S_F = \Delta S_F \times C_F \times i_F$$

$\Delta S_F$  is the released feeder capacity, kVA

$C_F$  is the cost of installed feeder, \$/kVA

$i_F$  is the annual fixed charge rate applicable to the feeder

# Financial Benefits due to Voltage Improvement

14

- The revenues to the utility are increased as a result of increased kilowatthour energy consumption due to the voltage rise produced on a system by the addition of the corrective capacitor banks
- The increase in revenues due to the increased kilowatthour energy consumption can be calculated as

$$\Delta S_{\text{BEC}} = \Delta \text{BEC} \times \text{BEC} \times \text{EC}$$

$\Delta S_{\text{BEC}}$  is the additional annual revenue due to increased kWh energy consumption, \$/year

$\Delta \text{BEC}$  is the additional kWh energy consumption increase

$\text{BEC}$  is the original (or base) annual kWh energy consumption, kWh/year

## Total Financial Benefits due to Capacitor Installations

15

- The total benefits due to the installation of capacitor banks can be summarized as

$$\begin{aligned}\sum \Delta S &= (\text{Demand reduction}) + (\text{Energy reduction}) + (\text{Revenue increase}) \\ &= (\Delta S_G + \Delta S_T + \Delta S_S + \Delta S_F) + \Delta S_{ACE} + \Delta S_{BEC}\end{aligned}$$

The total cost of the installed capacitor banks can be found from

$$\Delta EIC_c = \Delta Q_c \times IC_c \times i_c$$

$\Delta EIC_c$  is the annual equivalent of the total cost of installed capacitor banks, \$/year

$\Delta Q_c$  is the required amount of capacitor-bank additions, kvar

$IC_c$  is the cost of installed capacitor banks, \$/kvar

$i_c$  is the annual fixed charge rate applicable to capacitors

# Thank You

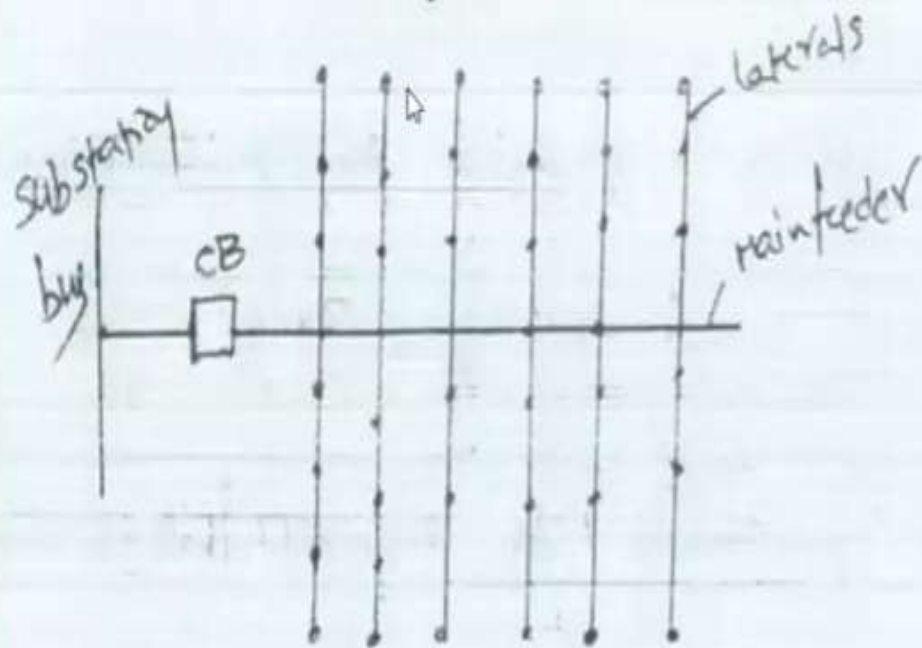
17

# VOLTAGE DROP & POWER LOSSE FOR RADIAL FEEDER WITH UNIFORMLY DISTRIBUTED LOADS



Radial feeder with uniformly distributed load :-

" voltage drop & power loss calculation "



• → Loads & distribution on T/f's

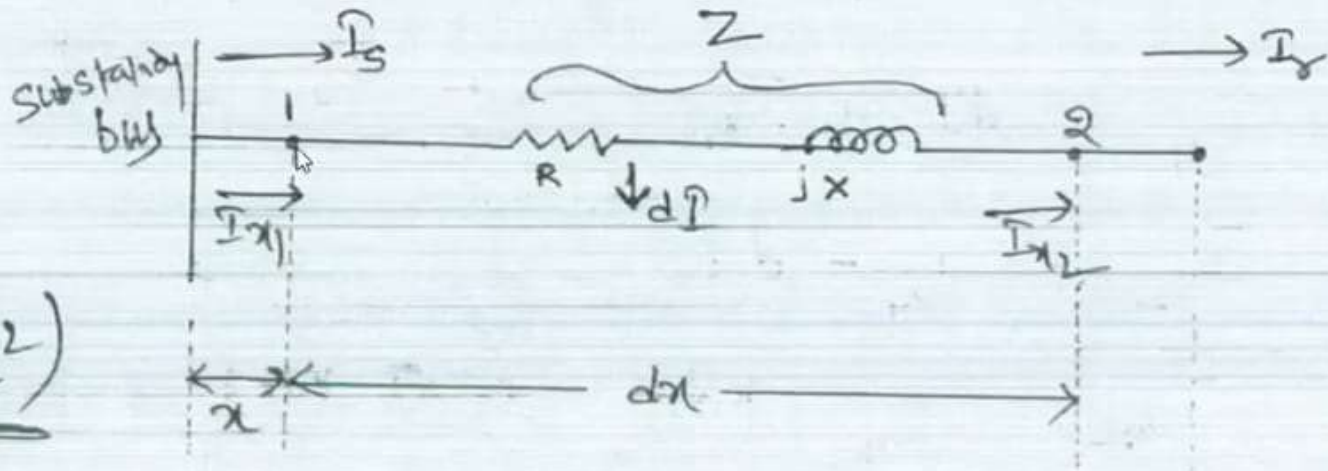
Fig(1)

→ Fig(1) consists of substation bus & Main feeder. The main feeder having branches which are called as laterals.

→ These laterals have "uniformly distributed loads" which are indicated by dots.

↳ "loads are distributed uniformly in all directions"

→ For mathematical calculations, fig(1) is represented as below fig(2)



fig(2)



→ The feeder having an impedance " $Z$ " and total length of feeder is " $l$ " ( $R + jX$ )

→ since load is uniformly distributed from  $x=0$  to  $x=l$ , the change of current with respect to distance is constant.

$$\frac{dI}{dx} = K$$

$$\Rightarrow dI = K dx$$

Consider

from fig(2), according to KCL, @ Point 1, entering = leaving  
currents current

$$I_{x_1} = dI + I_{x_2}$$

$$I_{x_2} = I_{x_1} - dI$$

$$\Rightarrow I_{x_2} = I_{x_1} - Kdx \quad (\because dI = Kdx)$$

For the total feeder, assume

$$I_s = I_r + Kl$$

[ sending end current =  
receiving end current  
+ some constant value  
w.r. to distance

①

when there is no distance of  $l$  i.e. when  $x = l$

the receiving end current also assumed to be zero.

i.e. when  $x = l$ ,  $I_x = 0$

Applying above conditions to eq (1)

$$\textcircled{1} \Rightarrow I_s = Kl$$

$$K = \frac{I_s}{l}$$

(if there is no length  $l$ ; there is no receiving end current)

for getting  $K$  value we are assuming like this

From equation (1)  $\Rightarrow I_x = I_s - kx \Rightarrow$

This <sup>is the</sup> equation for  
(total feeder) a total distance  
of "l"

For a particular distance of "x"

$\rightarrow$  equation can be written as  $I_x = I_s - kx$

substitute "k" value

$$I_x = I_S - \left(\frac{I_S}{l}\right)x$$

$$I_x = I_S \left(1 - \frac{x}{l}\right)$$

consider change in voltage is product of current & impedance w.r.to change in distance.

$$V = Z \int_{x=0}^l I_x dx$$

substitute " $I_x$ " value  $\rightarrow$

$$= Z \int_{x=0}^l I_S \left(1 - \frac{x}{l}\right) dx$$

$$= \frac{I_S}{S} Z \int_{x=0}^l \left(1 - \frac{x}{l}\right) dx$$



$$= I_S z \left[ \alpha - \frac{\alpha^2}{2l} \right] \Big|_{\alpha=0}^{\alpha}$$

$$= I_S z \left[ 1 - \frac{l}{2l} \right]$$

$$= I_S z \left( \frac{1}{2} \right)$$

$$V = \frac{1}{2} I_S z l$$

The differential power loss can be expressed as

$$dP_{LS} = I_x^2 \delta dx$$

$$P_{LS} = \int_{x=0}^l I_x^2 \delta dx$$

$x=0$

substitute  $I_x$

$$P_{LS} = \int_0^l I_s^2 \left(1 - \frac{x}{l}\right)^2 \delta dx$$

(These power losses considered as copper (variable losses) which varies with distance. No need to consider iron losses (constant).

$$P_{LS} = \frac{I_0^2}{5} \gamma \int_{x=0}^l \left(1 - \frac{x}{l}\right)^2 dx$$

$$= \frac{I_0^2}{5} \gamma \int_{x=0}^l \left(1 - \frac{2x}{l} + \frac{x^2}{l^2}\right) dx$$

$$= \frac{I_0^2}{5} \gamma \left[ x - \frac{2x^2}{2l} + \frac{x^3}{3l^2} \right] \Big|_{x=0}^l$$

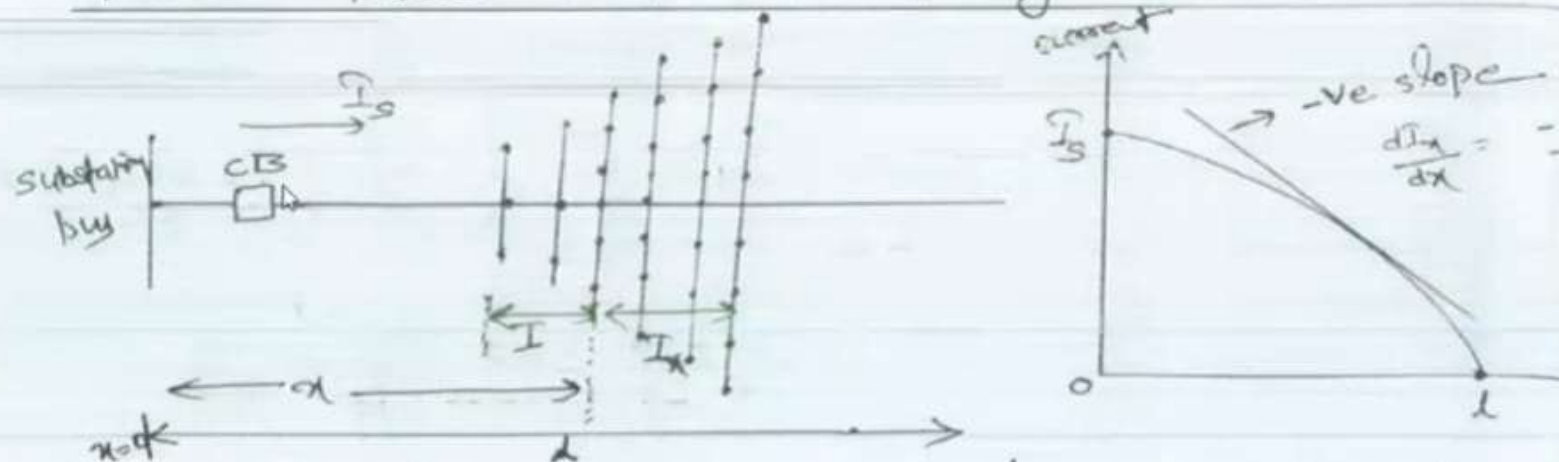
$$= \int_0^l \gamma \left[ l - \frac{2l^2}{2l} + \frac{l^3}{3l^2} \right]$$

$$= \int_0^l \gamma \left( \frac{l}{3} \right)$$

$$P_{CG} = \frac{\int_0^l \gamma l}{3}$$

## Radial feeder with non uniformly distributed load:

Radial feeder with non uniformly distributed load :-



→ Here the load is non uniformly distributed throughout the feeder.

→ Total length of feeder ' $l$ ', upto some point, distance is ' $x$ '.  $I$  &  $I_x$  are the two currents.

→ the current vs distance graph shows that the sending end current ( $I_s$ ) decreases with increasing the distance. so that the slope of curve is negative, which can be written

$$\frac{dI_x}{dx} = -\frac{I_s}{l}$$

(For calculation purpose, assume  $1/l = kx$ )

$$\therefore \frac{dI_x}{dx} = -I_S (kx)$$

$$dI_x = -k I_S x dx$$

For the entire feeder with  
total distance "l",  $I_S$  can be  
written as

$$I_S = \int_{x=0}^l dI_x$$

$$I_S = \int_{x=0}^l -k I_S x dx$$

$$I_S = -k I_S \frac{x^2}{2} \Big|_0^l$$

For some distance "x";

(upto 'x' distance the current is I  
from figure)

the current I can be written as

$$I = \int_0^x dI_x$$

$$I = \int_0^x -k I_S x dx$$

$$I = -k I_S \frac{x^2}{2} \Big|_0^x$$

$$I_S = -k I_S \frac{x^2}{2}$$

$$\Rightarrow k = -2/x^2$$

$$I = -k I_S \frac{x^2}{2}$$

substitute "k" value

$$I = \left(\frac{2}{x^2}\right) I_S \frac{x^2}{2}$$

$$I = \frac{I_S x^2}{x^2}$$



From figure  
relation b/w currents  
acc. KCL  $\Rightarrow$

$$I_S = I + I_x$$
$$I_x = I_S - I$$
$$I_x = I_S - \frac{I_S x^2}{1^2}$$

$$I_x = I_S \left( 1 - \frac{x^2}{1^2} \right)$$

we have

$$dV = I_x z dx$$

$$V = \int_{x=0}^l I_x z dx$$

substitute  $I_x$

$$V = \int_{x=0}^l I_S \left(1 - \frac{x^2}{l^2}\right) z dx$$

Power loss

$$dP_{LS} = I_x^2 r dx$$

$$P_{LS} = \int_{x=0}^l I_x^2 r dx$$

substitute  $I_x$

$$P_{LS} = \int_{x=0}^l \left[ I_S \left(1 - \frac{x^2}{l^2}\right) \right]^2 r dx$$

$$V = I_S Z \left[ \lambda - \frac{\lambda^3}{3l^2} \right] \Big|_{\lambda=0}^l$$

$$V = I_S Z \left( l - \frac{l^3}{3l^2} \right)$$

$$V = I_S Z \left( \frac{2l}{3} \right)$$

$$V = \frac{2}{3} I_S Z l$$

$$P_{LS} = \delta I_S^2 \int_{\lambda=0}^l \left( 1 - \frac{2\lambda^2}{l^2} + \frac{\lambda}{l} \right)$$

$$P_{LS} = \delta I_S^2 \left[ \lambda - \frac{2\lambda^3}{3l^2} + \frac{\lambda^2}{2l} \right]$$

$$P_{LS} = \delta I_S^2 \left( \frac{8l}{15} \right)$$

$$P_{LS} = \frac{8}{15} I_S^2 \delta l$$