

# Power System Analysis (20A02601T)

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# SYLLABUS

## Short circuit Analysis

- Per unit system of Representation - Per unit equivalent Reactance n/w of Three phase power system, numerical problems.
- Symmetrical fault analysis, short ckt current & MVA calculations, fault levels, Application of Series reactors, numerical problems
- Symmetrical Component Theory, Symmetrical component transformation, positive, negative and zero sequence components, voltage currents and impedances.
- Sequence n/w's, positive, negative and zero sequence n/w's, numerical problems
- Unsymmetrical fault analysis, LG, LL, LLG faults with and without fault impedance, numerical problems.

# Unsymmetrical Faults

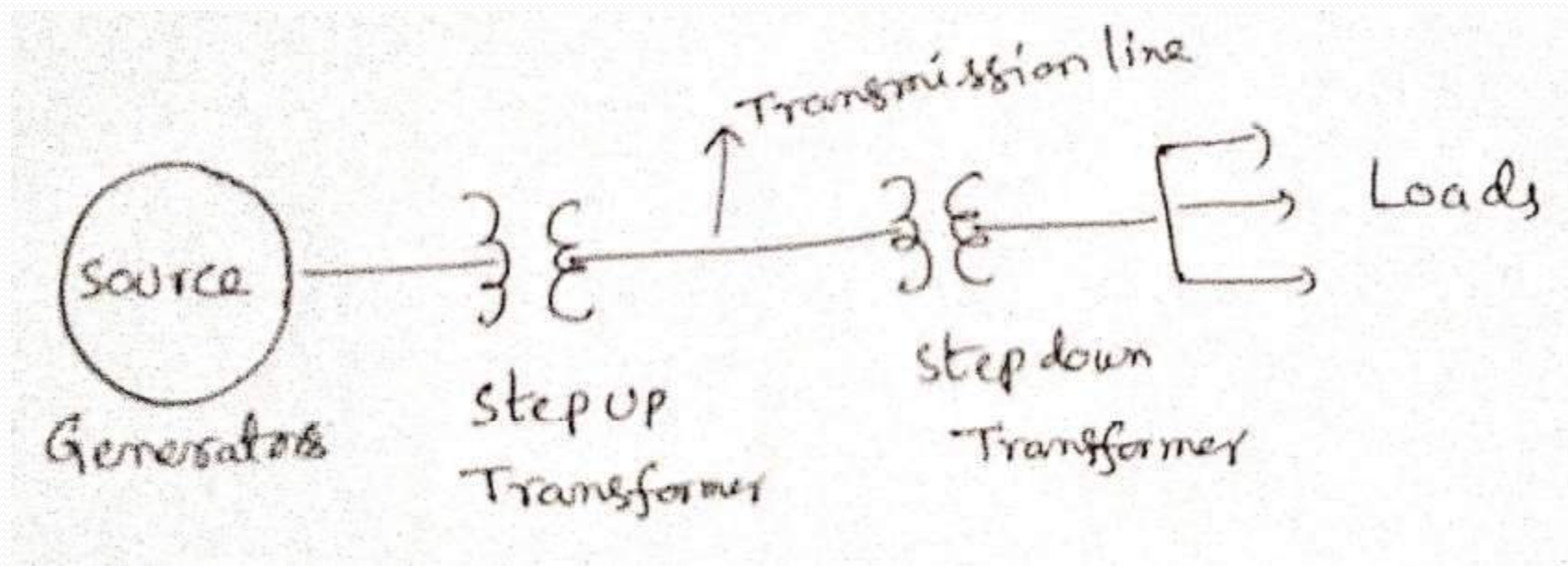
## Introduction

Now a days Electrical power system is growing in size and complexity in all sectors such as generation, transmission, distribution and load systems.

If any fault occurs in power system network which results in severe economic losses and reduces the reliability of the electrical system.

# Unsymmetrical Faults

Introduction ...



Under normal or safe operating conditions, the Electric equipments in a power system network operates at normal voltage and current ratings

- Once the Fault occurs in a power system, voltage and current values deviates from their nominal ranges.
- A Fault in an electric power system can be defined as , any abnormal condition of the system that involves the electrical failure of the equipment, such as , transformers, generators, bus bars, etc.

### General causes of Power System Faults:

- The causes of faults are numerous, e.g. 1. Lightning 2. Heavy winds 3. Trees falling across lines 4. Vehicles colliding with towers or poles 5. Birds shorting lines 6. Aircraft colliding with lines 7. Vandalism (Intentionally damaging property of other people 8. Small animals entering switchgear 9. Line breaks due to excessive loading.

### Additional Causes of Electrical Faults

- **Weather conditions:** It includes lightning strikes, heavy rains, heavy winds, salt deposition on overhead lines and conductors, snow and ice accumulation on transmission lines, etc. These environmental conditions interrupt the power supply and also damage electrical installations.
- **Equipment failures:** Various electrical equipments like generators, motors, transformers, reactors, switching devices, etc causes short circuit faults due to malfunctioning, ageing, insulation failure of cables and winding. These failures result in high current to flow through the devices or equipment which further damages it.
- **Human errors:** Electrical faults are also caused due to human errors such as selecting improper rating of equipment or devices, forgetting metallic or electrical conducting parts after servicing or maintenance, switching the circuit while it is under servicing, etc.
- **Smoke of fires:** Ionization of air, due to smoke particles, surrounding the overhead lines results in spark between the lines or between conductors to insulator. This flashover causes insulators to lose their insulating capacity due to high voltage.

### Effects of Power System Faults:

- **Over current flow:** When fault occurs it creates a very low impedance path for the current flow. This results in a very high current being drawn from the supply, causing tripping of relays, damaging insulation and components of the equipments.
- **Danger to operating personnel:** Fault occurrence can also cause shocks to individuals. Severity of the shock depends on the current and voltage at fault location and even may lead to death.
- **Loss of equipment:** Heavy current due to short circuit faults result in the components being burnt completely which leads to improper working of equipment or device. Sometimes heavy fire causes complete burnout of the equipments.
- **Disturbs interconnected active circuits:** Faults not only affect the location at which they occur but also disturbs the active interconnected circuits to the faulted line.
- **Electrical fires:** Short circuit causes flashovers and sparks due to the ionization of air between two conducting paths which further leads to fire as we often observe in news such as building and shopping complex fires.

# Unsymmetrical Faults

Introduction ...





# Unsymmetrical Faults

# TYPES

## TYPES OF FAULTS:

• Electrical faults in three-phase power system mainly classified into two types

1. Open circuit faults
2. Short circuit faults

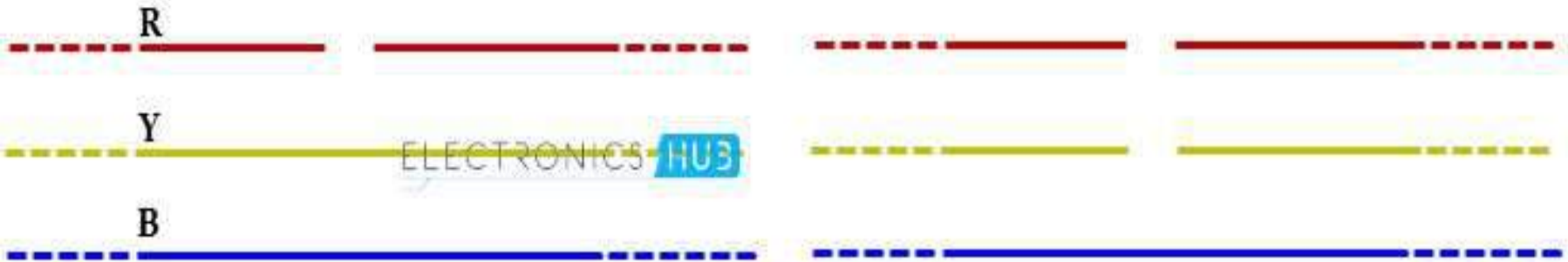
### 1. Open circuit faults:

- Open circuit faults are also called as series faults
- These faults occur due to the failure of one or more conductors.
- The figure shows the open circuit faults for single, two and three phases (or conductors) open condition.

# Unsymmetrical Faults

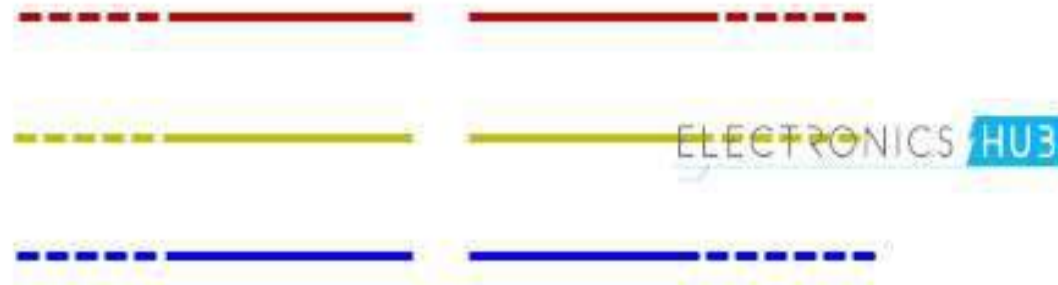
## Open circuit faults

### Open-circuit Faults



(a). Single-phase open-circuit

(a). Two-phase open-circuit



(a). Three-phase open-circuit

# Unsymmetrical Faults

Open circuit faults

## Effects:

- Abnormal operation of the system
- Danger to the personnel as well as animals
- Exceeding the voltages beyond normal values in certain parts of the network, which further leads to insulation failures and developing of short circuit faults.

### 2. Short Circuit Faults:

- A short circuit can be defined as an abnormal connection of very low impedance between two points of different potential, whether made intentionally or accidentally.
- Short circuit faults are also called as shunt faults.
- These are the most common and severe kind of faults, resulting in the flow of abnormal high currents through the equipment or transmission lines.
- If these faults are allowed to persist even for a short period, it leads to the extensive damage to the equipment

### Effects of Short Circuit Faults:

-These faults are caused due to the insulation failure between phase conductors or between earth and phase conductors or both.

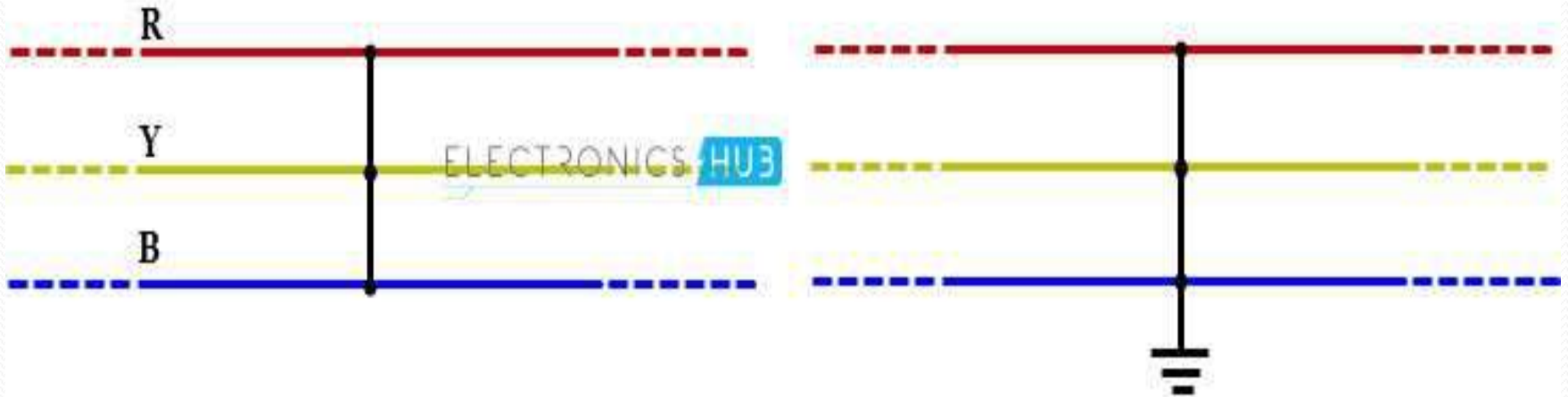
### -Types of Short Circuit Faults:

- The various possible short circuit fault conditions include three phase to earth, three phase clear of earth, phase to phase, single phase to earth, two phase to earth and phase to phase plus single phase to earth

# Unsymmetrical Faults

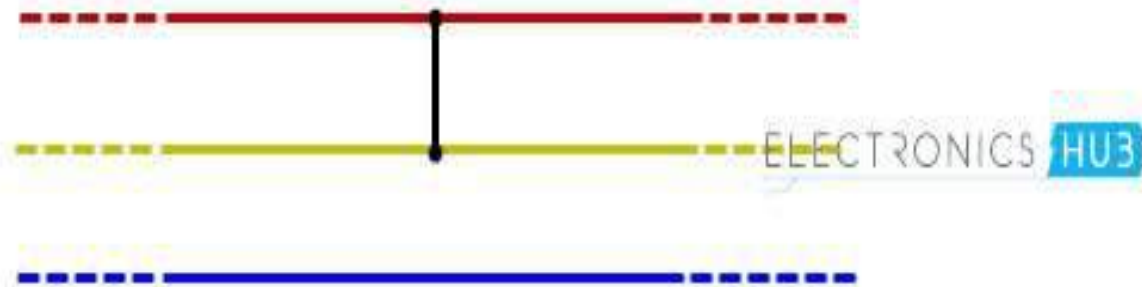
## Short Circuit Faults...

### Short-circuit Faults



(a). Three-phase clear of earth

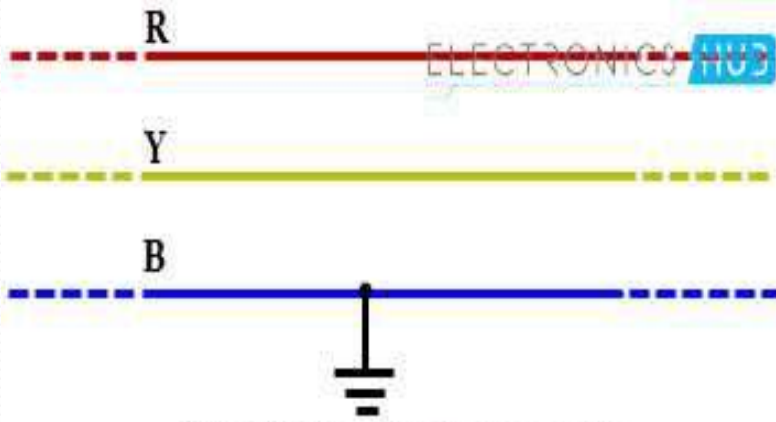
(b). Three-phase-to-earth



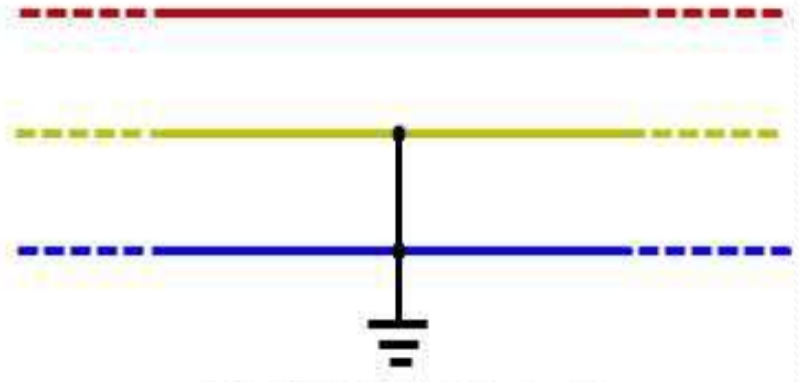
(c). Phase-to-phase

# Unsymmetrical Faults

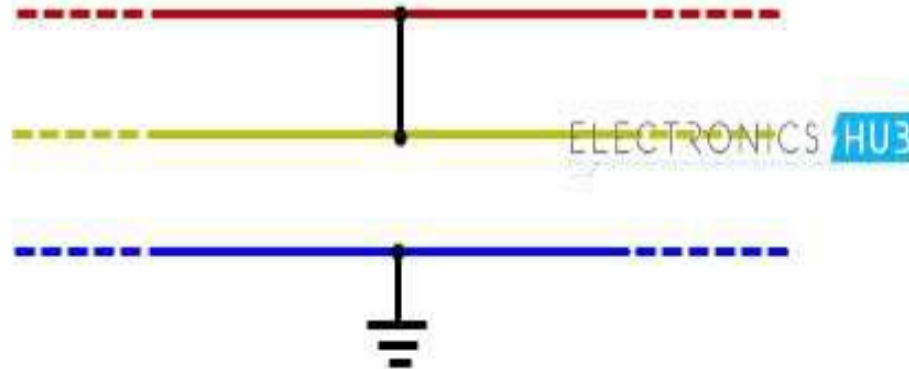
## Short Circuit Faults...



(d). Single-phase-to-earth



(e). Two-phase-to-earth



(f). Phase-to-phase plus  
single-phase-to-earth

### **Causes**

These may be due to internal or external effects

Internal effects include breakdown of transmission lines or equipment, aging of insulation, deterioration of insulation in generator, transformer and other electrical equipments, improper installations and inadequate design.

External effects include overloading of equipments, insulation failure due to lightning surges and mechanical damage by public.



### Effects

Arcing faults can lead to fire and explosion in equipments such as transformers and circuit breakers.

Abnormal currents cause the equipments to get overheated, which further leads to reduction of life span of their insulation.

The operating voltages of the system can go below or above their acceptance values that creates harmful effect to the service rendered by the power system.

The power flow is severely restricted or even completely blocked as long as the short circuit fault persists.

### Types of Shunt Faults:

Shunt Faults are classified into two types

#### 1. Unsymmetrical faults.

A) Three phase fault(LLL)

B) Three phase to ground fault (LLLG)

#### 2. Symmetrical faults.

A) LG

B) LL

C) LLG

### Types of Shunt Faults:

Shunt Faults are classified into two types

#### 1. Symmetrical faults:

- These are very severe faults and occur infrequently in the power systems. These are also called as balanced faults.
- Only 2-5 percent of system faults are symmetrical faults. If these faults occur, system remains balanced but results in severe damage to the electrical power system equipments.
- These are two types

A) Three phase fault(LLL)

B) Three phase to ground fault (LLLG)

### 2. Unsymmetrical faults:

- These are very common and less severe than symmetrical faults.
- These are also called unbalanced faults since their occurrence causes unbalance in the system. Unbalance of the system means that that impedance values are different in each phase causing unbalance current to flow in the phases.
- These are three types
  - A) LG
  - B) LL
  - C) LLG

### Unsymmetrical faults:

- In the analysis of Unsymmetrical faults, the following points are very important.
- 1. The Generated EMF is of positive sequence only
- 2. No current flow in the network other than due to fault
- 3. Phase R shall be taken as reference phase.
- 4. In each case of Unsymmetrical faults, EMF's per phases are denoted by  $E_R$ ,  $E_Y$  and  $E_B$  and the line voltage per phase are  $V_R$ ,  $V_Y$  and  $V_B$ .

### 1. Line to ground (L-G) Fault

- Line to ground fault (L-G) is most common fault and 65-70 percent of faults are of this type.
- These are two types
  - A) LG without fault impedance
  - B) LG with fault impedance



# Unsymmetrical Faults

LG Fault without  $Z_f$ ...

From the boundary conditions ( $I_Y=0$  and  $I_B=0$ )

The sequence currents in 'R' phase in terms of line current

$$\bar{I}_{R0} = \frac{1}{3} (\bar{I}_R + \bar{I}_Y + \bar{I}_B)$$

$$\boxed{\bar{I}_{R0} = \frac{1}{3} (\bar{I}_R)}$$

$$\bar{I}_{R1} = \frac{1}{3} (\bar{I}_R + a\bar{I}_Y + a^2\bar{I}_B)$$

$$\boxed{\bar{I}_{R1} = \frac{1}{3} \bar{I}_R}$$

$$\bar{I}_{R2} = \frac{1}{3} (\bar{I}_R + a^2\bar{I}_Y + a\bar{I}_B)$$

$$\boxed{\bar{I}_{R2} = \frac{1}{3} \bar{I}_R}$$

$$\therefore \boxed{\bar{I}_{R0} = \bar{I}_{R1} = \bar{I}_{R2} = \frac{1}{3} \bar{I}_R} \quad \text{---(1)}$$



# Unsymmetrical Faults

LG Fault without  $Z_f$ ...

$$\text{Fault current } I_f = I_R = 3 I_{R0} \quad \text{---(2)}$$

From the boundary conditions ( $V_R=0$ )

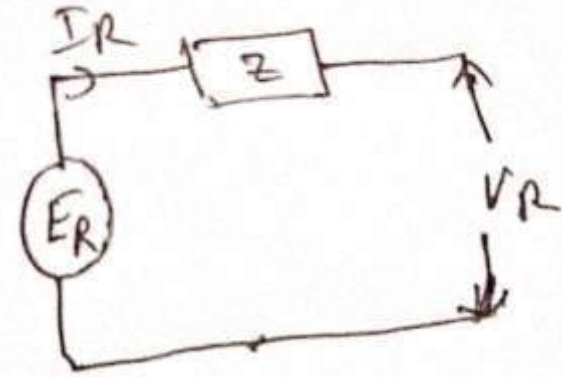
$$E_R = V_R + I_R Z$$

$$E_R = V_R + I_{R1} z_1 + I_{R2} z_2 + I_{R0} z_0$$

we know,  $V_R = 0$   
 $I_{R1} = I_{R2} = I_{R0} = \frac{1}{3} I_R$

$$E_R = \frac{1}{3} I_R (z_1 + z_2 + z_0)$$

Fault current ( $I_R$ ) or  $I_f = \frac{3 \bar{E}_R}{z_1 + z_2 + z_0} \quad \text{---(3)}$



# Unsymmetrical Faults

LG Fault with  $Z_f$ ...

## 2. Line to ground (L-G) Fault with Fault Impedance ( $Z_f$ )

Boundary conditions

$$I_Y = 0$$

$$I_B = 0$$

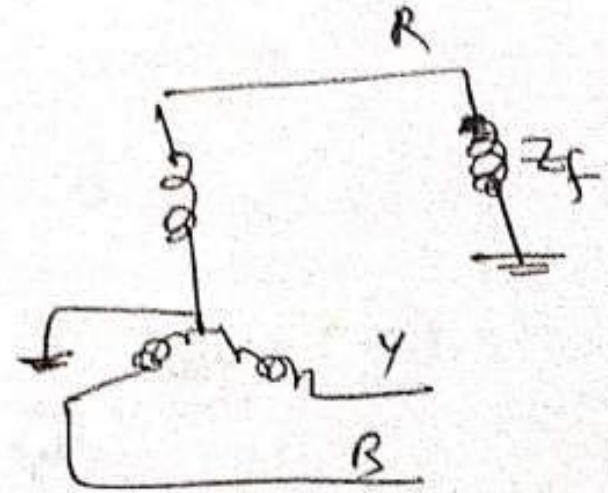
$$V_R = I_R Z_f$$

Sequence n/w equations

$$V_{R1} = \bar{E}_R - \bar{I}_1 z_1$$

$$V_{R2} = -\bar{I}_2 z_2$$

$$V_{R0} = -\bar{I}_0 z_0$$



# Unsymmetrical Faults

LG Fault with  $Z_f$ ...

From the boundary conditions ( $I_Y=0$  and  $I_B=0$ )

The sequence currents in 'R' phase in terms of line current

$$\bar{I}_{R0} = \frac{1}{3} (\bar{I}_R + \bar{I}_Y + \bar{I}_B)$$

$$\boxed{\bar{I}_{R0} = \frac{1}{3} (\bar{I}_R)}$$

$$\bar{I}_{R1} = \frac{1}{3} (\bar{I}_R + a\bar{I}_Y + a^2\bar{I}_B)$$

$$\boxed{\bar{I}_{R1} = \frac{1}{3} \bar{I}_R}$$

$$\bar{I}_{R2} = \frac{1}{3} (\bar{I}_R + a^2\bar{I}_Y + a\bar{I}_B)$$

$$\boxed{\bar{I}_{R2} = \frac{1}{3} \bar{I}_R}$$

$$\therefore \boxed{\bar{I}_{R0} = \bar{I}_{R1} = \bar{I}_{R2} = \frac{1}{3} \bar{I}_R} \quad \text{---(1)}$$

# Unsymmetrical Faults

LG Fault with  $Z_f$ ...

$$\text{Fault current } I_f = I_R = 3 I_{R_0} \quad \text{---(2)}$$

From the boundary conditions ( $V_R = I_R \cdot Z_f$ )

$$V_R = I_R Z_f$$

$$V_R = 3 I_{R_0} Z_f$$

$$V_{R_1} + V_{R_2} + V_{R_0} = 3 I_{R_0} Z_f$$

$$E_{R_1} - I_{R_1} z_1 - I_{R_2} z_2 - I_{R_0} z_0 = 3 I_{R_0} Z_f$$

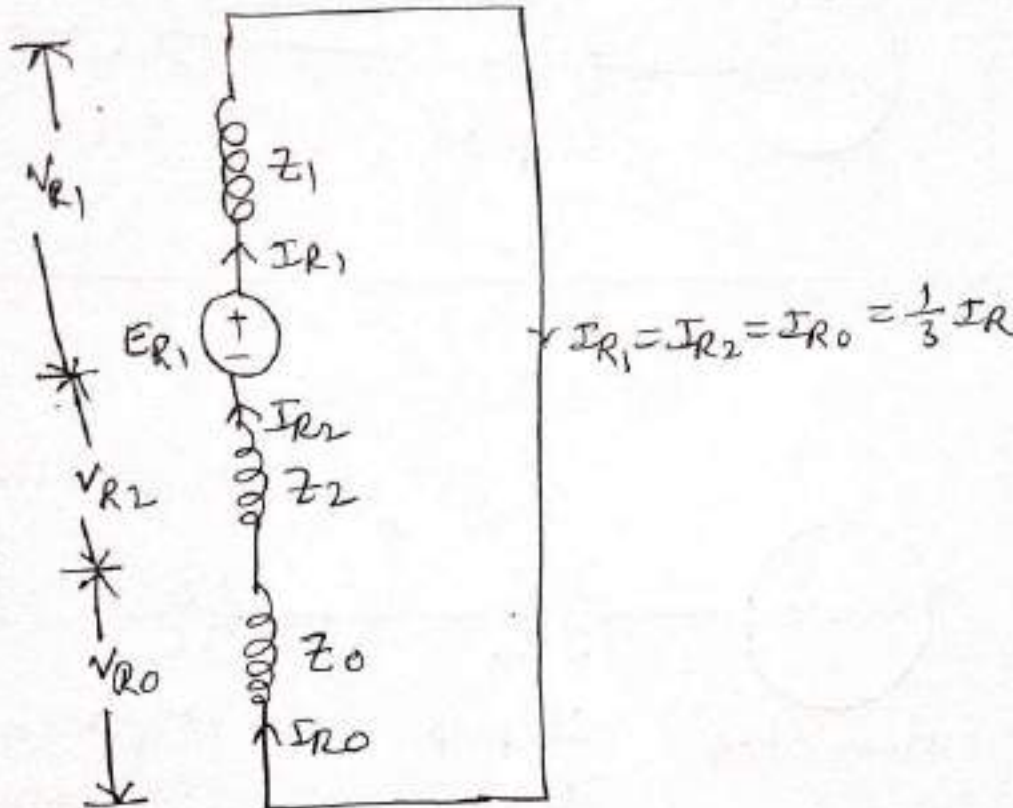
$$E_{R_1} = I_{R_1} z_1 + I_{R_2} z_2 + I_{R_0} (z_0 + 3 Z_f)$$

$$= \frac{1}{3} I_R (z_1 + z_2 + z_0 + 3 Z_f)$$

# Unsymmetrical Faults

LG Fault with  $Z_f$ ...

$$I_R = I_f = \frac{3 E_{R1}}{z_1 + z_2 + z_0 + 3 Z_f}$$



# Unsymmetrical Faults

LL Fault without Zf...

## 1. Line to Line (L-L) Fault without Fault Impedance(Zf)

Boundary conditions

$$V_Y = V_B$$

$$I_R = 0$$

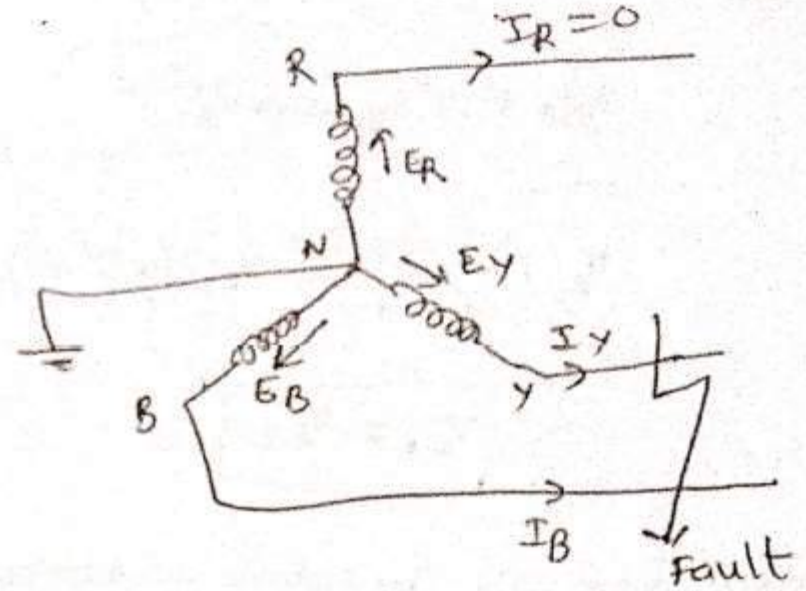
$$I_Y = -I_B \text{ i.e. } I_Y + I_B = 0$$

Sequence n/w equations

$$V_{R1} = \bar{E}_R - \bar{I}_1 z_1$$

$$V_{R2} = -\bar{I}_2 z_2$$

$$V_{R0} = -\bar{I}_0 z_0$$



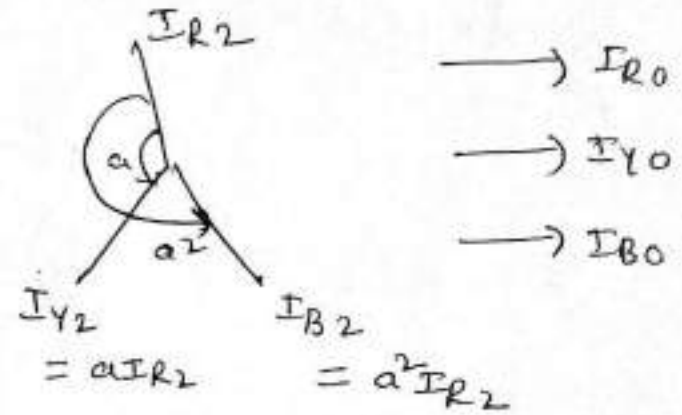
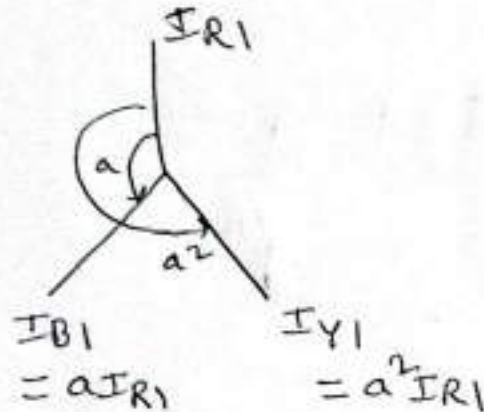
# Unsymmetrical Faults

LL Fault without  $Z_f$ ...

Analysis :-

From the boundary conditions ( $I_R = 0$ ,  $I_Y + I_B = 0$ )

$$I_Y + I_B = 0$$



$$I_{R0} + a^2 I_{R1} + a I_{R2} + I_{R0} + a I_{R1} + a^2 I_{R2} = 0$$

$$2 I_{R0} + I_{R1} (a + a^2) + I_{R2} (a + a^2) = 0$$

$$\text{here } I_{R0} = \frac{1}{3} (I_R + I_Y + I_B)$$

$\downarrow 0$        $\downarrow 0$

$$I_{R0} = 0$$

# Unsymmetrical Faults

LL Fault without  $Z_f$ ...

$$(I_{R1} + I_{R2})(a + a^2) = 0$$

$$I_{R1} + I_{R2} = 0$$

$$\boxed{I_{R1} = -I_{R2}} \longrightarrow (1)$$

From boundary condition ( $V_Y = V_B$ )

$$V_Y = V_B$$

$$V_{R0} + a^2 V_{R1} + a V_{R2} = V_{R0} + a V_{R1} + a^2 V_{R2}$$

$$V_{R1}(a^2 - a) = V_{R2}(a^2 - a)$$

$$\boxed{V_{R1} = V_{R2}} \longleftarrow (2)$$



# Unsymmetrical Faults

LL Fault without  $Z_f$ ...

$$E_{R1} - I_{R1} Z_1 = -I_{R2} Z_2$$

$$\left( \because I_{R2} = -I_{R1} \right)$$

$$E_{R1} = I_{R1} Z_1 + I_{R1} Z_2$$

$\rightarrow$  simplification prob

$$I_{R1} = \frac{E_{R1}}{Z_1 + Z_2}$$

$\rightarrow (3)$

$$0 = 9I$$

$$\text{Fault current } (I_f) = I_y = -I_B$$

$$= I_{R0} + a^2 I_{R1} + a I_{R2}$$

$$= 0 + a^2 I_{R1} - a I_{R2}$$

$$= I_{R1} (a^2 - a)$$

$$= (a^2 - a) \frac{E_{R1}}{Z_1 + Z_2}$$

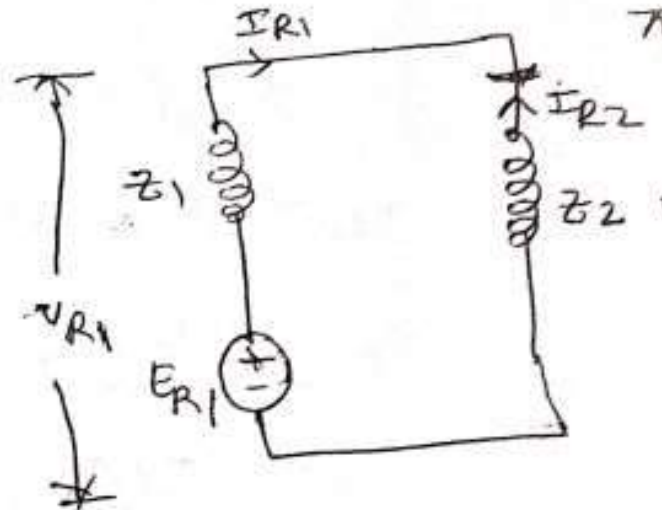
# Unsymmetrical Faults

LL Fault without  $Z_f$ ...

$$I_f = -j\sqrt{3} \frac{E_{R1}}{Z_1 + Z_2} \quad \rightarrow (4)$$

NOTE:-

\* In case of LL fault the sequence currents are  $I_{R0} = 0$   
 $I_{R1} = -I_{R2}$  then the equivalent circuit is



# Unsymmetrical Faults

LL Fault with  $Z_f$ ...

## 1. Line to Line (L-L) Fault with Fault Impedance ( $Z_f$ )

Boundary conditions

$$I_R = 0$$

$$I_Y + I_B = 0$$

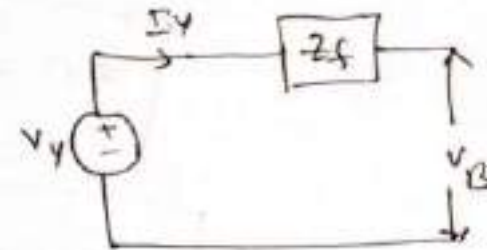
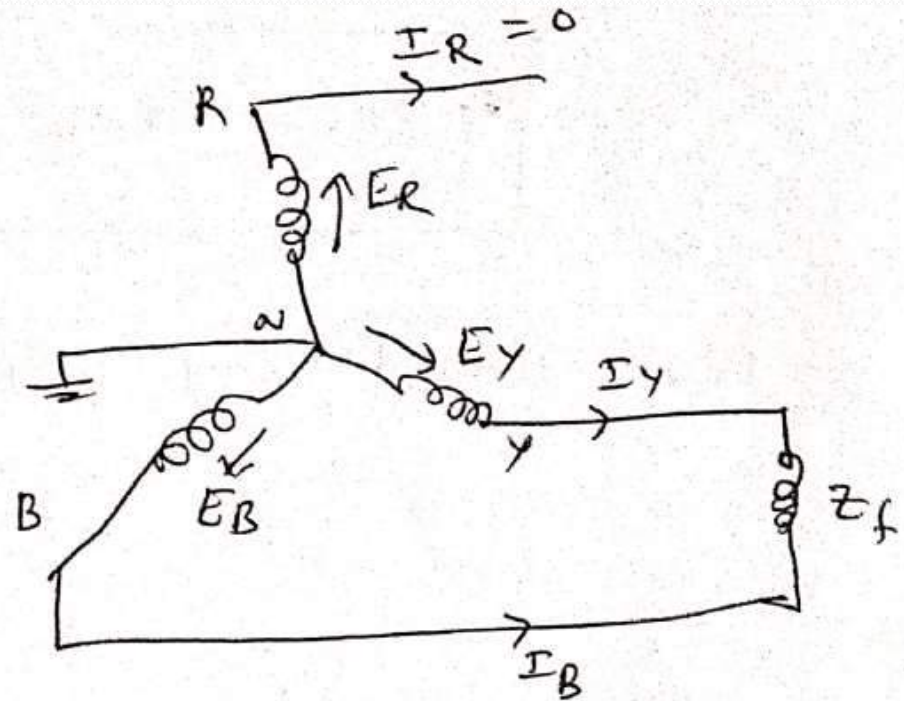
$$V_Y = V_B + I_Y Z_f$$

Sequence n/w equations

$$V_{R1} = \bar{E}_R - \bar{I}_1 z_1$$

$$V_{R2} = -\bar{I}_2 z_2$$

$$V_{R0} = -\bar{I}_0 z_0$$



$$V_Y = V_B + I_Y Z_f$$

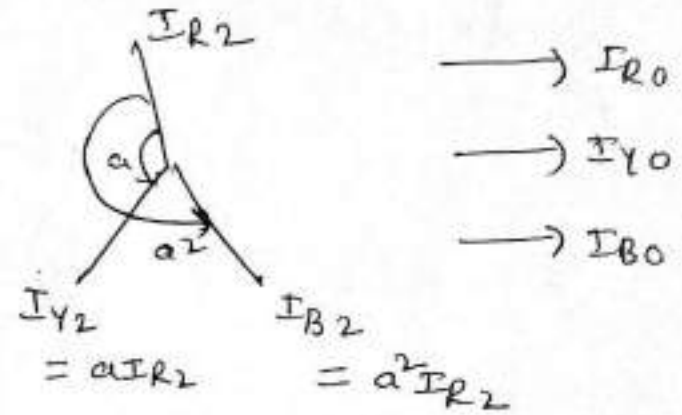
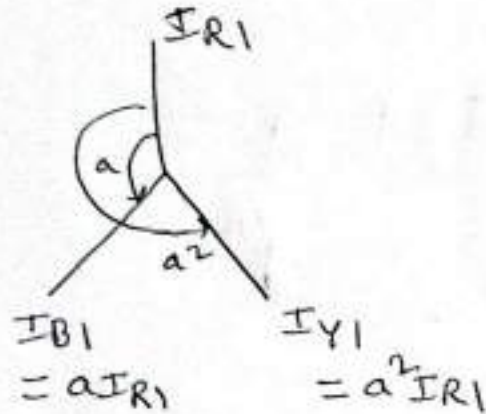
# Unsymmetrical Faults

LL Fault without  $Z_f$ ...

Analysis :-

From the boundary conditions ( $I_R = 0$ ,  $I_Y + I_B = 0$ )

$$I_Y + I_B = 0$$



$$I_{R0} + a^2 I_{R1} + a I_{R2} + I_{R0} + a I_{R1} + a^2 I_{R2} = 0$$

$$2 I_{R0} + I_{R1} (a + a^2) + I_{R2} (a + a^2) = 0$$

here  $I_{R0} = \frac{1}{3} (I_R + I_Y + I_B)$

$$I_{R0} = 0$$

# Unsymmetrical Faults

LL Fault with  $Z_f$ ...

$$(I_{R1} + I_{R2}) (a + a^2) = 0$$

$$I_{R1} + I_{R2} = 0$$

$$\boxed{I_{R1} = -I_{R2}} \longrightarrow (1)$$

From Boundary condition  $V_y = V_B + I_y Z_f$

$$V_{R0} + a^2 V_{R1} + a V_{R2} = V_{R0} + a V_{R1} + a^2 V_{R2} + I_y Z_f$$

$$V_{R1} (a^2 - a) + V_{R2} (a - a^2) = \underbrace{(I_{R0} + a^2 I_{R1} + a I_{R2})}_{I_y} Z_f$$

$$V_{R1} (a^2/a) - V_{R2} (a^2/a) = + I_{R1} Z_f (a^2/a^2)$$

$$V_{R1} = V_{R2} + I_{R1} Z_f$$

# Unsymmetrical Faults

LL Fault with  $Z_f$ ...

Boundary condition  $I_{R2} = 0$

$$E_{R1} - I_{R1} Z_1 = -I_{R2} Z_2 + I_{R1} Z_f$$

$$E_{R1} - I_{R1} Z_1 = I_{R1} Z_f \quad (I_{R2} = 0)$$

$$E_{R1} - I_{R1} Z_1 = I_{R1} Z_2 + I_{R1} Z_f$$

$$E_{R1} = I_{R1} Z_1 + I_{R1} Z_2 + I_{R1} Z_f + V = V$$

Series network

$$E_{R1} = I_{R1} (Z_1 + Z_2 + Z_f)$$

$$I_{R1} = \frac{E_{R1}}{Z_1 + Z_2 + Z_f} \quad (2)$$

Fault current ( $I_f$ ) =  $I_y = -I_B$

$$= I_{R0} + a^2 I_{R1} + a I_{R2}$$

# Unsymmetrical Faults

LL Fault with  $Z_f$ ...

$$= 0 + a^2 I_{R1} + a I_{R1}$$

$$I_f = (a^2 - a) I_{R1}$$

$$I_f = -j\sqrt{3} \times \frac{E_{R1}}{Z_1 + Z_2 + Z_f}$$

# Unsymmetrical Faults

LL-G Fault without Zf...

## 1. Double Line to Ground (LL-G) Fault without Fault Impedance(Zf)

Boundary conditions

$$I_R = 0$$

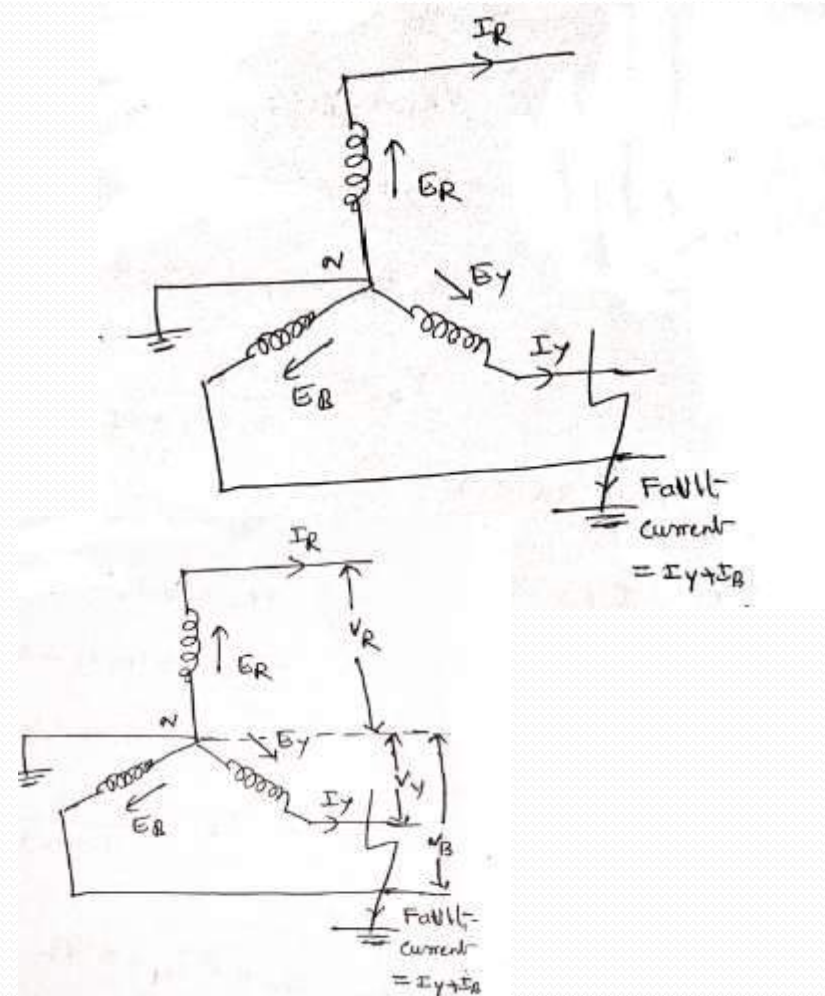
$$V_Y = V_B = 0$$

Sequence n/w equations

$$V_{R1} = \bar{E}_R - \bar{I}_1 z_1$$

$$V_{R2} = -\bar{I}_2 z_2$$

$$V_{R0} = -\bar{I}_0 z_0$$





# Unsymmetrical Faults

LL Fault without Zf...

From boundary condition ( $V_Y = V_B = 0$ )

We know that

$$V_{R0} = \frac{1}{3} (V_R + V_Y + V_B)$$

$$V_{R0} = \frac{1}{3} V_R$$

$$V_{R1} = \frac{1}{3} (V_R + aV_Y + a^2V_B)$$

$$V_{R1} = \frac{1}{3} V_R$$

$$V_{R2} = \frac{1}{3} (V_R + a^2V_Y + aV_B)$$

$$V_{R2} = \frac{1}{3} V_R$$

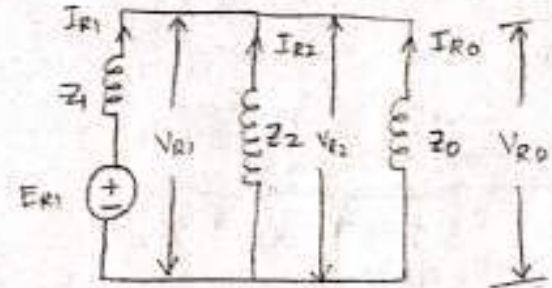
$$\therefore V_{R0} = V_{R1} = V_{R2} = \frac{1}{3} V_R \quad \text{--- (1)}$$

From boundary condition ( $I_R = 0$ )

$$I_R = 0$$

$$I_{R0} + I_{R1} + I_{R2} = 0$$

The equivalent circuit is



From circuit

$$I_{R1} = \frac{E_{R1}}{Z_{eq}}$$

$$I_{R1} = \frac{E_{R1}}{Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}} \quad \text{--- (2)}$$

$$I_{R2} = -I_{R1} \times \frac{Z_0}{Z_0 + Z_2} \quad \text{--- (3)}$$

$$I_{R0} = -I_{R1} \times \frac{Z_2}{Z_0 + Z_2} \quad \text{--- (4)}$$

# Unsymmetrical Faults

LL Fault without  $Z_f$ ...

Fault current

$$I_f = I_y + I_B$$

We know

$$I_{R0} = \frac{1}{3} (I_R + I_y + I_B)$$

$$I_y + I_B = 3 I_{R0}$$

$$\therefore I_f = 3 I_{R0}$$

$$= -3 \times \frac{E_{R1}}{Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}} \times \frac{Z_2}{Z_0 + Z_2}$$

$$= -3 \times \frac{E_{R1} (Z_0 + Z_2)}{Z_1 (Z_0 + Z_2) + Z_0 Z_2} \times \frac{Z_2}{Z_0 + Z_2}$$

$$I_f = \frac{-3 E_{R1} Z_2}{Z_1 (Z_0 + Z_2 + Z_0 Z_2)} \quad \rightarrow (5)$$

# Unsymmetrical Faults

## LL-G Fault with $Z_f$ ...

### 1. Double Line to Ground (LL-G) Fault with Fault Impedance( $Z_f$ )

Boundary conditions

$$I_R = 0$$

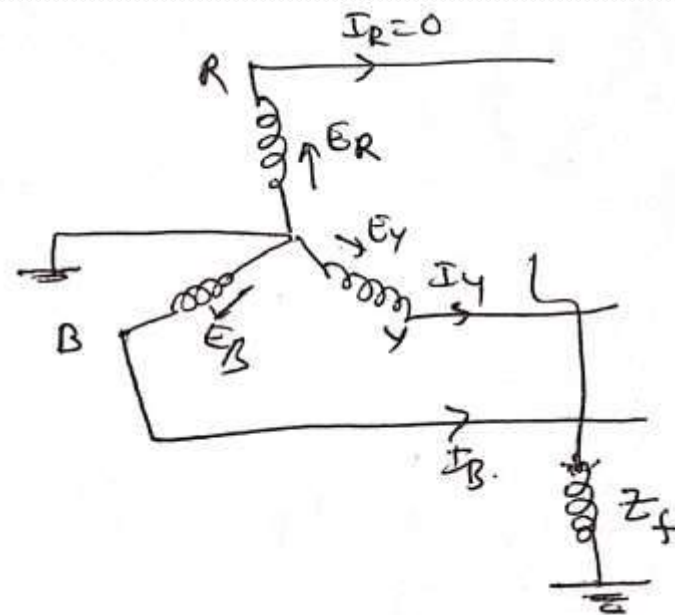
$$V_Y = V_B = (I_Y + I_B) Z_f$$

Sequence n/w equations

$$V_{R1} = \bar{E}_R - \bar{I}_1 z_1$$

$$V_{R2} = -\bar{I}_2 z_2$$

$$V_{R0} = -\bar{I}_0 z_0$$



# Unsymmetrical Faults

## LL-G Fault with $Z_f$ ...

Analysis :-

From the boundary condition of voltage

$$V_y = (I_y + I_B) Z_f$$

where  $I_y + I_B = 3 I_{R0}$

$$V_y = 3 I_{R0} Z_f \quad \text{--- (2)}$$

also  $V_y = V_B$

$$V_{R0} + a^2 V_{R1} + a V_{R2} = V_{R0} + a V_{R1} + a^2 V_{R2}$$

$$V_{R1} (a^2 - a) - V_{R2} (a^2 - a) = 0$$

$$(V_{R1} - V_{R2})(a^2 - a) = 0$$

$$V_{R1} - V_{R2} = 0$$

$$\boxed{V_{R1} = V_{R2}}$$

# Unsymmetrical Faults

## LL-G Fault with $Z_f$ ...

$$E_{R1} - I_{R1} Z_1 = -I_{R2} Z_2$$

$$I_{R2} = \frac{-(E_{R1} - I_{R1} Z_1)}{Z_2} \quad \rightarrow (4)$$

From Eqn (2)

$$V_y = 3 I_{R0} Z_f$$

$$V_{R0} + a^2 V_{R1} + a V_{R2} = 3 I_{R0} Z_f$$

but  $V_{R1} = V_{R2}$

$$V_{R0} + a^2 V_{R1} + a V_{R1} = 3 I_{R0} Z_f$$

$$V_{R0} + (a^2 + a) V_{R1} = 3 I_{R0} Z_f$$

$$V_{R0} - V_{R1} = 3 I_{R0} Z_f \quad \left[ \because a^2 + a = -1 \right]$$

$$V_{R1} = V_{R0} - 3 I_{R0} Z_f$$

$$E_{R1} - I_{R1} Z_1 = -I_{R0} Z_0 - 3 I_{R0} Z_f$$

$$I_{R0} = \frac{E_{R1} - I_{R1} Z_1}{Z_0 + 3 Z_f} \quad \rightarrow (5)$$

# Unsymmetrical Faults

LL-G Fault with  $Z_f$ ...

$$E_{R1} - I_{R1} z_1 = -I_{R0} z_0 - 3 I_{R0} z_f$$

$$I_{R0} = \frac{-(E_{R1} - I_{R1} z_1)}{z_0 + 3 z_f} \rightarrow (5)$$

Fault current

$$I_f = I_y + I_B = 3 I_{R0}$$

$$I_f = I_y + I_B = \frac{-3 (E_{R1} - I_{R1} z_1)}{z_0 + 3 z_f}$$

# Unsymmetrical Faults

LL-G Fault with  $Z_f$ ...


Note :- To calculate  $I_{R1}$

$$I_R = 0$$

$$I_{R0} + I_{R1} + I_{R2} = 0$$

$$-\frac{(E_{R1} - I_{R1}Z_1)}{Z_0 + 3Z_f} + I_{R1} - \frac{(E_{R1} - I_{R1}Z_1)}{Z_2} = 0$$

$$I_{R1} = \frac{E_{R1}}{Z_1 + Z_2} \frac{(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}$$



**PSA**  
**III UNIT**  
**POWER FLOW STUDIES -1**



# SYLLABUS

## **POWER FLOW STUDIES-I**

Necessity of Power Flow Studies – Data for Power Flow Studies – Derivation of Static Load Flow Equations – Load Flow Solutions using Gauss Seidel Method: Acceleration Factor, Load Flow Solution with and without P-V Buses, Algorithm and Flowchart. Numerical Load flow Solution for Simple Power Systems (Max. 3-Buses): Determination of Bus Voltages, Injected Active and Reactive Powers (Sample One Iteration only) and Finding Line Flows/Losses for the given Bus Voltages

# CONTENTS

1. What is Power Flow Studies?
2. Necessity of Power Flow Studies
3. Data for Power flow Studies
4. Classification of Iterative methods
5. Introduction to Gauss-Seidel Method without PV bus
6. Algorithm for Gauss-Seidel Method without PV bus
7. Flowchart for Gauss-Seidel Method without PV bus
8. Introduction to Gauss-Seidel Method with PV bus
9. Algorithm for Gauss-Seidel Method with PV bus
10. Flowchart for Gauss-Seidel Method without PV bus
11. Importance of Acceleration Factor
12. Problems on Gauss-Seidel Method with & without PV bus

# 1. What is Power Flow Studies?

- It is also known as Load flow study .
- Power flow study is defined as the Study or monitoring of power flow in the power system network and to obtain the steady state operation.
- Power flow analysis is the most important and essential approach to investigating the problems in power system operation and planning .
- Load flow study is a bread and butter for any power system engineer or electric energy system engineer.
- In fact, It gives you pulse of the system, what is happening in the system that is given by load flow studies.

# 1. What is Power Flow Studies? Contd..

- It is a numerical analysis of the power flow in an interconnected system.
- It is defined as the Study of set of non linear algebraic equations of the power system network for the purpose of to obtain
  - Magnitude of Voltage (V)
  - Phase angle of voltage ( $\delta$ )
  - Real Power (P) and
  - Reactive Power (Q)

## Linear vs. Non-linear

### Linear Equations

$$x + 2 = 6$$

$$3x - 7 = 23$$

$$10 - x = 11$$

$$x = 11 - 5x$$

### Non-linear Equations

$$x^2 - 3 = 6$$

$$2x^2 + 7 = 18$$

$$x^3 + x = 7$$

$$6x^4 + x^2 = 2x$$

# 1. What is Load Flow Studies?

Contd...

- The various steps involved in power flow study are as follows
1. Modelling of power system network.
  2. Representation of modelled network using non linear algebraic equations.
  3. Solving of non linear algebraic equations using iterative techniques.

## 2.Necessity of Power Flow Studies?

- It helps in continues monitoring of current state of the system.
- The Power flow studies involves the solution of the power system network under steady state operation subjected to certain inequality constraints under which the system operates.
- These constraints are in the form of load node voltages, reactive power generation of the generators, the tap settings of tap changing transformers under load etc.
- <C:\Users\dell\Desktop\circuit.pdf>
- From the line flow we can also determine the over and under load conditions.
- It helps in System loss minimization and transformer tap setting for economic operation.

## 2.Necessity of Power Flow Studies?

- It is required for Planning, Operation, Economic Scheduling & Exchange of power b/w utilities .
- Whether you do power study, stability study, economic operation this load flow study is very important.
- To analyze the effectiveness of alternative plans for future system expansion to meet the increased load demand.
- To determine the best location for capacitors or voltage regulators for improvement of voltage regulation.
- It helps in designing a new power system network.
- The load flow studies are required at various stages of transient or dynamic stability analysis.

## 2.Necessity of Load Flow Studies? Contd...

- Load flow analysis can provide a balanced steady state operation of the power system, without considering system transient processes. Hence, the mathematic model of load flow problem is a nonlinear algebraic equations without differential equations.
- The main objective of the Power flow studies to determine the Voltage magnitude and phase angle of voltages at each bus, real and reactive power injected at busses and also real and reactive power flows over transmission lines in the steady state by solving nonlinear algebraic equations.



### 3. Data for Load Flow Studies?

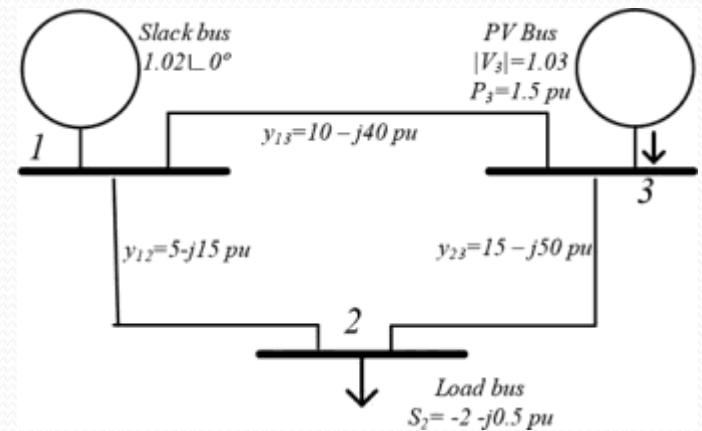
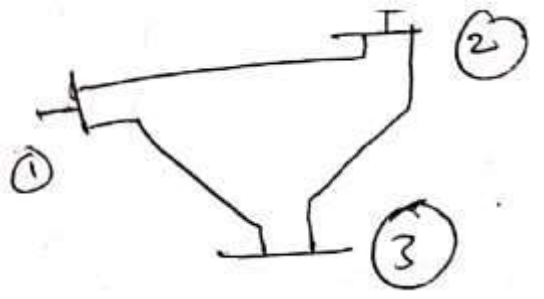
- Load flow is nothing but the steady state of the power system network.
- It is the Study of set of non linear algebraic equations of the power system network for the purpose of to obtain (or) The data obtained from the load flow studies are
  1. The magnitude of the voltage ( $V$ )
  2. Phase angle of the voltage ( $\delta$ )
  3. Active power ( $P$ )
  4. Reactive power ( $Q$ ) flow on transmission lines.
- **At any bus**, Out of these four quantities two quantities are specified and remaining two quantities are to be determined giving rise to three types of busses.

# 3. Data for Load Flow Studies?

## ➤ Classification of buses

There are three types of buses

1. Slack bus
2. Load bus
3. Generator bus



## Slack bus

➤ Slack is also called as reference bus to meet the power balance condition i.e provide power mismatch between generation and load plus losses.

$$\sum_{i=1}^N P_{Gi} = \sum_{i=1}^N P_{Di} + P_L$$

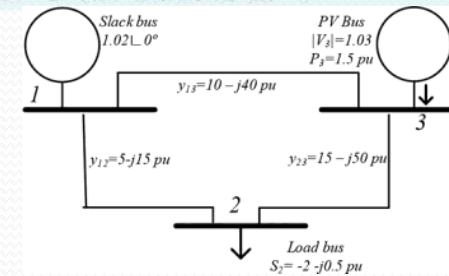
➤ Slack bus is usually identified as bus 1.

➤ The known (Specified) variables on this bus is  $|V|$  and  $\delta$  and the determined variables are  $P$  and  $Q$ .

# 3. Data for Load Flow Studies?

## Load bus

- It is also called as PQ bus or non-generator bus
- This non-generator bus which can be obtained from historical data records, measurement or forecast.
- The consumer power is met at this bus.
- the real and reactive power supply to a power system are defined to be positive, while the power consumed in a power system are defined to be negative.
- At this bus, P and Q are specified and the  $|V|$  and  $\delta$  are to be determined by solving load flow equations.



# 3. Data for Load Flow Studies?

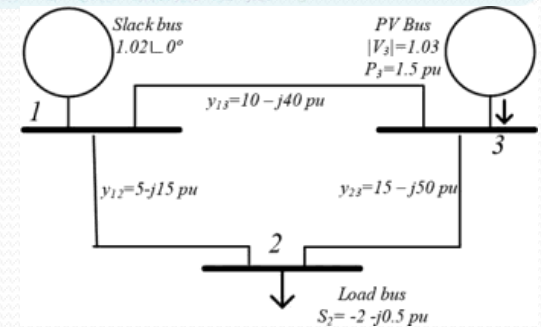
## Generator bus

➤ It is also called as voltage controlled bus or PV bus

➤ This Generator bus is connected to generator unit in which Output active power is controlled by prime mover and voltage can be controlled by adjusting the excitation of the generator.

➤ The known variable in this bus is P and |V| and the unknown is Q and  $\delta$

➤ In this generator bus, there must be limits for Q (Ex :  $3 \leq Q \leq 2$  MVAR )



### 3. Data for Load Flow Studies?

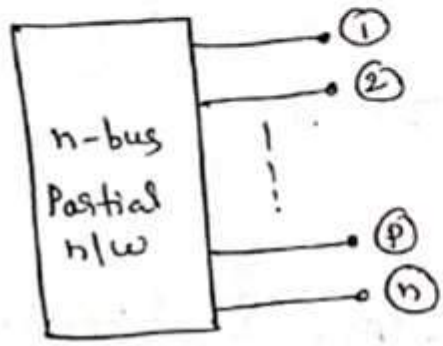
- Summary on Classification of buses (  $V$ ,  $\delta$ ,  $P$  and  $Q$  )

Type of Buses	Know or Specified Quantities	Unknown Quantities or Quantities to be determined.
Generation or P-V Bus	$P,  V $	$Q, \delta$
Load or P-Q Bus	$P, Q$	$ V , \delta$
Slack or Reference Bus	$ V , \delta$	$P, Q$

# 3. Static Load Flow Studies?

## Derivation of static load flow equations

Consider an  $n$ -bus system as shown in fig, with bus voltages  $V_1, V_2, \dots, V_n$  and bus currents  $I_1, I_2, \dots, I_n$  as shown in fig.



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[..\PSA previous year qp\15A02603062018.pdf](https://www.indiabix.com/question/psa/previous-year-questions/15A02603062018.pdf)

Performance equation in admittance form is

$$i = yV$$
$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_n \end{bmatrix} = \begin{matrix} & \begin{matrix} 1 & 2 & p & n \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ p \\ n \end{matrix} & \begin{bmatrix} Y_{11} & Y_{12} & Y_{1p} & Y_{1n} \\ Y_{21} & Y_{22} & Y_{2p} & Y_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{p1} & Y_{p2} & Y_{pp} & Y_{pn} \\ Y_{n1} & Y_{n2} & Y_{np} & Y_{nn} \end{bmatrix} \end{matrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_p \\ \vdots \\ V_n \end{bmatrix}$$

### 3. Static Load Flow Equations? Contd..

From the above matrix,

$$I_1 = Y_{11}V_1 + Y_{12}V_2 + \dots + Y_{1p}V_p + Y_{1n}V_n$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 + \dots + Y_{2p}V_p + Y_{2n}V_n$$

$$\vdots$$
$$I_p = Y_{p1}V_1 + Y_{p2}V_2 + \dots + Y_{pp}V_p + Y_{pn}V_n$$

In General,

$$I_p = \sum_{q=1}^n Y_{pq}V_q \quad \text{--- (1) where } p=1,2,\dots,n$$

The complex power injected into  $p^{\text{th}}$  bus is given as

$$S_p = P_p + jQ_p = V_p I_p^* \quad \text{--- (2)}$$

$$S_p^* = P_p - jQ_p = V_p^* I_p \quad \text{--- (3)}$$

### 3. Static Load Flow Equations? Contd..

sub eq (1) in eq (3)

$$P_p - jQ_p = V_p^* \sum_{q=1}^n Y_{pq} V_q \quad - (4) \quad , p = 1, 2, \dots, n$$

$$\left. \begin{aligned} \text{let } V_p^* &= |V_p| \angle -\delta_p \\ V_q &= |V_q| \angle \delta_q \\ Y_{pq} &= |Y_{pq}| \angle \theta_{pq} \end{aligned} \right\} - (5)$$

sub eq (5) in eq (4)

$$P_p - jQ_p = |V_p| |V_q| |Y_{pq}| \sum_{q=1}^n \angle \theta_{pq} + \delta_q - \delta_p \quad - (6)$$

where

$$P_p = \sum_{q=1}^n |V_p| |V_q| |Y_{pq}| \cos(\theta_{pq} + \delta_q - \delta_p) \quad - (7)$$

$$Q_p = - \sum_{q=1}^n |V_p| |V_q| |Y_{pq}| \sin(\theta_{pq} + \delta_q - \delta_p) \quad - (8)$$

These equations (7) & (8) are called static load flow equations also called as, Non linear algebraic equations



## 3. Static Load Flow Studies? Types..

### Methods of solving static load flow equations

➤ The different methods for load flow solutions are

1. Gauss method
2. Gauss Seidel method
3. Newton Raphson method
4. Decoupled method
5. Fast Decoupled method

# Gauss – Seidel Method

## Gauss – Seidel Method:

Gauss Seidel method is one of the common methods employed for solving power flow equations.

### *Advantages:*

Used for small size system

Simplicity in technique I,e Calculations are simple

Small computer memory requirement i.e Programming task is lesser

Less computational time per iteration

### *Disadvantages:*

Not suitable for larger systems

Slow rate of convergence resulting in larger number of iterations

Increase in the number of iterations with increase in the number of buses

### 3. G-S without PV bus

Gauss Seidal method for load flow solution without PV buses

---

Let  $n =$  total no. of buses

Take bus 1 as slack bus.

Assume PV buses are absent

Here there are  $(n-1)$  PQ buses [out of  $n$ ]

Assume initially there is a flat voltage profile except for a slack bus

i.e.  $V_p^{(0)} = 1 \angle 0^\circ \text{ pu}$ , where  $p = 2, 3 \dots n, \neq 1$

### 3. G-S without PV bus contd...

We know complex conjugate injected into  $p^{\text{th}}$  bus is given by

$$P_p - jQ_p = V_p^* I_p$$

$$I_p = \frac{P_p - jQ_p}{V_p^*} \quad \text{--- (1)}$$

but we know

$$I_p = \sum_{q=1}^n Y_{pq} V_q, \quad \text{where } p = 2, 3, 4, \dots, n$$

$$I_p = Y_{pp} V_p + \sum_{\substack{q=1 \\ q \neq p}}^n Y_{pq} V_q$$

$$Y_{pp} V_p = I_p - \sum_{q=1}^n Y_{pq} V_q$$

### 3. G-S without PV bus contd...

$$V_p = \frac{1}{Y_{pp}} \left[ I_p - \sum_{\substack{q=1 \\ q \neq p}}^n Y_{pq} V_q \right]$$

$$V_p = \frac{1}{Y_{pp}} \left[ \frac{P_p - jQ_p}{V_p^*} - \sum_{\substack{q=1 \\ q \neq p}}^n Y_{pq} V_q \right]$$

For Gauss Seidel method and for  $(k+1)$  iterations

$$V_p^{k+1} = \frac{1}{Y_{pp}} \left[ \frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{\substack{q=1 \\ q \neq p}}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^n Y_{pq} V_q^k \right] \quad (2)$$

From the above eqn.:

$$\frac{P_p - jQ_p}{Y_{pp}} = A_p, \quad \frac{Y_{pq}}{Y_{pp}} = B_{pq}, \quad q = 1, 2, \dots, n, \quad q \neq p$$

### 3. G-S without PV bus contd...

Then eq (2) becomes

$$V_P^{k+1} = \frac{A_P}{(V_P^k)^*} - \sum_{\substack{q=1 \\ q \neq P}}^{P-1} B_{Pq} V_q^{k+1} - \sum_{q=P+1}^n B_{Pq} V_q^k \quad (3)$$

→ This iterative process is continued till  $\frac{\Delta V}{V} < \epsilon$

$$\Delta V_P^k = V_P^{k+1} - V_P^k \leq \epsilon \downarrow 0.0001$$

# 3. Algorithm for G-S without PV bus

## ALGORITHM

1. Read the given data of a power system and form  $Y_{bus}$
2. Except slack bus, assume the flat voltage profile for all buses

$$V_p^0 = 1 + j0 \text{ for } p = 2, 3, \dots, n$$

3. Set iteration count  $k=0$  and  $(\Delta V_{max}) = \epsilon [0.0001]$

4. calculate 
$$A_p = \frac{P_p - jQ_p}{Y_{pp}} \quad \begin{array}{l} p = 2, 3, \dots, n \\ p \neq 1 \end{array}$$

$$B_{pq} = \frac{Y_{pq}}{Y_{pp}}, \quad \begin{array}{l} q = 2, \dots, n \\ q \neq p \end{array}$$

5. Set bus count  $p=1$

# 3. Algorithm for G-S without PV bus

## ALGORITHM

6. If it (bus) is a PQ bus, then

(i) calculate  $V_p^{k+1} = \frac{A_p}{(V_p^k)^x} - \sum_{\substack{q=1 \\ q \neq p}}^{p-1} B_{pq} V_q^{k+1} - \sum_{q=p+1}^n Y_{pq} V_q^x$

(ii) calculate  $\Delta V_p^k = V_p^{k+1} - V_p^k$

(iii) Assign new voltages to old

i.e.  $V_p^k = V_p^{k+1}$

otherwise goto next bus

7. Increment bus count i.e.  $P = P+1$

8. Check all the buses are taken into account. If yes goto next step otherwise goto step 6 and repeat



# 3. Algorithm for G-S without PV bus

## ALGORITHM

9. check convergence  
if  $\Delta V_{\max}$  i.e.  $(V_p^{k+1} - V_p^k) \leq \epsilon$  then goto next step  
otherwise increment the iteration count by 1 i.e.  
 $K = K + 1$ , go to step 5 & repeat
10. calculate the line power flows and slack bus power.

# 3. Flowchart for G-S without PV bus

## SUMMARY

For Gauss Seidel method and for  $(k+1)$  iterations

$$V_p^{k+1} = \frac{1}{Y_{pp}} \left[ \frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{\substack{q=1 \\ q \neq p}}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^n Y_{pq} V_q^k \right] \quad (2)$$

From the above eqn.;

$$\frac{P_p - jQ_p}{Y_{pp}} = A_p, \quad \frac{Y_{pq}}{Y_{pp}} = B_{pq}, \quad q = 1, 2, \dots, n, \quad q \neq p$$

### 3. Flowchart for G-S without PV bus

➤ <C:\Users\dell\Desktop\GS WITHOUT PV BUS.pdf>

# Problem on G-S without PV bus

➤ [PROBLEM ON G-S WITHOUT PV BUS.pdf](#)

### 3. Introduction to G-S with PV bus

#### Gauss Seidal method with PV buses

Let 'n' be the total no. of buses in a power system. out of

This,

$p=1$ , slack bus

$p=2, 3, 4, \dots, i$ , PQ bus

$p=i+1, i+2, \dots, n$ , PV bus

For PV bus, There are two conditions

1. The value of reactive power must be in the given range

$$Q_{p\min} < Q_p < Q_{p\max}$$

2. At this bus, the voltage magnitude is equal to the specified value.

$$|V_p| = |V_p|_{\text{spec}}$$

For PV bus,  $P$  &  $V$  are specified and  $Q$  &  $\delta$  are ~~to~~ determined quantities.

### 3. Introduction to G-S with PV bus

We know

$$P_p - jQ_p = V_p^* I_p$$

$$\frac{P_p - jQ_p}{V_p^*} = I_p$$

$$\frac{P_p - jQ_p}{V_p^*} = \sum_{q=1}^n Y_{pq} V_q$$

### 3. Introduction to G-S with PV bus

$$\frac{P_p - jQ_p}{V_p^*} = \sum_{q=1}^{p-1} Y_{pq} V_q + Y_{pp} V_p + \sum_{q=p+1}^n Y_{pq} V_q$$

$$P_p - jQ_p = V_p^* \left[ \sum_{q=1}^{p-1} Y_{pq} V_q + \sum_{q=p}^n Y_{pq} V_q \right]$$

For  $(k+1)^{\text{th}}$  iteration

$$Q_p = -\text{Im} \left[ (V_p^k)^* \left\{ \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^n Y_{pq} V_q^k \right\} \right]$$

The limits of  $Q_p^{k+1}$  are

case (1): If  $Q_p^{k+1} \leq Q_{p \min}$  then set  $Q_p^{k+1} = Q_{p \min}$

case (2): If  $Q_p^{k+1} \geq Q_{p \max}$  then set  $Q_p^{k+1} = Q_{p \max}$

case (3):  $Q_{p \min} \leq Q_p^{k+1} \leq Q_{p \max}$  then  $Q_p^{k+1} = Q_p^{k+1}$

### 3. Introduction to G-S with PV bus

➤ For PQ bus

Also we know

$$V_p^{k+1} = \frac{A_p}{(V_p^k)^*} - \sum_{\substack{q=1 \\ q \neq p}}^{p-1} B_{pq} V_q^{k+1} - \sum_{q=p+1}^n B_{pq} V_q^k$$

$$\text{where } A_p = \frac{P_p - jQ_p^{k+1}}{Y_{pp}}$$

$$\therefore \delta_p^{k+1} = \angle V_p^{k+1}$$



### 3. Algorithm for G-S with PV bus

Algorithm for G-S method with PV Buses

① Read the given data of power system and form  $Y_{bus}$

② Except the slack bus and PV bus, assume flat voltage profile  $V_p^0 = 1 + j0$  for  $p = 2, 3, \dots, i$

③ set iteration count  $k=0$  and  $\Delta V_{max} = \epsilon$

④ calculate  $A_p = \frac{P_p - jQ_p}{Y_{pp}}$ ,  $p = 2, 3, \dots, i$   
 $p \neq 1$  & PV bus

$$B_{pq} = \frac{Y_{pq}}{Y_{pp}}, \quad p = 2, \dots, n$$

$q \neq p$

⑤ set bus count  $p=1$

### 3. Algorithm for G-S with PV bus

Contd...

6. check for slack bus

if yes goto step 13

otherwise goto next step

7. check for PV bus

if yes

(i) calculate

$$\theta_p^{k+1} = -\text{Imp} \left[ (V_p^k)^* \left\{ \sum_{z=1}^{p-1} Y_{pz} V_z^{k+1} + \sum_{z=p}^n Y_{pz} V_z^k \right\} \right]$$

(ii) check for limits of  $\theta_p^{k+1}$  and set according to equation

if  $\theta_p^{k+1} \leq \theta_{p\min}$ , set  $\theta_p^{k+1} = \theta_{p\min}$

if  $\theta_p^{k+1} \geq \theta_{p\max}$ , set  $\theta_p^{k+1} = \theta_{p\max}$

if  $\theta_{p\min} \leq \theta_p^{k+1} < \theta_{p\max}$ , set  $\theta_p^{k+1} = \theta_p^{k+1}$

otherwise goto next step

### 3. Algorithm for G-S with PV bus

Contd...

8. calculate

$$A_p = \frac{P_p - jQ_p^{k+1}}{Y_{pp}}, \quad B_{pz} = \frac{Y_{pz}}{Y_{pp}}$$

9. calculate  $V_p^{k+1} = \frac{A_p}{(V_p^k)^*} - \sum_{\substack{z=1 \\ z \neq p}}^{p-1} B_{pz} V_z^{k+1} - \sum_{z=p+1}^n Y_{pz} V_z^k$

10. calculate  $\delta_p^{k+1} = \text{Angle of } V_p^{k+1}$

11. calculate  $\Delta V_{\max} = \Delta V_p^k = V_p^{k+1} - V_p^k$

12. substitute  $V_p^{k+1}$  in place of  $V_p^k$

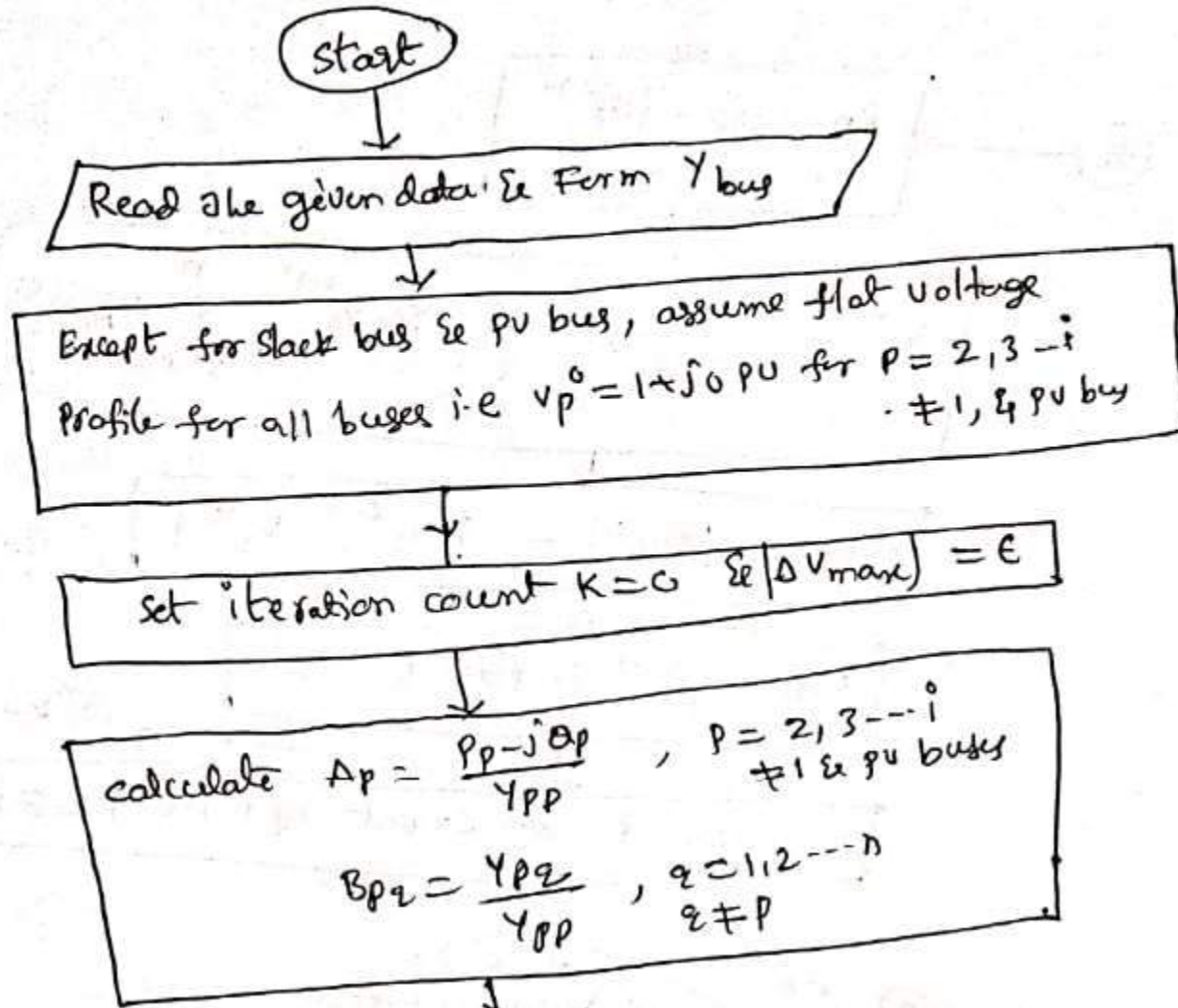
### 3. Algorithm for G-S with PV bus

Contd...

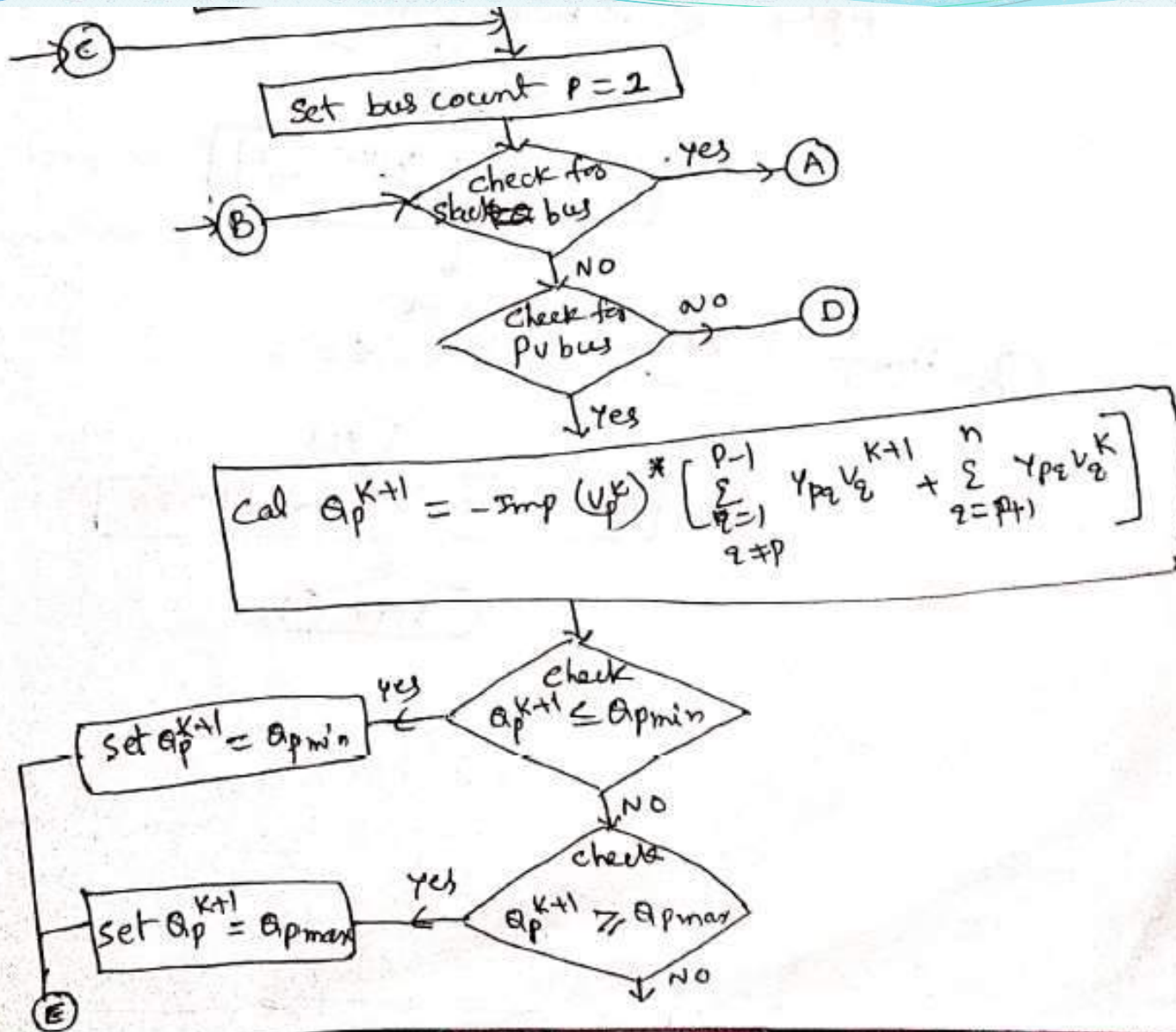
13. Increment bus count by 1 i.e.  $P = P + 1$
14. check if all buses are taken into account  
if yes goto next step  
otherwise goto step 6
15. check  
if  $|\Delta V_{max}| \leq \epsilon$   
if yes goto next step  
otherwise increment iteration count  $K = K + 1$  and  
goto step 5
16. calculate line power flows and slack bus power.

# 3. Flowchart for G-S with PV bus

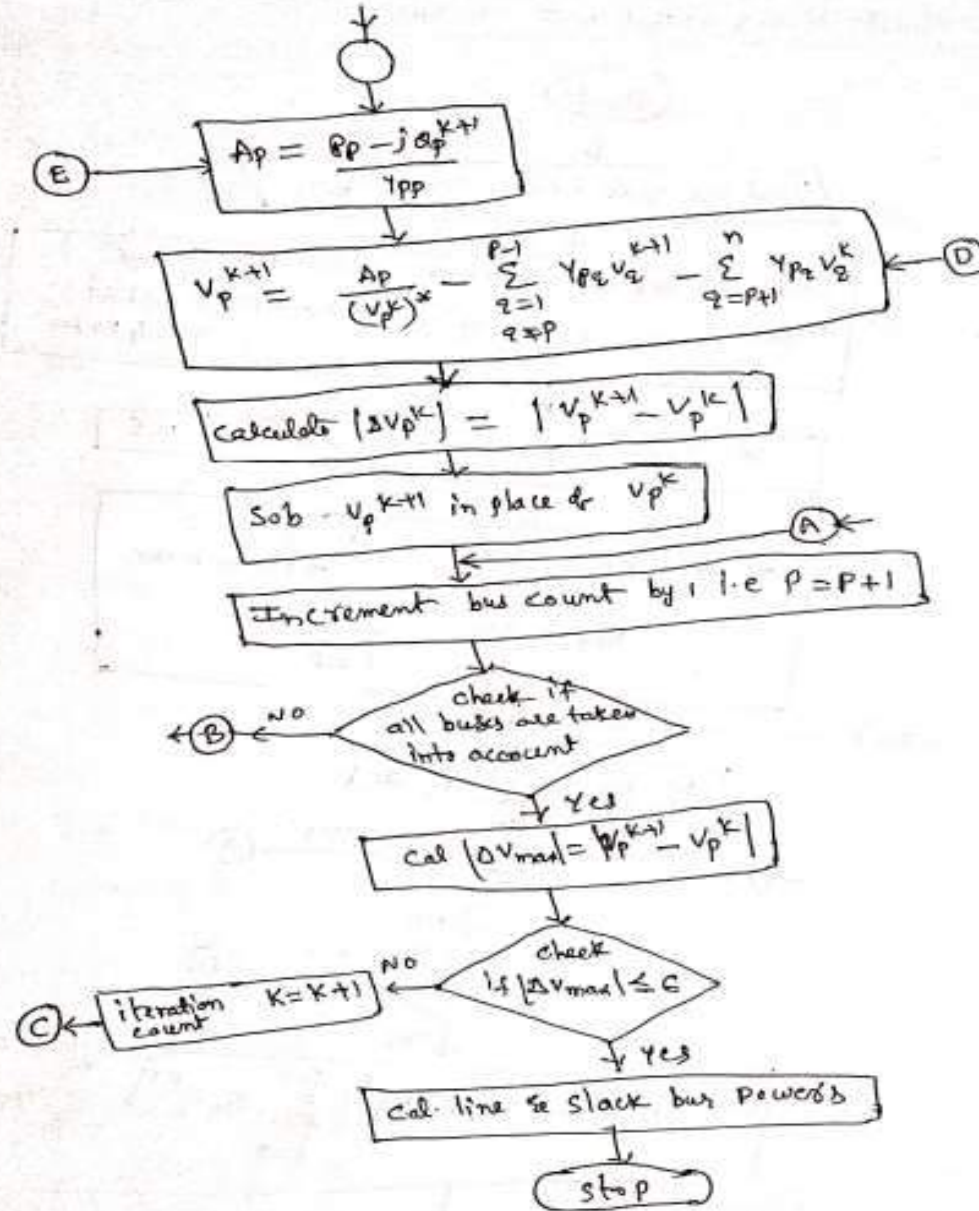
Flow chart of G-S method with PV bus



# 3. Flowchart for G-S with PV bus



# 3. Flowchart for G-S with PV bus



### 3. Acceleration factor

Acceleration factor and its importance in power system

- The Acceleration factor is a real constant number
- It is denoted by the letter ' $\alpha$ '
- It is used to improve the rate of convergence in the Gauss Seidel method or speed up the rate of convergence.

- The Accelerated value of the voltage for the  $p^{\text{th}}$  bus at  $(k+1)^{\text{th}}$  iteration is given as

$$V_p^{k+1}(\text{acce}) = V_p^k + \alpha (V_p^{k+1} - V_p^k)$$

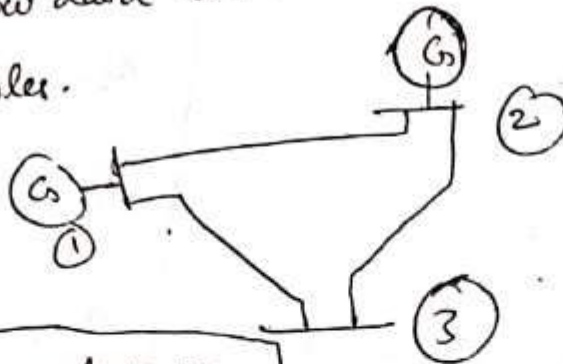
- Typical value of ' $\alpha$ ' lies b/w 1.4 to 1.6. Generally it is 1.6
- However, a wrong selection of ' $\alpha$ ' may lead to divergent or slow convergence.



# Problems

## Problems

- ① The load flow data for the ps-shown in fig is given in the following tables.



Bus code P-Q	Impedance $Z_{PQ}$
1-2	$0.08 + j0.24$
1-3	$0.02 + j0.06$
2-3	$0.06 + j0.18$

Bus code	Assumed bus vol	Generation		Load	
		megawatts	megavars	megawatts	megavars
1	$1.05 + j0$	0	0	0	0
2	$1.0 + j0$	20	0	50	20
3	$1.0 + j0$	0	0	60	25

# Problems

The voltage magnitude at bus 2 is to be maintained at 1.03 pu.  
The max & min reactive power limits of the generator at bus 2 are 35 & 0 megavars res. with bus 1 as slack bus, obtain the voltage at bus 3 using G-S method after first iteration.

Sol vol at bus 2,  $(V_2)_{spe} = 1.03 \text{ pu}$   
Reactive power limits at bus 2 are 35 mvar & 0 mvar  
i.e.  $0 \leq Q_2 \leq 35$

Bus 1 is a slack bus

voltage at bus 3 }  $V_3^1 = ?$   
after 1st iteration }

Assume Base mVA = 50 mVA.

The data in table 2 is converted into pu values.

# Problems

$$P_{G2} = \frac{20}{50} = 0.4 \text{ pu}$$

$$Q_{G2} = \frac{0}{50} = 0 \text{ pu}$$

$$P_{D2} = \frac{50}{50} = 1 \text{ pu}$$

$$Q_{D2} = \frac{20}{50} = 0.4 \text{ pu}$$

$$P_{G3} = \frac{0}{50} = 0$$

$$Q_{G3} = \frac{0}{50} = 0$$

$$P_{D3} = \frac{60}{50} = 1.2$$

$$Q_{D3} = \frac{25}{50} = 0.5 \text{ pu}$$

$$\therefore P_2 = P_{G2} - P_{D2} = 0.4 - 1 = -0.6 \text{ pu}$$

$$P_3 = P_{G3} - P_{D3} = 0 - 1.2 = -1.2 \text{ pu}$$

$$Q_3 = Q_{G3} - Q_{D3} = 0 - 0.5 = -0.5 \text{ pu}$$

To obtain nodal admittance matrix

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

# Problems

$$Y_{11} = Y_{12} + Y_{13}$$

$$= \frac{1}{Z_{12}} + \frac{1}{Z_{13}}$$

$$= \frac{1}{0.08 + j0.24} + \frac{1}{0.02 + j0.06}$$

$$= 1.25 - j3.75 + 5 - j1.5$$

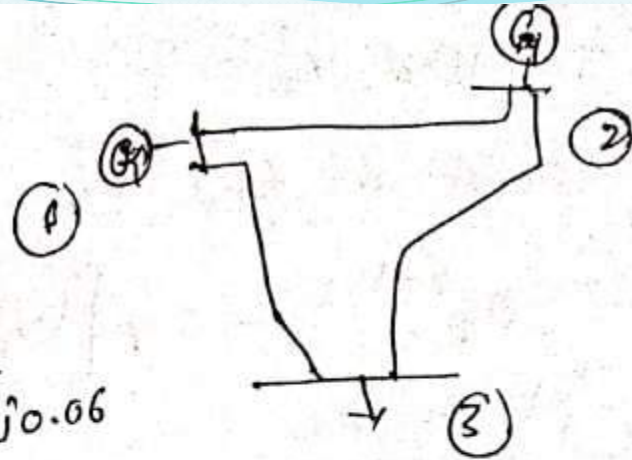
$$= 6.25 - j18.75$$

$$Y_{22} = Y_{22} + Y_{23} = \frac{1}{Z_{22}} + \frac{1}{Z_{23}}$$

$$= 2.91 - j8.75$$

$$Y_{33} = Y_{31} + Y_{32} = \frac{1}{Z_{31}} + \frac{1}{Z_{32}}$$

$$= 6.66 - j6.5$$



# Problems

$$Y_{12} = Y_{21} = -y_{12} = -\frac{1}{z_{12}} = \frac{-1}{0.08 + j0.24} = -1.25 + j3.75$$

$$Y_{23} = Y_{32} = -y_{23} = -1.66 + j5$$

$$Y_{31} = Y_{13} = -y_{13} = -5 + j15$$

$$Y_{bus 2} = \begin{bmatrix} 6.25 - j18.75 & -1.25 + j3.75 & -5 + j15 \\ -1.25 + j3.75 & 2.91 - j8.75 & -1.66 + j5 \\ -5 + j15 & -1.66 + j5 & 6.66 - j6.5 \end{bmatrix}$$

Assuming a flat voltage profile

$$V_1 = V_1^0 = 1.05 + j0 \text{ pu (slack bus)}$$

$$V_2^0 = 1 + j0 \text{ pu}$$

$$|V_2|_{spe} = 1.03 \text{ pu}$$

$$V_3^0 = 1 + j0 \text{ pu}$$

# Problems

$$Q_p^{c+1} = \text{Im} \left\{ (V_p^c)^* \left[ \sum_{q=1}^{p-1} Y_{pq} V_q^{c+1} + \sum_{q=p}^n Y_{pq} V_q^c \right] \right\}$$

$$Q_{2 \text{ cal}} = - \text{Im} \left\{ (V_{2 \text{ spec}}^0)^* \left[ Y_{21} V_1^1 + Y_{22} V_{2 \text{ spec}}^0 + Y_{23} V_3^0 \right] \right\}$$

$$= - \text{Im} \left\{ 1.03 \left[ (-1.25 + j3.75) \times 1.05 + (2.917 - j8.75) \times 1.03 + (-1.667 + j5) \times 1 \right] \right\}$$

$$= - \text{Im} (0.0257 - j0.07725)$$

$$= 0.07725 \text{ p.u.}$$

Bus 2 acted as generated bus since  $Q_{2 \text{ cal}}$  is within specified limits

$$\therefore V_2^1 = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_{2 \text{ cal}}}{(V_2^0)^*} - Y_{21} V_1^1 - Y_{23} V_3^0 \right]$$

$$= \frac{1}{2.917 - j8.75} \left[ \frac{(-0.3 - j0.07725)}{1.03} - (-1.25 + j3.75) \times 1.05 - (-1.667 + j5) \times 1 \right]$$

$$= \frac{1}{2.917 - j8.75} [2.68824 - j9.0125]$$

# Problems

$$= 1.01915 - j0.0325 = 1.0196 \angle -1.828^\circ \text{ p.u.}$$

$$V_2^1 = V_{2\text{spec}}^1 \angle \delta_2^1 = 1.03 \angle -1.828^\circ = (1.02947 - j0.0329) \text{ p.u.}$$

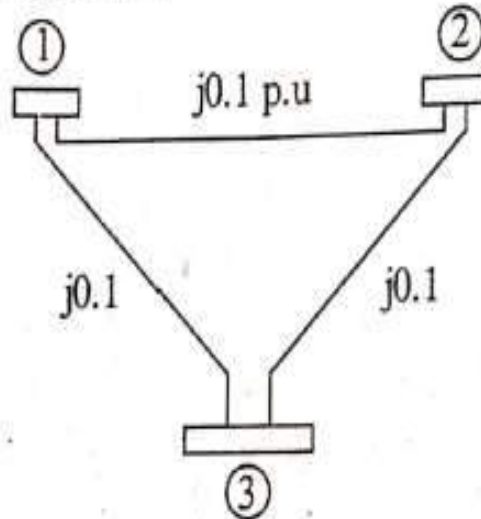
$$V_3^1 = \frac{1}{Y_{33}} \left[ \frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31} V_1^1 - Y_{32} V_2^1 \right]$$

$$= \frac{1}{6.667 - j6.5} \left[ \frac{-0.6 + j0.25}{1.0} - (-5 + j1.5) \times 1.05 - (-1.667 + j5) \times (1.03) \angle -1.828^\circ \right]$$

$$= (0.96627 - j0.03696) \text{ p.u.}$$

# Problems

Consider the 3-bus system shown in figure. The p.u line reactances are indicated on the figure, the line resistances are negligible. The magnitude of all the 3-bus voltages are specified to be 1.0 p.u. The bus powers are specified in the following table.



Bus	Real demand	Reactive demand	Real generation	Reactive generation
1	$P_{D1} = 1.0$	$Q_{D1} = 0.6$	$P_{G1} = ?$	$Q_{G1}(\text{unspecified})$
2	$P_{D2} = 0$	$Q_{D2} = 0$	$P_{G2} = 1.4$	$Q_{G2}(\text{unspecified})$
3	$P_{D3} = 0$	$Q_{D3} = 1.0$	$P_{G3} = 0$	$Q_{G3}(\text{unspecified})$

Carry out the load flow solution using G.S method upto one iteration.taking 1<sup>st</sup> bus as slack bus



# Problems

Given data is

$$V_1^0 = V_2^0 = V_3^0 = 1.0 \text{ p.u.} = 1 + j0 \text{ p.u.}$$

Bus 1 is a slack bus.

Carry out load flow solution using G-S method upto one iteration.

To obtain Nodal Admittance matrix,

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$$\begin{aligned} Y_{11} &= Y_{12} + Y_{13} = \frac{1}{Z_{12}} + \frac{1}{Z_{13}} \\ &= \frac{1}{j0.1} + \frac{1}{j0.1} \\ &= -j10 - j10 \\ &= -j20 \end{aligned}$$

$$\begin{aligned} Y_{22} &= Y_{12} + Y_{23} = \frac{1}{Z_{12}} + \frac{1}{Z_{23}} \\ &= \frac{1}{j0.1} + \frac{1}{j0.1} \\ &= -j10 - j10 \\ &= -j20 \end{aligned}$$

# Problems

$$\begin{aligned} Y_{33} &= Y_{13} + Y_{23} = \frac{1}{Z_{12}} + \frac{1}{Z_{23}} \\ &= \frac{1}{j0.1} + \frac{1}{j0.1} \\ &= -j10 - j10 \\ &= -j20 \end{aligned}$$

$$Y_{12} = Y_{21} = -Y_{12} = -\frac{1}{Z_{12}} = -\frac{1}{j0.1} = j10$$

$$Y_{23} = Y_{32} = -Y_{23} = -\frac{1}{Z_{23}} = -\frac{1}{j0.1} = j10$$

$$Y_{31} = Y_{13} = -Y_{13} = -\frac{1}{Z_{13}} = -\frac{1}{j0.1} = j10$$

# Problems

∴ Nodal admittance matrix,

$$Y_{\text{bus}} = \begin{bmatrix} -j20 & j10 & j10 \\ j10 & -j20 & j10 \\ j10 & j10 & -j20 \end{bmatrix}$$

Assuming a flat voltage profile,

$$V_1^0 = V_1^1 = \dots = V_1^c = V_1 = 1 + j0 \text{ p.u. (slack bus)}$$

$$V_2^0 = 1 + j0 \text{ p.u.}$$

$$V_3^0 = 1 + j0 \text{ p.u.}$$

To find  $V_2^1$ , first  $Q_2^1$  is calculated using

$$Q_p^{c+1} = \text{Im}(V_p^c)^* \left[ \sum_{q=1}^{p-1} Y_{pq} V_q^{c+1} + \sum_{q=p}^n Y_{pq} V_q^c \right]$$

For first iteration,  $c = 0$

Here,  $p = 2$  and  $n = 3$

$$\begin{aligned} \therefore Q_2^1 &= \text{Im} \left\{ (V_2^0)^* \left[ Y_{21} V_1^1 + Y_{22} V_2^0 + Y_{23} V_3^0 \right] \right\} \\ &= \text{Im} \{ (1 - j0) [j10(1 + j0) + (-j20)(1 + j0) + j10(1 + j0)] \} \\ &= \text{Im} \{ (1 - j0) [j10 - j20 + j10] \} \\ &= \text{Im}(j0) \end{aligned}$$

$$\therefore Q_2^1 = 0 \text{ p.u.}$$

# Problems

Voltage at bus 2 can be calculated by,

$$V_p^{c+1} = \frac{1}{Y_{pp}} \left[ \frac{P_p - jQ_p}{(V_p^c)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{c+1} - \sum_{q=p+1}^n Y_{pq} V_q^c \right]$$

$$\begin{aligned} V_2^1 &= \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21} V_1^1 - Y_{23} V_3^0 \right] \\ &= \frac{-1}{j20} \left[ \frac{1.4 - j0}{1 - j0} - j10(1 + j0) - j10(1 + j0) \right] \\ &= \frac{-1}{j20} [1.4 - j0 - j10 - j10] \\ &= \frac{-1}{j20} (1.4 - j20) \\ &= 1 + j0.07 \text{ p.u.} \\ &= 1.002 \angle 4.004^\circ \text{ p.u.} \end{aligned}$$

Voltage at bus 3,

$$\begin{aligned} V_3^1 &= \frac{1}{Y_{33}} \left[ \frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31} V_1^1 - Y_{32} V_2^1 \right] \\ &= \frac{-1}{j20} \left[ \frac{0 - j1}{1 - j0} - j10(1 + j0) - j10(1 + j0.07) \right] \\ &= \frac{-1}{j20} (0 - j1 - j10 + 0.7 - j10) \\ &= \frac{-1}{j20} (0.7 - j21) \\ &= 1.05 + j0.035 \text{ p.u.} \\ &= 1.05 \angle 1.909^\circ \text{ p.u.} \end{aligned}$$

# Problems

The voltages at the end of first iteration are,

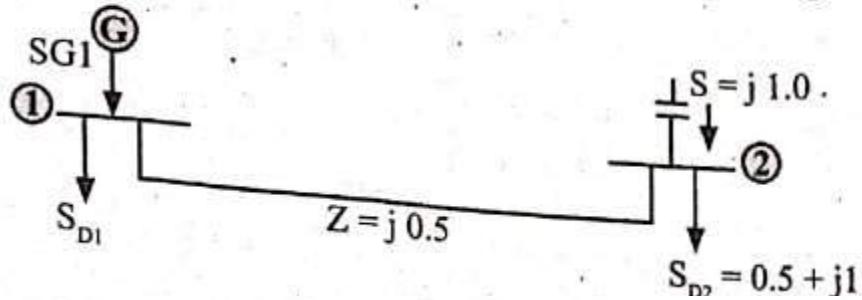
$$V_1^1 = 1 + j0 \text{ p.u.} = 1 \angle 0^\circ \text{ p.u.}$$

$$V_2^1 = 1 + j0.07 \text{ p.u.} = 1.002 \angle 4.004^\circ \text{ p.u.}$$

$$V_3^1 = 1.05 + j0.035 \text{ p.u.} = 1.05 \angle 1.909^\circ \text{ p.u.}$$

# Problems

Obtain the voltage at bus 2 for the simple system shown below, using the Gauss - Seidel method, if  $V_1 = 1 \angle 0^\circ$  p.u.



Given system is,

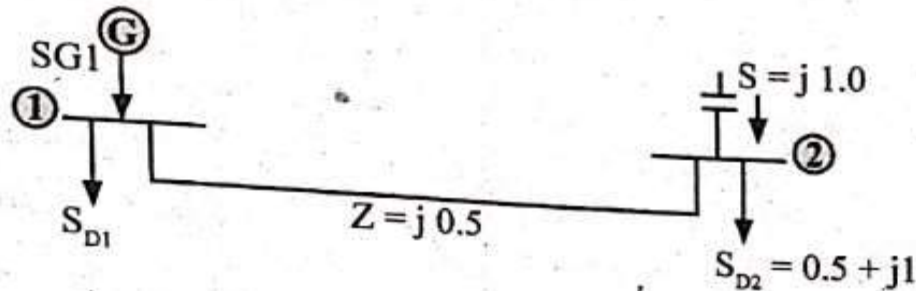


Figure (1)

Also given,

$$V_1 = 1 \angle 0^\circ \text{ p.u.} = 1 + j0 \text{ p.u.}$$

From given system, we have,

$$S_{D2} = 0.5 + j1 \text{ p.u.}$$

Reactance,  $Z = j0.5 \text{ p.u.}$

Reactive Power,  $S = j1.0 \text{ p.u.}$

To determine,

The voltage at bus 2,  $V_2 = ?$

As shown in given system that, the capacitor at bus 2, injects a reactive power of  $S = 1.0 \text{ p.u.}$

# Problems

Therefore the complex power injection at bus 2 is given by,

$$\begin{aligned} S_2 &= S - S_{D_2} \\ &= j1.0 - (0.5 + j1.0) \\ &= -0.5 \text{ p.u} \end{aligned}$$

To obtain nodal admittance matrix,

$$Y_{\text{bus}} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$\begin{aligned} Y_{11} = Y_{22} &= \frac{1}{Z_{12}} \quad (\because Z_{12} = Z = j0.5) \\ &= \frac{1}{j0.5} \\ &= -j2 \end{aligned}$$

$$\begin{aligned} \text{Also, } Y_{12} = Y_{21} &= -Y_{12} = \frac{-1}{Z_{12}} \\ &= \frac{-1}{j0.5} = j2 \end{aligned}$$

$\therefore$  Nodal admittance matrix,

$$Y_{\text{bus}} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

# Problems

Since  $V_1$  is specified it is a constant through all the iterations. Let the initial voltage at bus 2 be,

$$V_2^0 = 1 + j0.0 = 1 \angle 0^\circ \text{ p.u}$$

Now we have,

$$V_2^{(K+1)} = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{(V_2^{(K)})^*} - Y_{21} V_1 \right]$$

For first iteration,  $K = 0$

$$\therefore V_2^{(0+1)} = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21} V_1 \right]$$

$$V_2^1 = \frac{1}{-j2} \left[ \frac{-0.5}{1 \angle 0^\circ} - (j2 \times 1 \angle 0^\circ) \right] \quad (\because P_2 = 0 \text{ and } Q = 0.5 \text{ p.u})$$

$$= j0.5(-0.5 - j2)$$

$$= 1.0 - j0.25$$

$$= 1.030776 \angle -14.036^\circ$$

For,  $K = 1$ ,

$$V_2^{1+1} = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{(V_2^1)^*} - Y_{21} V_1 \right]$$

$$V_2^2 = \frac{1}{-j2} \left[ \frac{-0.5}{1.030776 \angle 14.036^\circ} - (j2 \times 1 \angle 0^\circ) \right]$$

$$= j0.5(-0.47058 - j1.88236)$$

$$= 0.94118 - j0.23529$$

$$= 0.970145 \angle -14.036^\circ$$



# Problems

For,  $K = 2$ ,

$$\begin{aligned}V_2^3 &= \frac{1}{-j2} \left[ \frac{-0.5}{0.970145 \angle 14.036^\circ} - (j2 \times 1 \angle 0^\circ) \right] \\&= j0.5 [-0.49999 - j1.87500] \\&= 0.9375 - j0.24999 \\&= 0.970258 \angle -14.930^\circ\end{aligned}$$

For  $K = 3$ ,

$$\begin{aligned}V_2^4 &= \frac{1}{-j2} \left[ \frac{-0.5}{0.970258 \angle 14.930^\circ} - (j2 \times 1 \angle 0^\circ) \right] \\&= j0.5 [-0.49793 - j1.867231] \\&= 0.933615 - j0.248965 \\&= 0.966240 \angle -14.931^\circ\end{aligned}$$

For  $K = 4$ ,

$$\begin{aligned}V_2^5 &= \frac{1}{-j2} \left[ \frac{-0.5}{0.966240 \angle 14.931^\circ} - (j2 \times 1 \angle 0^\circ) \right] \\&= j0.5 (-0.49999 - j1.866671) \\&= 0.933335 - j0.24999 \\&= 0.96624 \angle -14.994^\circ\end{aligned}$$

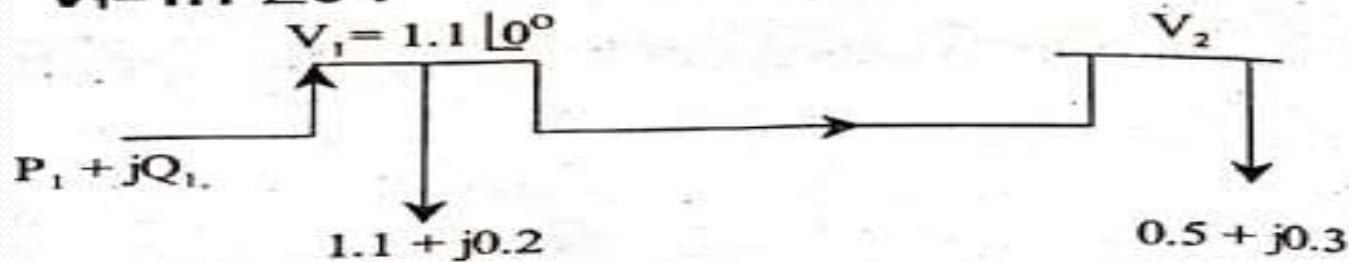
Since the difference in the voltage magnitude for last two iterations is less than  $10^{-6}$  p.u, thus the iterations can be stopped  $\therefore$  The voltage at bus 2 after five iteration is obtain by,

$$V_2 = 0.96624 \angle -14.994^\circ \text{ p.u}$$

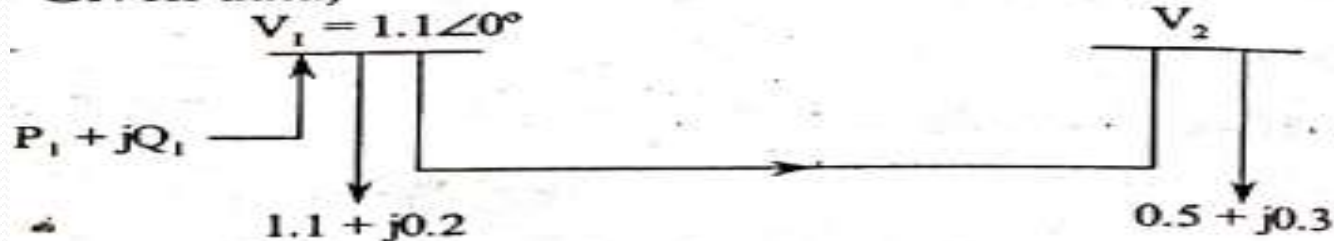
# Problems

A 2-bus system has been shown in figure. Determine the voltage at bus 2 by G.S. method after 2 iterations.

$Y_{11} = Y_{22} = 1.6 \angle -80^\circ$  p.u.;  $Y_{21} = Y_{12} = 1.9 \angle 100^\circ$  p.u.;  
 $V_1 = 1.1 \angle 0^\circ$ .



Given data,



$$Y_{11} = Y_{22} = 1.6 \angle -80^\circ = 0.277 - j1.575 \text{ p.u.}$$

$$Y_{12} = Y_{21} = 1.9 \angle 100^\circ = -0.329 + j1.871 \text{ p.u.}$$

$$V_1 = 1.1 \angle 0^\circ = 1.1 + j0 \text{ p.u.}$$

Voltage at bus 2 = ? after 2 iterations using G.S. method.  
 Assuming bus 1 to be a slack bus.

# Problems

$$\Rightarrow \begin{aligned} V_1^0 &= V_1^1 = V_1^2 = \dots = V_1^c = V_1 = 1.1 \angle 0^\circ \\ &= 1.1 + j0 \text{ p.u.} \\ V_2^0 &= 1 + j0 \text{ p.u.} \end{aligned}$$

To obtain the nodal admittance matrix,

$$\begin{aligned} Y_{\text{bus}} &= \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \\ &= \begin{bmatrix} 0.277 - j1.575 & -0.329 + j1.871 \\ -0.329 + j1.871 & 0.277 - j1.575 \end{bmatrix} \end{aligned}$$

Given,

$$P_1 + jQ_1 = 1.1 + j0.2$$

$$P_2 + jQ_2 = 0.5 + j0.3$$

Voltage at bus 2 can be calculated by

$$V_p^{c+1} = \frac{1}{Y_{pp}} \left[ \frac{P_p - jQ_p}{(V_p^c)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{c+1} - \sum_{q=p+1}^n Y_{pq} V_q^c \right]$$

For first iteration,  $c = 0$

Here,  $p = 2$  and  $n = 2$

$$\begin{aligned} \therefore V_2^1 &= \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21} V_1^1 \right] \\ &= \frac{1}{0.277 - j1.575} \left[ \frac{0.5 - j0.3}{1 - j0} - (-0.329 + j1.871)(1.1 + j0) \right] \\ &= \frac{1}{0.277 - j1.575} (0.5 - j0.3 + 0.3619 - j2.058) \\ &= \frac{1}{0.277 - j1.575} (0.8619 - j2.358) \\ &= 1.5455 + j0.2754 \text{ p.u.} \end{aligned}$$

# Problems

$$= 1.5699 \angle 10.103^\circ \text{ p.u.}$$

For second iteration,  $c = 1$

Again  $p = 2$  and  $n = 2$

∴ Voltage at bus 2 after second iteration,

$$\begin{aligned} V_2^2 &= \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{(V_2^1)^*} - Y_{21} V_1^2 \right] \\ &= \frac{1}{0.277 - j1.575} \left[ \frac{0.5 - j0.3}{1.5455 - j0.2754} - (-0.329 + j1.871)(1.1 + j0) \right] \\ &= \frac{1}{0.277 - j1.575} (0.347 - j0.132 + 0.3619 - j2.058) \\ &= \frac{1}{0.277 - j1.575} (0.7089 - j2.19) \\ &= 1.4255 + j0.199 \text{ p.u.} = 1.439 \angle 7.948^\circ \text{ p.u.} \end{aligned}$$

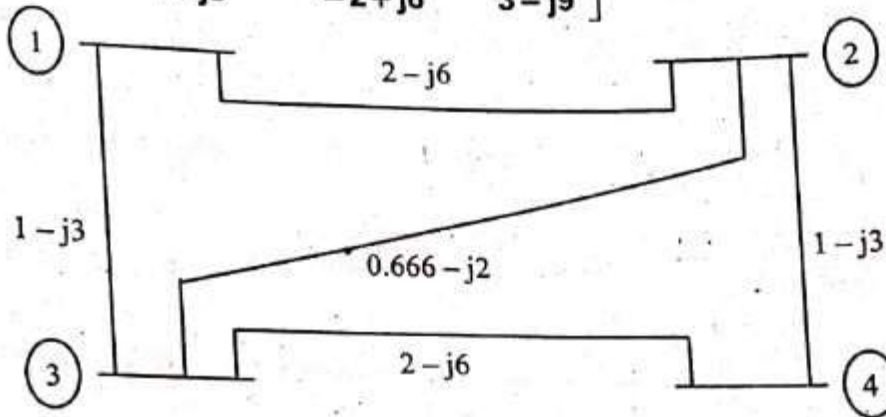
∴ Voltage at bus 2 after second iteration,

$$V_2^2 = 1.4255 + j0.199 \text{ p.u.} = 1.439 \angle 79.48^\circ \text{ p.u.}$$

# Problems

The system shown in figure, the bus admittance matrix is,

$$Y_{BUS} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.666-j11 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix} \text{ p.u}$$



Figure

With  $P_2 = 0.5$  p.u,  $Q_2 = -0.2$  p.u,  $P_3 = -1$  pu,  $Q_3 = 0.5$  p.u and  $P_4 = 0.3$  p.u,  $Q_4 = -0.1$  p.u and  $V_1 = 1.04 \angle 0$  p.u. Determine the value of  $V_2$  that is produced by the first iteration of the G-S method.

Given that,

$$Y_{BUS} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.666-j11 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}$$

$$P_2 = 0.5 \text{ p.u}$$

$$Q_2 = -0.2 \text{ p.u}$$

$$P_3 = -1 \text{ p.u}$$

$$Q_3 = 0.5 \text{ p.u}$$

$$P_4 = 0.3 \text{ p.u}$$

$$Q_4 = -0.1 \text{ p.u}$$

# Problems

$$\begin{aligned} V_1 &= 1.04 \angle 0^\circ \quad (\text{Slack bus}) \\ \Rightarrow V_1^0 &= V_1^1 = V_1^2 = V_1^k = V_1 = 1.04 + j0 \text{ p.u} \\ V_2^0 &= 1.04 + j0 \text{ p.u} \end{aligned}$$

By Gauss-Seidel iteration method, we have,

$$V_p^{c+1} = \frac{1}{Y_{pp}} \left[ \frac{P_p - jQ_p}{(V_p^c)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{c+1} - \sum_{q=p+1}^n Y_{pq} V_q^c \right]$$

Now, from the given data we have,

First iteration,  $c = 0$  [ $\because$  Iteration starts from zero]

Number of buses,  $n = 4$

Voltage at 2<sup>nd</sup> bus,  $p = 2$

Substituting all the values in equation (1) we get,

$$V_2^{0+1} = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{(V_2^0)^*} - \sum_{q=1}^{2-1} Y_{2q} V_q^{0+1} - \sum_{q=2+1}^4 Y_{2q} V_q^0 \right]$$

$$V_2^1 = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21} V_1^1 - \sum_{q=3}^4 Y_{2q} V_q^0 \right]$$

# Problems

$$V_2^1 = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - (Y_{23}V_3^0 + Y_{24}V_4^0) \right]$$

$$V_2^1 = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0 - Y_{24}V_4^0 \right]$$

$$V_2^1 = \frac{1}{3.666 - j11} \left[ \frac{0.5 - j(-0.2)}{(1.04 + j0)^*} - (-2 + j6)(1.04 + j0) - (-0.666 + j2)(1.04 + j0) - (-1 + j3)(1.04 + j0) \right]$$

$$V_2^1 = \frac{1}{3.666 - j11} \left[ \frac{0.5 + j0.2}{1.04 - j0} - (-2 + j6)(1.04) - (-0.666 + j2)(1.04) - (-1 + j3)(1.04) \right]$$

$$V_2^1 = \frac{1}{3.666 - j11} \left[ \frac{0.5 + j0.2}{1.04} - (-2.08 + j6.24) - (-0.69264 + j2.08) - (-1.04 + j3.12) \right]$$


$$V_2^1 = \frac{1}{3.666 - j11} [0.4807 + j0.1923 + 2.08 - j6.24 + 0.69264 - j2.08 + 1.04 - j3.12]$$

$$V_2^1 = \frac{1}{3.666 - j11} [4.29334 - j11.2477]$$

$$V_2^1 = 1.0373 + j0.0445$$

(or)

$$V_2^1 = 1.0382 \angle 2.4567^\circ$$



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**POWER FLOW STUDIES -2**



# SYLLABUS

## **POWER FLOW STUDIES-II**

Newton Raphson Method in Rectangular and Polar Co-Ordinates Form: Load Flow Solution with or without PV Buses- Derivation of Jacobian Elements, Algorithm and Flowchart. Decoupled and Fast Decoupled Methods.- Comparison of Different Methods – DC Load Flow

# 1. Newton Raphson Method

## Newton Raphson method

Newton Raphson method is a Fastest and most reliable method and also it is most powerful technique, as compared to Gauss Seidel method.

→ This Newton Raphson method is a quadratic rate of convergence, whereas  $G$ - $S$  method is a linear rate of convergence.

→ This method does not require acceleration factor also insensitive to the selection of slack bus.

→ Time per iteration is less.

→  $N$ - $R$  method can be applied to the load flow solution in a number of ways. The most commonly used methods are

(1) Rectangular coordinates

(2) Polar coordinates

# N-R Rectangular Method

(1) N-R method using Rectangular coordinates

(A) Derivation of load flow equations

In this method, the load flow equations are expressed in rectangular form.

consider 'n' bus power system

At bus 'p', the complex conjugate power is given by

$$P_p - jQ_p = V_p^* I_p, \quad \text{where } I_p = \sum_{q=1}^n Y_{pq} V_q$$

$$P_p - jQ_p = V_p^* \sum_{q=1}^n Y_{pq} V_q \quad - (1)$$

$$\left. \begin{aligned} \text{let } V_p^* &= e_p - jf_p \\ V_q &= e_q + jf_q \\ Y_{pq} &= G_{pq} - jB_{pq} \end{aligned} \right\} - (2)$$

Substituting eq (2) in eq (1)

# N-R

# Rectangular Method

$$\begin{aligned} P_p - jQ_p &= (e_p - jf_p) \sum_{q=1}^n (G_{pq} - jB_{pq}) (e_q + jf_q) \\ &= \sum_{q=1}^n (e_p - jf_p) \left[ (G_{pq} - jB_{pq}) (e_q + jf_q) \right] \\ &= \sum_{q=1}^n (e_p - jf_p) \left[ G_{pq} e_q + B_{pq} f_q - jB_{pq} e_q + jG_{pq} f_q \right] \\ &= \sum_{q=1}^n \left[ e_p (G_{pq} e_q + B_{pq} f_q) + e_p \left[ -jB_{pq} e_q + jG_{pq} f_q \right] \right. \\ &\quad \left. - jf_p \left[ G_{pq} e_q + B_{pq} f_q \right] - jf_p \left[ -jB_{pq} e_q + jG_{pq} f_q \right] \right] \\ &= \sum_{q=1}^n \left[ e_p (G_{pq} e_q + B_{pq} f_q) + f_p (G_{pq} f_q - B_{pq} e_q) \right] \\ &\quad - j \sum_{q=1}^n \left[ f_p (G_{pq} e_q + B_{pq} f_q) - e_p (G_{pq} f_q - B_{pq} e_q) \right] \end{aligned}$$

# N-R

# Rectangular Method

The real part of equation (4) is

$$P_p = \sum_{q=1}^n \left[ e_p (e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq}) \right] \quad (5)$$

The imaginary part of eq (4) is

$$Q_p = \sum_{q=1}^n \left[ f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq}) \right] \quad (6)$$

$$\text{Also } (V_p)^2 = e_p^2 + f_p^2 \quad (7)$$

The three set of equations i-e (5), (6) and (7) are called load flow equations and these equations are non linear equations.

# N-R Rectangular method without PV bus

(B) When PV buses are Absent: Bus 1 is slack bus and remaining all are PV buses  
Newton Raphson method is an iterative method which approximates

The set of non linear equations to set of linear equations

using Taylor's Series.

In Taylor's Series

Let unknown quantities be  $x_1, x_2, \dots, x_n$

specified quantities be  $y_1, y_2, \dots, y_n$

These are related by set of non linear equations

$$y_1 = f_1(x_1, x_2, \dots, x_n)$$

$$y_2 = f_2(x_1, x_2, \dots, x_n)$$

$$\vdots$$
$$y_n = f_n(x_1, x_2, \dots, x_n)$$

# N-R Rectangular method without PV bus

To solve these non linear equations, we start with an approximate solution i.e  $x_1^0, x_2^0, \dots, x_n^0$ . here '0' represents zeroth iteration.

Assume corrections required  $\Delta x_1^0, \Delta x_2^0, \dots, \Delta x_n^0$  and the 1<sup>st</sup> bus is assumed as slack bus.

The equations of  $y_1$  will be

$$y_1 = f_1(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \dots, x_n^0 + \Delta x_n^0)$$

$$= f_1(x_1^0, x_2^0, \dots, x_n^0) + \Delta x_1^0 \frac{\partial f_1}{\partial x_1} + \Delta x_2^0 \frac{\partial f_1}{\partial x_2} + \dots + \Delta x_n^0 \frac{\partial f_1}{\partial x_n} + \phi$$

where  $\phi$  higher order terms which are eliminated by NR method

In matrix form,

$$\begin{bmatrix} y_1 - f_1(x_1^0, x_2^0, \dots, x_n^0) \\ y_2 - f_2(x_1^0, x_2^0, \dots, x_n^0) \\ \vdots \\ y_n - f_n(x_1^0, x_2^0, \dots, x_n^0) \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \dots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix} \quad \text{--- (8)}$$

# N-R Rectangular method without PV bus

$$B = [J] C \quad \text{--- (9)}$$

where  $J$  is the first derivation matrix known as Jacobian matrix

when referred to power system problem, the above set of linearized equations become,

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_n \\ \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \Delta Q_n \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \dots & \frac{\partial P_2}{\partial e_n} & \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} & \dots & \frac{\partial P_2}{\partial f_n} \\ \frac{\partial P_3}{\partial e_2} & \frac{\partial P_3}{\partial e_3} & \dots & \frac{\partial P_3}{\partial e_n} & \frac{\partial P_3}{\partial f_2} & \frac{\partial P_3}{\partial f_3} & \dots & \frac{\partial P_3}{\partial f_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial P_n}{\partial e_2} & \frac{\partial P_n}{\partial e_3} & \dots & \frac{\partial P_n}{\partial e_n} & \frac{\partial P_n}{\partial f_2} & \frac{\partial P_n}{\partial f_3} & \dots & \frac{\partial P_n}{\partial f_n} \\ \hline \frac{\partial Q_2}{\partial e_2} & \frac{\partial Q_2}{\partial e_3} & \dots & \frac{\partial Q_2}{\partial e_n} & \frac{\partial Q_2}{\partial f_2} & \frac{\partial Q_2}{\partial f_3} & \dots & \frac{\partial Q_2}{\partial f_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial Q_n}{\partial e_2} & \frac{\partial Q_n}{\partial e_3} & \dots & \frac{\partial Q_n}{\partial e_n} & \frac{\partial Q_n}{\partial f_2} & \frac{\partial Q_n}{\partial f_3} & \dots & \frac{\partial Q_n}{\partial f_n} \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \vdots \\ \Delta e_n \\ \Delta f_2 \\ \vdots \\ \Delta f_n \end{bmatrix} \quad \text{--- (10)}$$

In short form it can be written as

$$\Delta P_p = \sum_{q=2}^n \frac{\partial P_p}{\partial e_q} \Delta e_q + \sum_{q=2}^n \frac{\partial P_p}{\partial f_q} \Delta f_q \quad \text{--- (11)}$$

$$\Delta Q_p = \sum_{q=2}^n \frac{\partial Q_p}{\partial e_q} \Delta e_q + \sum_{q=2}^n \frac{\partial Q_p}{\partial f_q} \Delta f_q \quad \text{--- (12)}$$



# N-R Rectangular method without PV bus

i.e

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & | & J_2 \\ \hline J_3 & | & J_4 \end{bmatrix} \begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix} \quad \text{--- (13)}$$

The elements of Jacobian matrix can be derived from local flow equations (5) & (6)

The real part of equation (4) is

$$P_p = \sum_{q=1}^n \left[ e_p (e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq}) \right] \quad \text{--- (5)}$$

The imaginary part of eq (4) is

$$Q_p = \sum_{q=1}^n \left( f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq}) \right) \quad \text{--- (6)}$$

# N-R Rectangular method without PV bus

The real part of equation (4) is

$$P_p = \sum_{q=1}^n \left[ e_p (e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq}) \right] \quad (5)$$

The imaginary part of eq (4) is

$$Q_p = \sum_{q=1}^n \left( f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq}) \right) \quad (6)$$

J<sub>1</sub>

The diagonal elements of J<sub>1</sub> are

$$\frac{\partial P_p}{\partial e_p} = 2e_p G_{pp} + f_p / B_{pp} - f_p B'_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n (e_q G_{pq} + f_q B_{pq}) \quad (14)$$

$$= 2e_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n (e_q G_{pq} + f_q B_{pq}) \quad (15)$$

The off diagonal elements of J<sub>1</sub> are

$$\frac{\partial P_p}{\partial e_q} = e_p G_{pq} - f_p B_{pq}, \quad q \neq p \quad (16)$$

# N-R Rectangular method without PV bus

The real part of equation (4) is

$$P_p = \sum_{q=1}^n \left[ e_p (e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq}) \right] \quad (5)$$

The imaginary part of eq (4) is

$$Q_p = \sum_{q=1}^n \left( f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq}) \right) \quad (6)$$

J<sub>2</sub>

The diagonal elements of J<sub>2</sub> are

$$\frac{\partial P_p}{\partial f_p} = e_p B_{pp} + 2f_p G_{pp} - e_p B_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n (f_q G_{pq} - e_q B_{pq}) \quad (17)$$

The off diagonal element of J<sub>2</sub> are

$$\frac{\partial P_p}{\partial f_q} = e_p B_{pq} + f_p G_{pq}, \quad q \neq p \quad (18)$$

# N-R Rectangular method without PV bus

The real part of equation (4) is

$$P_p = \sum_{q=1}^n \left[ e_p (e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq}) \right] \quad (5)$$

The imaginary part of eq (4) is

$$Q_p = \sum_{q=1}^n \left[ f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq}) \right] \quad (6)$$

J<sub>3</sub>

The diagonal elements of J<sub>3</sub> are

$$\frac{\partial Q_p}{\partial e_p} = f_p G_{pp} - f_p G_{pp} + 2e_p B_{pp} - \sum_{\substack{q=1 \\ q \neq p}}^n (f_q G_{pq} - e_q B_{pq}) \quad (10)$$

The off diagonal elements of J<sub>3</sub> are

$$\frac{\partial Q_p}{\partial e_q} = e_p B_{pq} + f_p G_{pq}, \quad q \neq p \quad (20)$$

# N-R Rectangular method without PV bus

The real part of equation (4) is

$$P_p = \sum_{q=1}^n \left[ e_p (e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq}) \right] \quad (5)$$

The imaginary part of eq (4) is

$$Q_p = \sum_{q=1}^n \left[ f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq}) \right] \quad (6)$$

J<sub>4</sub>

The diagonal elements of J<sub>4</sub> are

$$\begin{aligned} \frac{\partial Q_p}{\partial f_p} &= 2f_p B_{pp} + e_p / G_{pp} - e_p / G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n (e_q G_{pq} + f_q B_{pq}) \\ &= 2f_p B_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n (e_q G_{pq} + f_q B_{pq}) \quad (21) \end{aligned}$$

The off-diagonal elements of J<sub>4</sub> are

$$\frac{\partial Q_p}{\partial f_q} = -e_p G_{pq} + f_p B_{pq}, \quad q \neq p \quad (22)$$

$$\therefore \frac{\partial Q_p}{\partial f_q} = -\frac{\partial Q_q}{\partial f_p}$$

$$\therefore (J_4)_{pq} = -(J_4)_{qp}$$

Symmetry

# N-R Rectangular method without PV bus

(C) when PV buses are present

out of 'n' buses, 1<sup>st</sup> bus is slack bus, and remaining are PQ and PV buses. Then the set of equations be written as

$$(11) \quad \begin{bmatrix} \Delta P \\ \Delta Q \\ (\Delta V_p)^2 \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ \cdots & \cdots \\ J_3 & J_4 \\ \cdots & \cdots \\ J_5 & J_6 \end{bmatrix} \begin{bmatrix} \Delta e \\ \cdots \\ \Delta f \end{bmatrix} \quad (23)$$

(The elements of  $J_5$  &  $J_6$  are ~~from~~ derived from eq (7))

# N-R Rectangular method without PV bus

J<sub>5</sub>

The diagonal elements of J<sub>5</sub> are

$$\frac{\partial |V_p|^2}{\partial e_p} = 2e_p \quad (24)$$

The off diagonal elements of J<sub>5</sub> are

$$\frac{\partial |V_p|^2}{\partial e_q} = 0, \quad q \neq p \quad (25)$$

J<sub>6</sub>

The diagonal elements of J<sub>6</sub> are

$$\frac{\partial |V_p|^2}{\partial f_p} = 2f_p$$

The off diagonal elements of J<sub>6</sub> are

$$\frac{\partial |V_p|^2}{\partial f_q} = 0, \quad q \neq p$$

$$|V_p|^2 = e_p^2 + f_p^2 \quad (7)$$

# N-R Rectangular method without PV bus

The next better solution will be

$$e_p^1 = e_p^0 + \Delta e_p^0$$

$$f_p^1 = f_p^0 + \Delta f_p^0$$

These values are used in the next iteration.

In General, the better estimates for bus voltages will be

$$e_p^{k+1} = e_p^k + \Delta e_p^k$$

$$f_p^{k+1} = f_p^k + \Delta f_p^k$$

This process is repeated till the largest element in the residual column vector is less than  $\epsilon$ .



# N-R Rectangular method without PV bus

IV UNIT\NR RECTANGULAR\Algorithm for NR  
WITHOUT PV rectangular.pdf

# N-R Rectangular method without PV bus

IV UNIT\NR RECTANGULAR\NR Flowchart without pv bus  
(RECTA).pdf

# N-R Rectangular method with PV bus

IV UNIT\NR RECTANGULAR\Algorithm for NR rec with  
PV.pdf

# N-R Rectangular method with PV bus (Flowchart)

IV UNIT\NR RECTANGULAR\Flowchart for NR rec with  
PV.pdf

# N-R Polar Coordinates method

## N-R method using polar coordinate

- In this formulation, the load flow equations are expressed in polar form.
- The total no. of equations in rectangular coordinate version are  $2(n-1)$ , whereas in polar coordinate version are  $2(n-1) - g$ , where 'g' is generator bus. Thus, the use of polar form results in lesser no. of equations and smaller size of Jacobian as compared with the rectangular form.

# N-R Polar Coordinates method

## Load flow equations

we know, The complex conjugate of power is given by

$$P_p - jQ_p = V_p^* I_p \quad (1)$$

$$P_p - jQ_p = V_p^* \sum_{q=1}^n Y_{pq} V_q \quad (2)$$

$$\text{let } V_p^* = |V_p| \angle -\delta_p$$

$$V_q = |V_q| \angle \delta_q$$

} (3)

$$V_p = |V_p| \angle \delta_p$$

$$Y_{pq} = G_{pq} - jB_{pq} \Rightarrow Y_{pq} = |Y_{pq}| \angle -\theta_{pq}$$

substitute eq (3) in eq (2)

$$P_p - jQ_p = \sum_{q=1}^n |V_p| |V_q| |Y_{pq}| \angle (\theta_{pq} + \delta_q - \delta_p) \quad (4)$$

# N-R Polar Coordinates method

real part of the equation is

$$P_p = \sum_{q=1}^n |V_p| |V_q| |Y_{pq}| \cos(\theta_{pq} + \delta_q - \delta_p) \quad \text{--- (5)}$$

$$P_p = \sum_{q=1}^n |V_p| |V_q| |Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q) \quad \text{--- (5)}$$

The imaginary part is

$$Q_p = \sum_{q=1}^n |V_p| |V_q| |Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \quad \text{--- (6)}$$

eq (5) & (6) are called load flow equations.

# N-R Polar Coordinates method without PV bus

(A) when PV buses are absent

For a given power system n/w, there are 'n' no. of buses. Assuming bus 1 is a slack bus and all remaining buses are taken as load buses.

The differential equations which relate the change in real and reactive power to change the magnitude & phase angle of bus voltage

$$\Delta P_p = \sum_{q=2}^n \frac{\partial P_p}{\partial \delta_q} \Delta \delta_q + \sum_{q=2}^n \frac{\partial P_p}{\partial |V_q|} \Delta |V_q| \quad \text{--- (7)}$$

P, Q, V,  $\delta$

$$\Delta Q_p = \sum_{q=2}^n \frac{\partial Q_p}{\partial \delta_q} \Delta \delta_q + \sum_{q=2}^n \frac{\partial Q_p}{\partial |V_q|} \Delta |V_q| \quad \text{--- (8)}$$



# N-R Polar Coordinates method without PV bus

In matrix form

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_n \\ \hline \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \Delta Q_n \end{bmatrix} = \begin{matrix} 2 \\ 3 \\ \vdots \\ n \\ \hline 2 \\ 3 \\ \vdots \\ n \end{matrix} \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \dots & \frac{\partial P_2}{\partial \delta_n} & | & \frac{\partial P_2}{\partial |V_2|} & \frac{\partial P_2}{\partial |V_3|} & \dots & \frac{\partial P_2}{\partial |V_n|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \dots & \frac{\partial P_3}{\partial \delta_n} & | & \frac{\partial P_3}{\partial |V_2|} & \frac{\partial P_3}{\partial |V_3|} & \dots & \frac{\partial P_3}{\partial |V_n|} \\ \vdots & \vdots & \ddots & \vdots & | & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial \delta_2} & \frac{\partial P_n}{\partial \delta_3} & \dots & \frac{\partial P_n}{\partial \delta_n} & | & \frac{\partial P_n}{\partial |V_2|} & \frac{\partial P_n}{\partial |V_3|} & \dots & \frac{\partial P_n}{\partial |V_n|} \\ \hline \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \dots & \frac{\partial Q_2}{\partial \delta_n} & | & \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} & \dots & \frac{\partial Q_2}{\partial |V_n|} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \dots & \frac{\partial Q_3}{\partial \delta_n} & | & \frac{\partial Q_3}{\partial |V_2|} & \frac{\partial Q_3}{\partial |V_3|} & \dots & \frac{\partial Q_3}{\partial |V_n|} \\ \vdots & \vdots & \ddots & \vdots & | & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n}{\partial \delta_2} & \frac{\partial Q_n}{\partial \delta_3} & \dots & \frac{\partial Q_n}{\partial \delta_n} & | & \frac{\partial Q_n}{\partial |V_2|} & \frac{\partial Q_n}{\partial |V_3|} & \dots & \frac{\partial Q_n}{\partial |V_n|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \vdots \\ \Delta \delta_n \\ \hline \Delta |V_2| \\ \Delta |V_3| \\ \vdots \\ \Delta |V_n| \end{bmatrix} \quad (9)$$

In a simple way.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & | & J_2 \\ \hline J_3 & | & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \quad (10)$$

The elements of Jacobian matrix can be derived from the bus load power equations.

# N-R Polar Coordinates method without PV bus

$$P_p = \sum_{q=1}^n |V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q) \quad (5)$$

eq(5) can be rewritten as

$$P_p = |V_p|^2 Y_{pp} \cos \theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n |V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q) \quad (11)$$

J<sub>1</sub>

The diagonal elements of J<sub>1</sub> are

$$\frac{\partial P_p}{\partial \delta_p} = - \sum_{\substack{q=1 \\ q \neq p}}^n |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \quad (12)$$

The off diagonal elements of J<sub>1</sub> are

$$\frac{\partial P_p}{\partial \delta_q} = |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q), \quad q \neq p \quad (13)$$

# N-R Polar Coordinates method without PV bus

eq(5) can be rewritten as

$$P_p = |V_p|^2 Y_{pp} \cos \theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n |V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q) \quad (11)$$

J<sub>2</sub>

The diagonal elements of J<sub>2</sub> are

$$\frac{dP_p}{dV_p} = 2|V_p| Y_{pp} \cos \theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n |V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q) \quad (14)$$

The off diagonal elements of J<sub>2</sub> are

$$\frac{dP_p}{dV_q} = \cancel{|V_p|^2 Y_{pp} \cos \theta_{pp}} + |V_p Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q), \quad q \neq p \quad (15)$$

# N-R Polar Coordinates method without PV bus

$$Q_p = \sum_{q=1}^n |V_p| |V_q| |Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \quad (6)$$

Eq (6) can be rewritten as

$$Q_p = |V_p|^2 Y_{pp} \sin \theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \quad (16)$$

J<sub>3</sub>

The elements of J<sub>3</sub> (diagonal) are

$$\frac{\partial Q_p}{\partial \delta_p} = \sum_{\substack{q=1 \\ q \neq p}}^n |V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q) \quad (17)$$

The off diagonal elements of J<sub>3</sub> are

$$\frac{\partial Q_p}{\partial \delta_q} = - |V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q), \quad q \neq p \quad (18)$$

# N-R Polar Coordinates method without PV bus

Eq (6) can be rewritten as

$$Q_p = |V_p|^2 Y_{pp} \sin \theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \quad (16)$$

J<sub>4</sub>

The diagonal elements of J<sub>4</sub> are

The off diagonal elements of J<sub>4</sub> are

The off diagonal elements of J<sub>4</sub> are

$$\frac{\partial Q_p}{\partial V_q} = |V_p Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q), \quad q \neq p \quad (20)$$

$$\frac{\partial Q_p}{\partial |V_q|} = |V_p Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q), \quad q \neq p \quad (20)$$

# N-R Polar Coordinates method without PV bus

The off diagonal elements of  $J_1$  are

$$\frac{\partial P_p}{\partial \delta_q} = |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q), \quad q \neq p \quad (13)$$

The off diagonal elements of  $J_2$  are

$$\frac{\partial P_p}{\partial |V_q|} = |V_p Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q), \quad q \neq p \quad (14)$$

The off diagonal elements of  $J_3$  are

$$\frac{\partial \theta_p}{\partial \delta_q} = -|V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q), \quad (15)$$

The off diagonal elements of  $J_4$  are

$$\frac{\partial \theta_p}{\partial |V_q|} = |V_p Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q), \quad q \neq p \quad (20)$$

# N-R Polar Coordinates method without PV bus

It may be noted that, do not see the symmetry in the Jacobian, if polar coordinates are used. However, if replace  $\Delta |V|$  by  $\frac{\Delta |V|}{|V|}$  in eq (10), it modifies as

$$\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \frac{\Delta |V|}{|V|} \end{bmatrix} \quad \text{--- (21)}$$

The equations for ~~diagonal~~<sup>Jacobian</sup> elements are

~~Diagonal~~ H  
diagonal element

$$H_{pp} = \frac{\partial P_p}{\partial \delta_p} = \text{Same as eq (12)}$$

off dia

$$H_{pq} = \frac{\partial P_p}{\partial \delta_q} = \text{Same as eq (13)}$$

# N-R Polar Coordinates method without PV bus

J

The diagonal ( $J_{pp}$ ) & off diagonal elements ( $J_{pq}$ ) are same as eq (17) & (18).

N

The diagonal elements of N

$$N_{pp} = \frac{\partial P_p}{\frac{\partial |V_p|}{|V_p|}} = \frac{\partial P_p}{\partial |V_p|} |V_p|$$

from eq (14)

$$N_{pp} = \frac{\partial P_p}{\partial |V_p|} |V_p| = 2|V_p|^2 Y_{pp} \cos \theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n |V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q) \quad (22)$$

The off diagonal elements of N

$$N_{pq} = \frac{\partial P_p}{\frac{\partial |V_q|}{|V_q|}} = \frac{\partial P_p}{\partial |V_q|} |V_q| = |V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q), \quad q \neq p \quad (22)$$



# N-R Polar Coordinates method without PV bus

L

The diagonal elements of  $L$  are

$$L_{pp} = \frac{\partial \theta_p}{\partial |V_p|} = \frac{\partial \theta_p}{\partial |V_p|} |V_p|$$

$$= 2|V_p|^2 Y_{pp} \sin \theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q)$$

The off-diagonal elements of  $L$  are

$$L_{pq} = \frac{\partial \theta_p}{\partial |V_q|} |V_q| = |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q),$$

$$q \neq p$$

(24)

Symmetry

$$H_{pq} = +L_{pq}$$

# N-R Polar Coordinates method without PV bus

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & | & J_2 \\ -J_3 & | & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

$$\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \frac{\Delta |V|}{|V|} \end{bmatrix}$$

# N-R Polar method with PV bus

(B) when PV buses are present

→ Now consider when PV buses are included in 'n' bus power systems.

→ For PV bus, the reactive power  $Q_p$  is not specified and  $V_p$  specified.

We know

$$\begin{bmatrix} \Delta P_p \\ \Delta Q_p \end{bmatrix} = \begin{bmatrix} A & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \frac{\Delta |V|}{|V|} \end{bmatrix}$$

In the above equations, for PV buses,  $\Delta Q_p$  does not appear in the left side and  $\frac{\Delta |V|}{|V|}$  does not appear on the right side.

# N-R Polar method with PV bus

$$\begin{bmatrix} \Delta P \\ 0 \end{bmatrix} = \begin{bmatrix} H & N \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \frac{\Delta |V_q|}{|V_q|} \end{bmatrix} \quad \text{--- (25)}$$

only two terms  $H_{pq}$ ,  $N_{pq}$  are present

$$H_{pq} = \frac{\partial P_p}{\partial \delta_q} \quad , \quad N_{pq} = \frac{\partial P_p}{\frac{\partial |V_q|}{|V_q|}} \quad \text{--- (26)}$$

# Advantages of FDLF

## Merits of Fast Decoupled Load Flow Method

1. It is the fastest of all the load flow methods to obtain the load flow solution.
2. The programming is very simple.
3. The memory requirement is less.
4. Number of iterations are independent of size of the system.
5. The time required per iteration is less.
6. Number of iterations required are less i.e., 1 or 2 iterations only.
7. Efficiently used for both smaller and larger systems.
8. Rate of convergence characteristics is faster than other methods.
9. It can efficiently be used for both larger and smaller systems.

# Disadvantages of FDLF

## Demerits of Fast Decoupled Load Flow Method

1. System configuration changes are easily effected and when the solutions are adjusted then the number of iterations are increased.
2. When ever the value of  $\alpha$  is changed, the array  $B_p$  has to be reformulated and inverted until the power regulating phase shifting transformers are present.

# Comparison of different load flow methods

Derive D.C power flow equations.

We know that,

The complex power is given as,

$$S_p = P_p + jQ_p$$

Where,

$$P_p = |V_p| \sum_{q=1}^n |V_q| |Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q)$$

$$Q_p = |V_p| \sum_{q=1}^n |V_q| |Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q)$$

Considering only the real part, we have,

$$P_p = |V_p| \sum_{q=1}^n |V_q| |Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q)$$

The above equation can be modified as,

$$\Rightarrow P_p = |V_p| \sum_{q=1}^n |V_q| [G_{pq} \cos(\theta_p - \theta_q) + B_{pq} \sin(\theta_p - \theta_q)]$$

# Comparison of different load flow methods

Now, let us assume that, the resistance of a transmission line be zero, all the voltage magnitudes be unity and the cosine of angle  $(\theta_p - \theta_q) \approx 1$  and sine of angle  $(\theta_p - \theta_q) \approx \theta_p - \theta_q$ . When the resistance of transmission line is small, the conductance will also be small. Hence,  $G = 0$ , therefore equation (2) becomes,

$$P_p = |1| \sum_{q=1}^n |1| [(0)(1) + B_{pq}(\theta_p - \theta_q)]$$
$$\Rightarrow P_p = \sum_{q=1}^n B_{pq}(\theta_p - \theta_q) \quad \dots (3)$$

Where,

$$B_{pq} = \frac{1}{X_{pq}}$$

The equation (3), represents the expression for the D.C power flow.



**PSA**  
**V UNIT**  
**POWER SYSTEM STABILITY**  
**ANALYSIS**

# SYLLABUS

## **POWER SYSTEM STABILITY ANALYSIS**

Elementary Concepts of Steady State, Dynamic and Transient Stabilities - Description of: Steady State Stability Power Limit, Transfer Reactance, Synchronizing Power Coefficient, Power Angle Curve and Determination of Steady State Stability and Methods to Improve Steady State Stability - Derivation of Swing Equation - Determination of Transient Stability by Equal Area Criterion, Application of Equal Area Criterion, Critical Clearing Angle Calculation. Solution of Swing Equation by 4th Order Runge Kutta Method (up to 2 iterations) - Methods to improve Stability - Application of Auto Reclosing and Fast Operating Circuit Breakers.

# 1. Elementary Concepts of Stability

## Power system stability

consider a power system that has a no. of synchronous machines (alternators) operating in parallel. For the power system to remain stable, the various synchronous machines in the power system should remain in synchronism.

Suppose the system is subjected to any kind of disturbance, then it should be capable of bringing the system to a normal or stable condition by developing the restoring force. This capability of a system to return to the original position on occurrence of disturbance is called "stability".

# 1. Elementary concepts of Stability

In General, the stability can be defined as The

"Ability of a system to maintain synchronism even if it is subjected to disturbances."

(or)

The ability of a system to reach a normal or stable operation after being subjected to disturbances is called stability.

# 1. Elementary concepts of Stability

## Types of stability

Depending upon the magnitude of disturbance, the stability is divided into three types.

- ① steady state stability
- ② Transient stability
- ③ Dynamic stability

# 1. Elementary concepts of Stability

## ① steady state stability :

" It is the ability of power system to maintain synchronism after being subjected to the small & gradual disturbances is known as steady state stability.

→ here the small & gradual disturbances are mainly due to change in load, change in generation, change in the speed of prime mover etc.

→ steady state stability refers to inherent stability that exists without the aid of automatic control devices.

# 1. Elementary concepts of Stability

## (2) Transient stability

"The ability of a power system to maintain synchronism or normal operation after being subjected to the sudden & large disturbances is known as Transient stability."

→ large & sudden disturbances are due to tripping of generators, sudden change in load, switching operation and faults, tripping of lines etc.

→ The action of voltage regulators and turbine governors is not included in transient stability.

# 1. Elementary concepts of Stability

## (3) Dynamic stability

"The ability of power system to maintain synchronism after transient stability period till the system attains a new steady state equilibrium condition, is known as dynamic stability."

→ It is ~~concerned with~~ an extension of steady state stability.

→ The dynamic stability is concerned with small disturbance that lasts for a long time including the automatic control devices.

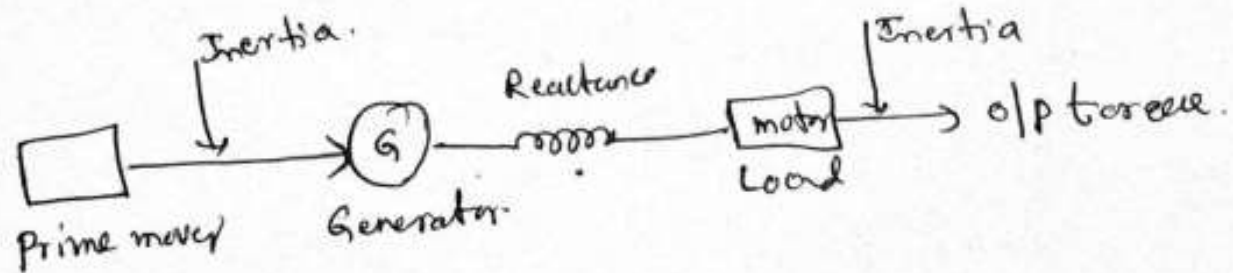


# 1. Elementary concepts of Stability

- Dynamic stability refers to the artificial stability given to an inherently unstable system by automatic control devices.
- In this stability, the disturbances are due to short-circuit, loss of generation or loss of load etc.
- This stability can be significantly improved through the use of Power system stabilizers and study has to be carried out 5-10 sec & sometimes upto 30 seconds.

# 1. Elementary concepts of Stability

Essential factors in the stability problem



There are two types of factors.

(1) Mechanical

(i) Prime mover input torque

(ii) Inertia of prime mover & Generator

(iii) Inertia of motor & shaft load

(2) Electrical factors

(i) Internal voltage of syn. Generator

(ii) Reactance of the system including generator, line & motor

(iii) Internal vol of syn. motor.

# 1. Elementary concepts of Stability

## Stability limit

(3)

The maximum power that can be transferred by the power system from source to load under stable conditions is known as "stability limit".

Depending upon the magnitude of disturbances [small, large, slower]

There are three types of stability limits.

(i) steady state stability limit :

The maximum power that can be transferred by the system from source to load under stable conditions, even though the system is subjected to small & gradual disturbances is known as "steady state stability limit".

In order to maintain steady state stability, every system should operate below this limit.

# 1. Elementary concepts of Stability

## (ii) Transient state stability limit :

The maximum power that can be transferred by the system from source to load under stable conditions, even though the system is subjected to sudden & large disturbances due to load changes is known as "Transient state stability limit."

## (iii) Dynamic state stability limit

The maximum power that can be transferred by the system from source to load under stable conditions, even though the system is subjected to small disturbances that lasts for a long time is known as "Dynamic state stability limit."

# Methods to improve SSS

Explain the methods to improve the steady state stability power

We know the maximum power equation i.e. the equation for maximum steady state stability limit

$$P_r = \frac{V_s V_r}{X}$$

(max)

- ① The steady state stability limit can be increased by decreasing the value of line reactance ( $X$ )
- ② The value of line reactance can be reduced
  - (i) by using parallel lines instead of single line
  - (ii) By using bundled conductors  $[L \downarrow, X_L \uparrow = 2\pi f L \downarrow]$
  - (iii) By using series capacitors for over head lines
  - (iv) By series reactors for underground cables
  - (v) By adding synchronous machines in parallel

# Methods to improve SSS

- (3) By increasing sending end & receiving end voltages
- (4) By using higher excitation voltages
- (5) By using auto reclosing circuit breakers.

# Power Angle Curve

Power angle curve of synchronous machine

The steady state power equation is given by

$$P_r = \frac{V_r V_s}{X} \sin \delta \quad (1)$$

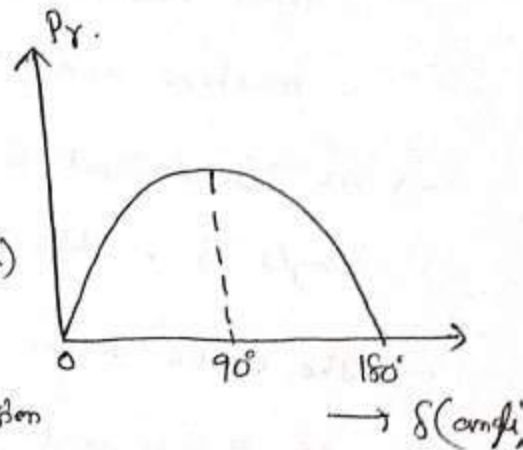
For drawing Power angle curve,

the following are the assumptions

(1) neglect the armature resistance ( $R_a$ ) of synchronous machine

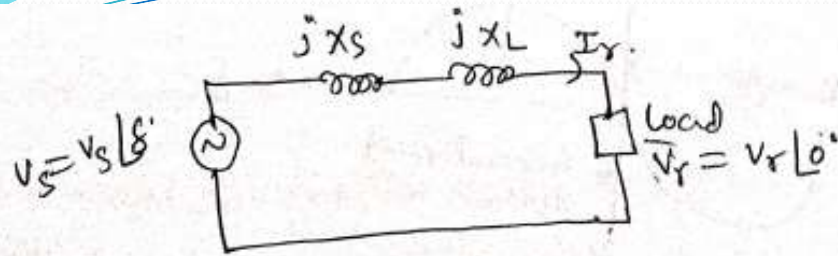
(2) Neglect the resistance of Transmission line

(3) neglect the Shunt admittance ( $\alpha$ ) of transmission line



Power angle curve.

# Power Angle Curve



From the fig.  $\bar{V}_s = V_s \angle \delta' =$  Sending end voltage / phase.

$\bar{V}_r = V_r \angle \delta'' =$  Receiving end voltage.

$jX_s =$  synchronous reactance / phase

$jX_L =$  reactance of Transmission line / phase.

$jX = j(X_s + X_L) =$  Transfer reactance / phase.

Now,

$$P_r = \frac{V_s V_r}{X} \sin \delta \quad \text{--- (2)}$$



# Power Angle Curve

Now,

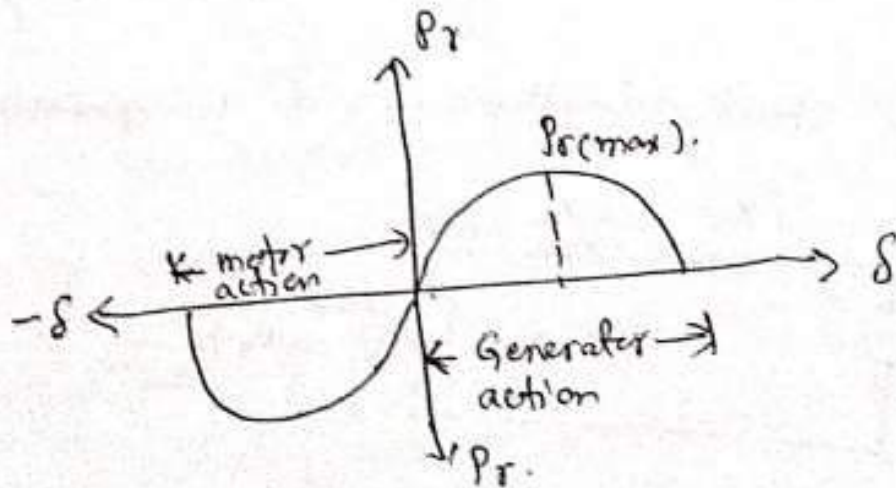
$$P_r = \frac{V_s V_r}{X} \sin \delta \quad \text{--- (2)}$$

- (i) For +ve values of ' $\delta$ ',  $P_r$  is +ve, i.e. power is transferred from source to load then the synchronous machine acts as a "generator" [ $0^\circ < \delta < 180^\circ$ ]
- (ii) For -ve values of ' $\delta$ ',  $P_r$  is -ve i.e. power is flow from receiving end to sending end then the synchronous machine acts as "motor". [ $-180^\circ < \delta < 0^\circ$ ]

# Power Angle Curve

→ As the amount & direction of power flow depends on angle ' $\delta$ '. This angle is called power angle.

→ The curve drawn b/w power angle load angle ( $\delta$ ) is as power angle curve.



→ maximum power is transferred when  $\delta = 90^\circ$

→ The typical value of ' $\delta$ ' is about  $30^\circ$  to  $45^\circ$

# Transfer Reactance

Transfer reactance ( $x$ )

(6)

The reactance present b/w the generator point and load point is known as "transfer reactance".

→ it is denoted by ' $x$ '

$$\rightarrow x = x_s + x_L$$

where ↓

$x_s$  = reactance of syn. machine

$x_L$  = reactance of Trll line.

→ The power transferred through transmission line is given by

$$P_r = \frac{V_s V_r}{x} \sin \delta \quad \text{--- (1)}$$

For fixed values of  $V_s$ ,  $V_r$  &  $\delta$

# Transfer Reactance

$$P_r \propto \frac{1}{X}$$

∴ Power transferred from source to load is inversely proportional to reactance ( $X$ ).

→ The value of Transfer reactance ( $X$ ) ~~can be~~ is reduced to improve the stability of power system from source to load.

→ The value of ' $X$ ' can be reduced by following methods.

- By using two parallel lines instead of single line
- By using bundled conductors
- By using series capacitors for overhead lines.
- By using series reactors for underground cables.

# Synchronizing Power Coefficient

## Synchronizing power coefficient

The synchronous power transferred by power system (or) power transferred by a syn. machine connected to infinite bus is given by

$$P = P_e = P_r = \frac{V_s V_r}{x_s} \sin \delta \quad \text{--- (1) } \Rightarrow \text{cylindrical}$$

where  $\delta \rightarrow$  load angle & it is variable

$$\text{Synchronous power} = P$$

$$\text{Synchronizing power} = P_s$$

Synchronizing power

$$\text{Coefficient} = P_{sy}$$

# Synchronizing Power Coefficient

Def. of  $P_{sy}$

It is the rate at which  $P$  varies with  $\delta$  (or)

$$P_{sy} = \frac{dP}{d\delta}$$
$$= \frac{d}{d\delta} \left[ \frac{V_s V_r}{x_s} \sin \delta \right]$$

$$P_{sy} = \frac{V_s V_r}{x_s} \cos \delta \quad \text{--- (2)}$$

eq (2) is called synchronizing power coefficient (or) stability factor (or) stiffness factor.

# Synchronizing Power Coefficient

## Significance

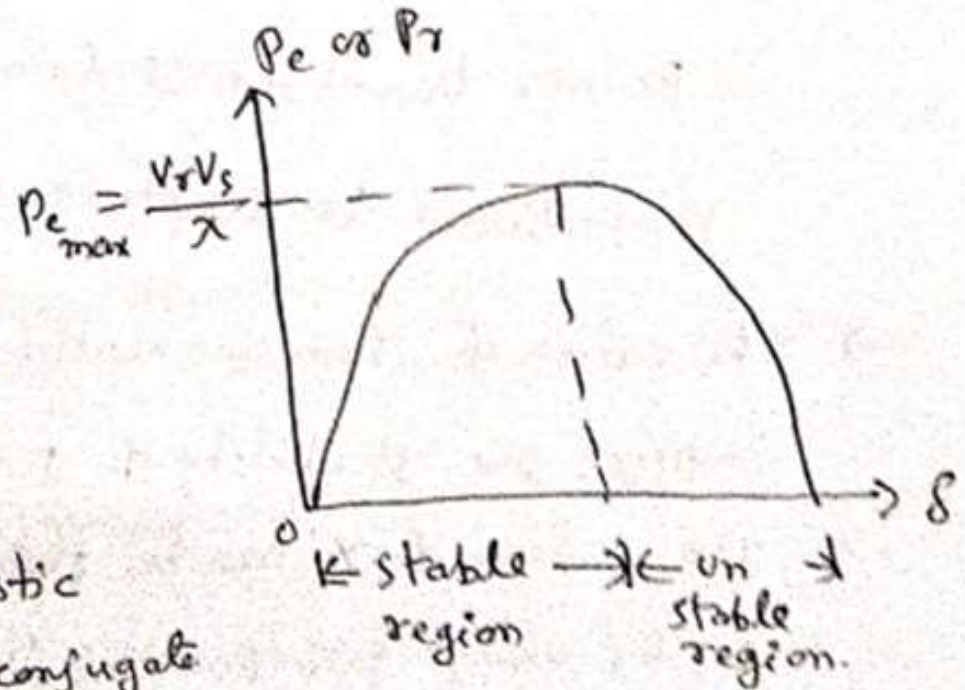
case (i)  $[0 < \delta < 90^\circ]$

For  $0 < \delta < 90^\circ$ ,

$$\frac{dP_e}{d\delta} = +ve \text{ value.}$$

- The system is stable
- The roots of characteristic equation are complex conjugate

- The response of the system is damped oscillatory



# Synchronizing Power Coefficient

Case (2) :  $\delta = 90^\circ$

(9)

when  $\delta = 90^\circ$ ,  $\frac{dP_e}{d\delta} = 0$

- The system is critically stable
- The roots of characteristic equation are real and equal

Case (3) :  $90^\circ < \delta < 180^\circ$

$\frac{dP_e}{d\delta}$  is -ve

- The system is unstable
- The roots of characteristic equation are real & unequal.



# Synchronizing Power Coefficient

Note:

In order to achieve the maximum power, eq (2) must be equal to zero. i.e

$$\frac{dP_e}{d\delta} = 0$$

$$\frac{V_s V_r}{X} \cos \delta = 0$$

$$\boxed{\delta = 90}$$

$$\therefore \boxed{P_{e(max)} = \frac{V_s V_r}{X}}$$

- Note:
- (1) Synchronizing power coefficient as taken is the degree of stability.
  - (2) Based on the value of synchronizing power coefficient, the stability analysis of a system can be done.

# Methods to improve SSS

Methods to improving steady state stability

The following methods are used to improve the steady state stability

(1) Use of Breaking Resistors

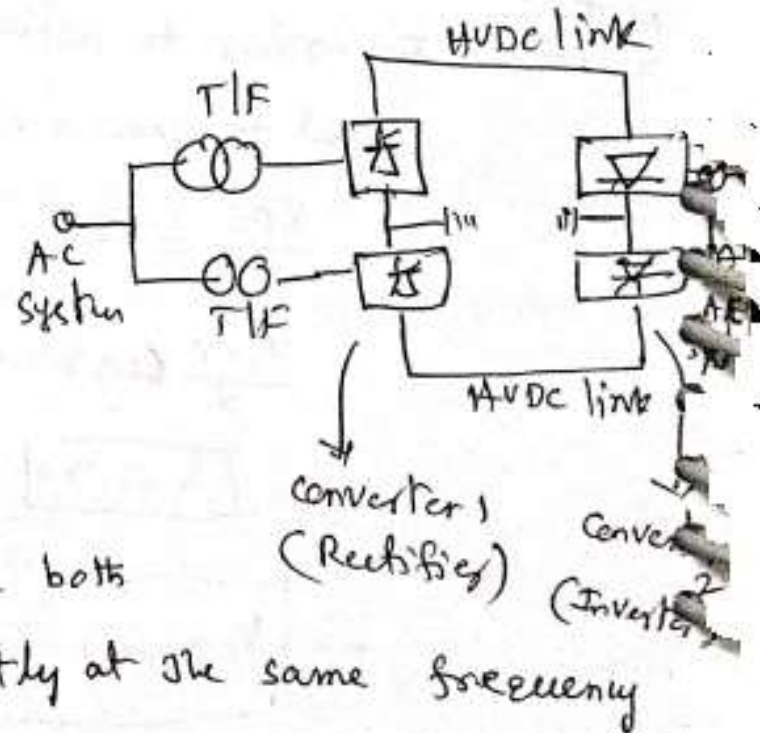
Breaking resistors reduce the load on the generators during fault when large load on the system is lost. In this way stability can be maintained.

# Methods to improve SSS

## (2) Use of HVDC links

Using large amount of HVDC links with Thyristors improve stability.

- By using HVDC links, if any fault occurs on one system, the stability of other link (system) will not be effected.
- Here DC link is asynchronous i.e both the links (systems) are not exactly at the same frequency like in an AC link.



# Methods to improve SSS

## (3) Use of Full load rejection technique

- In certain cases, it is difficult to maintain stability even after using fast valving along with high speed clearing time

→ So, by using full load rejection technique the system stability can be maintained.

→ In full load rejection technique, the system can be reloaded and resynchronized after fault occurs.

## (iv) By pass valving:

- By using by pass valving, the mechanical power input to the turbine can be reduced to improve stability.

- The control scheme senses the difference b/w the mechanical i/p and reduced the electrical o/p during fault and then it ~~opens~~ <sup>closes</sup> the closing of the turbine valve thereby decreasing the power o/p.

# Methods to improve SSS

## (v) Short circuit current limiters

- These are used to limit the short ckt current in the distribution lines & long transmission lines & also modifies the transfer impedance during fault condition there by raising the system stability.

## (vi) Fast acting automatic voltage Regulators

- By using Fast acting Automatic voltage regulators, we can obtain the satisfactory operation of a Syn-Generator at high load angles and also during transient conditions of a complex power system.

# Methods to improve SSS

## (vii) Single pole switching

- By using this method, we can de energise only the faulted phase of the transmission system instead of the whole system, by arranging properly the protection scheme & breakers, because most of the faults are L.G. faults only on transmission lines.

# SWING EQUATION

\* Imp

## Swing equation

Def:

It is defined as "an equation relating the relative motion of the rotor angle with respect to its stator field as a function of time"

→ It is great important for the study of transient stability.

→ mathematically, it is given as

$$m \frac{d^2 \delta}{dt^2} = P_m - P_e$$

# SWING EQUATION

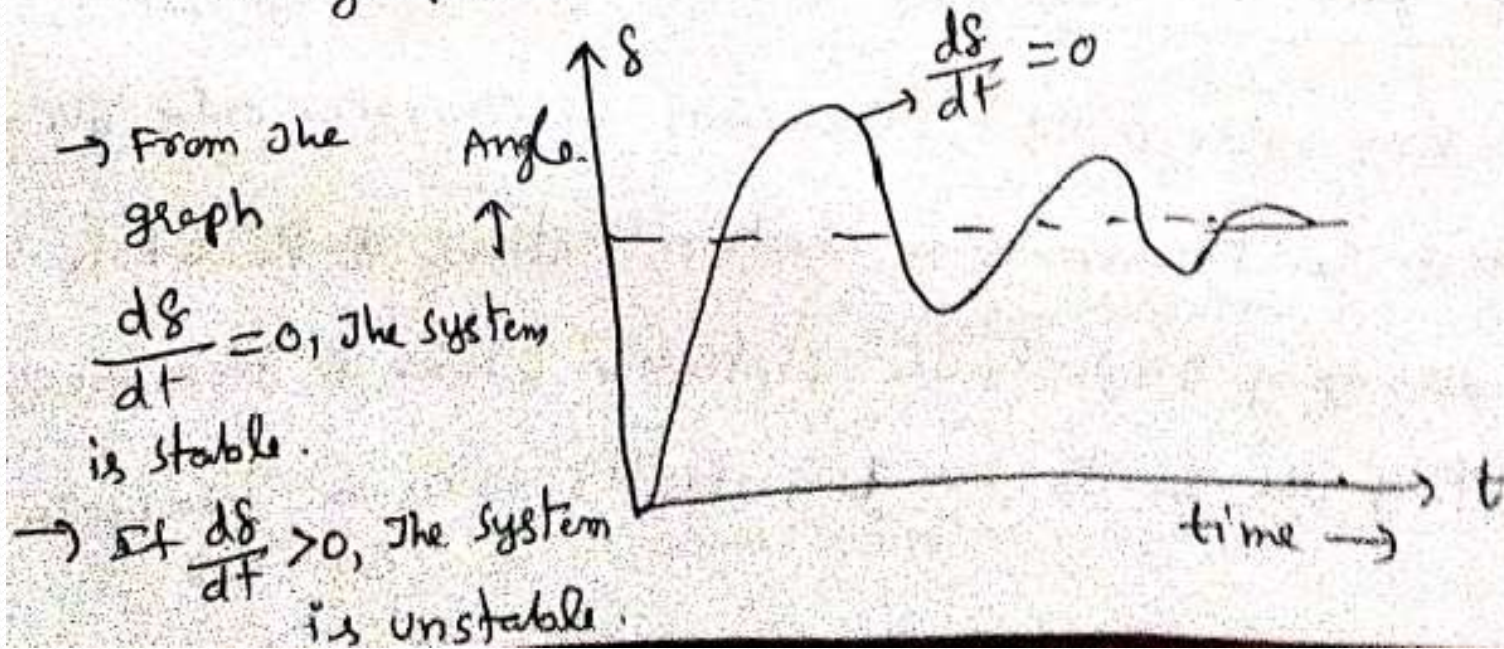
## Importance

- it is greater importance for study of Transient stability
- The swing equation is used to determine the stability of a rotating synchronous machine within a power system.
- when swing equation is solved, the expression for ' $\delta$ ' is obtained which is the function of time. The graph of this solution is known as "swing curve" of a machine.
- By investigating the swing curves of overall machines connected to the system we can know whether the machine continue in synchronous or not after a disturbance.



# SWING EQUATION

→ The graph represents variation of ' $\delta$ ' wrt ' $t$ '



→ Hence, the swing equation indicates whether the rotor should accelerate or decelerate whenever there is an imbalance b/w mechanical i/p & electrical o/p. (9)

# SWING EQUATION

State the assumptions made in deriving swing equation of

single machine connected to infinite bus

- ① mechanical i/p for a generator and mechanical load on a motor is assumed to be constant
- ② Iron, friction and windage losses are neglected.
- ③ Resistance of a transmission line & synchronous machines are neglected.
- ④ Damping terms produced by the damper windings of syn. m/c is ignored.
- ⑤ Time constants (sub transient & armature) are neglected.
- ⑥ Voltage behind the transient reactances are assumed to be constant throughout the analysis.

# SWING EQUATION

- There are three types of transient disturbances occurring in

The study of power system are

(1) Load increase

(2) switching operations

(3) Faults with subsequent ext isolation.

The problem of transient stability can be overcome by

solving

1. Swing equation

2. Equal area criterion.

# Applications of Equal Area Criterion

## Applications of equal area criterion

There are three applications for equal area criterion

These are

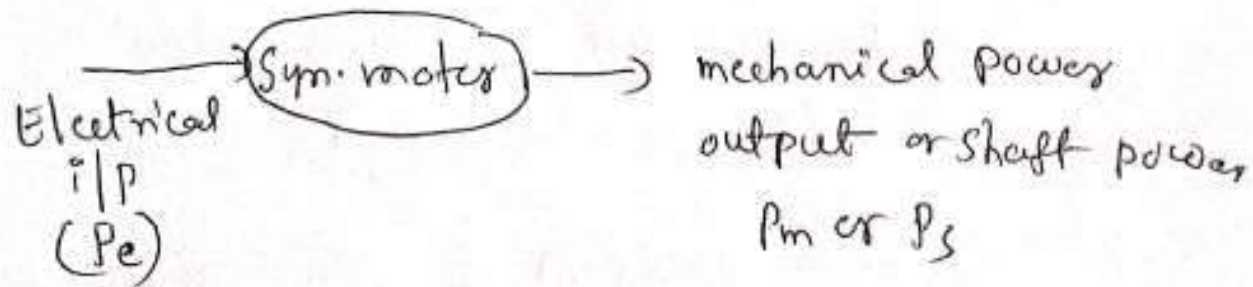
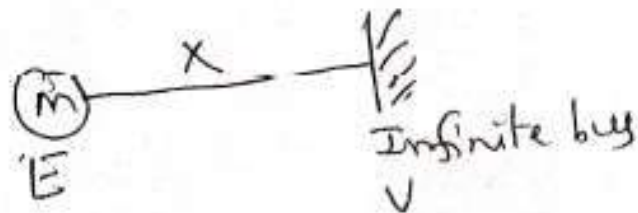
- (1) Sudden load increases
- (2) Switching out of one of the line (or) opening of one of the parallel lines
- (3) Fault cleared after some time.

## 4. Determination of Critical Clearing Angle Calculation

# Applications of Equal Area Criterion

(1) sudden load increases

Let us consider a Syn. motor is connected to an infinite bus bar.

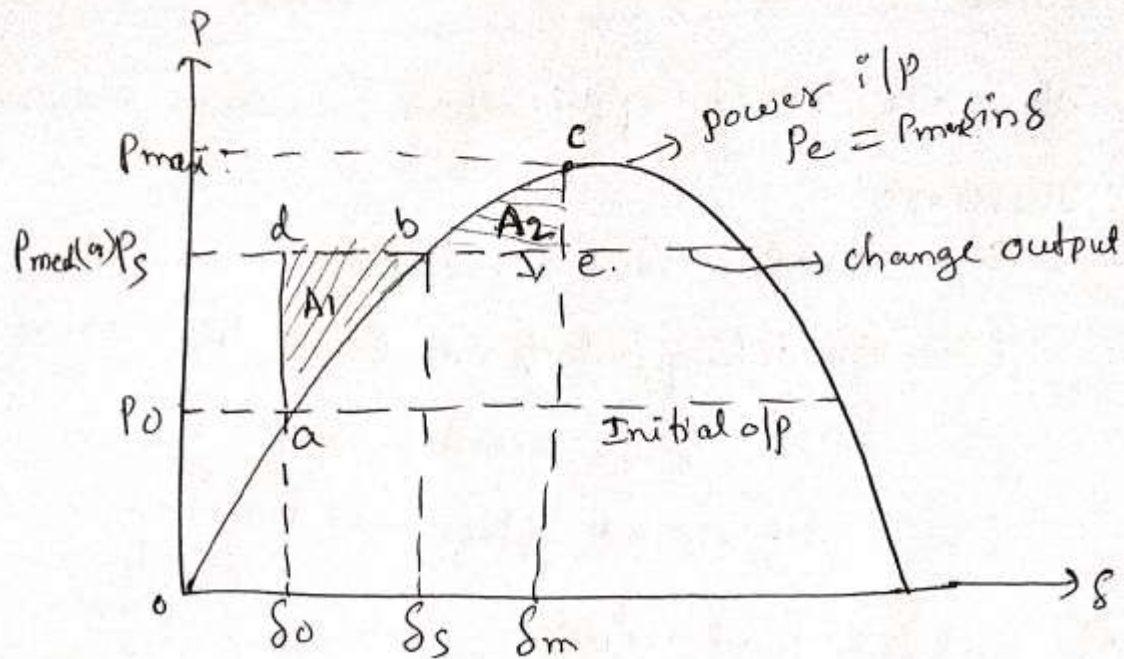


Whenever a sudden change in load, the behaviour of load angle ( $\delta$ ) can be studied using following three points.

# Applications of Equal Area Criterion

- (a) Torque or load angle does not change; if the rotor runs at synchronous speed.
- (b) Load angle decreases, if the rotor speed is greater than the syn. speed.
- (c) Load angle increases, if the rotor speed is decreases i.e. it is less than syn. speed.

# Applications of Equal Area Criterion



At point 'a'

At initial condition :

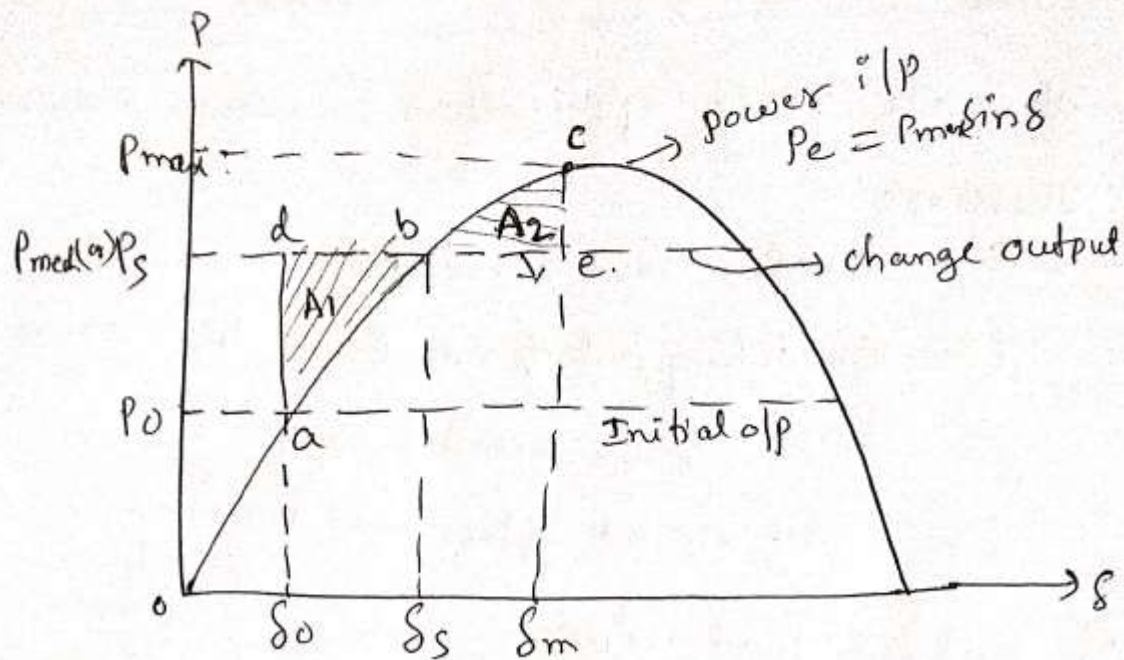
- Power input = power output

$$(P_e = P_s)$$

- rotor speed ( $\omega$ ) = synchronous speed ( $\omega_s$ )

- load angle ( $\delta$ ) =  $\delta_0$

# Applications of Equal Area Criterion



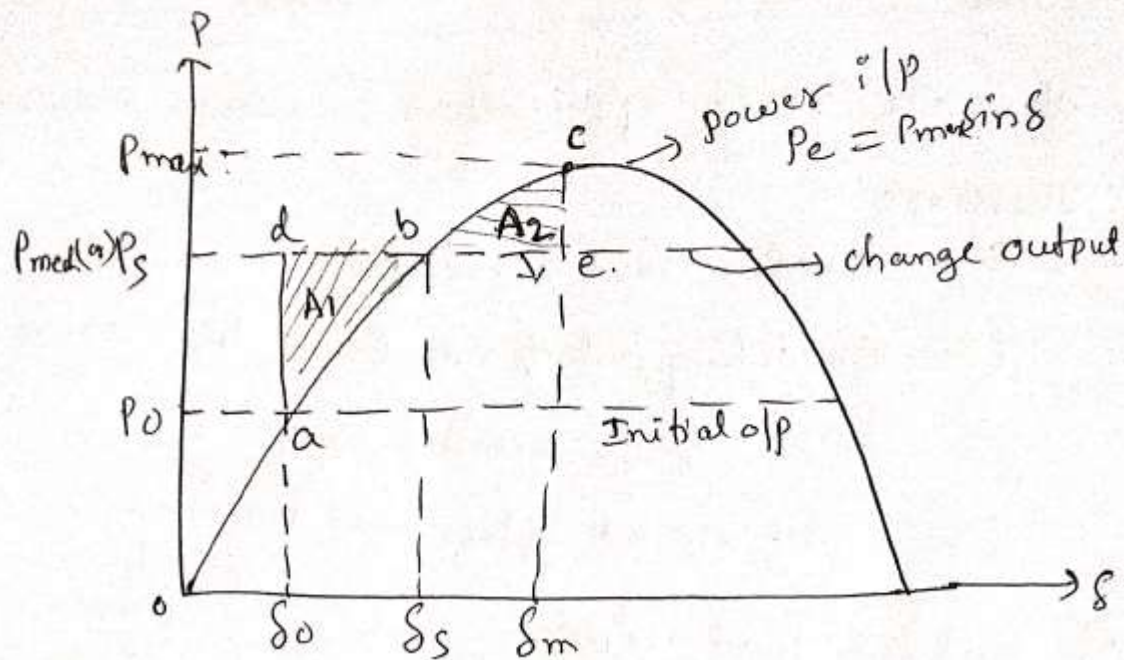
Due to sudden increasing in load

- when load increases, load angle increases then rotor speed decreases.

- $P_e < P_s$
- $\omega < \omega_s$
- $\delta > \delta_0$
- deceleration (rotor)



# Applications of Equal Area Criterion



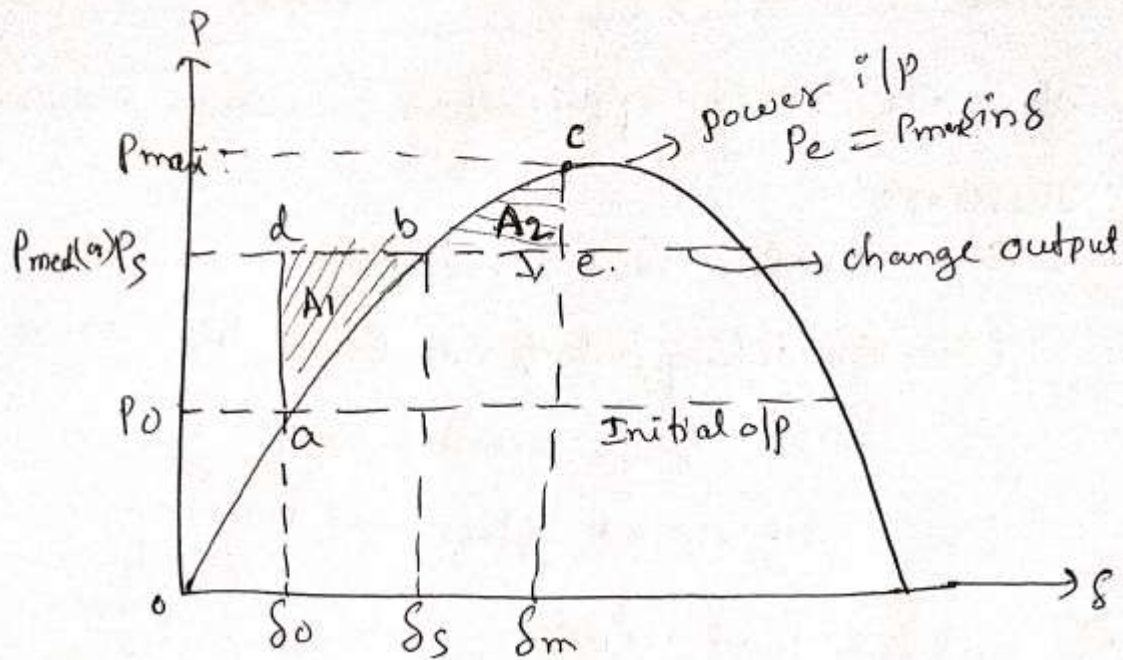
From A to B

- $P_e < P_s$
- $\omega < \omega_s$
- $s > s_0$
- Deceleration (rotor)

At point B

- $P_e = P_s$
- Decelerating force is zero but due to inertia of rotor motion.
- $\omega < \omega_s$  (minimum rotor speed)
- $s$  goes on increasing.

# Applications of Equal Area Criterion



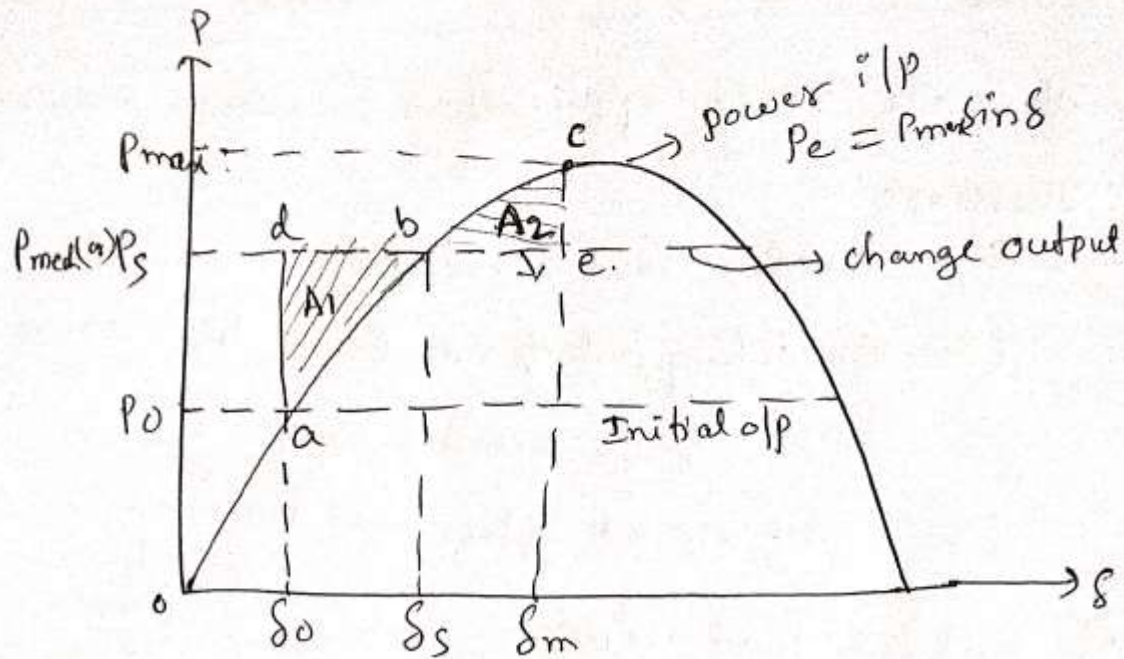
From B to c

- $P_e > P_s$
- Torque angle  $\delta$  increases
- $\omega < \omega_s$
- rotor acceleration

At point 'c'

- $P_e > P_s$
- Load angle increases,  $\delta = \delta_{max}$
- $\omega = \omega_s$
- Rotor accelerate.

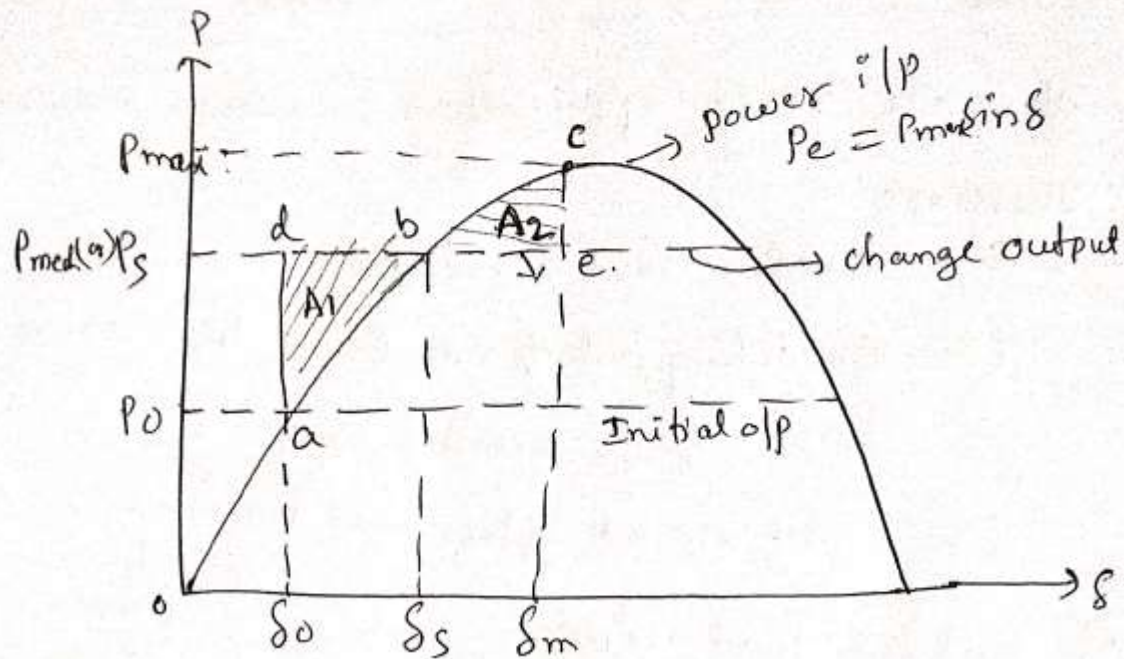
# Applications of Equal Area Criterion



From C to B

- $P_e > P_s$
- load angle starts decreasing, the speed goes on increasing till it reaches B'
- $\omega > \omega_s$
- rotor acceleration.

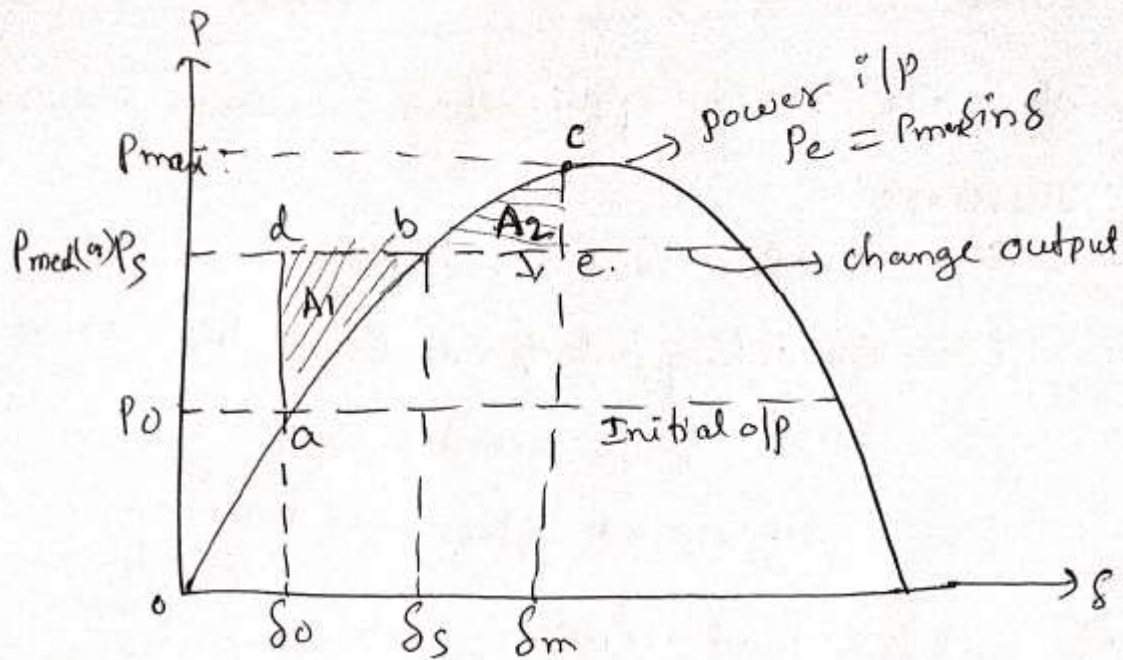
# Applications of Equal Area Criterion



At point B

- $P_e = P_s$
- Accelerating force is zero, but due to inertia of the rotor
  - $\omega > \omega_s$  [max. speed of rotor]
- $s = s_s$

# Applications of Equal Area Criterion



From B to A

- $P_e < P_s$  or  $P_s > P_e$
- rotor starts decelerating
- $\omega > \omega_s$
- $\delta$  starts decreasing

At Point-A

- $P_e < P_s$
- $\delta$  decreasing
- $\omega = \omega_s$
- rotor is at syn. speed

# Applications of Equal Area Criterion

(2) Switching out of one of the parallel lines

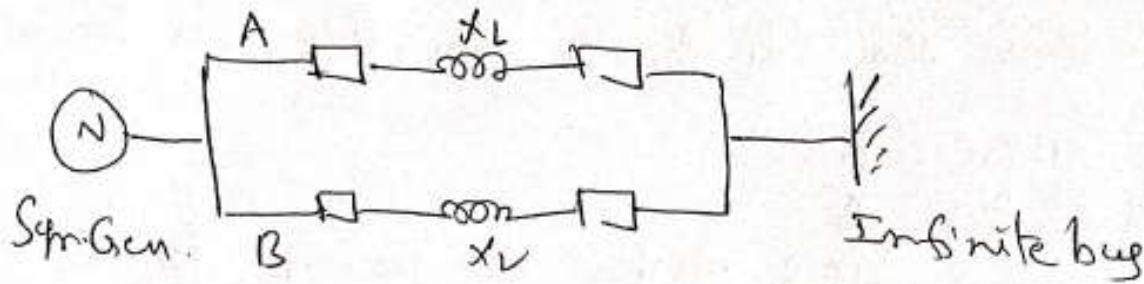


Fig (a)

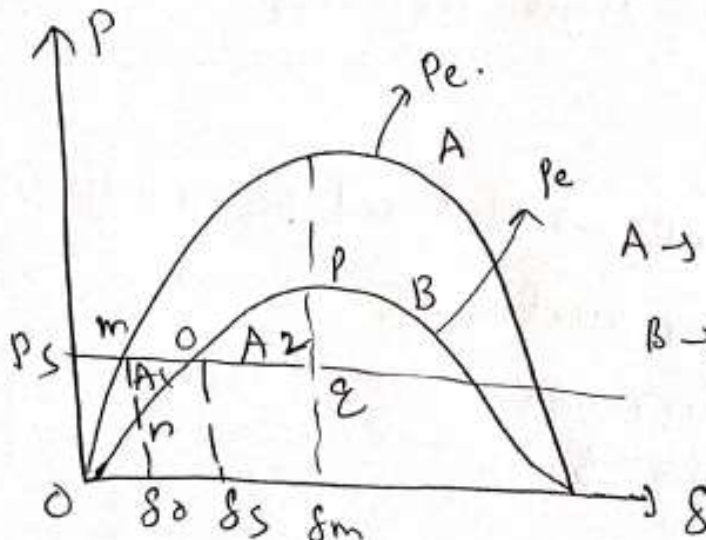
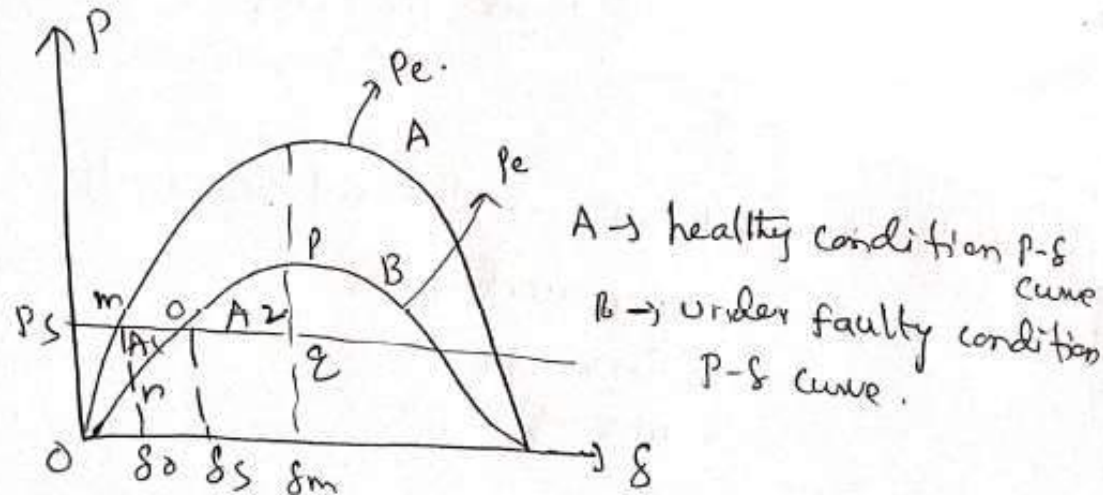


Fig (b)

$\rightarrow$  (G)  $\rightarrow$  Electrical Power o/p  
 mechanical power i/p  
 $(P_s)$   $(P_e)$

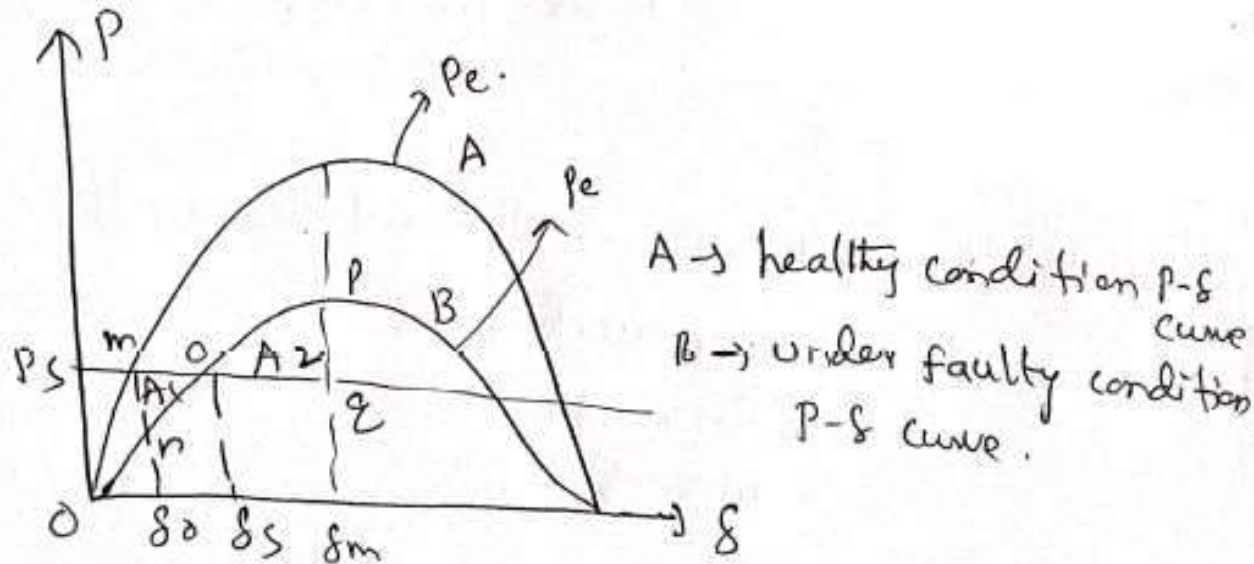
A  $\rightarrow$  healthy condition P- $\delta$  curve  
 B  $\rightarrow$  under faulty condition P- $\delta$  curve.

# Applications of Equal Area Criterion



From Fig(a), A Syn-Generator is connected to an infinite bus through two parallel feeders whenever a fault occurs at line 'B', (00) opening of the trail line which results increase in equivalent reactance and hence decrease in the max. power transferred. Because of this, the Syn-Generator lose synchronism even though the load could be supplied over the remaining line under steady state condition.

# Applications of Equal Area Criterion

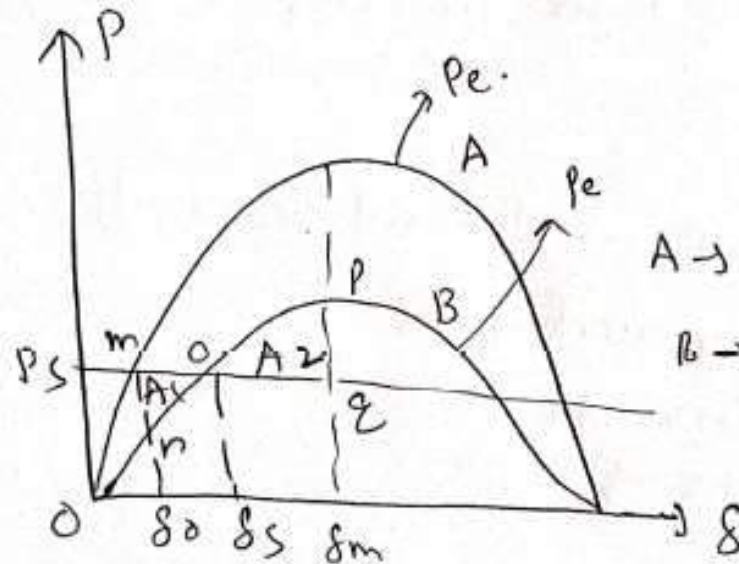


on curve B.  
At point 'n'

Since mechanical power  $i/p$  is constant which is higher than the power  $o/p$  [ $P_s > P_e$ ], rotor accelerates and hence load angle increases



# Applications of Equal Area Criterion



A → healthy condition P-δ curve  
 B → under faulty condition P-δ curve.

From n to o:

mech. i/p power  $P_s > P_e \rightarrow$  electrical power o/p  
 rotor accelerates  
 $\delta$  increases.  
 $\omega > \omega_s$ .

At 'o':

$$P_s = P_e$$

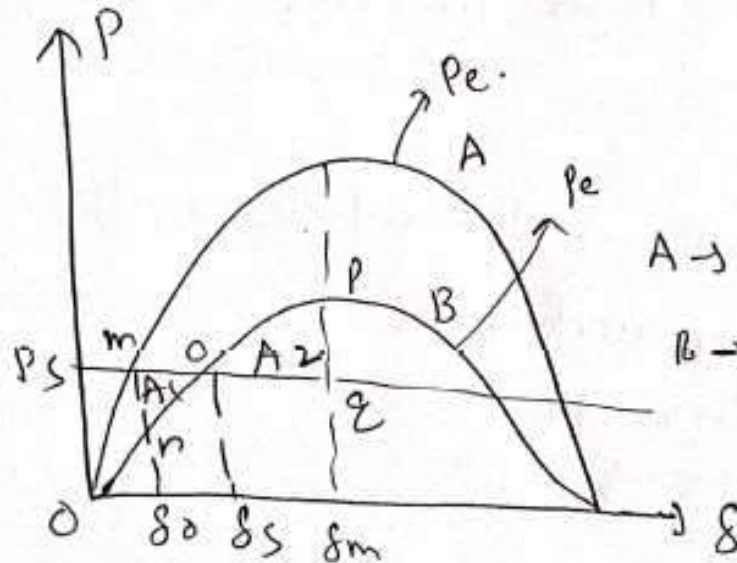
accelerating power becomes zero, but due to inertia.

$\omega > \omega_s$ . (max. speed).

$$\delta = \delta_s$$

Speed continues to increase.

# Applications of Equal Area Criterion



A → healthy condition P-δ curve  
 B → under faulty condition P-δ curve.

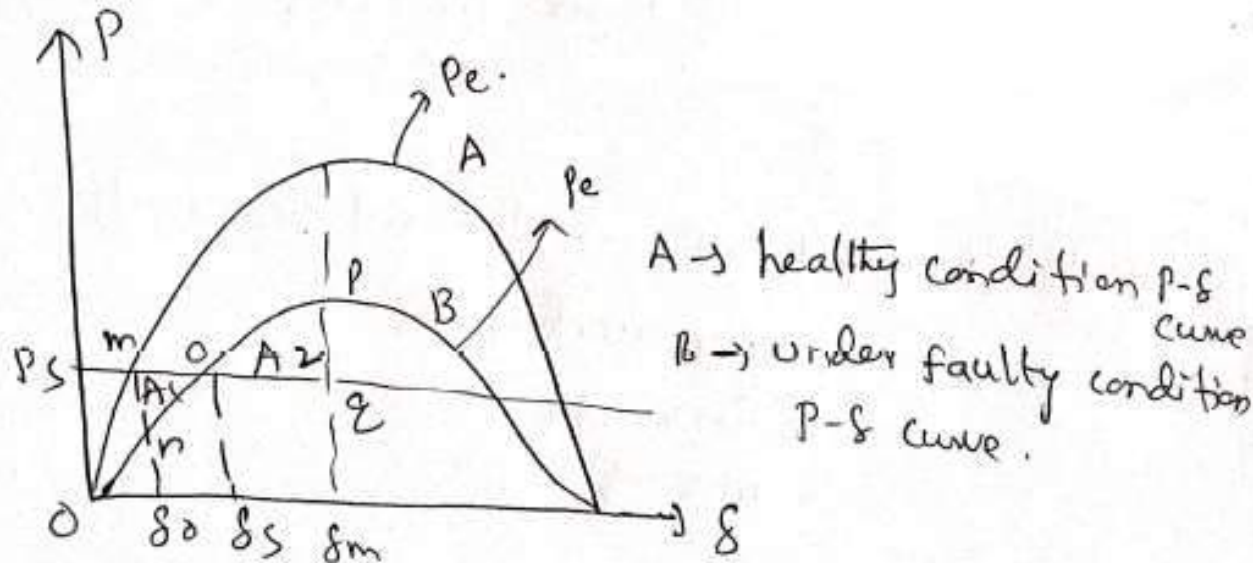
From 0 to P

$P_s < P_e$   
 rotor starts decelerating  
 $\omega > \omega_s$   
 $\delta$  increases

At point P

$P_s < P_e$   
 rotor speed decelerates  
 $\omega = \omega_s$   
 $\delta = \delta_m$

# Applications of Equal Area Criterion



From p to o

$$P_s < P_e$$

rotor deceleration

$\delta$  decreases

$$\omega < \omega_s$$

At 'o'

$$P_s = P_e$$

rotor deceleration stops. but due to inertia

$$\omega < \omega_s \quad (\text{min speed})$$

$$\delta = \delta_s$$

# Factors Affecting Transient Stability

Discuss the various factors that affects the transient stability of a power system.

Factors affecting transient stability :-

- i) Prime mover input torque
- ii) Inertia of prime mover and generator
- iii) Inertia of motor and shaft load.
- iv) Shaft load output torque
- v) Internal voltages of synchronous generators
- vi) Reactance of the system including generator, motor and line etc.,
- vii) internal voltage of motor.

# Methods to Improve Transient Stability

Methods to improve transient stability:-

$$P_r = \frac{V_s V_r}{x} \sin \delta$$

- **Increase of system voltages**:- It increases the value of maximum power and transient stability.
- **Use of high speed excitation systems**:- It increases the value of generated voltage which leads to increase in maximum power and transient stability.
- **Use of high speed governors**:- which can quickly adjust generator input to load.
- **Use of high speed circuit breakers**:- It reduces the severity of faults and protects against the lightning which leads to increase in transient stability.

# Methods to Improve Transient Stability

Methods to improve transient stability:-

## Use of auto-reclosers:-

Automatic reclosing of circuit breakers are known as auto-reclosers or auto-reclosing. Most of the faults (About 80-90% ) on transmission and distribution lines are transient in nature and are self-clearing. By auto-reclosing and rapid switching, the fault is isolated as fast as in 2 cycles and then the circuit breaker reclosers after a suitable time interval. Auto-reclosers increase the decelerating area and transient stability.

## Use of automatic voltage regulators:-

If the excitation system is controlled by an automatic voltage regulator, then the voltage regulator controls the field current and generated voltage which leads to increase in maximum power and transient stability.

# Methods to Improve Transient Stability

- **Reduction of transfer reactance**:- It increases the value of maximum power and transient stability.
- **Use of breaking resistors**:- Breaking resistor is connected at or near the generator bus for stability improvement where large load is suddenly lost or clearing is delayed
- **Load shedding**:- It is applied for distribution systems or major industrial loads. Load shedding reduces the losses and increases the stability.
- **Bypass valving**:- In this method, stability of the system can be increased by decreasing the mechanical input to the turbine.
- **Use of high inertia machines.**
- **Use of HVDC Links .**
- **Use of switching of series capacitors.**

# R K Method

- [V UNIT PSS\RK Method 4th Order.pdf](#)